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Application of atomic decomposition to gear damage detection

Zhipeng Feng[a,b], Fulei Chu[a,*]

[a]*Department of Precision Instruments, Tsinghua University, Beijing 100084, China*
[b]*Institute of Vehicular Engineering, University of Science and Technology Beijing, Beijing 100083, China*

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Abstract

Atomic decomposition can represent arbitrary signals in an overcomplete dictionary sparsely and adaptively, and it can match the local structure of signals very well. Therefore, it possesses advantages over traditional basis-expansion-based signal analysis methods, in extracting characteristic waveforms from complicated mechanical vibration signals. Periodic impulses characterize damaged gear vibration. In order to extract the transient features of gear vibration, atomic decomposition methods, including method of frames (MOF), best orthogonal basis (BOB), matching pursuit (MP) and basis pursuit (BP), are used in the analysis of vibration signals from both healthy and faulty gearboxes. With a compound dictionary specially designed to match the local structure of signals, the meshing frequency and its harmonics, impulses and transient phenomena of the damaged gear vibration signals are extracted simultaneously. Furthermore, from the time–frequency plots of atomic decomposition, the gear tooth damage is recognized easily according to the periodic impulses. By comparing with traditional time–frequency analysis methods, e.g. short time Fourier transform and continuous wavelet transform, it is found that atomic decomposition is more effective in simultaneously extracting the impulses and harmonic components of damaged gear vibration signals.

1. Introduction

Gears, as important mechanisms for transmitting power or rotation, are widely used in machinery. The smooth operation and high efficiency of gears are necessary for the normal running of machinery. Therefore, gear damage detection is a main topic in the field of condition monitoring and fault diagnosis.

Due to the intrinsic dynamic characteristics and complicated ambient excitations, the on-site measured gear vibration signals are frequently characterized by complexity and nonstationarity, for example: the tooth meshing effect, amplitude and phase modulation phenomena inherent with gear pair transmission; the multiple signal components originated from different excitations, complicated propagation, and dynamic coupling between mechanical components; the time varying characteristics due to the operating conditions; and especially the transient impulses induced by gear damage.

In order to effectively extract the meaningful information from vibration signals for gear damage detection, various signal processing methods are employed. The well-known traditional approaches include the

*Corresponding author.
E-mail address: chufl@mail.tsinghua.edu.cn (F. Chu).

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spectrum, sidebands, and envelope demodulation analysis in the frequency domain, as well as the cepstrum analysis and time synchronous average in the time domain [1]. To extract information from nonstationary gear vibration signals caused by faults or operation conditions, various time–frequency analysis methods have been employed as well. Staszewski et al. [2] applied Wigner–Ville distribution to the detection of local tooth faults in spur gears. Wang and McFadden [3,4] used both discrete and continuous wavelet transforms to detect abnormal transients generated in the early stage of gear damage. Regarding the rhythmic property of gear rotation, cyclostationarity analysis is tried in gear vibration signal processing. Capdessus et al. [5] found that the vibration of gear systems showed second-order cyclostationarity, and applied it to the early diagnosis of spalling in gear teeth. Zhu et al. [6] studied the application and effectiveness of cyclostationarity from the first order to the third order in gearbox condition monitoring. By virtue of the fine time–frequency resolution, Loutridis [7] utilized empirical mode decomposition in Hilbert–Huang transform to detect damage in gears. Recently, blind source separation is tested in gear diagnosis as well. Roan et al. [8] applied a nonlinear adaptive independent component analysis algorithm to gear tooth failure detection. The above-mentioned methods possess their own merits, as well as shortcomings.

Atomic decomposition [9–15], recently developed in the signal processing community, including method of frames (MOF) [9,10], best orthogonal basis (BOB) [11], matching pursuit (MP) [12–14], and basis pursuit (BP) [15], etc., shows advantages such as sparsity and fine resolution over traditional basis-expansion-based signal processing methods. With a specially designed waveform dictionary adapted to the local structure of signals, atomic decomposition can extract different features such as impulses, harmonic oscillations, and chirping phenomena, etc. So far, atomic decomposition methods have been studied in the machine diagnosis community. Liu et al. [16] employed MP with time–frequency atoms to detect the localized defects of rolling element bearings. Yang et al. [17] extracted the vibration characteristics of defective rolling element bearings by means of BP. Shi et al. [18] analyzed the transient vibration of rotating machinery based on adaptive time–frequency decomposition with Gaussian chirplets.

In engineering applications, a large percentage of gear faults are induced by localized gear damage. Typical localized damage types include pits, chips, and cracks on gear tooth surface. With such damage existing on gears, the gear tooth meshing will become not as smooth as the normal one, causing impulses to occur. Furthermore, during the running of damaged gears, impulses will be produced repetitively due to the gear rotation, with the period depending on the number of damaged teeth and their distribution over the gear. In a word, periodic or quasi-periodic impulses characterize the vibration of damaged gears, and provide an intuitively understandable indicator of localized damage. So in this sense, the effective extraction of impulses from gear vibration signals is of great importance for gear damage detection.

Although traditional FFT-based signal processing methods play an important role in gear vibration diagnosis, they require that the signals satisfy a strict assumption of stationarity. While in engineering applications, on-site acquired gear vibration signals are often nonstationary, due to the fault-induced time varying property, and especially the transient impulses caused by damaged tooth meshing. Most of all, in basis-expansion-based methods, the characteristic impulses, which are a key indicator of gear damage, may be diluted over a large number of basis functions, so that mistakes can be resulted in. Atomic decomposition represents arbitrary signals with the atoms best matching the characteristic signal structure, without any redundant intermediate transform, so as to extract the characteristic information directly from the signals, and to avoid the information loss and preserve the symptom information as much as possible. With a specially designed time–frequency dictionary, atomic decomposition can extract the impulses, harmonic vibration, and other transient phenomena directly, and allow for further time–frequency analysis. In this sense, it provides a new approach to investigate the nonstationary and multi-component gear vibration signals and extract the inherent damage symptoms.

In this paper, atomic decomposition methods are utilized to analyze the vibration signals and recognize the tooth damage of gearboxes. In Section 2, the dictionary and the atomic decomposition methods, including MOF, BOB, MP, and BP, are briefly introduced. In Section 3, the vibration signals of a healthy gearbox and a faulty one in a pumping station are analyzed. With a specially designed compound dictionary, the characteristic periodic impulses inherent in the vibration signals are extracted, by means of MP and BP. Accordingly, from the time–frequency plot, the tooth damage is identified more easily than from the spectrum. Finally, some conclusions are drawn in Section 4.

2. Atomic decomposition

Basis expansions in orthogonal bases, such as Fourier and wavelet bases, play an important role in signal analysis. Basis functions influence signal representations significantly. If the basis functions are similar to the main components of a signal, the signal can be reconstructed perfectly via only a few basis functions, i.e. the decomposition is sparse. Otherwise, the signal can only be recovered via a large or even infinite number of basis functions, and consequently the information contained in the signal will be diluted over much of the basis functions. In this sense, the traditional basis expansions are not flexible enough for modeling arbitrary signals encountered in engineering application. For instance, Fourier analysis provides a poor representation of time localized signals, and wavelet analysis is not well adapted to represent high frequency signals in a narrow bandwidth. This shortcoming is attributed to the attempt to represent arbitrary signals with a limited set of basis functions in a fixed form.

Atomic decomposition generalizes basis expansions, for sparsely representing signals and best matching their local structure. It can recover arbitrary signals in an overcomplete dictionary. Although the decomposition in the overcomplete dictionary is not unique, it provides adaptability to select the best representation, and allows for the compact representation of signals. According to different principles, various methods have been developed for atomic decomposition, such as MOF, BOB, MP, and BP. In these approaches, a dictionary is a collection of parametric waveforms, and an atom is a parametric waveform. In order to best match the local structure of arbitrary signals, a variety of overcomplete waveform dictionaries are constructed correspondingly, e.g. wavelet packets, cosine packets [11], Gabor atoms [12,13], damped sinusoids [19], chirplets [20,21], FM^{m}lets [22], and dopplerlets [23].

2.1. Dictionary and atom

The dictionary extends the basis. It is a collection of parametric waveforms, namely a library of functions, which might not satisfy the condition of orthonormality required by basis. Usually, the dictionary used in atomic decomposition is overcomplete and redundant, i.e. the number of waveforms in the dictionary is larger than the length of the signal analyzed, and some elements in the dictionary can be represented in terms of other ones.

A parametric waveform in the dictionary is called an atom. Arbitrary signals can be reconstructed by superposing a series of atoms associated with certain physical meaning, and the characteristics of signals can be interpreted in terms of the properties of atoms, e.g. in the time–frequency analysis of signals, the energy, location and variation of signal components is represented by the building block associated with the constructing atom.

In order to match the local structure of signals, the dictionary must be carefully designed adapting to the signal properties. At present, various dictionaries have been constructed. For example, the Dirac dictionary is simply a collection of Dirac functions, which is suitable to analyze impulses. The Fourier dictionary, a typical frequency dictionary, is a collection of sinusoidal waveforms with their frequencies sampled more finely, and is effective in analyzing harmonic oscillations. The wavelet dictionary, a representative of time-scale dictionaries, is a collection of translations and dilations of a basic mother wavelet, together with translations of a father wavelet, and it is efficient in analyzing signals with constant proportional bandwidth. In order to study the property of complicated signals from different points of view, various time–frequency dictionaries are proposed, such as wavelet packets, cosine packets, chirplets, FM^{m}lets, and dopplerlets, etc. Wavelet packets is a time–frequency dictionary frequently used, which includes a standard orthogonal wavelet dictionary, Dirac dictionary, and a collection of waveforms spanning over a range of bandwidths and durations.

2.2. Signal decomposition in overcomplete dictionary

Signal decomposition represents a signal s as a superposition of elementary waveforms ϕ_γ

$$s = \sum_{\gamma \in \Gamma} \alpha_\gamma \phi_\gamma = \Phi\alpha \tag{1}$$

or in an approximation form

$$s = \sum_{i=1}^{m} \alpha_{\gamma_i} \phi_{\gamma_i} + r_m, \tag{2}$$

where α is coefficient, γ is a parameter associated with elementary waveforms, Γ is a parameter collection of elementary waveforms, Φ is the matrix form of elementary waveform collection, and r_m is a residue.

Unlike traditional basis expansions, e.g. Fourier expansion in an orthogonal basis, atomic decomposition represents arbitrary signals in an overcomplete dictionary, and the decomposition result is not unique. Theoretically, finding the best decomposition in an overcomplete dictionary is a complicated process as follows. Firstly, compute the inner product coefficients of a certain signal with all the atoms in the dictionary. Then, sort the atoms by the coefficients. Finally, approximate the signal with the first m atoms. Although it provides adaptability and sparsity in signal representation, finding the best signal decomposition is an NP-hard problem and very time-consuming. Fortunately, a sub-optimal solution via optimization methods is acceptable and feasible in application. Presently, several atomic decomposition approaches in overcomplete dictionaries have been presented, including MOF, BOB, MP, and BP.

2.2.1. Method of frames

The MOF [9,10] selects the representation with minimum l_2 norm of coefficients, i.e.

$$\min \|\alpha\|_2 \quad \text{subject to} \quad \Phi\alpha = s. \tag{3}$$

The solution to MOF is unique, while it is not sparsity preserving, and is intrinsically resolution limited. The computational complexity of MOF is of the order $O(N \ln N)$, where N is the signal length.

2.2.2. Best orthogonal basis

The BOB [11] method is specially designed for the wavelet packets and cosine packets dictionaries. It finds the orthogonal basis minimizing an additive entropy measure of coefficients, i.e.

$$\min\left\{E[s(B)] | \text{ortho basis } B \subset D\right\}. \tag{4}$$

In some cases, BOB delivers near-optimal sparse representations. When the signal has a sparse representation in an orthogonal basis from the library, BOB will work well. However, when the signal is composed of a moderate number of highly nonorthogonal components, it may not deliver sparse representations. In this sense, the demand for BOB to find an orthogonal basis prevents it from finding a highly sparse representation. The computational complexity of BOB is of the order $O(N \ln N)$.

2.2.3. Matching pursuit

The MP [12,13] is a stepwise greedy approximation algorithm. Starting from a null initial model, it iteratively builds up an approximation by adjoining at each stage an atom which best correlates with the current residue. In each iteration, MP selects an atom that best minimizes the signal residue

$$\min \|r_m\|_2. \tag{5}$$

MP is essentially a nonlinear optimization problem without analytical solution. It can be solved by means of some optimization routines, such as zooming algorithm, Newton–Raphson [12–14], and genetic algorithm [24–26].

When the dictionary is orthogonal, MP works well. Otherwise, the situation becomes less clear. Because the algorithm is dependent on the local signal structure, it might choose wrong atoms in the first few iterations, and consequently, spend most time correcting the mistakes made in the first few iterations. The computational complexity of MP is of the order $C_{\mathrm{MP}} N \ln N$, where C_{MP} is the number of the atoms selected.

As a general signal decomposition algorithm with a profound mathematic foundation, MP has been widely studied and applied in the signal processing community. Some refinements are presented to improve it, such as orthogonal MP [27,28], optimized orthogonal MP [29,30], and high-resolution MP [31].

2.2.4. Basis pursuit

The BP [15] chooses the decomposition with minimum l_1 norm of coefficients, i.e.

$$\min\|\alpha\|_1 \quad \text{subject to } \Phi\alpha = s. \tag{6}$$

BP is a convex nonquadratic optimization principle, rather than an algorithm. In essence, it is equivalent to linear program. Some global optimization rules, such as simplex algorithm and interior point method, can be employed to solve it.

Compared with MOF, BOB, and MP, BP is featured by super-resolution and better sparsity, whereas it suffers from slower computational speed. The computational complexity of BP is of the order $C_{\mathrm{BP}}N\ln N$, where C_{BP} is the number of atoms selected.

3. Gear vibration signal analysis

The gear vibration is dominated by the tooth pair meshing effect and the gear rotation. Tooth deflection under load, initial machining errors, and wear, are manifested at the tooth meshing frequency and its harmonics. Localized faults, such as cracks and spalls, give a wide range of harmonics and sidebands throughout the spectrum. However, distributed faults due to eccentricity and distortion give stronger harmonics grouped around zero frequency and sidebands around the harmonics of tooth meshing frequency.

Sidebands are generated by amplitude modulation and/or phase modulation as a result of faulty teeth meshing. In many investigations, the sideband structure in the spectrum is accepted as a symptom of the tooth damage, and the localization of the damage can be identified by the spacing between the sideband and the meshing frequency. However, because of inevitable manufacturing and assembling errors, the sidebands may exist in the spectrum even if the gear is normal. Therefore, it is sometimes difficult to identify the reason of the gear malfunction by means of the spectral analysis only.

Impulses characterize vibration of the damaged gear. Theoretically, periodic impulses, including the intensity and the time interval, are the robust symptom of tooth damage. Unfortunately, it is not easy to extract impulses directly by means of traditional basis expansion methods, either the Fourier or the wavelet transform. Instead of representing an impulse with infinite sinusoids in the Fourier expansion, the atomic decomposition is capable of extracting impulses inherent in arbitrary signals directly, as long as the employed dictionary includes suitable atoms, like the Dirac dictionary, to match them. In this sense, the atomic decomposition provides a new approach to analyze gear vibration and identify tooth damage.

3.1. Specification of gearbox and measurement

The on-site measured vibration signals of the gearboxes in two pump sets presented in the Ref. [32] are analyzed in this paper to demonstrate the effectiveness of the above-mentioned atomic decomposition methods. The pump sets are identical: the pump is driven by an electromotor with a gearbox as speed reduction set, and the two pump sets have similar power consumption, age and amount of running hours. One pump set shows severe gear damage (pits, i.e. surface cracks due to unequal load and wear, the damaged areas spread in the center of each gear tooth surface, refer to Ref. [32] for details about the gear damage), whereas the other set runs normally.

The number of teeth in the first gear wheel is 13, and the driving shaft speed is 997 rev/min (namely the rotational frequency is 16.6 Hz), therefore the gear meshing frequency is $13*997/60 = 216$ Hz. The vibration is measured with seven accelerometers at different positions near the structural elements, such as the shaft, the gears, and the bearings. The data is filtered by an analog low-pass filter with cutoff frequency of 5000 Hz, and sampled at 12,800 Hz. In the following section, the vibration signals from the accelerometers radially mounted near the driving shaft are analyzed to demonstrate the performance of atomic decomposition.

3.2. Time–frequency analysis

Due to the motion property, i.e. rotation and teeth meshing, the damaged gear vibration consists mainly of periodic impulses and harmonic oscillations. In the time domain, it may be approximately an amplitude

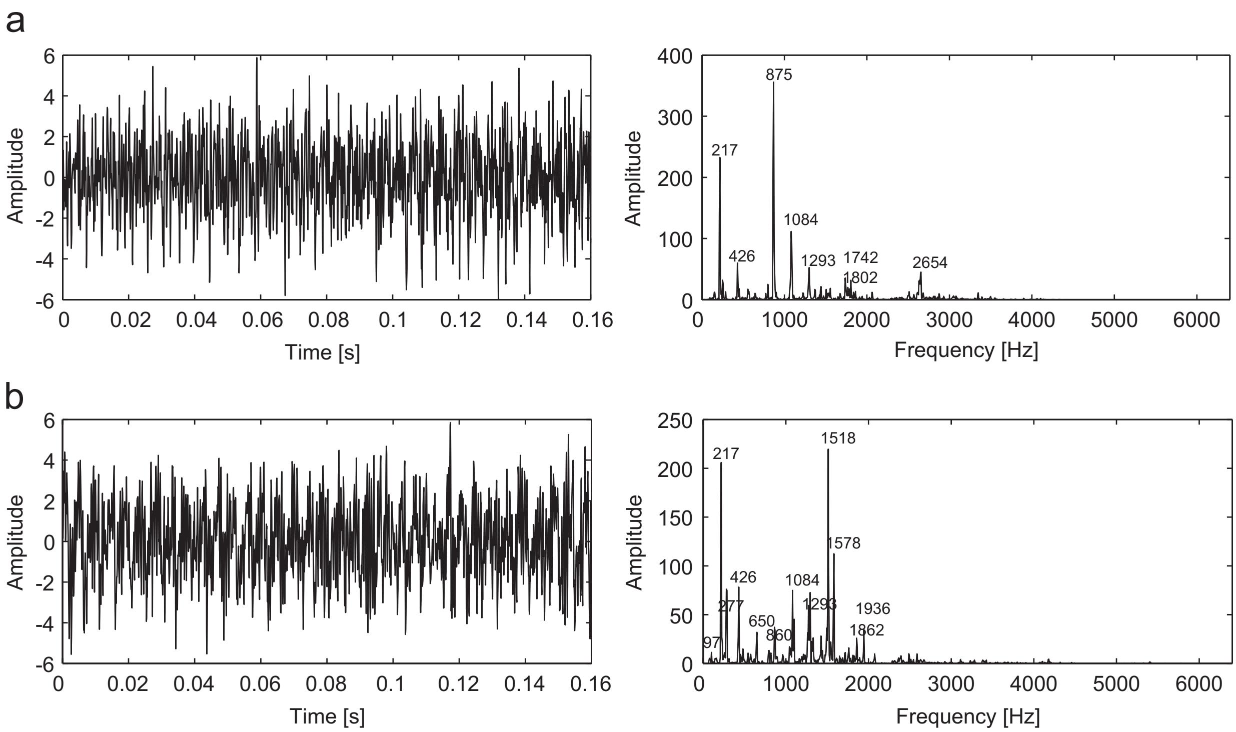

Fig. 1. Waveform and power spectrum: (a) healthy gear and (b) faulty gear.

modulation and/or phase modulation. Meanwhile, in the frequency domain, the gear meshing frequency and its harmonics, the gear pair rotational frequencies, the sidebands grouped around the harmonics, and other frequency components may co-exist in the spectrum. In this sense, gear vibration signals are of multiple components, and each component has a unique feature. Consequently, it is more physically meaningful to extract each component directly and interpret the corresponding vibration behavior separately.

Gear tooth imperfections, due to manufacturing inaccuracy, eccentricity, and elastic deformation, are inevitable. Therefore, it is not easy to identify whether a gear has fault or not, only according to the spectral analysis, except when a severe damage occurs. The vibration signal waveforms and power spectrums of the healthy and faulty gearboxes are shown in Figs. 1(a) and (b), respectively. From the waveforms, no significant difference can be found between the healthy and faulty gearboxes. The meshing frequency, its harmonics, and sidebands co-exist in the power spectrum of both the healthy and the faulty gearboxes. For the healthy one, the frequency components range from 0 to 3000 Hz. While for the faulty one, the signal concentrates in a relatively low frequency band of 0–2000 Hz, and the amplitude at each frequency bin shows a significant difference from the healthy gearbox. However, the power spectrum displays no obvious signature similar to the symptom of any typical gear fault, so that it is not easy to identify the fault reason only by means of the power spectrum.

Instead of approximating a signal with a given type of basis, and analyzing it in only either the time or frequency domain, as the traditional Fourier analysis does, the atomic decomposition extracts the characteristic waveforms consisted in the signal directly, and allows for further analysis in the time–frequency joint domain, thus ensuring more clear and meaningful interpretation of the information contained in the signal.

3.2.1. Compound time–frequency dictionary

It is impossible to know the exact waveform of gear vibration signal components, but empirical knowledge and prior visual inspection of signals help in constructing a dictionary to approximate the characteristic structures by means of atomic decomposition. The dictionary should be carefully designed in order to not only

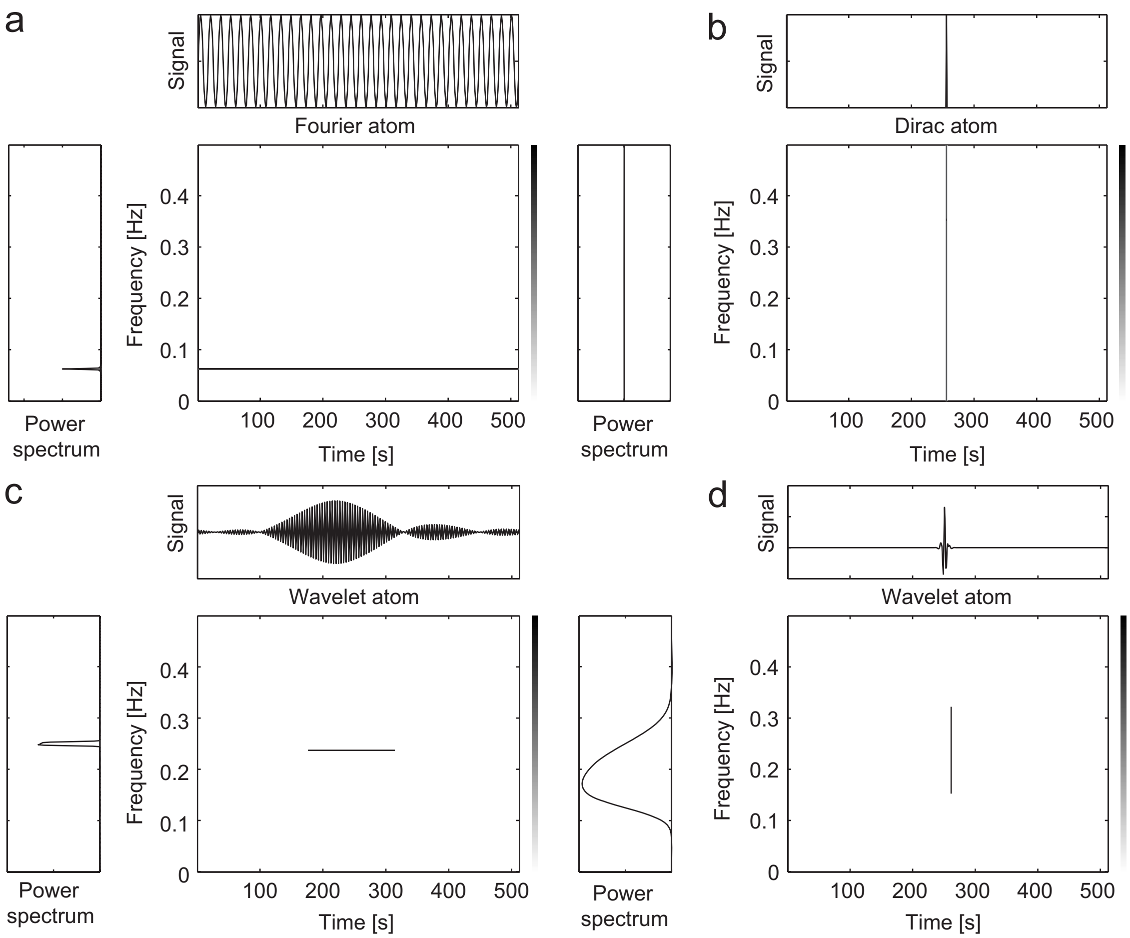

Fig. 2. Time–frequency plots of typical atoms: (a) Fourier atom, (b) Dirac atom, (c) wavelet atom and (d) wavelet atom.

best match the characteristics of gear vibration, but also ensure the implementation of atomic decomposition algorithms. Regarding the properties of gear vibration signals and the requirement of fine time-frequency resolution, a compound dictionary consisting of the Fourier dictionary, Dirac dictionary, standard orthogonal symlet-8 wavelets dictionary, and a collection of the waveforms spanning over a range of bandwidths and durations, which are employed to pursue the harmonic oscillations, impulses, and other transient phenomena of gear vibration, respectively, is constructed for the atomic decomposition. The time–frequency plots of some typical atoms, such as the Fourier, Dirac, and wavelet atoms, are shown in Figs. 2(a)–(d), respectively (the signal waveform is shown on the top, its power spectrum density at left, and a gray bar or color bar denoting the time–frequency distribution amplitude at right. It is the same for the display of other time–frequency analysis results).

3.2.2. Time–frequency analysis based on atomic decomposition

The vibration signals of the healthy and faulty gearboxes (for the reason of computational complexity and limited PC memory, the truncated signals of the first 512 points are used in the time–frequency analysis) are analyzed by means of atomic decomposition methods based on the compound time–frequency dictionary, and the time–frequency plots of the MOF, BOB, MP, and BP decompositions are shown in Figs. 3–6, respectively.

MOF extracts the meshing frequency and its harmonics, the impulses, and other transient vibration simultaneously, as shown in Fig. 3. For the healthy gearbox, some impulses appear in the time–frequency distribution, but the interval between consecutive impulses is not regular. In engineering application,

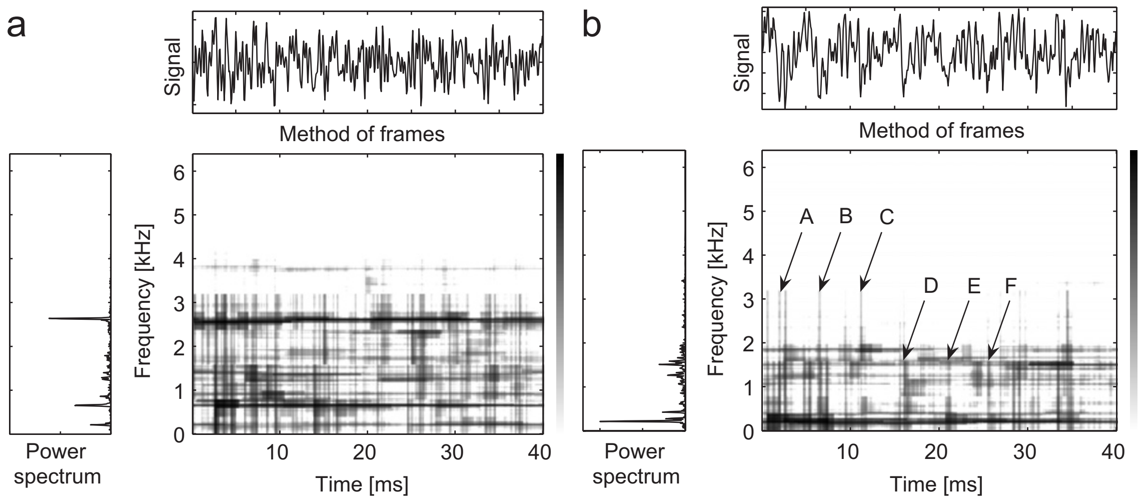

Fig. 3. Time–frequency plot of MOF: (a) healthy gear and (b) faulty gear.

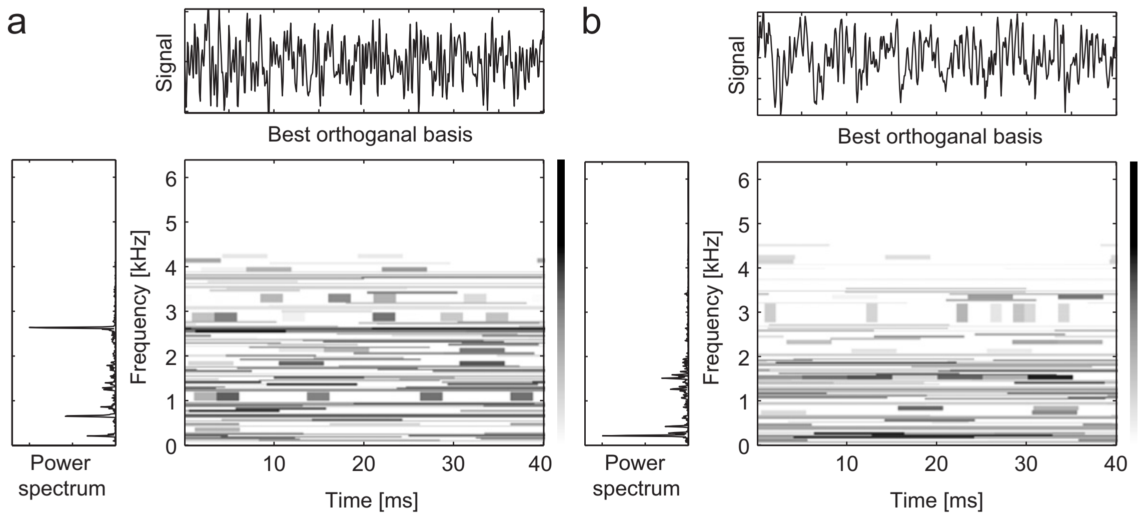

Fig. 4. Time–frequency plot of BOB: (a) healthy gear and (b) faulty gear.

degenerate lubrication or load fluctuation may cause the momentary break down of oil film between sliding gear tooth surfaces, and accordingly result in impulses even though the gear is healthy. In this sense, these irregular impulses do not indicate any gear damage. While for the faulty gearbox, some quasi-periodic impulses appear in the time–frequency distribution, with a regular interval of about 4–5 ms (e.g. the interval between the consecutive impulses A–C, as well as that between D–F). Theoretically, if localized damage spreads evenly over all the gear teeth, then when each gear teeth pair goes into mesh, the oil film between sliding gear tooth surfaces will break down, resulting in an impulse. Furthermore, if the gear pair rotates at a strictly constant speed and runs under a strictly constant load, an impulse train will appear with a period equal to the gear meshing period. While the on-site condition is undoubtedly different from the ideal one, so the consecutive impulses A-F do not constitute an ideal periodic impulse train. The non-consecutiveness (e.g. the absence of some impulses at expected instants after impulse F) of these impulses is possibly attributed to the complicated gear-shaft-bearing-casing vibration propagation which weakens and introduces interferences to vibration signals, as well as the inevitable instantaneous load fluctuation and the uneven distribution of

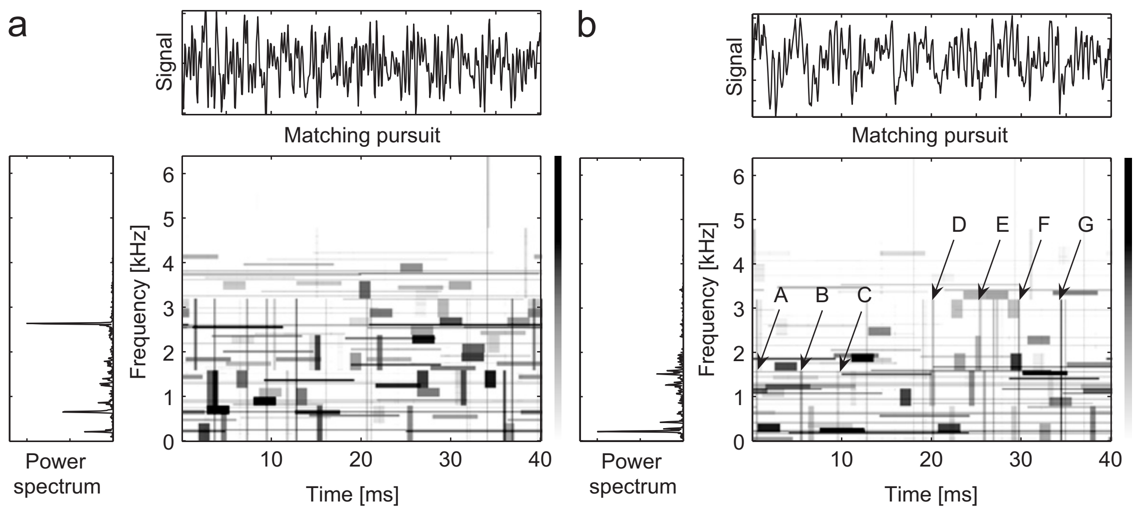

Fig. 5. Time–frequency plot of MP: (a) healthy gear and (b) faulty gear.

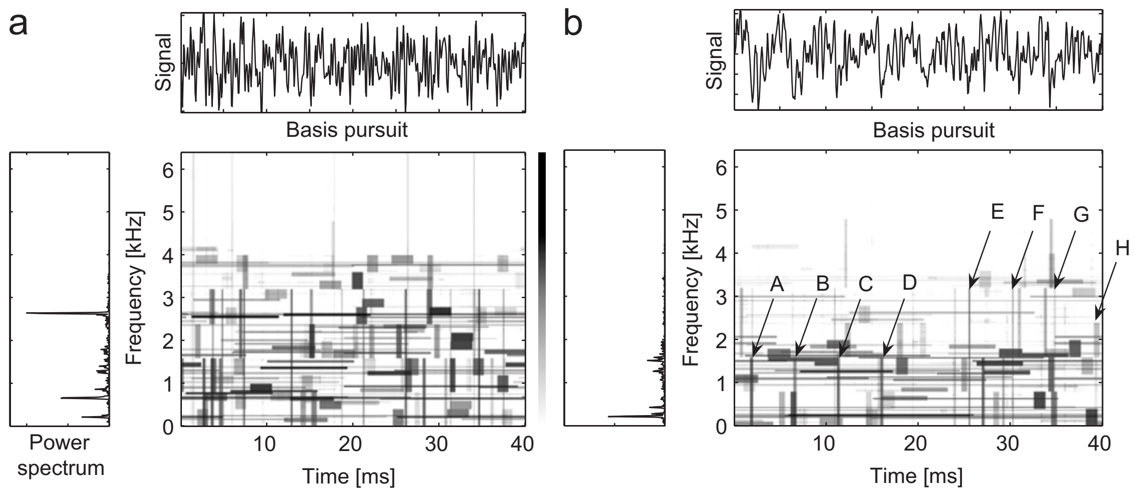

Fig. 6. Time–frequency plot of BP: (a) healthy gear and (b) faulty gear.

localized damage over all the gear teeth which may result in irregular impulses. Due to the inevitable instantaneous fluctuation of gear rotational speed, the intervals between these consecutive impulses may differ a little from each other. While these intervals are in the vicinity of the gear meshing period—4.6 ms, and approximately correspond to the gear tooth meshing frequency of 216 Hz. In this sense, the nearly periodic consecutive impulses indicate that localized damage exists on most of the gear teeth, which accords well with the real condition of the faulty gearbox. Although the gear damage can be detected by means of MOF, it should be noticed that the time–frequency plot consists of too many tiling blocks due to the worse sparsity preserving property of MOF, influencing the further analysis, which is not expected in application.

BOB is specially designed for signal decomposition based on the wavelet packets and the cosine packets dictionaries. It successfully extracts the harmonic components corresponding to the peaks in power spectrum, and some transient phenomena, but totally misses the characteristic impulses, as shown in Fig. 4. In this sense, it is unsuitable to analyze the characteristic impulses of damaged gear vibration.

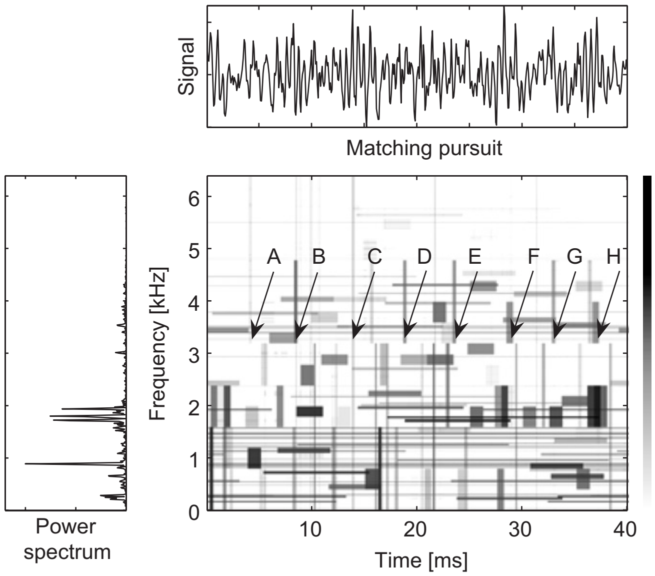

Fig. 7. Time–frequency plot of MP.

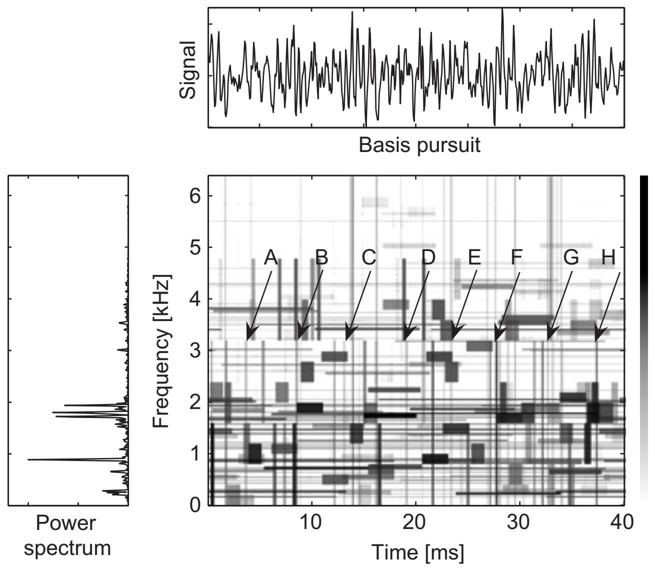

Fig. 8. Time–frequency plot of BP.

MP identifies the meshing frequency and its harmonics, the impulses, and other transient vibration successfully. As shown in Fig. 5, the decomposition is sparser than that of MOF. For the healthy gearbox, some impulses appear in the time–frequency distribution, but the interval between consecutive impulses is irregular. While for the faulty one, the significant impulses are nearly evenly spaced, with a regular interval of about 4–5 ms (e.g. the interval between the consecutive impulses A–C, as well as that between D–G) which approximately corresponds to the gear tooth meshing period—4.6 ms implying that localized damage spreads over the gear teeth.

Similar to MP, BP identifies the meshing frequency and its harmonics, characteristic impulses, and other transient vibration clearly, as shown in Fig. 6. For the healthy gearbox, some impulses appear in the time–frequency distribution, but the interval between consecutive impulses is not regular. While for the faulty one, grouped periodic impulses are extracted, exhibiting a significant difference from the healthy one. The time interval between the consecutive impulses A–D, as well as that between E–H, equals 4–5 ms approximately, which equals approximately the gear tooth meshing period—4.6 ms, indicating the existence of localized damage on gear tooth surface.

To further demonstrate the effectiveness of atomic decomposition in analyzing gear vibration signals, another dataset of the faulty gearbox running at the same speed but under a different load is analyzed, and the time–frequency plots of MP and BP are shown in Figs. 7 and 8, respectively. It can be seen that the meshing frequency and its harmonics, periodic impulses, and other transient vibration are extracted simultaneously. In particular, the consecutive impulses appear in the time–frequency plane near-periodically, and the interval approximately equals 4–5 ms, for example, in the time–frequency plot of MP, the interval between the impulses A and B, that between C–E, and as well as that between F–H; in the time–frequency plot of BP, the interval between the impulses A–C, as well as that between D–H. The interval corresponds approximately to the gear tooth meshing period–4.6 ms, implying that an impulse is produced when each gear teeth pair goes

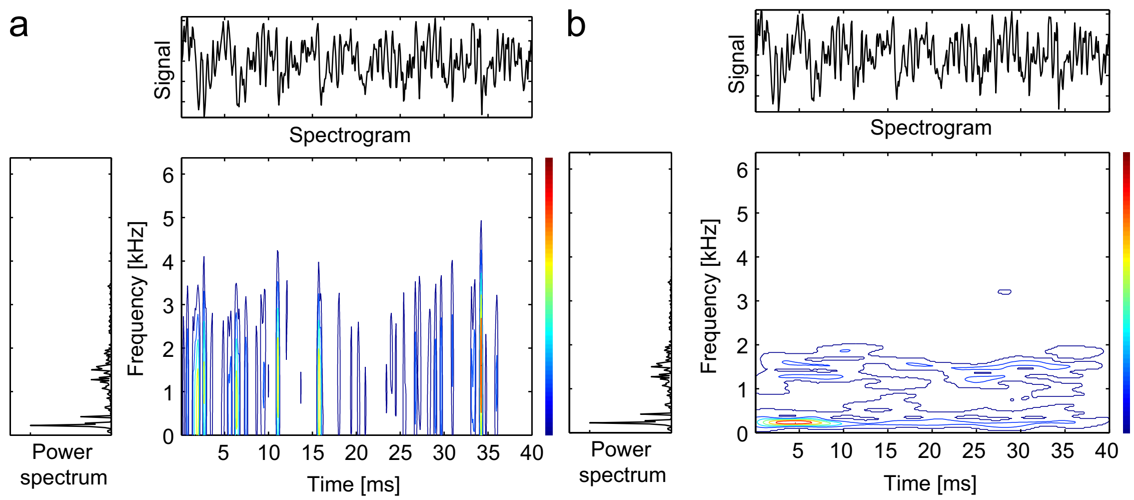

Fig. 9. Spectrograms: (a) Hamming window-5 and (b) Hamming window-64.

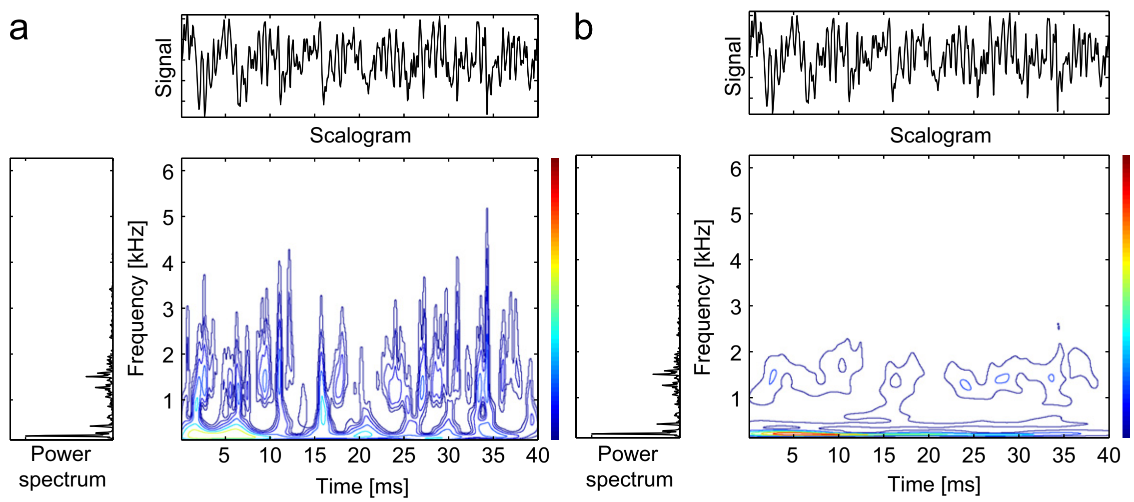

Fig. 10. Scalograms: (a) Morlet-5 and (b) Morlet-50.

into mesh, and accordingly indicates that localized damage exists on the gear teeth. This again verifies the feasibility of MP and BP in gear damage detection.

As mentioned above, such atomic decomposition methods as MOF, BOB, MP, and BP, successfully extract the meshing frequency, its harmonics, and some transient vibration. Most of all, MOF, MP and BP show their efficiency in simultaneously extracting the characteristic impulses of damaged gear vibration, while BOB fails in doing this. In terms of sparsity, MP and BP outperform MOF, resulting in the clearer time–frequency plots for gear diagnosis applications.

3.2.3. Comparison with traditional time–frequency analysis

To compare with traditional basis-expansion-based methods, the faulty gearbox vibration signal is analyzed by means of short time Fourier transform and continuous wavelet transform, and the spectrogram and scalogram are shown in Figs. 9 and 10, respectively.

Some Hamming smoothing windows of different lengths are used in the short time Fourier transform. It is found that a shorter window emphasizes on the time localization, and is more effective in resolving the characteristic impulses, as shown in Fig. 9(a) (in which the window length is about 1% of the signal length), but totally misses the meshing frequency and its harmonics. A longer window highlights the frequency resolution, and can identify the meshing frequency and its harmonics, but completely misses the characteristic impulses, as shown in Fig. 9(b) (in which the window length is one-eighth of the signal length).

In order to extract the characteristic impulses in the signal, the Morlet wavelet is used in the continuous wavelet transform. Among the wavelets with different parameters, it is found that the Morlet-5 wavelet highlights the local events in the signal, and is effective in extracting the impulses, as shown in Fig. 10(a), but fails to identify the meshing frequency and its harmonics. The Morlet-50 wavelet emphasizes the frequency components, as shown in Fig. 10(b), but fails to extract the characteristic impulses.

As linear transforms, both short time Fourier transform and wavelet transform are subject to Heisenberg uncertainty principle, i.e. the time localization and frequency resolution cannot be obtained at their highest simultaneously, either of them can only be enhanced at the expense of the other one, so that their time–frequency resolution is limited. In addition, as basis-expansion-based methods, the basis in either Fourier or wavelet transform is fixed, therefore they lack adaptability in simultaneously matching the complicated components inherent in gear vibration signals, such as the meshing frequency and its harmonics, impulses, and other transient vibration. In this sense, they are inferior to atomic decomposition methods, especially MP and BP.

In summary, the atomic decomposition shows advantages over the traditional basis expansion in the analyses of gear vibration signals. Usually, an on-site measured gear vibration signal is complicated and of multiple components, due to the inevitable manufacturing and assembling errors, and gear tooth deflection under load. Various frequency components may co-exist in the spectrum, such as the rotational frequencies of gear pair, meshing frequency, their harmonics, and sidebands. Because of the spectrum complexity, it is not easy to identify the fault reason via spectral analysis only. While with a compound dictionary adapted to matching the impulses, harmonics and other transient oscillations inherent in the signals, the transient behaviors characteristic of gear vibration are extracted simultaneously, via the atomic decomposition methods. Furthermore, by virtue of the time–frequency plot, the periodic impulses are identified clearly, and accordingly the fault reason is interpreted well. Compared with the traditional basis-expansion-based methods, the atomic decomposition is far more effective in simultaneously extracting the impulses and other transient phenomena of gear vibration.

4. Conclusions

Different from traditional basis expansion, which has a strict requirement of orthonormality on basis functions, atomic decomposition represents arbitrary signals in an overcomplete dictionary, which is pre-constructed according to signal properties. Theoretically, it possesses advantages over traditional basis-expansion-based signal analysis methods, especially in analyzing complicated signals and extracting information contained, and also in representing arbitrary signals sparsely and adaptively.

In general, gear vibration signals consist of complicated components, such as rotational frequency, meshing frequency and its harmonics, impulses, and other transient phenomena. Periodic impulses characterize damaged gear vibration. In order to extract the impulse feature of damaged gear vibration, atomic decomposition methods, including MOF, BOB, MP, and BP, are used to analyze the vibration signals of both healthy and faulty gearboxes, with a compound dictionary specially designed to match the local structure of the signals. BOB does not seem to work effectively in extracting the characteristic impulses, while MP and BP seem to have the most potential in extracting not only the meshing frequency and its harmonics but also the characteristic impulses. Furthermore, via the time–frequency plot of the decomposition, the gear tooth damage is identified effectively according to the interval of periodic impulses. By comparing atomic decomposition with short time Fourier transform and continuous wavelet transform, it is found that the better time–frequency resolution and adaptability to signal components enable atomic decomposition methods to outperform traditional basis-expansion-based methods in analyzing gear vibration signals.

Atomic-decomposition-based time–frequency analysis is suitable for qualitative detection of gear damage. While for quantitative evaluation of gear condition, the way to extract a feature from atomic decomposition result to quantify gear damage degree is still a topic worthy of further research.

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References

[1] R.B. Randall, A new method of modeling gear faults, *Journal of Mechanical Design* 104 (1982) 259–267.
[2] W.J. Staszewski, K. Worden, G.R. Tomlinson, Time–frequency analysis in gearbox fault detections using the Wigner–Ville distribution and pattern recognition, *Mechanical System and Signal Processing* 11 (1997) 673–692.
[3] W.J. Wang, P.D. McFadden, Application of orthogonal wavelets to early gear damage detection, *Mechanical Systems and Signal Processing* 9 (1995) 497–507.
[4] W.J. Wang, P.D. McFadden, Application of wavelets to gear box vibration signals for fault detection, *Journal of Sound and Vibration* 192 (1996) 927–937.
[5] C. Capdessus, M. Sidahmed, J.L. Lacoume, Cyclostationary process: application in gear faults diagnosis, *Mechanical Systems and Signal Processing* 14 (3) (2000) 371–385.
[6] Z.K. Zhu, Z.H. Feng, F.R. Kong, Cyclostationarity analysis for gearbox condition monitoring: approaches and effectiveness, *Mechanical Systems and Signal Processing* 19 (2005) 467–482.
[7] S.J. Loutridis, Damage detection in gear systems using empirical mode decomposition, *Engineering Structures* 26 (2004) 1833–1841.
[8] M.J. Roan, J.G. Erling, L.H. Sibul, A new, non-linear, adaptive, blind source separation approach to gear tooth failure detection and analysis, *Mechanical Systems and Signal Processing* 16 (5) (2002) 719–740.
[9] I. Daubechies, The wavelet transform, time–frequency localization and signal analysis, *IEEE Transactions on Information Theory* 36 (5) (1990) 961–1005.
[10] I. Daubechies, *Ten Lectures on Wavelets*, SIAM, Philadelphia, PA, 1992.
[11] R.R. Coifman, M.V. Wickerhauser, Entropy-based algorithms for best basis selection, *IEEE Transactions on Information Theory* 38 (1992) 713–718.
[12] S. Mallat, Z. Zhang, Matching pursuit with time–frequency dictionaries, *IEEE Transactions on Signal Processing* 41 (1993) 3397–3415.
[13] S. Qian, D. Chen, Signal representation using adaptive normalized Gaussian functions, *Signal Processing* 36 (1) (1994) 1–11.
[14] S. Mallat, *A Wavelet Tour of Signal Processing*, Academic Press, San Diego, CA, 1998.
[15] S.S. Chen, D.L. Donoho, M.A. Saunders, Atomic decomposition by basis pursuit, *SIAM Review* 43 (1) (2001) 129–159.
[16] B. Liu, S.F. Ling, R. Gribonval, Bearing failure detection using matching pursuit, *NDT&E International* 35 (2002) 255–262.
[17] H. Yang, J. Mathew, L. Ma, Fault diagnosis of rolling element bearings using basis pursuit, *Mechanical Systems and Signal Processing* 19 (2005) 341–356.
[18] D.F. Shi, F. Tsung, P.J. Unsworth, Adaptive time–frequency decomposition for transient vibration monitoring of rotating machinery, *Mechanical Systems and Signal Processing* 18 (2004) 127–141.
[19] M.M. Goodwin, M. Vetterli, Matching pursuit and atomic signal models based on recursive filter banks, *IEEE Transactions on Signal Processing* 47 (7) (1999) 1890–1902.
[20] A. Bultan, A four-parameter atomic decomposition of chirplets, *IEEE Transactions on Signal Processing* 47 (1999) 731–744.

[21] Q. Yin, S. Qian, A. Feng, A fast refinement for adaptive Gaussian chirplet decomposition, *IEEE Transactions on Signal Processing* 50 (2002) 1298–1306.
[22] H. Zou, Q. Dai, R. Wang, Y. Li, Parametric TFR via windowed exponential frequency modulated atoms, *IEEE Signal Processing Letters* 8 (2001) 140–142.
[23] H. Zou, Y. Chen, J. Zhu, Q. Dai, G. Wu, Y. Li, Steady-motion-based Dopplerlet transform application to the estimation of range and speed of a moving sound source, *IEEE Journal of Oceanic Engineering* 29 (3) (2004) 887–905.
[24] A.R. Ferreira da Silva, Evolutionary-based methods for adaptive signal representation, *Signal Processing* 81 (2001) 927–944.
[25] A.R. Ferreira da Silva, Approximation with evolutionary pursuit, *Signal Processing* 83 (2003) 465–481.
[26] A.R. Ferreira da Silva, Atomic decomposition with evolutionary pursuit, *Digital Signal Processing* 13 (2003) 317–337.
[27] Y.C. Pati, R. Rezaiifar, P.S. Krishnaprasad, Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition, *Proceeding of the 27th Asilomar Conference on Signals, Systems, Computers*, 1993.
[28] G. Davis, S. Mallat, M. Avellaneda, Adaptive greedy approximations, *Constructive Approximation* 13 (1997) 57–98.
[29] L. Rebollo-Neira, D. Lowe, Optimized orthogonal matching pursuit approach, *IEEE Signal Processing Letters* 9 (4) (2002) 137–140.
[30] M. Andrle, L. Rebollo-Neira, E. Sagianos, Backward-optimized orthogonal matching pursuit approach, *IEEE Signal Processing Letters* 11 (9) (2004) 705–708.
[31] S. Jaggi, W.C. Karl, S. Mallat, A.S. Willsky, High resolution pursuit for feature extraction, *Applied and Computational Harmonic Analysis* 5 (1998) 428–449.
[32] A. Ypma, Learning methods for machine vibration analysis and health monitoring, PhD Dissertation, TU Delft, 2001. Vibration datasets are from: ⟨http://www.aypma.nl/PhD/pump_sets.html⟩.