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Short Communication

Critical velocity of fluid-conveying pipes resting on two-parameter foundation

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Abstract

Analytical expressions are derived for computation of critical velocity of a fluid flowing in a pipeline and resting on a two-parameter foundation like the Pasternak foundation. Fourier series and Galerkin methods have been utilized in computing the results for three simple boundary conditions, namely: pinned–pinned, pinned–clamped and clamped–clamped. Results are presented for varying values of both the foundation stiffness parameters and comparison is made with available literature for the case of the second parameter equal to zero, and new results are presented for varying values of the second foundation parameter. Interesting conclusions are drawn on the effect of the foundation parameters on the critical flow velocity of the pipeline.
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1. Introduction

The technology of transporting fluids, especially petroleum liquids, through long pipelines, which cover different types of terrain, has evolved over the years. The velocity of the fluid in a pipeline transporting fluids imparts energy to the pipeline making it to vibrate. It is well established from published literature that there exists a critical velocity of the fluid near which the natural frequency of the pipeline tends to zero. This is the required condition for buckling of the pipeline. Literature abounds with analyses, which give information on the influence of boundary conditions on the stability of fluid conveying pipes. Interest in studying the dynamic behaviour of such fluid conveying pipes was stimulated when excessive transverse vibrations were observed and subsequently analysed first by Ashley and Haviland in 1950 [1] and later by Housner in 1952 [2]. Housner considered a simply supported beam model for the pipeline and analysed it using a series solution approach and showed the existence of a critical flow velocity for a pipeline, which could cause buckling. In 1955, Long [3] studied the influence of clamped–clamped and clamped–pinned boundary conditions on the critical velocity. In 1966, Gregory and Paidoussis [4] presented results on the dynamic behaviour of a cantilevered pipe conveying fluid. All the above studies did not consider elastic support conditions.

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Nomenclature

$A\rho$	mass of pipe/unit length
C_{ij}	integration constants
E	modulus of elasticity
I	moment of inertia
k_1	winkler foundation stiffness/unit length
k_2	shear foundation constant/unit length
L	length of the pipe
m	mass of pipe/unit length
M	total mass of pipe plus fluid/unit length
v	steady flow velocity of fluid
V	non-dimensional flow velocity parameter
V_{cr}	critical velocity parameter
w	lateral displacement of the pipe
x	dimension along the length of pipe

Greek symbols

β	non-dimensional mass-ratio parameter
γ_1	non-dimensional Winkler foundation parameter
γ_2	non-dimensional shear foundation parameter
λ_r	beam frequency parameter
ψ_r	beam eigenfunctions
σ_r	frequency function
ω_j	jth mode of vibration
Ω	non-dimensional frequency parameter

When a pipeline rests on an elastic medium such as a soil, a model of the soil medium must be included in the governing differential equation. A very common structural model of the soil medium is the Winkler model, in which soil is represented by a series of constant stiffness, closely spaced linear springs. This model is extensively used in engineering analysis because of its simplicity and also because it is possible to obtain closed-form solutions for uniform stiffness. In 1970, Stein and Tobriner [5] studied the vibrations of a fluid-conveying pipe resting on an elastic foundation. Lottati and Kornecki, in 1986 [6], studied the influence of the elastic foundation on the stability of the pipeline. Later, in 1992, Dermendjian-Ivanova [7] investigated the behaviour of a fluid conveying pipe resting on an elastic foundation and obtained the critical fluid velocity. In 1993, Raghava Chary et al. [8] presented a detailed analysis of fluid conveying pipes resting on elastic foundation. In a recent paper, Doaré and de Langre [9] studied instability of fluid conveying pipes on Winkler-type foundation. The focus in their paper was on instability of infinitely long fluid conveying pipes using wave propagation approach, wherein results are interpreted in terms static neutrality as criteria for pinned–pinned, clamped–clamped ends and dynamic neutrality for clamped–free ends. All these studies modelled the elastic foundation as a Winkler model.

A real soil medium, however is more complex in its elastic behaviour than what the above model considers. The Winkler model assumes that the deformation of the foundation is only in the loaded region and hence implies a deformation discontinuity between the loaded and unloaded parts. Also, this model is inadequate when a lift-off takes place between the soil and the structure. To address such deficiencies, many researchers suggested an interaction between the springs of the Winkler model to obtain a more realistic model of the soil. Hence, two-parameter foundation models were developed, of which, the Pasternak model is considered closer to the soil behaviour than other models—for example, see Dutta and Roy [10]. In the Pasternak model, an incompressible shear layer is introduced between the Winkler springs and the pipe surface. The springs are connected to this shear layer, which is capable of resisting only transverse shear, thus allowing for "shear interaction" between the Winkler springs. Pipelines, especially those carrying petroleum products, traverse varied terrains like sand, gravel, mud and rock. The Pasternak model is considered to be closer to these real media. Analysis of fluid conveying pipes has been extensively performed for the case of one-parameter elastic foundation models like the Winkler model, and there is a good amount of literature on the behaviour of beams on two-parameter foundations. However, to the best of authors' knowledge, no study has been published dealing with the behaviour of fluid-conveying pipes resting on a two-parameter elastic foundation. It is therefore felt necessary to study the dynamics and stability of fluid conveying pipes resting on two-parameter foundation such as Pasternak foundation for pinned–pinned, clamped–clamped and clamped–pinned ends.

In this paper, the work of previous authors [5–8] has been extended suitably to include the influence of a two-parameter foundation model on the vibration and stability characteristics of the fluid conveying pipes. Results are presented showing the variation for various values of the foundation stiffness parameters.

2. Equation of motion

The differential equation of motion for lateral displacement $w(x,t)$ of a uniform fluid-conveying pipe resting on a Winkler-type elastic foundation is given by

$$EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + \rho A v^2\frac{\partial^2 w}{\partial x^2} + 2\rho A v\frac{\partial^2 w}{\partial x \partial t} + k_1 w = 0. \quad (1)$$

The symbols in the above equation are defined in the nomenclature. In this equation, the elastic medium is modelled on the Winkler-type foundation. The equation of motion for a fluid-conveying pipe resting on a two-parameter foundation becomes:

$$EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + (\rho A v^2 - k_2)\frac{\partial^2 w}{\partial x^2} + 2\rho A v\frac{\partial^2 w}{\partial x \partial t} + k_1 w = 0. \quad (2)$$

In Eq. (2) above, k_2 represents the additional parameter defining the foundation, usually termed as the shear constant of the foundation. The model is shown in Fig. 1. Eq. (2) is now solved for three simple boundary conditions.

2.1. Pinned–pinned pipe

The boundary conditions for a pinned–pinned pipe are

$$w(0,t) = w(L,t) = 0,$$
$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0. \quad (3)$$

Taking the solution of Eq. (2) which satisfies the boundary conditions Eq. (3) as

$$w(x,t) = \sum_{n=1,3,5,\ldots} a_n \sin\frac{n\pi x}{L}\sin\omega_j t + \sum_{n=2,4,6,\ldots} a_n \sin\frac{n\pi x}{L}\cos\omega_j t, \quad j = 1,2,3,\ldots, \quad (4)$$

where ω_j represents the natural frequency of the jth mode of vibration. Substitution of Eq. (4) into Eq. (2) and expanding in a Fourier series we have an equation of the form:

$$\left[\mathbf{K} - \omega_j^2\mathbf{MI}\right]\{\mathbf{a}\} = 0, \quad (5)$$

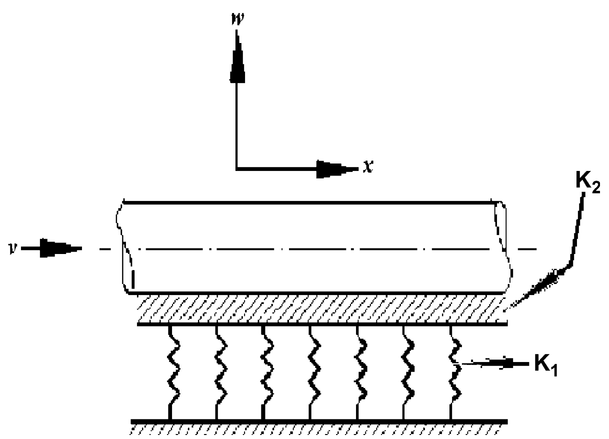

Fig. 1. Model of a fluid-conveying pipe resting on a two-parameter foundation.

where **K** is the stiffness matrix whose elements are enumerated in Ref. [8] and will not be repeated here, **I** is the identity matrix and $\mathbf{a}^{\mathrm{T}} = \{a_1, a_2, \ldots, a_n\}$. Retaining the first two terms of the above equation, and setting the determinant equal to zero, we get

$$\Omega_j^4 - \left[\left(\frac{256}{9}\beta - 5\pi^2\right)(V^2-\gamma_2) + 17\pi^4 + 2\gamma_1\right]\Omega_j^2 + \left[4\pi^4(V^2-\gamma_2)^2 - (V^2-\gamma_2)(5\pi^2\gamma_1 + 20\pi^6) + (16\pi^8 + 17\pi^4\gamma_1 + \gamma_1^2)\right] = 0. \quad (6)$$

In Eq. (6), the following non-dimensional parameters have been used:

$$\beta = \frac{\rho A}{M}; \quad \Omega_j = \omega_j L^2\sqrt{\frac{M}{EI}}, \quad j = 1, 2, 3, \ldots; \quad V = vL\sqrt{\frac{\rho A}{EI}}; \quad \gamma_1 = \frac{k_1 L^4}{EI}; \quad \gamma_2 = \frac{k_2 L^2}{EI}.$$

When the fluid velocity reaches a certain value V_{cr}, the fundamental natural frequency becomes zero. Hence, setting $\Omega_j = 0$ in Eq. (6), we obtain

$$\left[4\pi^4(V^2-\gamma_2)^2 - (V^2-\gamma_2)(5\pi^2\gamma_1 + 20\pi^6) + (16\pi^8 + 17\pi^4\gamma_1 + \gamma_1^2)\right] = 0. \quad (7)$$

Solving Eq. (7) for V, we obtain the critical flow velocity for the pinned–pinned case. Doaré and de Langre [9], have used Eq. (8) below, for computing the critical velocity, considering only the Winkler foundation. This equation is based on the relations for a column under compressive load [11]

$$V_{\mathrm{cr}} = N\pi\left(1 + \frac{\gamma_1}{(N\pi)^4}\right)^{1/2}, \quad (8)$$

where N is the smallest integer satisfying $N^2(N+1)^2 \geqslant \gamma_1/\pi^4$.

2.2. Pinned–clamped and clamped–clamped pipe

The boundary conditions for a pinned–clamped pipe are

$$w(0,t) = w(L,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0. \quad (9)$$

And those for a clamped–clamped pipe are

$$w(0,t) = w(L,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = \frac{\partial w(L,t)}{\partial x} = 0. \quad (10)$$

We assume the deflection of the pipe to be of the form

$$w(x,t) = \Re\left[\phi_n\left(\frac{x}{L}\right)e^{i\omega t}\right]. \quad (11)$$

In Eq. (11), $\Re$ denotes the real part, $\phi_n(x/L)$ is a series of beam eigenfunctions $\psi_r(\xi)$ given by

$$\psi_r(\xi) = \cosh(\lambda_r\xi) - \cos(\lambda_r\xi) - \sigma_r(\sinh(\lambda_r\xi) - \sin(\lambda_r\xi)), \quad r = 1, 2, 3, \ldots, n; \quad \xi = \left(\frac{x}{L}\right), \quad \sigma_r = \frac{\cosh\lambda_r - \cos\lambda_r}{\sinh\lambda_r - \sin\lambda_r}. \quad (12)$$

In the above equation, λ_r is the frequency parameter of the pipe without fluid flow, which is considered as a beam, and it's values [12] are:

$\lambda_1 = 3.926602$ and $\lambda_2 = 7.068583$ for the pinned–clamped case and $\lambda_1 = 4.730041$ and $\lambda_2 = 7.853205$ for the clamped–clamped case.

Substituting Eq. (11) in the equation of motion Eq. (2) gives

$$L_n = EI\frac{\partial^4 \phi}{\partial x^4} + (\rho A v^2 - k_2)\frac{\partial^2 w}{\partial x^2} + 2\mathrm{i}\omega\rho A v\frac{\partial \phi}{\partial x} + (k_1 - M\omega^2)\phi = 0. \tag{13}$$

Following the method given in Ref. [8], using Galerkin's method and minimizing the mean square of the residual L_n over the length of the pipe and using only the first two terms, we have the following equations in V.

For the pinned–clamped case:

$$(C_{11}C_{12} - C_{12}C_{21})(V^2-\gamma_2)^2 + (V^2-\gamma_2) \times\left[(\lambda_1^4+\gamma_1)C_{22} + (\lambda_2^4+\gamma_1)C_{11}\right] + \left[(\lambda_1^4+\gamma_1)(\lambda_2^4+\gamma_1)\right] = 0. \tag{14}$$

For the clamped–clamped case:

$$(C_{11}C_{12})(V^2-\gamma_2)^2 + (V^2-\gamma_2)\left[(\lambda_1^4+\gamma_1)C_{22} + (\lambda_2^4+\gamma_1)C_{11}\right] + \left[(\lambda_1^4+\gamma_1)(\lambda_2^4+\gamma_1)\right] = 0. \tag{15}$$

In Eqs. (14) and (15), the constants C_{11}, etc., are integral values, which are enumerated in Ref. [8]. Solving the above equations for V, we obtain the critical flow velocities for the pinned–clamped and clamped–clamped cases, respectively. In Doaré and de Langre [9], Eqs. (16) and (17) below have been used for obtaining the critical flow velocity for the clamped–clamped boundary conditions, considering the Winkler foundation model only

$$V_{\mathrm{cr}} = 2\pi\left(1 + \frac{3\gamma_1}{(2\pi)^4}\right)^{1/2}. \tag{16}$$

Eq. (16) is used for $\gamma_1 \leqslant (84/11)\pi^4$, and Eq. (17) below,

$$V_{\mathrm{cr}} = \pi\left(\frac{N^4+6N^2+1}{N^2+1} + \frac{\gamma_1}{\pi^4(N^2+1)}\right)^{1/2} \tag{17}$$

otherwise. Here, N is the smallest integer satisfying $N^4+2N^3+3N^2+2N+6 \geqslant \gamma_1/\pi^4$.

3. Results and discussion

In the present work, for the pinned–pinned case, the first two terms of the equation resulting from using Fourier series have been considered in obtaining the numerical results. For the clamped–clamped case, the present work has used the assumed modes in the Galerkin method, again retaining the first two terms while Doaré and de Langre [9], have used Eq. (8) for the pinned–pinned case and Eqs. (16) and (17) for the clamped–clamped case. For both the boundary conditions, they have considered the Winkler foundation model only. Since the mode shapes of the pipe will not appreciably change with fluid flow, the modes that are assumed in the present work are for a pipe or beam without fluid flow.

3.1. Case 1: $\gamma_2 = 0$, γ_1 varying

It is useful to compare the results of the present work for the condition where $\gamma_2 = 0$, which represents the Winkler foundation model, with those of Doaré and de Langre [9]. Tables 1 and 2 show the comparison. It is seen that for all the boundary conditions, the variation in the results is not significant, especially for lower values of the Winkler parameter, even though only the first two terms of the respective equations have been considered.

In Figs. 2 and 3, here, a comparison is made with Fig. 3 of Doaré and de Langre [9], for the pinned–pinned and the clamped–clamped boundary conditions, respectively, for the condition where the parameter γ_2 equals zero. Eq. (8) has been used for pinned–pinned case and Eqs. (16) and (17) for the clamped–clamped case. As shown in Fig. 2, for the value of the shear parameter γ_2 equal to zero, there is very good agreement with the curve given in Fig. 3 of Doaré and de Langre [9] for the pinned–pinned case, up to a value of $\gamma_1 = 4500$. Higher values of γ_1 give higher values of critical velocity as compared to the work of Doaré and de Langre [9]. This deviation could be attributed to the use of only the first two terms of Eq. (5). As the value of γ_1 is

Table 1
Values of the critical velocity parameter for various values of γ_1 with $\gamma_2 = 0.0$ for the pinned–pinned case

γ_1	Doaré and de Langre [9]	Present work	% Variation
1.00E+00	3.1577	3.15768	0.0
1.00E+01	3.2989	3.29891	0.0
1.00E+02	4.4723	4.47233	0.0
2.00E+02	5.4894	5.48943	0.0
3.00E+02	6.3455	6.34555	0.0
4.00E+02	7.0435	7.04347	0.0
5.00E+02	7.2211	7.22105	0.0
6.00E+02	7.3944	7.39436	0.0
7.00E+02	7.5637	7.5637	0.0
8.00E+02	7.7293	7.72934	0.0
9.00E+02	7.8915	7.89149	0.0
1.00E+03	8.0504	8.05039	0.0
1.10E+03	8.2062	8.2062	0.0
1.30E+03	8.5093	8.50928	0.0
1.50E+03	8.8019	8.80192	0.0
1.70E+03	9.0851	9.08515	0.0
2.00E+03	9.4942	9.49416	0.0
2.50E+03	10.1392	10.13924	0.0
3.00E+03	10.7457	10.74566	0.0
3.50E+03	11.3196	11.31965	0.0
4.00E+03	11.5697	11.8659	2.6
4.50E+03	11.8105	12.38809	4.9
5.00E+03	12.0464	12.88914	7.0
5.50E+03	12.2778	13.37143	8.9
6.00E+03	12.505	13.83691	10.7
7.00E+03	12.9473	14.72381	13.7

increased, more and more modes should be taken into consideration [13]. Fig. 3 shows the comparison for the clamped–clamped case. In this case also, for higher values of γ_1 there is a deviation in the results obtained here, due to the same reason.

3.2. Case 2: γ_1, γ_2 varying

In Tables 3–5, the effect of the shear parameter γ_2 on the critical velocity is clearly brought out. The comparison is made for two values of the Winkler parameter $\gamma_1 = 10.0$ and 1.0E4. It is seen that, percentage-wise, compared to the value of $\gamma_2 = 0.0$, there is a very high increase in the value of V_{cr} for increasing values of γ_2. This increase is somewhat lower for the pinned–clamped and the clamped–clamped conditions. In Figs. 4–6, the influence of γ_2 on the critical velocity parameter of the pipe for the three boundary conditions is shown for various values of γ_1. There is not any perceptible change in the behaviour of the pipe until the shear constant of the two-parameter foundation γ_2 takes a value of 10.0. The critical velocity increases slightly for the value of γ_2 of 10.0. For a value of γ_2 of 100.0, there is a sharp jump in the value of the critical velocity parameter and this trend continues for increasing values of γ_2, as shown in the figures. Another observation from these plots is that, for lower values of γ_2, there is a sharp increase in the value of critical velocity for the Winkler foundation constant γ_1 values greater than 10.0. The critical velocity does not seem to be effected by the value of the Winkler constant γ_1 for higher values of γ_2.

3.3. Case 3: $\gamma_1 = 0.0$, γ_2 varying and $\gamma_2 = 0.0$, γ_1 varying

Finally, a comparison of the individual effects of each of the two foundation parameters on the critical velocity parameter, when the other is equivalent to zero, is shown in Fig. 7, for the pinned–pinned case.

Table 2
Values of the critical velocity parameter for various values of γ_1 with $\gamma_2 = 0.0$ for the clamped–clamped case

γ_1	Doaré and de Langre [9]	Present work	% Variation
1.00E+00	6.2892	6.38505	1.5
1.00E+01	6.3434	6.44208	1.6
1.00E+02	6.8613	6.98684	1.8
2.00E+02	7.3944	7.54614	2.1
3.00E+02	7.8915	8.06676	2.2
4.00E+02	8.3591	8.55576	2.4
5.00E+02	8.8019	9.01828	2.5
6.00E+02	9.2235	9.45821	2.5
7.00E+02	9.6266	9.87856	2.6
8.00E+02	8.9447	9.9984	11.8
9.00E+02	9.2235	10.10641	9.6
1.00E+03	9.4942	10.21328	7.6
1.10E+03	9.7573	10.31904	5.8
1.30E+03	10.2634	10.52738	2.6
1.50E+03	10.5512	10.73167	1.7
1.70E+03	10.7415	10.93215	1.8
2.00E+03	11.0209	11.22615	1.9
2.50E+03	11.4713	11.69976	2.0
3.00E+03	11.9048	12.15492	2.1
3.50E+03	12.323	12.59364	2.2
4.00E+03	12.7274	13.01758	2.3
4.50E+03	13.1194	13.42815	2.4
5.00E+03	13.5001	13.82653	2.4
5.50E+03	13.7824	14.21375	3.1
6.00E+03	13.9649	14.5907	4.5
7.00E+03	14.3231	15.31678	6.9

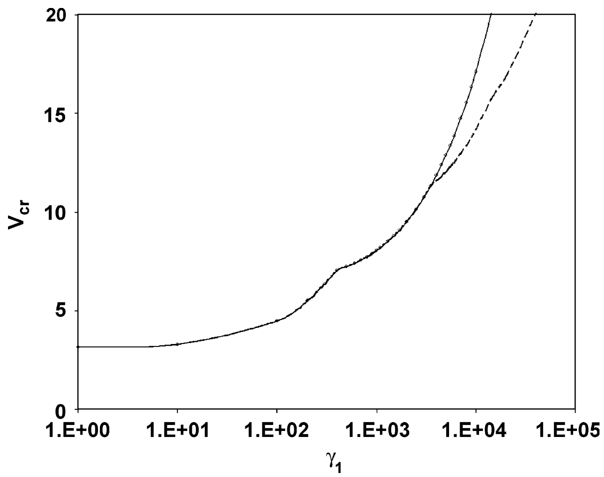

Fig. 2. Comparison of results for $\gamma_2 = 0.0$, with Fig. 3. of Doaré and de Langre [9] —○—, pinned–pinned pipe (present work); – – –, pinned–pinned pipe [9].

The top curve shows that there is a sharp increase in the critical velocity when there is a progressive increase in the value of γ_2 beyond 100.0. This curve represents the case where γ_1 is near zero. The bottom curve shows the variation of critical velocity with γ_1 when γ_2 is near zero. It can be observed that the influence of the shear constant of the two-parameter foundation is more than that of the Winkler constant on the critical velocity.

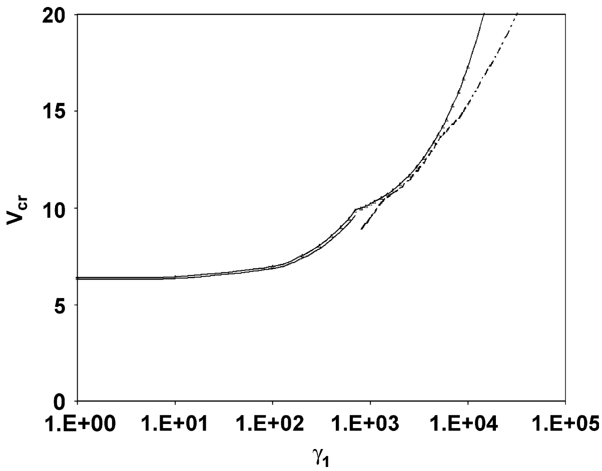

Fig. 3. Comparison of results for $\gamma_2 = 0.0$, with Fig. 3 of Doaré and de Langre [9] —△—, clamped–clamped pipe (present work); ———, clamped–clamped pipe—Eq. (16); —· ·–, clamped–clamped pipe—Eq. (17).

Table 3
Pinned–pinned case: Variation in V_{cr} for $\gamma_1 = 10.0$ and 1.0E4

γ_2	$V_{\text{cr-10.0}}$	% Variation	$V_{\text{cr-1.0E4}}$	% Variation
1.0E−06	3.2989	0.0	17.1108	0.0
1.0E−04	3.2989	0.0	17.1108	0.0
1.0E+01	4.4586	35.2	17.4006	1.7
1.0E+02	10.4823	217.8	19.8187	15.8
1.0E+03	31.7786	863.3	35.9552	110.1
5.0E+03	70.7875	2045.8	72.751	325.2
1.0E+04	100.054	2933.0	101.4533	492.9

Table 4
Pinned–clamped case: variation in V_{cr} for $\gamma_1 = 10.0$ and 1.0E4

γ_2	$V_{cr-10.0}$	% Var.	$V_{cr-1.0E4}$	% Var.
1.0E−06	4.5908	0.0	16.9195	0.0
1.0E−04	4.5908	0.0	16.9195	0.0
1.0E+01	5.5745	21.4	17.2125	1.7
1.0E+02	11.0034	139.7	19.6537	16.2
1.0E+03	31.9542	596.0	35.8646	112.0
5.0E+03	70.8595	1443.5	72.7067	329.7
1.0E+04	100.105	2080.5	101.4213	499.4

Table 5
Clamped–clamped case: variation in V_{cr} for $\gamma_1 = 10.0$ and 1.0E4

γ_2	$V_{cr-10.0}$	% Var.	$V_{cr-1.0E4}$	% Var.
1.0E−06	6.4420	0.0	17.3133	0.0
1.0E−04	6.4420	0.0	17.3133	0.0
1.0E+01	7.1763	11.4	17.5997	1.7
1.0E+02	11.895	84.7	19.9937	15.5
1.0E+03	32.2722	401.0	36.0520	108.2
5.0E+03	71.0035	1002.2	72.7993	320.5
1.0E+04	100.207	1455.5	101.487	486.2

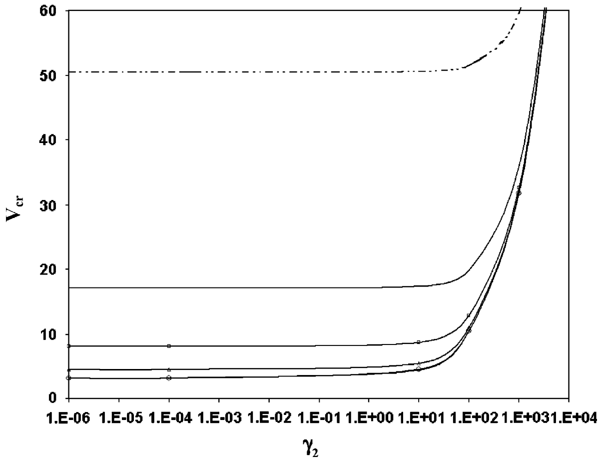

Fig. 4. Pinned–pinned pipe: Variation of V_{cr} with γ_2 for various values of γ_1; —×—, $\gamma_1 = 0$; —○—, $\gamma_1 = 1.0$; —△—, $\gamma_1 = 100.0$; —□—, $\gamma_1 = 1000.0$; ——, $\gamma_1 = 10000.0$; —· ·–, $\gamma_1 = 9.9\text{E}+4$.

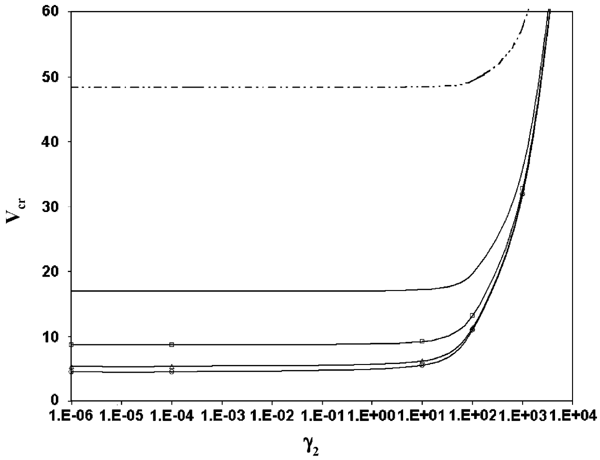

Fig. 5. Pinned–clamped pipe: Variation of V_{cr} with γ_2 for various values of γ_1; —×—, $\gamma_1 = 0$; —○—, $\gamma_1 = 1.0$; —△—, $\gamma_1 = 100.0$; —□—, $\gamma_1 = 1000.0$; ——, $\gamma_1 = 10000.0$; —· ·–, $\gamma_1 = 9.9\text{E}+4$.

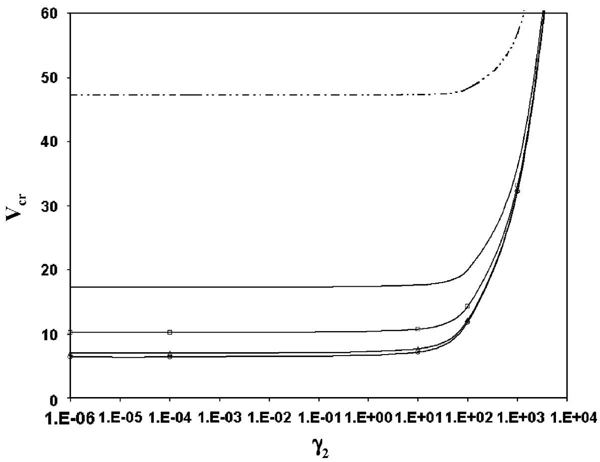

Fig. 6. Clamped–clamped pipe: Variation of V_{cr} with γ_2 for various values of γ_1; —×—, $\gamma_1 = 0$; —○—, $\gamma_1 = 1.0$; —△—, $\gamma_1 = 100.0$; —□—, $\gamma_1 = 1000.0$; ——, $\gamma_1 = 10000.0$; —· ·–, $\gamma_1 = 9.9\text{E}+4$.

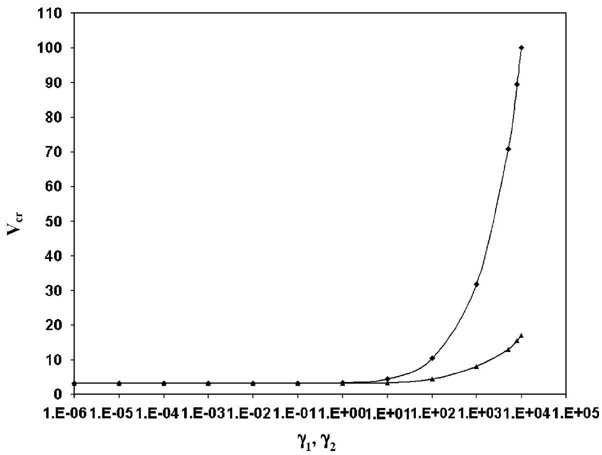

Fig. 7. Pinned–pinned pipe: Comparison of the effect of γ_1 and γ_2 on V_{cr}; —◆—, $\gamma_1 = 0.0$, —▲—, $\gamma_2 = 0.0$.

4. Conclusions

The critical flow velocity of fluid-conveying pipes has been computed for three simple boundary conditions—pinned–pinned, pinned–clamped and clamped–clamped, when such a pipe is resting on a two-parameter elastic medium like the Pasternak foundation. Results have been presented for varying values of both the foundation parameters. From the foregoing discussion, we can conclude the following:

(a) A comparison shows that the results from the present study are satisfactorily close to the results obtained by earlier researchers Doaré and de Langre [9], for the case where γ_2, the shear foundation parameter, equals zero, even though only two terms are considered for the computations. They have given results for pinned–pinned and clamped–clamped boundary conditions. In the present work, a single expression for the critical flow velocity is used to cover the entire range of foundation parameter values, while Doaré and de Langre [9] have used two equations to compute the critical flow velocity parameter for different ranges of the foundation parameter γ_1, for the clamped–clamped conditions.

(b) Results are also given for the pinned–clamped boundary condition. From the expressions for critical flow velocity parameter, one can compute the values of the parameter for conditions like $\gamma_2 = 0.0$ (only Winkler foundation), $\gamma_1 = 0.0$ (absence of Winkler foundation) and both γ_1 and γ_2 varying.

(c) New results are presented for a fluid-conveying pipe resting on a two-parameter foundation. The effect of the second parameter on the critical flow velocity is investigated. The results show that the influence of the shear parameter γ_2, cannot be ignored. The variation in the critical flow velocity is higher in the presence of γ_2.

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