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Minimal models for disk brake squeal

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Abstract

Numerous publications on the modeling of disk brake squeal can be found in the literature. Recent publications describe the onset of disk brake squeal as an instability of the trivial solution resulting from the non-conservative friction forces even for a constant friction coefficient. Therefore, a minimal model of disk brake squeal must contain at least two degrees of freedom. A literature review of minimal models shows that there is still a lack of a minimal model describing the basic behavior of disk brake squeal which can easily be associated to an automotive disk brake.

Therefore, a new minimal model of a disk brake is introduced here, showing an obvious relation to the technical system. In this model, the vibration of the disk is taken into account, as it plays a dominant role in brake squeal. The model is analyzed with respect to its stability behavior, and consequences in using it in the optimization of disk brake systems are discussed.

1. Introduction

It is commonly accepted by engineers and scientists working in the field of brake noise, that squeal in a disk brake is initiated by an instability due to the friction forces leading to self-excited vibrations. The self-excited brake system then oscillates, reaching a limit cycle. The reason for the onset of instability has been put forward on different reasons, for example, the change of the friction characteristic with the speed of the contact points [1–3] or the change of the relative orientation of the disk and the friction pads leading to a modification of the friction force [4] and a flutter instability which is found even with a constant friction coefficient [5–14]. A broad overview on the phenomenon and the modeling of disk brake squeal is given in Ref. [15]. Some of these models and mechanisms have, however, not been validated by physical experiments. It is of course well known that a negative slope in the friction characteristic leads to instability and self-excitation. It is, however, also known from laboratory experiments that there may be instability and self-excitation leading to squeal even in the absence of a negative slope of the friction characteristic. The authors believe that the flutter instability which may even occur with a constant coefficient of friction in most cases is a more realistic cause of brake squeal.

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Brakes are commonly modeled as multibody systems (MBS) or using finite element (FE) methods resulting in models with high numbers of degrees of freedom [7–9]. Nevertheless, for a basic understanding of the excitation mechanism, the influence of system parameters and for active control of brake squeal [7–10], models with a low number of degrees of freedom are more convenient. In the literature a number of such models can be found, containing two or three degrees of freedom; the models by Shin et al. [3], Hoffmann et al. [11], Popp et al. [12] and Brommundt [13] will be discussed in the following literature review of minimal models.

The result of this review is that a minimal model easily to be associated to an automotive disk brake and representing the basic behavior of disk brake squeal is still missing. A new two-degree-of-freedom model for disk brake squeal is developed in this paper, complementing the existing models in the sense described above. The basic excitation mechanism as found in this model is implemented in MBS [7–9] and can be implemented in FE models assuming a velocity-independent friction characteristic. This makes the new minimal model valuable for a better understanding of the basis of the brake squeal phenomenon.

2. Literature review of minimal models

2.1. The model by Shin et al.

The model by Shin et al. [3] is a recent example of models probably found in hundreds of publications, describing the onset of self-excited vibrations by a falling friction characteristic. A model, based on this assumption, can produce self-excited vibrations even in the case of one degree of freedom. Vibrations of the disk (note that symmetric disks have double eigenfrequencies) play a dominant role in the case of brake squeal. Since squealing very often is mono-frequent, a discretization of the disk using two eigenmodes corresponding to one eigenfrequency close to the frequency of squeal gives a suitable description of the problem. The authors of the paper [3] intend to give a detailed discussion of damping effects in the pads and in the disk, also considers nonlinear effects and do not focus on the derivation of minimal models for brake squeal. Nevertheless, it is typical for describing self-excited vibrations based on a falling friction characteristic.

The system shown in Fig. 1 represents the pad and the disk as single-degree-of-freedom systems connected through a sliding friction interface. Provided that the relative velocity between the pad and disk is always positive and considering the friction characteristic given in Fig. 2, the resulting equations of motion can be written as

$$m_1\ddot{x}_p + c_1\dot{x}_p - N\alpha(\dot{x}_p - \dot{x}_d) + k_1 x_p = N(\mu_s - \alpha v_0), \tag{1}$$

$$m_2\ddot{x}_d + c_2\dot{x}_d - N\alpha(\dot{x}_d - \dot{x}_p) + k_2 x_d = -N(\mu_s - \alpha v_0). \tag{2}$$

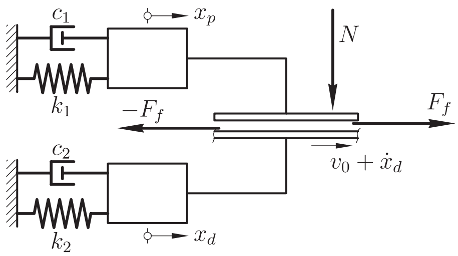

Fig. 1. Minimal model by Shin et al.

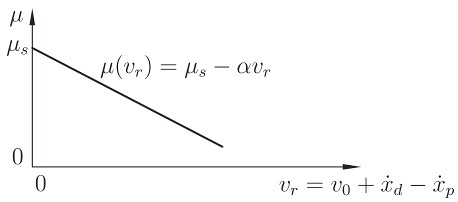

Fig. 2. Minimal model by Shin et al.: friction characteristic.

The stability analysis for the linear system can be performed using the Hurwitz criterion. Obviously the trivial solution may become unstable, as there are negative damping terms due to the falling friction characteristic.

An actual automotive disk brake does, however, look quite different than this system. In fact, there are virtually no obvious similarities between the two. Although the assumption of a negative friction characteristic is somewhat artificial, it is introduced here to produce negative damping terms and therefore instabilities to be interpreted as squealing. Other authors have therefore dealt with models of friction oscillators assuming a constant or even an increasing friction characteristic, which then have at least two degrees of freedom. Some of these models are discussed in the following.

2.2. The model by Hoffmann and Gaul

In Ref. [11], Hoffmann and Gaul present a minimal model for clarifying the physical mechanisms underlying the mode-coupling instability of self-excited friction induced oscillations. It can therefore also be interpreted as a model for disk brake squeal. The model is shown in Fig. 3. A conveyor belt with constant velocity v_B is pushed with a constant normal force F_N against a block modeled as a particle m. The block is suspended by two linear springs with stiffnesses k_1 and k_2 and, in addition, a third linear spring with stiffness k_3 representing the normal contact stiffness between the block and the moving belt. A Coulomb-type friction force F_F with constant μ is assumed. Considering small perturbations around the steady sliding state and approximating the friction force by $F_F = \mu k_3 y$ the resulting equations of motion are

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} - \mu k_3 \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

with

$$k_{11} = k_1 \cos^2 \alpha_1 + k_2 \cos^2 \alpha_2,$$

$$k_{12} = k_{21} = k_1 \sin \alpha_1 \cos \alpha_1 + k_2 \sin \alpha_2 \cos \alpha_2,$$

$$k_{22} = k_1 \sin^2 \alpha_1 + k_2 \sin^2 \alpha_2 + k_3.$$

Comparing these equations with Eqs. (1) and (2), it can be seen that velocity proportional terms are not present here. Nevertheless, instability of the trivial solution can occur due to the asymmetry in the displacement proportional terms, which is discussed broadly in Ref. [11].

Also this model looks quite different from an actual disk brake, but, as was pointed out in Ref. [11], this was not the authors' main concern.

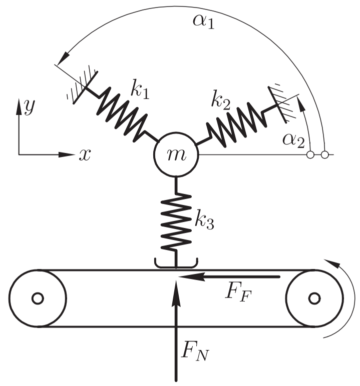

Fig. 3. Minimal model by Hoffmann and Gaul.

2.3. The model by Popp et al.

Popp et al. also give a model with two degrees of freedom (see e.g. in Ref. [12]). As the model by Hoffmann and Gaul, it does also not require a falling characteristic of the friction coefficient for instability. Fig. 4 shows this model. It has a direct resemblance to the two-degree-of-freedom model by North [5] and may be considered as its extension. In Ref. [12] Popp's model is described as resulting from a flexible disk performing structural vibrations, discretized via two sinusoidal modes and focussing only on the section of the disk in contact with the pads. The generalized coordinates are the displacement x and the rotation φ of that section. The pads are located at a distance s from the center of mass and the elastic and damping properties of the disk are represented by springs with stiffnesses c_1, c_2 and dampers with constants d_1, d_2, respectively. The equations of motion are given in Ref. [12] as

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} c_1 + c_3 & -c_3 s \\ -c_3(s - \mu h) & c_2 + c_3(s^2 - \mu h s) \end{bmatrix}\begin{bmatrix} x \\ \varphi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{4}$$

These equations of motion are derived under the assumption, that the pads always stay in contact with the disk. A preload N_0 on both pads is therefore necessary, which would change the equations of motion to

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} c_1 + c_3 & -c_3 s \\ -c_3(s - \mu h) & c_2 + c_3(s^2 + \mu h s) + 2N_0(h(1+\mu^2) + \mu s) \end{bmatrix}\begin{bmatrix} x \\ \varphi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{5}$$

It can be seen, that in contrast to the model of Hoffmann and Gaul, the preload does now effect the equations of motion, as they cause a φ-proportional term.

A stability analysis of the model without preload is also given in Ref. [12] showing the necessary conditions for instability

$$s \neq 0 \quad \text{and} \quad \mu > \frac{s}{h} > 0. \tag{6}$$

From the equations of motion (4) it is also obvious that there must be a non-vanishing distance s for the occurrence of instability.

In Popp's model it is far easier to recognize similarities with an actual disk brake than in the previous models. From experiments it can clearly be observed, that the vibrations of the disk play an essential role in the occurrence of squeal. Therefore, vibrations of the disk should be taken into consideration in a minimal model of disk brake squeal.

Nevertheless the question remains, how to interpret the distance s. The problem to be described is a disk fixed in its center and performing vibrations in permanent contact with two pads. The location of these pads can, e.g. be described by polar coordinates including a distance r from the center and an angle ψ for the spatial orientation. It is obvious that a change of the angle ψ should not affect the stability behavior of the model,

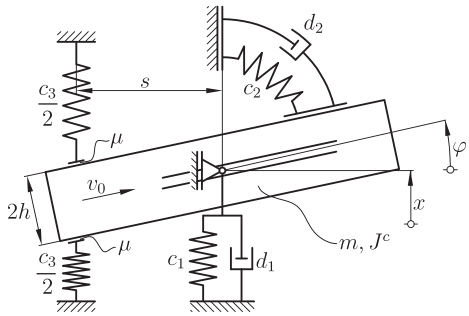

Fig. 4. Minimal model by Popp et al.

which should be invariant to ψ. This is not the case in the model described here, as in the linearized case s is given by $s = \psi r$.

2.4. Models with three degrees of freedom

In Ref. [13] Brommundt describes a three-degree-of-freedom model shown in Fig. 5. This model is somewhat similar to that of Hoffmann and Gaul (but was published earlier) and associates an additional degree of freedom to the conveyor belt. In Ref. [13], it is shown, that even in case of a monotonously increasing friction characteristic instability can occur in this model. In the equations of motion of that model, also asymmetric displacement-proportional terms are present.

In Ref. [14] Schmieg and Vielsack also give a three-degree-of-freedom model for brake squeal, resulting in equations of motion with asymmetric displacement proportional terms which can produce instabilities of the trivial solution. This model is much closer to the one of Hoffmann and Gaul than to that of Popp, as it does not focus on vibrations of the disk, but on vibrations of the pads and the caliper.

3. The new two-degree-of-freedom model for disk brake squeal

The previous discussion of models from the literature has shown, that a simple minimal model easily to be associated with automotive disk brakes is still missing. As far as the authors know the model by Popp et al. comes closest to that aim. The need of a distance s for getting instability, the missing preload N_0, which is essential for the contact between pads and disk and the exclusive modeling of a section of the disk, give room for further improvement of this model.

In what follows, the authors suggest a new two-degree-of-freedom model, in which some of the salient features of a disk brake are captured in a rather obvious way.

3.1. General description of the new model

The new model depicted in Fig. 6 consists of a rigid disk with thickness h and central inertia tensor

$$\mathbf{\Theta} = \begin{bmatrix} \Theta & 0 & 0 \\ 0 & \Theta & 0 \\ 0 & 0 & \Phi \end{bmatrix} \tag{7}$$

with respect to the body fixed d_i frame. The disk is hinged in a spherical joint in its center of mass and visco-elastically supported by rotational springs (rotational stiffness k_t) and rotational dampers (damping coefficient d_t) such that it can perform wobbling motions while rotating with constant angular speed about its nominal rotation axis.

Experimental investigations of real automotive disk brakes clearly show that the disk does usually not perform a rigid body wobbling motion, rather undergoing an elastic vibration, e.g. with three to five nodal diameters in the frequency range from 1 to 10 kHz. In the context of this paper, the wobbling motion of a disk

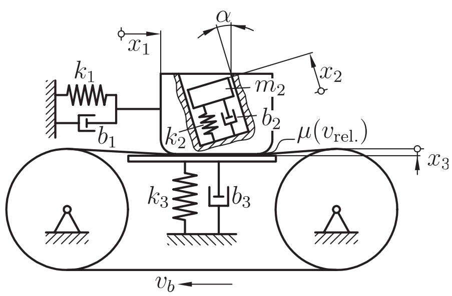

Fig. 5. Model by Brommundt.

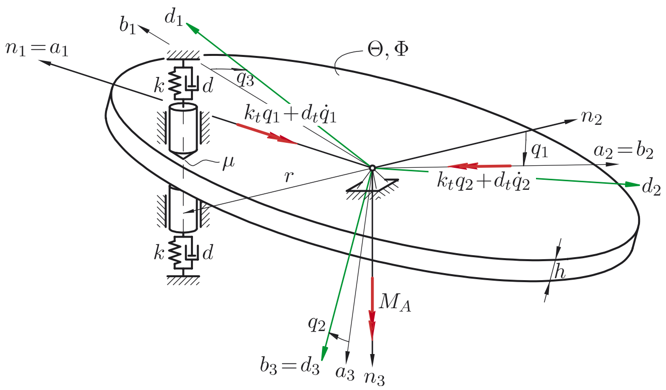

Fig. 6. Disk brake model with wobbling disk.

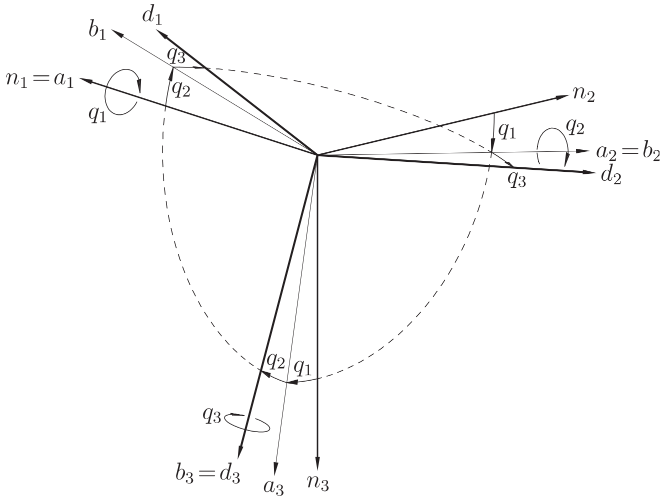

Fig. 7. Coordinate systems.

was chosen in order to produce a *minimal* model easily to be understood. For the purpose of the stability discussion given later, the system parameters were chosen such that the inertia and elastic properties of the wobbling disk represent two orthogonal elastic eigenmodes with three nodal diameters of a real disk, experimentally investigated by the authors in Refs. [7–10]. It is shown in Ref. [16], that the equations of motion derived by assuming a rotating elastic Kirchhoff plate discretized by two orthogonal modes corresponding to one eigenfrequency are almost the same to those derived for the present model. In this context, it is also shown that a proper discussion of the kinematics of that problem requires substantial additional effort, making the elastic disk less suitable for the simplest type of a minimal model.

A Newtonian (inertial) coordinate system is defined by the Cartesian coordinate system with unit vectors $\mathbf{n}_i$ $(i = 1, 2, 3)$. The disk is in contact with two pads with preload N_0, mounted at distance r from the center of the disk. The preload is considered to be large enough so that there is always contact between the pads and the disk. The pads are fixed in the $\mathbf{n}_1, \mathbf{n}_2$-plane and can move only in the $\mathbf{n}_3$-direction, with restoring forces generated by two springs with stiffness k and with two dampers with damping constant d. The orientation of the disk is described by the Cardan-angles q_i $(i = 1, 2, 3)$ and two intermediate coordinate systems $\mathbf{a}_i$ and $\mathbf{b}_i$ $(i = 1, 2, 3)$ are introduced (see Fig. 7). The resulting angular velocity of the disk with respect to the Newtonian system is then given by

$$ {}^{N}\boldsymbol{\omega}^{D} = \dot{q}_1 \mathbf{n}_1 + \dot{q}_2 \mathbf{a}_2 + \dot{q}_3 \mathbf{b}_3. \tag{8} $$

It is assumed that there is a non-holonomic constraint such that

$$^{N}\boldsymbol{\omega}^{D} \cdot \mathbf{n}_3 = \Omega = \text{const.} \tag{9}$$

holds and the corresponding constraint torque is $\mathbf{M}_A = M_A\mathbf{n}_3$.

The general analysis of the nonlinear model is performed using the commercial software `Autolev` based on Kane's algorithm for the derivation of equations of motion, which is described in Ref. [17].

3.2. Determination of the contact points

As the pads are fixed in the $\mathbf{n}_1, \mathbf{n}_2$-plane, the contact points can be described on the one hand by the position vectors

$$\mathbf{p}_1 = -r\,\mathbf{n}_2 + \left(h_1 - \frac{h}{2}\right)\mathbf{n}_3, \tag{10}$$

$$\mathbf{p}_2 = -r\mathbf{n}_2 + \left(h_2 + \frac{h}{2}\right)\mathbf{n}_3, \tag{11}$$

where h_1 and h_2 are the corresponding displacements of the pads with respect to the static equilibrium and therefore the displacements of the springs. The pads are assumed to be located at distance r in negative $\mathbf{n}_2$-direction from the center of the disk. Generally, any arbitrary point in the $\mathbf{n}_1, \mathbf{n}_2$-plane with distance r from the center can be chosen without any influence on the stability results discussed in Section 4.

On the other hand, the contact points must be points on the surface of the disk, and therefore the position vectors have to fulfill the relations

$$\mathbf{p}_1 = d_{11}\mathbf{d}_1 + d_{12}\mathbf{d}_2 - \frac{h}{2}\mathbf{d}_3, \tag{12}$$

$$\mathbf{p}_2 = d_{21}\mathbf{d}_1 + d_{22}\mathbf{d}_2 + \frac{h}{2}\mathbf{d}_3. \tag{13}$$

Eqs. (10)–(13) give six relations for the six unknowns h_1, h_2, d_{11}, d_{12}, d_{21} and d_{22} which can therefore be calculated as functions of q_1, q_2 and q_3. The expressions for the solutions in the general nonlinear case are, as all following results, quite lengthy and therefore omitted in this paper. After linearizing for small angles q_1 and q_2, the position vectors can be expressed in the intermediate $\mathbf{b}_i$-frame as

$$\mathbf{p}_1 = \frac{h}{2}q_2\mathbf{b}_1 + \left(-r - \frac{h}{2}q_1\right)\mathbf{b}_2 - \frac{h}{2}\mathbf{b}_3, \tag{14}$$

$$\mathbf{p}_2 = -\frac{h}{2}q_2\mathbf{b}_1 + \left(-r + \frac{h}{2}q_1\right)\mathbf{b}_2 + \frac{h}{2}\mathbf{b}_3 \tag{15}$$

and the two linearized displacements are

$$h_1 = h_2 = -rq_1. \tag{16}$$

3.3. Determination of the relative velocity of the contact points

The velocity of the two material points on the surface of the disk in actual contact with the pad, can be determined by

$$\mathbf{v}_1 = {}^{N}\boldsymbol{\omega}^{D} \times \mathbf{p}_1, \tag{17}$$

$$\mathbf{v}_2 = {}^{N}\boldsymbol{\omega}^{D} \times \mathbf{p}_2, \tag{18}$$

as the velocity of the center of the disk is zero. The velocity of the contact points of the pads are obtained by differentiating the position vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ from Eqs. (12) and (13) with respect to time in the inertial system

$$\mathbf{v}_1' = \frac{{}^N\mathrm{d}}{\mathrm{d}t}\,\mathbf{p}_1, \tag{19}$$

$$\mathbf{v}_2' = \frac{{}^N\mathrm{d}}{\mathrm{d}t}\,\mathbf{p}_2. \tag{20}$$

The direction of the friction force is then given by the unit vector of the relative velocity of the two bodies in contact. The directions of the friction forces acting on the respective *pads* are therefore given by

$$\mathbf{r}_1 = \frac{\mathbf{v}_1 - \mathbf{v}_1'}{|\mathbf{v}_1 - \mathbf{v}_1'|} \tag{21}$$

and

$$\mathbf{r}_2 = \frac{\mathbf{v}_2 - \mathbf{v}_2'}{|\mathbf{v}_2 - \mathbf{v}_2'|}. \tag{22}$$

These two unit vectors can be linearized for small angles q_1, q_2 as well as for small angular velocities $\dot{q}_1, \dot{q}_2$ and are then given by

$$\mathbf{r}_1 = -\mathbf{b}_1 - \frac{h}{2r}\left(\frac{\dot{q}_1}{\dot{q}_3} + q_2\right)\mathbf{b}_2, \tag{23}$$

$$\mathbf{r}_2 = -\mathbf{b}_1 + \frac{h}{2r}\left(\frac{\dot{q}_1}{\dot{q}_3} + q_2\right)\mathbf{b}_2 \tag{24}$$

expressed in the intermediate $\mathbf{b}_i$-frame (linearized unit vectors).

3.4. Determination of the contact forces

The resulting forces acting on the upper pad sketched in Fig. 8 and are given by

$$\mathbf{F}_{p1} = F_{t1}\mathbf{r}_1 - F_{n1}\mathbf{d}_3 + N_0\mathbf{n}_3 - k\,h_1\mathbf{n}_3 - d\,\dot{h}_1\mathbf{n}_3 + H_{11}\mathbf{n}_1 + H_{12}\mathbf{n}_2, \tag{25}$$

while the forces on the lower pad are given by

$$\mathbf{F}_{p2} = F_{t2}\mathbf{r}_2 + F_{n2}\mathbf{d}_3 - N_0\mathbf{n}_3 - k\,h_2\mathbf{n}_3 - d\,\dot{h}_2\mathbf{n}_3 + H_{21}\mathbf{n}_1 + H_{22}\mathbf{n}_2, \tag{26}$$

where H_{ij} are forces due the supports (constraints) of the pads in the $\mathbf{n}_1, \mathbf{n}_2$-plane. The four contact forces F_{t1}, F_{t2}, F_{n1} and F_{n2} are determined by Eqs. (27)–(30) which are given by the assumed Coulomb friction law

$$F_{t1} = \mu F_{n1}, \tag{27}$$

$$F_{t2} = \mu F_{n2} \tag{28}$$

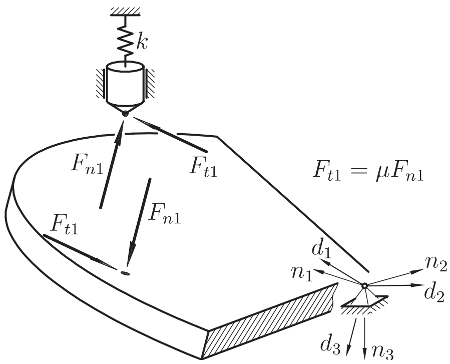

Fig. 8. Contact forces at the upper pad.

with constant friction coefficient μ, and the balance of forces in $\mathbf{n}_3$-direction for both pads

$$\mathbf{F}_{p1} \cdot \mathbf{n}_3 = 0, \tag{29}$$

$$\mathbf{F}_{p2} \cdot \mathbf{n}_3 = 0. \tag{30}$$

The resulting contact forces on the disk are then calculated from Eqs. (27)–(30) as

$$\mathbf{F}_{D1} = -F_{t1}\mathbf{r}_1 + F_{n1}\mathbf{d}_3, \tag{31}$$

$$\mathbf{F}_{D2} = -F_{t2}\mathbf{r}_2 - F_{n2}\mathbf{d}_3. \tag{32}$$

3.5. Equations of motion

The resulting torque on the disk is given by the torque due the viscoelastic suspension, the constraint torque M_A and the torque of the contact forces with respect to the center of the disk. It can be written as

$$\mathbf{M} = -k_t q_1 \mathbf{n}_1 - k_t q_2 \mathbf{a}_2 - d_t \dot{q}_1 \mathbf{n}_1 - d_t \dot{q}_2 \mathbf{a}_2 + M_A \mathbf{n}_3 + \mathbf{p}_1 \times \mathbf{F}_{D1} + \mathbf{p}_2 \times \mathbf{F}_{D2}. \tag{33}$$

The balance of angular momentum

$$\frac{{}^N\mathrm{d}}{\mathrm{d}t} \mathbf{\Theta}^N \omega^D = \mathbf{M} \tag{34}$$

and the non-holonomic constraint (9) lead to a constant driving torque

$$M_A = 2\mu r N_0 \tag{35}$$

and the equations of motion for a two-degree-of-freedom model for disk brake squeal

$$\begin{bmatrix} \Theta & 0 \\ 0 & \Theta \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\mu N_0 \frac{h^2}{r\Omega} + 2dr^2 + d_t & \Phi\Omega \\ -\Phi\Omega - \mu dhr & d_t \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$
$$+ \begin{bmatrix} k_t + 2kr^2 + N_0 h & \frac{1}{2}\mu N_0 \frac{h^2}{r} \\ -\mu(khr + 2N_0 r) & k_t + (1+\mu^2)N_0 h \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{36}$$

As mentioned in Section 3.2, the pads can be placed at any arbitrary point in the $\mathbf{n}_1, \mathbf{n}_2$-plane with distance r from the center, without any influence on the stability results, as can be shown by a corresponding coordinate transformation.

3.6. Discussion of the equations of motion

The equations of motion (36) shall now be discussed in more detail. From the equations of motion it is obvious that the velocity-dependent terms contain both gyroscopic terms, not associated to any energy dissipation, as well as damping terms. The gyroscopic terms do not come as a surprise, due to the wobbling motion of the rotating disk. Additionally to the terms caused by the damping of the brake pads and the wobbling disk, there is a linear damping term in the first equation, resulting just from kinematic relations and the friction forces. For the displacement proportional terms, it can be observed that there are asymmetric coupling terms, making the system a candidate for self-excited vibrations, which will be discussed in more detail in the following section. It should be noted, that the linearization of Eq. (34) is only possible if the speed of the disk Ω is high enough to rule out stick–slip phenomena at the contact points. That means, that the directions of the friction forces (21) and (22) have to be well defined for all times.

4. Stability analysis

In order to study the stability of Eq. (36) realistic parameters have to be carefully chosen as the stability results described in the following show a high sensitivity to those parameters. Extensive experimental investigations were undertaken for parameter identification on a test rig at TU Darmstadt with a state-of-the-art, mass produced, brake. Corresponding parameters were identified for several multibody models. In Ref. [10], the authors presented a four-degree-of-freedom model which was successfully used to actively suppress squeal on the test rig with that system parameters. Corresponding parameters were also identified in Ref. [18] in the authors' lab, where the following parameters for the two-degree-of-freedom model are extracted:

$$h = 0.02\,\mathrm{m},\quad k_t = 1.88\times10^7\,\mathrm{N\,m},\quad N_0 = 3.00\,\mathrm{kN},\quad \Theta = 0.16\,\mathrm{kg/m^2},$$

$$r = 0.13\,\mathrm{m},\quad k = 6.00\times10^6\,\mathrm{N/m},\quad \Omega = 5\pi\,\mathrm{s^{-1}},\quad \Phi = 2\Theta,$$

$$\mu = 0.6,\quad d_t = 0.1\,\mathrm{N\,m\,s},\quad d = 5.00\,\mathrm{N\,s/m},$$

which are representative of the authors' experimental work.

Some of the parameters will be varied to show their influence on the stability of the trivial solution. The *ansatz*

$$\mathbf{q}(t) = \hat{\mathbf{q}}e^{\lambda t} \tag{37}$$

is substituted in the equation of motion (36) to find the eigenvalues λ of the linearized system. Fig. 9 shows the root locus of the eigenvalues for varying speed of the disk. Above a certain critical speed Ω_{crit} there exist eigenvalues with positive real part, the trivial solution becomes unstable and the system shows self-excited vibrations which can be interpreted as squeal. Since brake squeal mainly occurs at low speed and relatively low braking pressure, e.g. a roll-out in front of a red traffic light, the critical speed of the system is an important parameter in matters of brake squeal.

One of the usual modifications carried out by the manufacturers to improve a squealing brake are changes in the brake pads. Fig. 10 shows the critical speed Ω_{crit} for varying stiffness k of the brake pads. One can see that the critical speed increases for decreasing stiffness of the brake pads. In contrast to that, the critical speed does not significantly depend on the friction coefficient μ within this model.

From Eq. (35) follows the driving torque $M_A = 2\mu r N_0$. This means that the brake torque and therefore the braking moment depend on the product $\mu r N_0$. This raises the question whether the brake torque should be generated by a low braking pressure (N_0) and a high friction coefficient (μ) or, vice versa, with view to the avoidance of squeal. Fig. 11 shows the influence of the combination of these two parameters on the critical speed. From this picture it is obvious that brake torque should be generated by a high normal force and a

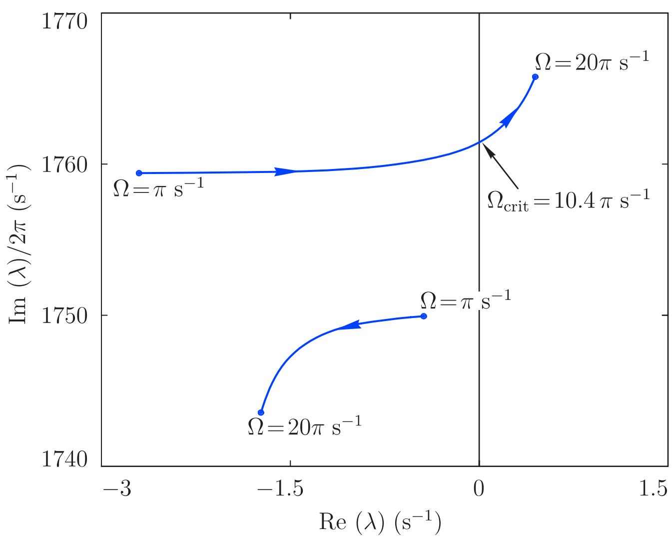

Fig. 9. Root locus of the eigenvalues for varying Ω (upper half-plane shown only).

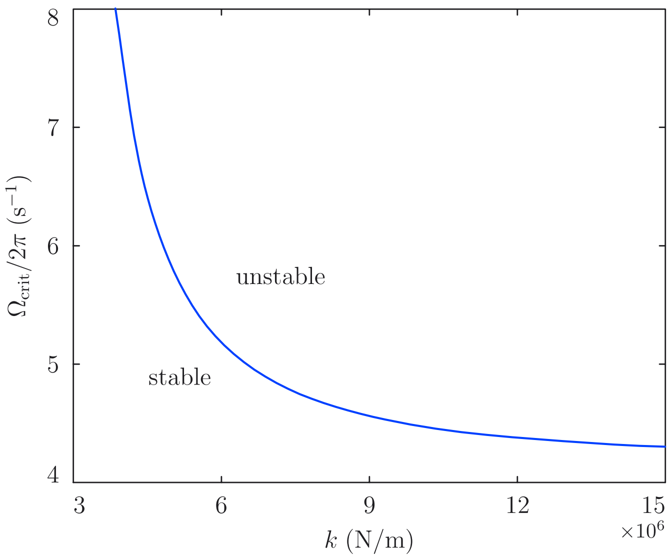

Fig. 10. Critical speed for varying k.

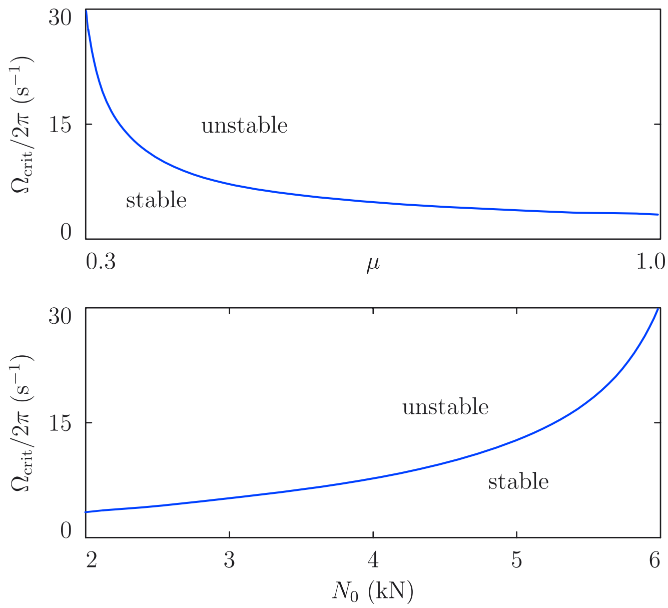

Fig. 11. Critical speed for varying μ and N_0 with constant braking torque.

relatively low coefficient of friction in order to avoid brake squeal. However, a low friction coefficient limits the capability of the brake system, in particular for emergency stops.

Another important design parameter is the radius r of the brake disk. The capability of the brake increases with increasing radius of the brake disk, but the radius of the disk is limited by the dimensions of the wheel rim, and its mass is part of the unsprung mass of the vehicle. Fig. 12 shows the dependence of the critical speed on r and N_0, with the product rN_0 held constant. In contrast to Fig. 11, there exists a minimum of the critical speed, i.e. an unfavorable combination of r and N_0.

It should finally be mentioned that all the stability results strongly depend on the chosen parameters. Especially the viscous damping term due to the kinematics of the frictional contact in Eq. (36), and thus the geometry of the system (r and h) plays an important role.

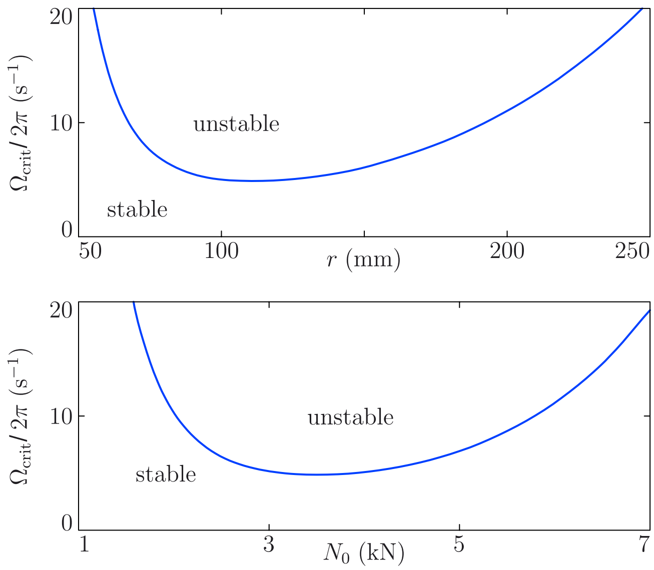

Fig. 12. Critical speed for varying r and N_0 with constant braking torque.

5. Conclusions

Minimal models containing two or three degrees of freedom, especially the models by Shin [3], Hoffmann and Gaul [11], Popp [12] and Brommundt [13] were discussed in this paper. This discussion showed that a minimal model describing the basic behavior of disk brake squeal which can easily be associated with a disk brake, is still missing. A new minimal model was therefore introduced, containing a wobbling disk in point contact with two pads. The parameters of the wobbling disk were chosen such that it represents two orthogonal modes of an elastic disk corresponding to the same eigenfrequency.

The equations of motion were derived and linearized using the commercial software `Autolev`. The model can be easily associated with a disk brake and its stability behavior is (in contrast to Ref. [12]) independent from the circumferential position of the pads. The analysis regarding the stability of the trivial solution shows the influence of the main parameters of the system. As basic parameters, those identified in Ref. [18] for a mass produced brake were used.

The minimal model presented in this paper represents a system easy to visualize. As seen in Chapter 3, the derivation of the nonlinear equations of motion is somewhat complicated but straight forward. The minimal model gives an insight into an excitation mechanism of disk brake squeal, which can also be observed in MBS or FE models with a high number of degrees of freedom.

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