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Short Communication

Crack detection of arch dam using statistical neural network based on the reductions of natural frequencies

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Abstract

This paper presents the numerical simulation and the model experiment upon a hypothetical concrete arch dam for the research of crack detection based on the reduction of natural frequencies. The influence of cracks on the dynamic property of the arch dam is analyzed. A statistical neural network is proposed to detect the crack through measuring the reductions of natural frequencies. Numerical analysis and model experiment show that the crack occurring in the arch dam will reduce natural frequencies and can be detected by using the statistical neural network based on the information of such reduction.

1. Introduction

It is well known that the occurrence of structural damage like cracks will induce changes in structural dynamic properties. These changes can be utilized by the vibration-based method to detect the structural damage. Hence, the crack occurring in an arch dam is expected to be detectable with vibration-based method. However, environmental factors such as water level of the reservoir will also affect the dynamic properties of dam. It is therefore crucial to understand if the changes due to crack are obvious enough to be measured under the influence of environmental noises and to indicate the crack happening.

Fanelli et al. [1] carried out vibration measurements of Talvacchia Dam in Italy. A scattering of resonance frequencies was observed with a pronounced trend related to temperature effects. Loh and Wu [2] studied the dynamic characteristics of Fei-Tsui arch dam using the seismic response data as well as the ambient vibration data. The identified first two modal frequencies of the dam decrease with the increase in the dam reservoir water level. Darbre and Proulx [3,4] presented the results of a continuous ambient vibration recording program carried out on the 250 m arch dam of Mauvoisim. The results showed that the resonance frequencies shift with the water level. The stiffening of the dam due to increasing hydrostatic pressure is more important than the added hydrodynamic masses for lower water levels, while this trend is reversed for higher water levels.

In this paper, we pay attention to the detection of the cracks occurring in an arch dam based on measurements of natural frequency. The research is carried out by means of numerical simulation and model

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experiment. The validity of simulation of crack adopted in FEM and the effectiveness of the statistical neural network for crack detection are confirmed.

2. The objective arch dam

A hypothetical concrete arch dam particularly designed is sketched in Fig. 1. It is a single curvic arch dam with the height of 80 m. It has a crest length of 209 m, a base thickness of 11 m and a crest thickness of 6 m.

An arch dam model with the scale of 1:100 is constructed for model experiment, as shown in Fig. 2. The dam body is made of cement mortar, while the basement, the abutment and the riverbed are made of concrete. The size of the model is given in Fig. 3.

A three-dimensional finite-element model including the dam, basement, abutment and riverbed is developed for numerical simulation by means of the software ANSYS. The size of numerical model is the same as the hypothetical dam, while the material parameters are taken the same as the physical model, as shown in Table 1.

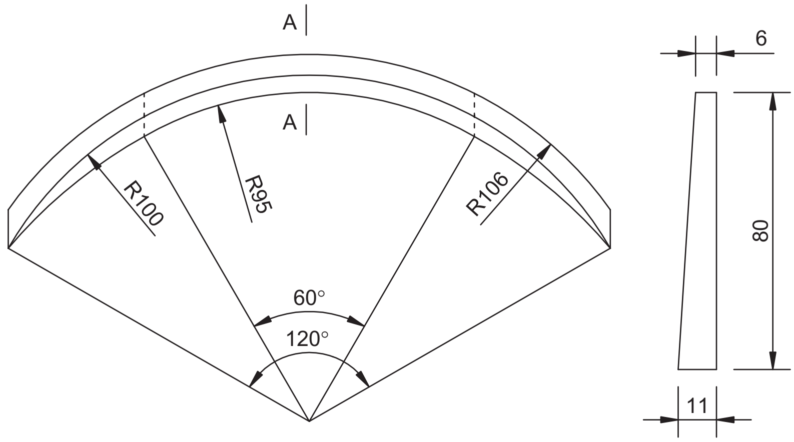

Fig. 1. The objective arch dam.

Fig. 2. Experimental model of the arch dam.

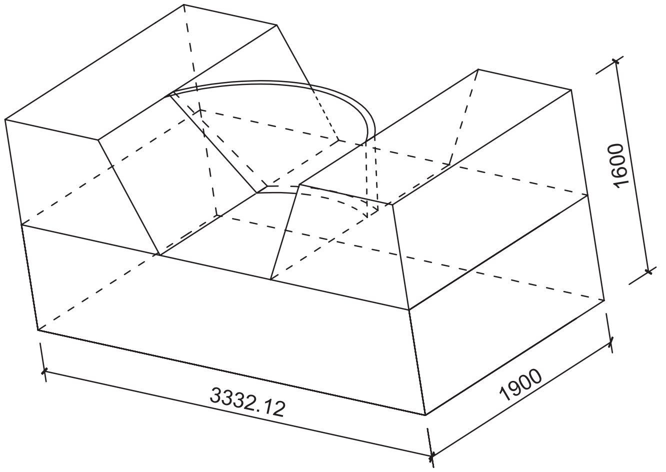

Fig. 3. Size of the experimental model.

Table 1
Main parameter values of experimental model and numerical model

Parameters	Experimental model	Numerical model
Dam height	80 cm	80 m
Crest thickness	6 cm	6 m
Base thickness	11 cm	11 m
Density of dam material	2100 kg/m^3	2100 kg/m^3
Density of base material	2200 kg/m^3	2200 kg/m^3
Dynamic modulus of dam material	16500 MPa	16500 MPa
Dynamic modulus of base material	21000 MPa	21000 MPa

3. Influence of cracks on natural frequencies of the dam

3.1. Numerical simulation

The proper simulation of crack is very important for numerical study of crack damage detection of a concrete dam. Generally, the presence of crack makes the response of the dam to external stimulations nonlinear. Crack models such as the link element, contact element, etc. have been proposed to simulate various nonlinearities [5–7], in particular in seismic analysis of structures. The nonlinearities may however be ignored in crack damage detection based on the vibration method because the excited vibration is relatively small. So the linear model should be appropriate in numerical simulation of crack detection. This will be justified by the model experiment.

One crack is assumed in the numerical model to investigate the influence of crack on natural frequencies of the arch dam. The crack faces are assumed stress-free without contact to each other. Three different depths of the crack, viz. one quarter, half and three quarters of the thickness, are considered as shown in Fig. 4.

The first six frequencies of the arch dam with and without crack are computed and listed in Table 2. The relative reductions of natural frequencies due to the crack are shown in Table 3. It can be seen that the reductions of natural frequencies due to the 0.25-depth crack (the crack with depth of one quarter of thickness) are very small, and the maximum is only 0.25% at the first mode frequency. However, the natural frequencies are obviously reduced due to the 0.75-depth crack (the crack with depth of three quarters of thickness) and the maximum relative reduction of the first frequency reaches 5.96%.

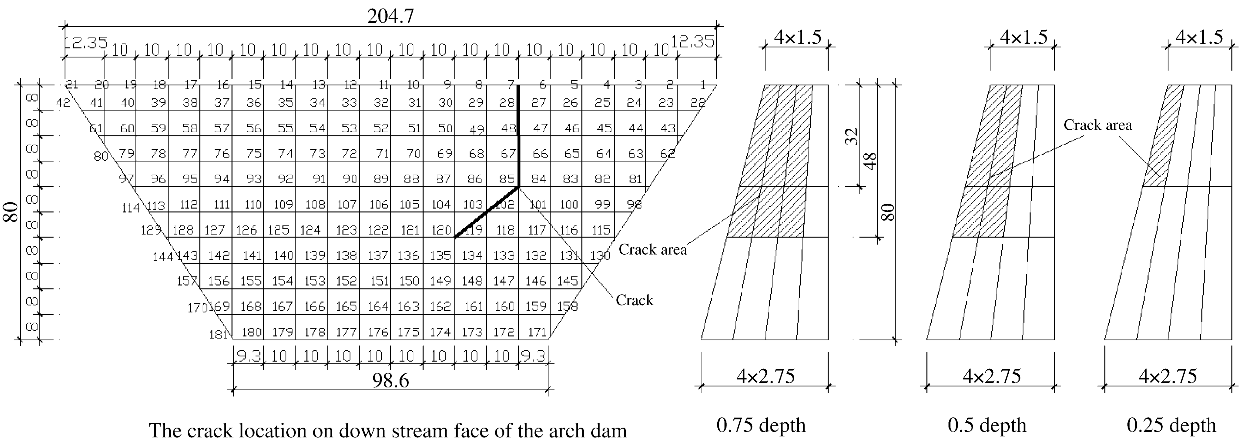

Fig. 4. The crack simulated in the numerical model.

Table 2
Computed natural frequencies of the first 6 modes (Hz)

Mode no.	No crack	0.25-depth crack	0.5-depth crack	0.75-depth crack
1	3.1481	3.1402	3.0848	2.9605
2	3.5491	3.5416	3.4886	3.3834
3	4.9589	4.9505	4.9392	4.9004
4	6.3298	6.3260	6.2712	6.1304
5	6.4967	6.4863	6.4321	6.2745
6	7.4366	7.4321	7.3722	7.3288

Table 3
Relative reductions of computed natural frequencies (%)

Mode no.	0.25-depth crack	0.5-depth crack	0.75-depth crack
1	0.25	2.01	5.96
2	0.21	1.70	4.67
3	0.17	0.40	1.18
4	0.06	0.93	3.15
5	0.16	0.99	3.42
6	0.06	0.87	1.45

3.2. Modal testing

The crack is also simulated in the physical model of the arch dam. As shown in Fig. 5, the location of the crack in the physical model is not exactly the same as, but closely near to the location as simulated in the numerical model. Two crack depths of half and three quarters of thickness are separately formed on the physical model. So, three cases, i.e. no crack, 0.5-depth crack and 0.75-depth crack are considered in our experiments.

The modal tests are carried out upon the physical model before and after the presence of crack in the dam. In the test, an accelerometer is set on the fixed location of point No. 32 shown in Fig. 5. The excitation is an impulse created by a force hammer (the hammer with a force transducer). Each point shown in Fig. 5 except point No. 32 is an excitation point in turn in the test. The transfer function between each excitation point and

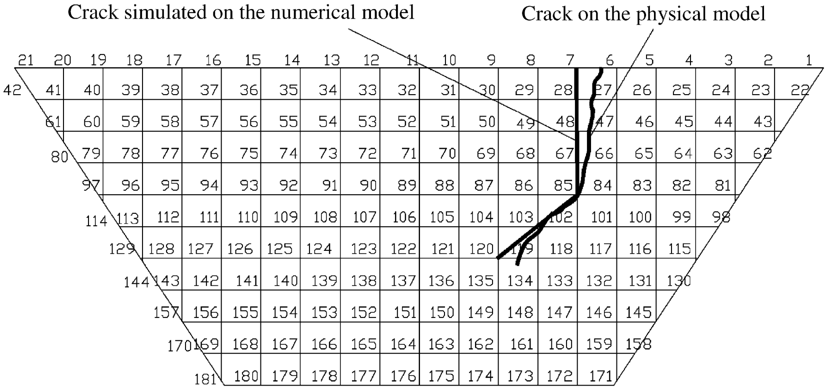

Fig. 5. The crack location on the physical model.

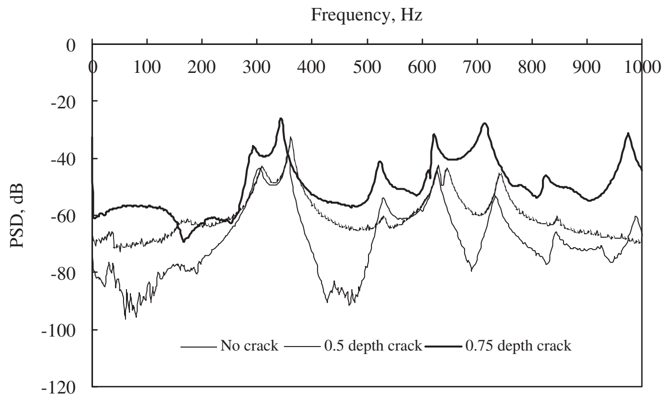

Fig. 6. Typical PSD curves.

Table 4
Mean values of natural frequencies obtained in model experiment (Hz)

Mode no.	No crack	0.5-depth crack	0.75-depth crack
1	307.06	302.21	291.64
2	360.55	356.29	343.51
3	530.92	529.99	522.72
4	630.06	627.76	618.85
5	644.08	635.71	—
6	741.45	732.99	714.57

the recording point No. 32 is obtained accordingly. The modal parameters of the model dam then can be identified.

About 50 modal tests have been done for each case. The typical power spectral density curves for the three cases are shown in Fig. 6. The mean values of the frequencies obtained from 50 tests are given in Table 4, and the frequency reductions of physical model due to the crack are listed in Table 5. Similar to the numerical analysis, the frequency reductions in experiment are obvious in the case of 0.75-depth crack.

Fig. 7 shows the comparison of frequency reductions between the test results and the FEM results. The relative frequency reductions due to the crack obtained from tests agree well with those obtained by FEM

Table 5
Relative reductions of tested natural frequencies (%)

Mode no.	0.5-depth crack	0.75-depth crack
1	1.58	5.02
2	1.18	4.73
3	0.18	1.54
4	0.36	1.78
5	1.30	—
6	1.14	3.62

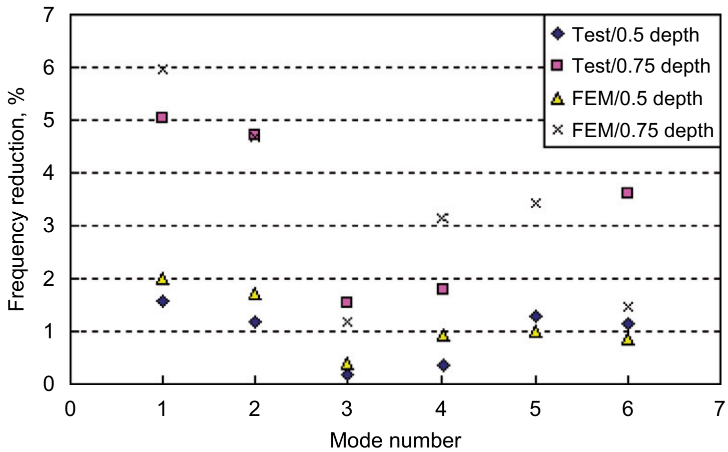

Fig. 7. Comparison of frequency reduction.

especially at the first three modes. This result implies that the influence of crack's nonlinearities under small excitation is slight and can be ignored for the present problem. Simulating a crack as two free surfaces facing each other is appropriate and the linear analysis is feasible in crack detection study.

4. Crack detection using statistical neural network

4.1. Statistical neural network [8]

Based on the probability density function (PDF), the statistical neural network (SNN) produces a comparison of two random variables for decision. Fig. 8 shows the PDF of two random variables X and Y.

The probability of X smaller than Y is

$$S_1 = \int_{-\infty}^{+\infty} \int_{x}^{+\infty} g(y)\,\mathrm{d}y f(x)\,\mathrm{d}x = \int_{-\infty}^{+\infty} f(x) \int_{x}^{+\infty} g(y)\,\mathrm{d}y\,\mathrm{d}x \tag{1}$$

and X larger than Y is

$$S_2 = \int_{-\infty}^{+\infty} f(x) \int_{-\infty}^{x} g(y)\,\mathrm{d}y\,\mathrm{d}x, \tag{2}$$

plus the probability is

$$S_1 + S_2 = \int_{-\infty}^{+\infty} f(x) \int_{x}^{+\infty} g(y)\,\mathrm{d}y\,\mathrm{d}x + \int_{-\infty}^{+\infty} f(x) \int_{-\infty}^{x} g(y)\,\mathrm{d}y\,\mathrm{d}x = 1. \tag{3}$$

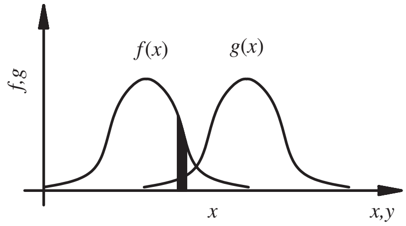

Fig. 8. The probability density function of X and Y.

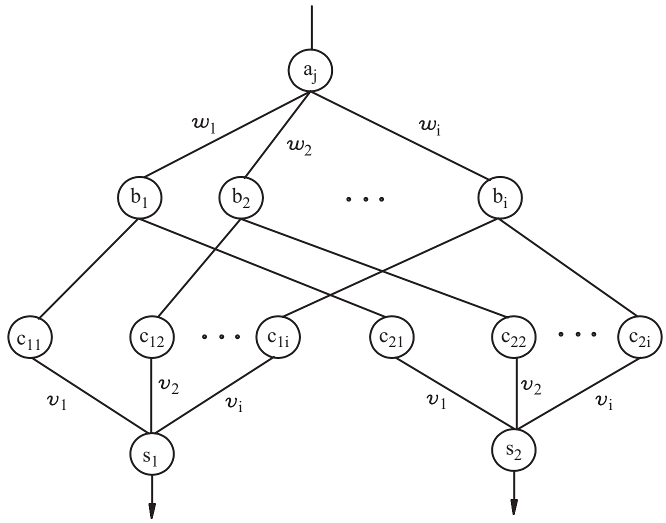

Fig. 9. Architecture of statistical neural network.

SNN makes comparison through the computation of S_1 and S_2 in discrete manner. A statistical neural network is a four-layer feed-forward neural network, which consists of one input layer, two hidden layers and one output layer. Fig. 9 illustrates a most simple architecture of a statistical neural network, considering one node of input layer.

For the neuron of the first hidden layer,

$$b_i = f(w_i a - \theta_i), \quad f(x) = \begin{cases} -1, & x < 0, \\ 0, & 0 \leqslant x < 1, \\ 1, & x \geqslant 1, \end{cases} \quad i = 1, 2, \ldots, n, \tag{4}$$

where, w_i presents the connection weight between the input layer and the first hidden layer, θ_i is threshold of node i in the first hidden layer.

The weight w_i and the threshold θ_i are defined as

$$\begin{aligned} w_i &= \frac{1}{\delta}, \quad \delta = \frac{a_{\max} - a_{\min}}{n}, \\ \theta_i &= \frac{a_{\min} + (i-1)\delta}{\delta}, \qquad i = 1, 2, \ldots, n \\ &= \frac{a_{\min}}{\delta} + i - 1, \end{aligned} \tag{5}$$

in which $a_{\max}$ and $a_{\min}$ are the upper and lower limits of the random variable used for training. For the second hidden layer,

$$\begin{aligned} C_{1i} &= g(-b_i), \\ C_{2i} &= g(b_i), \end{aligned} \qquad g(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leqslant 0, \end{cases} \tag{6}$$

$$S_1 = \sum_{i=1}^{n} v_i C_{1i}, \tag{7}$$

$$S_2 = \sum_{i=1}^{n} v_i C_{2i}, \tag{8}$$

where v_i is a weight of the second hidden layer to the output layer, which takes a PDF value of the random variable in a discrete type.

The crack is supposed to occur if smaller natural frequencies are measured when compared with the previous measurement. Owing to the measurement errors or noises, each mode frequency is a scattering series. So, the SNN is employed for crack detection in this paper. The natural frequencies of the intact dam are employed for training the SNN. When computing the weights, δ in expression (5) is the discrete interval of the frequency, while $a_{\max}$ and $a_{\min}$ take the possible maximum and minimum values of frequency. When the frequency series of a mode fed to the trained SNN as input data, the SNN will output S_1 and S_2. If the frequencies of N modes are used for crack detection, the outputs of SNN will be

$$O1 = \sum_{k=1}^{N} S_{k1}, \tag{9}$$

$$O_2 = \sum_{k=1}^{N} S_{k2}. \tag{10}$$

The concept ‘‘possibility’’ is defined to represent the reliability of the detection of crack occurring as follows:

$$\text{possibility} = \frac{O_1 - O_2}{O_1} 100\%. \tag{11}$$

If $O_1 > O_2$, it means that the result is positive and will give out the crack occurrence possibility. If $O_1 \leqslant O_2$, a negative value is obtained, meaning that no crack is detected.

4.2. Numerical simulation of crack detection

For simulating the measurement errors, 1000 frequency data for each mode in each case are generated by adding normally distributed random sequence with zero mean and certain variance noise to the computed frequency as

$$f_R = f(1 + \varepsilon R) \tag{12}$$

Table 6
Parameters used for training SNN in numerical simulation

Mode number	$a_{\max}$	$a_{\min}$	δ	n
1	3.3370	2.7820	0.0111	50
2	3.7620	3.1820	0.0116	50
3	5.2564	4.6064	0.0130	50
4	6.7096	5.7646	0.0189	50
5	6.8865	5.8965	0.0198	50
6	7.8828	6.8878	0.0199	50

in which ε is the noise level, R is the normally distributed random sequence with zero mean and unit variance and f is the computed frequency.

According to the frequency series of each mode, the values of $a_{\max}$, $a_{\min}$ and δ are given in Table 6 and the weights w_i, v_i and the threshold θ_i are computed consequently for training the SNN.

Table 7
Possibility of crack occurring detected by SNN

Noise level (%)	Largest error (%)	Possibility of crack occurring (%)		
		0.25-depth crack	0.5-depth crack	0.75-depth crack
0.1	0.3	80.1	89.8	100.0
0.5	1.5	27.5	77.1	99.0
0.86	2.58	21.9	73.4	95.4
1.0	3.0	15.4	48.9	94.2
2.0	6.0	8.0	35.1	89.5

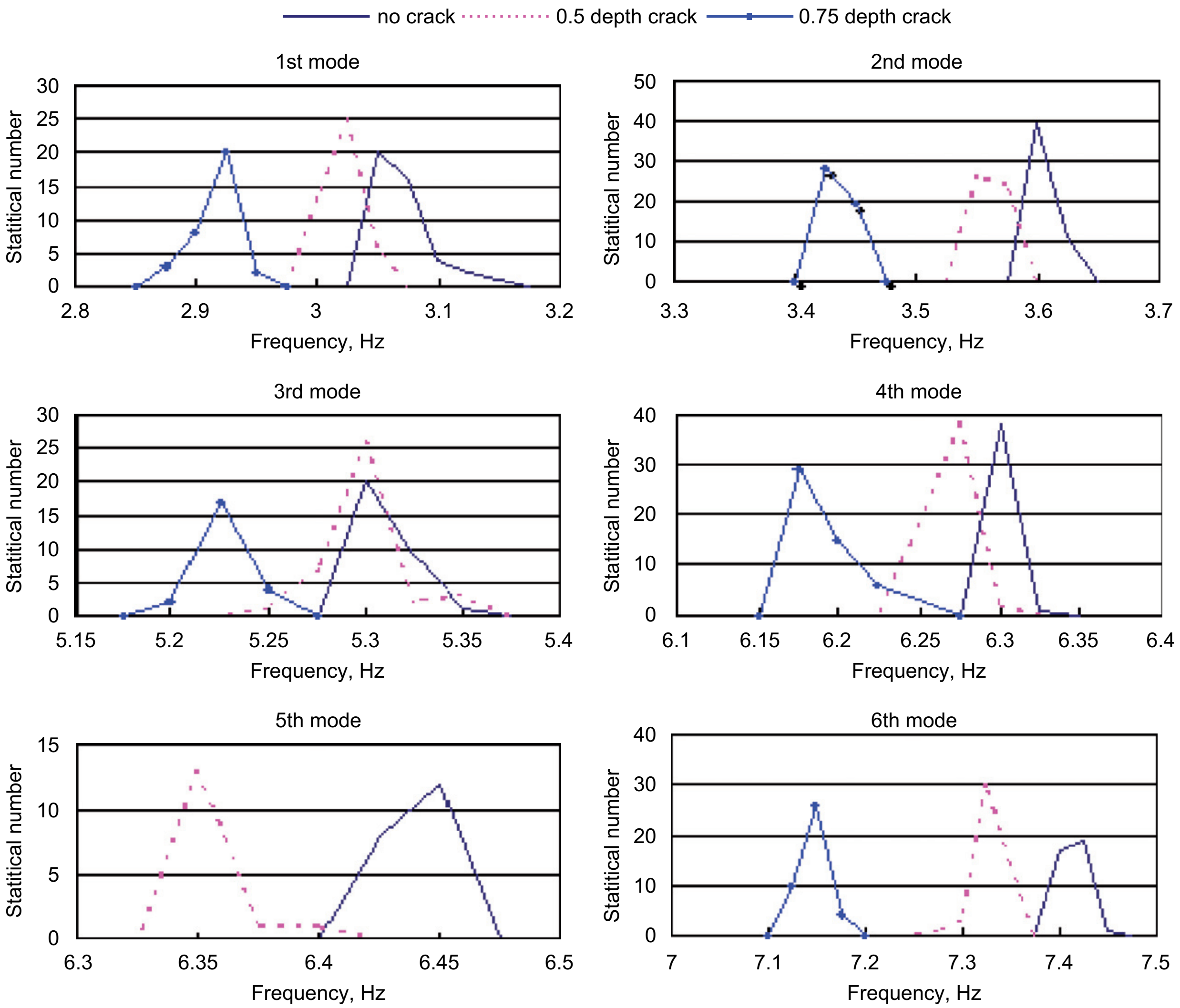

Fig. 10. The distribution of measured frequencies.

Table 8
Parameters used for training SNN with measurement

Mode no.	a_{max}	a_{min}	δ	n
1	3.2	2.8	0.025	50
2	3.7	3.3	0.025	50
3	5.5	5.1	0.025	50
4	6.4	6.0	0.025	50
5	6.6	6.2	0.025	50
6	7.5	7.1	0.025	50

Table 7 lists the possibility of crack occurring detected by SNN from frequencies of the first 6 modes for three cases of 0.25-depth crack, 0.5-depth crack and 0.75-depth crack. Five noise levels with the largest errors from 0.3% to 6.0% are considered. In the case of 0.25-depth crack, high possibility is obtained only when the noise is on the lowest level of 0.1%. However, in the case of 0.75-depth crack, high possibility is obtained on all the five noise levels. The high possibility is also obtained in the case of 0.5-depth crack when the noise is the same as, or lower than, the noise of real test with the largest error of 2.58%.

4.3. *Crack detection from measurement data*

For comparison with the computed frequencies, the measured frequencies need to be multiplied by the similarity factor of 1/100. Fig. 10 shows the distribution of frequencies in three cases of no crack, 0.5-depth crack and 0.75-depth crack. Different from the numerical simulation data, the measurement frequencies are not normally distributed. Similar to numerical simulation, the frequencies in the case of 0.75-depth crack obviously shift from the no crack case.

Table 8 lists the a_{max}, a_{min} and δ values that are employed for training the SNN and computed from measured frequencies.

The possibilities of crack occurring given by the SNN with measurement data are 98.0% for 0.5-depth crack, and 100% for 0.75-depth crack.

5. Conclusions

According to the numerical simulation and model experiment on the hypothetical concrete arch dam, the natural frequencies are obviously reduced due to the crack in the cases of 0.5-depth crack and 0.75-depth crack. From the measured scattering frequencies of the first six modes, the crack can be detected by SNN with high possibility in these two cases. The numerical simulation for 0.25-depth crack shows that the crack is too small to detect. The modeling of crack in FEM as traction-free faces is confirmed through comparison of frequency reduction between numerical simulation and model experiment.

In this paper, the modal tests are carried out on a small-dimension model in lab conditions without the influence of environmental factors like water level and temperature as in situ measurement. To avoid the influences of water level and temperature, selecting the frequencies measured in similar environmental conditions for comparison will be a good method. To understand the noise level of in situ measurement and further confirm the feasibility of crack detection, the ambient vibration test and forced vibration test upon real concrete arch dams are required.

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