

A general method for chaos synchronization and parameters estimation between different systems

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Abstract

For two different chaotic systems, which have the drive system with unknown parameters and the response system with known parameters, a unified mathematical expression for the controller and adaptive laws of parameters is proposed to realize complete synchronization and parameters estimation. The suggested method proves to be globally and asymptotically stable by means of the invariance principle of differential equations and the control technique of partial system states. The synchronizations between Lorenz system and Liu system, hyperchaotic Chen system and hyperchaotic new system are taken as two illustrative examples to demonstrate the effectiveness of this method.

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1. Introduction

In the past decades, the investigation of chaos synchronization has attracted a lot of attention due to its great potential applications in secure communication, chemical reactions, and biological systems [1]. Chaos synchronization refers to a process wherein two (or many) chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to coupling or forcing. So far, a wide variety of approaches have been proposed for the synchronization of chaotic systems that include adaptive control [2], observer-based control [3], variable structure control [4], backstepping control [5], active control [6], nonlinear control [7], and so on. However, most of the methods mentioned above are designed to synchronize two identical chaotic systems with known parameters. In fact, in many practical worlds such as laser array, biological systems and cognitive processes, it is hardly the case that the structure of drive and response systems can be assumed to be identical. Moreover, the parameters of some systems cannot be exactly known a priori, and the effect of these uncertainties will destroy the synchronization and even break it. Therefore, it is essential to investigate synchronization of two different chaotic systems in the presence of unknown parameters [8,9].

On the other hand, an interesting application of chaos synchronization is to estimate parameters of a system from time series when complete information about parameters is hard to receive. Recently, a lot of effort has

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been devoted to it; for example, in Ref. [10] an adaptive controller with parameters identification is designed to synchronize a class of chaotic systems with unknown parameters, and in Ref. [11] an approach of adaptive synchronization and parameter identification is proposed to study an uncertain Rössler hyperchaotic system. But as some researches [12–14] show that there exist incorrect statements and problematic proofs in several papers about parameters estimation, parameters estimation based on synchronization is worth being further investigated.

Motivated by the above discussions, in this paper, we will consider the complete synchronization and parameters estimation of different chaotic systems when parameters of drive and response systems are unknown and known, respectively. Different from previous study [8], a unified mathematical expression for the controller and parameters update law is designed based on the invariance principle of differential equations [15] and the control technique of partial system states [16]. With this method, chaos synchronization and parameters identification can be achieved simultaneously. Two illustrative examples are presented to show the effectiveness of the proposed scheme.

2. Design of the general method

Let a chaotic system be given as

$$\dot{x} = f(x) + F(x)\hat{\theta}, \quad (1)$$

where $x \in R^n$ is the state vector, $f(x) \in R^n$, $F(x) \in R^{n \times m}$. $\hat{\theta} \in R^m$ is the unknown parameter vector. And let $\Omega \subset R^n$ be a bounded closed set that contains the whole attractor of Eq. (1). Relation (1) represents the drive system, and the response system with a controller $U \in R^n$ is introduced as follows:

$$\dot{y} = g(y) + U, \quad (2)$$

where $y \in R^n$, $g(y) \in R^n$. Suppose y is defined on a bounded set S , and $S \supset \Omega$.

The goal is to design a controller U with an adaptive parameter vector $\tilde{\theta}$, which is able to synchronize the states of both the drive and the response systems and, meanwhile, identify the unknown parameter vector $\hat{\theta}$ via $\tilde{\theta}$. If we define the error vector as $e = y - x$, the dynamic equation of synchronization error can be expressed as

$$\dot{e} = g(y) - f(x) - F(x)\hat{\theta} + U. \quad (3)$$

Hence, the objective of synchronization is to make $\lim_{t \rightarrow \infty} \|e(t)\| = 0$.

For convenience, the notation $G > 0$ denotes that G is a positive definite matrix, while I denotes an identity matrix. Also, the following assumptions are introduced.

Assumption 1. Suppose there exists a row transformation Δ that can make $\Delta e = \begin{bmatrix} M \\ N \end{bmatrix}$, and furthermore, under the transformation Δ ,

$$\Delta F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix} \quad \text{and} \quad \Delta(g(y) - g(x)) = \begin{bmatrix} -AM + p(M, N, x, y) \\ w(M, N, x, y) \end{bmatrix}, \quad (4)$$

where $M = [e_{i_1}, e_{i_2}, \dots, e_{i_s}]^T \in R^s$ and $N = [e_{i_{s+1}}, e_{i_{s+2}}, \dots, e_{i_{s+q}}]^T \in R^q$ ($s + q = n$), $F_1(x) \in R^{s \times m}$, $F_2(x) \in R^{q \times m}$, $A \in R^{s \times s}$. p and w are two functions satisfying $p(0, 0, 0, 0) = 0$ and $w(0, 0, 0, 0) = 0$.

Assumption 2. There exist $G > 0$ and $H = \text{diag}[h_j] > 0$, $j = 1, \dots, q$, such that

$$M^T G p(M, N, x, y) + N^T H w(M, N, x, y) \leq M^T \phi M + N^T H \psi N, \quad (5)$$

where ϕ is a symmetric matrix satisfying that $GA + A^T G - 2\phi > 0$, $\psi = \text{diag}[\psi_j]$, $j = 1, \dots, q$.

Remark 1. Note that choosing an appropriate A is very important in Assumption 1. From the authors' experience, $-A$ is often taken as a stable matrix (whose real parts of eigenvalues are all negative). The systems satisfying Assumption 2 include the majority of typical chaotic systems such as Lorenz system, Chen system, Chua's Oscillator, Unified chaotic system, etc. The reader can see Ref. [16] for detailed proofs. In examples, we will give the proofs of inequality (5) for other chaotic systems.

Theorem 1. *If we choose a diagonal matrix $\varepsilon = \text{diag}[e_i] \in R^{n \times n}$ and a vector function $\tilde{\theta} \in R^m$ such that the following two conditions are satisfied:*

- (1) *if e_i is an element of N , namely $e_i \in \{e_{i_{s+1}}, e_{i_{s+2}}, \dots, e_{i_{s+q}}\}$, then $\dot{e}_i = -\beta_i e_i^2$, $\beta_i > 0$; otherwise $\dot{e}_i \equiv 0$.*
- (2) *$\tilde{\theta}$ is updated according to the law*

$$\dot{\tilde{\theta}} = - \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} GM \\ HN \end{bmatrix}, \tag{6}$$

then with the controller

$$U = f(x) - g(x) + F(x)\tilde{\theta} + \varepsilon e, \tag{7}$$

The error dynamical system (3) is globally asymptotically stable at the origin, i.e., the drive system (1) and response system (2) are globally synchronized asymptotically.

Proof. Let $\theta = \tilde{\theta} - \hat{\theta}$, $\bar{\varepsilon} = \text{diag}[e_{i_{s+1}}, e_{i_{s+2}}, \dots, e_{i_{s+q}}]$. Substituting Eq. (7) in to Eq. (3), the error system can be rewritten as

$$\dot{e} = g(y) - g(x) + F(x)\theta + \varepsilon e. \tag{8}$$

Under the transformation Δ , we get

$$\begin{aligned} \dot{M} &= -AM + p(M, N, x, y) + F_1\theta, \\ \dot{N} &= w(M, N, x, y) + F_2\theta + \bar{\varepsilon}N. \end{aligned} \tag{9}$$

Construct a Lyapunov function in the form of

$$V = M^T GM + N^T HN + \theta^T \theta + \sum_{j=1}^q \frac{h_j}{\beta_{i_{s+j}}} (\varepsilon_{i_{s+j}} + L)^2, \tag{10}$$

where L is a constant bigger than $\max\{|\psi_i|, i = 1, \dots, q\}$, i.e. $L > \max\{|\psi_i|, i = 1, \dots, q\}$. Differentiating V with respect to time t ,

$$\begin{aligned} \dot{V} &= M^T G(-AM + p(M, N, x, y) + F_1\theta) + (-AM + p(M, N, x, y) + F_1\theta)^T GM \\ &\quad + 2N^T H(w(M, N, x, y) + F_2\theta + \bar{\varepsilon}N) + 2\theta^T \dot{\theta} - 2 \sum_{j=1}^q \frac{h_j}{\beta_{i_{s+j}}} (\varepsilon_{i_{s+j}} + L) \beta_{i_{s+j}} N_j^2 \\ &= -M^T(GA + A^T G)M + 2M^T Gp(M, N, x, y) + 2N^T Hw(M, N, x, y) \\ &\quad + 2\theta^T F_1^T GM + 2\theta^T F_2^T HN - 2\theta^T \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} GM \\ HN \end{bmatrix} + 2N^T H\bar{\varepsilon}N \\ &\quad - 2 \sum_{j=1}^q h_j (\bar{\varepsilon}_j + L) N_j^2 \\ &\leq -M^T(GA + A^T G - 2\phi)M - 2N^T H(LI - \psi)N \leq 0. \end{aligned} \tag{11}$$

In inequality (11), we have used Assumption 2 and the condition $L > \max\{|\psi_i|, i = 1, \dots, q\}$. It is obvious that $\dot{V} = 0$ if and only if $e_i = 0, i = 1, 2, \dots, n$. According to the well-known LaSalle invariance principle [15], the trajectories of the error dynamical system, starting with arbitrary initial values, converge asymptotically to the largest invariant set $E = \{(e, \theta, \varepsilon) : e = 0, \theta = \theta^0, \varepsilon = \varepsilon^0\}$ contained in $\dot{V} = 0$ as $t \rightarrow \infty$, which implies that the two systems (1) and (2) are globally synchronized asymptotically. \square

Remark 2. From the above proof, one knows that θ will approach a constant vector θ^0 when synchronization is achieved. Therefore, if $\theta^0 = 0$, then $\tilde{\theta}$ will adapt itself to the unknown parameter vector $\hat{\theta}$, which implies that

synchronization-based parameter estimation can be realized. Note that, from Eq. (8), on the largest invariant set E , it is easy to get the following equation:

$$F(x)\theta^0 = 0. \quad (12)$$

Because x is a chaotic time series, $F_{ij}(x)(1 \leq i \leq n, 1 \leq j \leq m)$ are varying with time and usually have no relations with each other. These cause $\theta^0 = 0$ to be the unique solution of Eq. (12). Thus, chaotic behavior plays an important role in this type of parameters estimation.

Remark 3. In comparison with previous methods for chaos synchronization of different systems [8,9,17,18], the present method has the following advantages: (1) it is a general method based on the rigorous mathematics proofs, (2) it can judge which linear feedback terms can be omitted and, therefore, it is relatively simple, (3) the linear feedback strength ε_i is adaptive, which does not require to be determined a priori, (4) the problem of parameters estimation is strictly demonstrated, and it is shown this method can successfully recover all unknown parameters of a given chaotic system.

3. Examples

In this section, two examples and corresponding numerical simulations are given to illustrate the validity of the proposed method. In Example 1, synchronization and parameters estimation between Lorenz system with unknown parameters and Liu system are discussed. Because of having more complex dynamic behaviors than chaotic systems, hyperchaotic systems can enhance the security of the messages to be transmitted in secure communication. In Example 2, synchronization of hyperchaotic Chen system with unknown parameters and a new hyperchaotic system is considered, and the corresponding parameters are identified.

Example 1. Consider Lorenz system with unknown parameters as the drive system

$$\begin{aligned} \dot{x}_1 &= \hat{\theta}_1(x_2 - x_1), \\ \dot{x}_2 &= \hat{\theta}_2 x_1 - x_2 - x_1 x_3, \\ \dot{x}_3 &= -\hat{\theta}_3 x_3 + x_1 x_2, \end{aligned} \quad (13)$$

and the response system is the controlled Liu system [19]

$$\begin{aligned} \dot{y}_1 &= 10(y_2 - y_1), \\ \dot{y}_2 &= 40y_1 - y_1 y_3 + U, \\ \dot{y}_3 &= -2.5y_3 + 4y_1^2. \end{aligned} \quad (14)$$

Rewrite system (13) and system (14) in the form of Eqs. (1) and (2) as follows:

$$\dot{x} = f(x) + F(x)\hat{\theta}, \quad (15)$$

where

$$f(x) = \begin{bmatrix} 0 \\ -x_2 - x_1 x_3 \\ x_1 x_2 \end{bmatrix}, \quad F(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix}, \quad \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]^T,$$

is the unknown parameter vector.

$$\dot{y} = g(y) + U, \quad (16)$$

where

$$g(y) = \begin{bmatrix} 10(y_2 - y_1) \\ 40y_1 - y_1y_3 \\ -2.5y_3 + 4y_1^2 \end{bmatrix}.$$

We choose the row transformation

$$\Delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

such that

$$\Delta(g(y) - g(x)) = \begin{bmatrix} -10e_1 + 10e_2 \\ -2.5e_3 + 4(x_1 + y_1)e_1 \\ 40e_1 - (y_1e_3 + e_1x_3) \end{bmatrix} \quad \text{and} \quad \Delta F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix} = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & 0 & -x_3 \\ 0 & x_1 & 0 \end{bmatrix}. \quad (17)$$

Comparing Eq. (17) with Eq. (4), one can see that

$$A = \begin{bmatrix} 10 & 0 \\ 0 & 2.5 \end{bmatrix}, \quad M = [e_1, e_3]^T, \quad N = [e_2]^T, \quad p(M, N, x, y) = \begin{bmatrix} 10e_2 \\ 4(x_1 + y_1)e_1 \end{bmatrix}$$

$$w(M, N, x, y) = 40e_1 - (y_1e_3 + e_1x_3), \quad F_1(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & 0 & -x_3 \end{bmatrix}, \quad F_2(x) = [0 \quad x_1 \quad 0].$$

Furthermore, choosing

$$a \geq \max\{|x_1|, |y_1|\}, \quad b \geq \max\{|x_3|\}, \quad G = \begin{bmatrix} a^2 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = 1,$$

we have

$$\begin{aligned} &M^T G p(M, N, x, y) + N^T H w(M, N, x, y) \\ &= 4(x_1 + y_1)e_1e_3 + [10a^2 + (40 - x_3)]e_1e_2 - y_1e_2e_3 \\ &\leq 4a \left(2ae_1^2 + \frac{1}{2a}e_3^2 \right) + [10a^2 + (40 - x_3)]e_1e_2 - y_1e_2e_3 \\ &\leq M^T \phi M + N^T \psi N, \end{aligned}$$

where

$$\phi = \begin{bmatrix} 9a^2 & 0 \\ 0 & 2.2 \end{bmatrix}, \quad \psi = \left[\frac{(10a^2 + b + 40)^2 + 5a^4}{4a^2} \right], \quad \text{and} \quad GA + A^T G - 2\phi > 0.$$

Therefore, Assumptions 1 and 2 are satisfied. According to Theorem 1, the controller is taken as

$$U = f(x) - g(x) + F(x)\tilde{\theta} + \varepsilon e = \begin{bmatrix} (10 - \tilde{\theta}_1)(x_1 - x_2) \\ (\tilde{\theta}_2 - 40)x_1 - x_2 + \varepsilon_2e_2 \\ -4x_1^2 + x_1x_2 + (2.5 - \tilde{\theta}_3)x_3 \end{bmatrix}, \quad (18)$$

and adaptive laws of parameters are chosen as

$$\dot{\varepsilon}_2 = -\beta_2\varepsilon_2^2, \quad (\beta_2 > 0), \quad (19)$$

$$\dot{\hat{\theta}} = - \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} GM \\ HN \end{bmatrix} = \begin{bmatrix} a^2(x_1 - x_2)e_1 \\ -x_1e_2 \\ x_3e_3 \end{bmatrix}. \tag{20}$$

From Eq. (12), on the largest invariant set, we can get

$$(x_2 - x_1)\theta_1^0 = 0, \quad x_1\theta_2^0 = 0, \quad x_3\theta_3^0 = 0, \tag{21}$$

because x_i ($i = 1, 2, 3$) are chaotic time series, $\theta^0 = (\theta_1^0, \theta_2^0, \theta_3^0)^T$ is zero vector, which indicates the true values of $\hat{\theta}$ can be estimated by Eq. (20).

In the simulations, we choose the unknown parameter vector $\hat{\theta} = (10, 28, 8/3)^T$. Fig. 1(a)–(b) display phase portraits of two chaotic attractors, from which we can take $a = 20, b = 50$. Under the initial conditions $x(0) = (-1, 1, 5)^T, y(0) = (-0.9, 1.2, 6.3)^T, \hat{\theta}(0) = (0, 0, 0)^T, \varepsilon_2(0) = 0$, and the coefficient $\beta_2 = 1$, Figs. 2–4 show time evolution curves of errors, parameters ε_2 and $\theta = \hat{\theta} - \theta$, respectively. Obviously, with time passing, synchronization errors converge to zero, parameter ε_2 tends to a constant, and the fact that θ tend to zero implies $\hat{\theta}$ are able to approach the unknown parameters $\hat{\theta}$. These results show that the proposed scheme can be effective to achieve chaos synchronization and parameters estimation.

Example 2. Consider hyperchaotic Chen system with unknown parameters as the drive system

$$\begin{aligned} \dot{x}_1 &= \hat{\theta}_1(x_2 - x_1) + x_4, \\ \dot{x}_2 &= \hat{\theta}_2x_1 - x_1x_3 + \hat{\theta}_3x_2, \\ \dot{x}_3 &= x_1x_2 - \hat{\theta}_4x_3, \\ \dot{x}_4 &= x_2x_3 + \hat{\theta}_5x_4, \end{aligned} \tag{22}$$

and the response system is the controlled hyperchaotic new system [20]

$$\begin{aligned} \dot{y}_1 &= 35(y_2 - y_1) + y_2y_3, \\ \dot{y}_2 &= 25y_1 - y_1y_3 - y_2 - y_4, \\ \dot{y}_3 &= y_1y_2 - \frac{8}{3}y_3, \\ \dot{y}_4 &= 71.2y_1 + 0.5y_2y_3 + y_4. \end{aligned} + U \tag{23}$$

Rewrite system (22) and system (23) in the form of Eqs. (1) and (2) as follows:

$$\dot{x} = f(x) + F(x)\hat{\theta}, \tag{24}$$

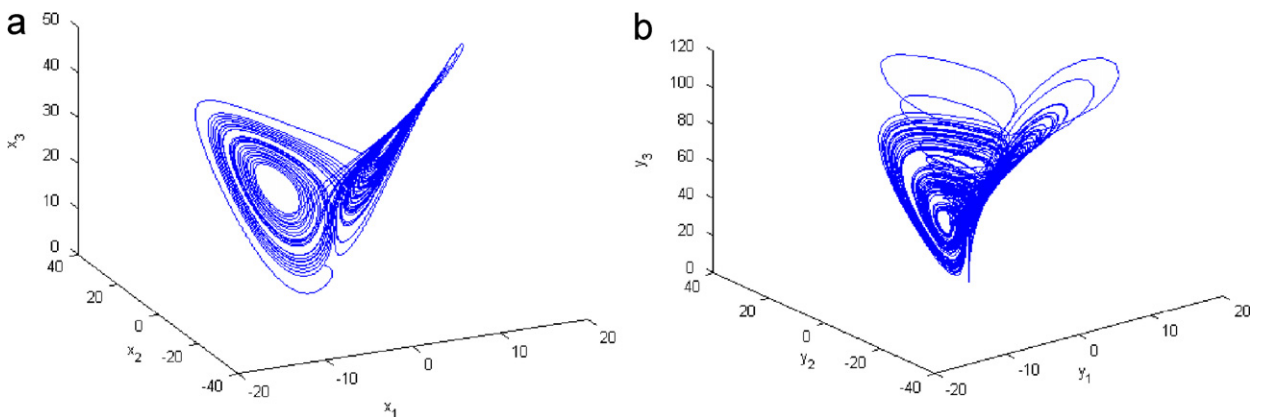


Fig. 1. Phase portraits of chaotic attractors: (a) Lorenz system and (b) Liu system.

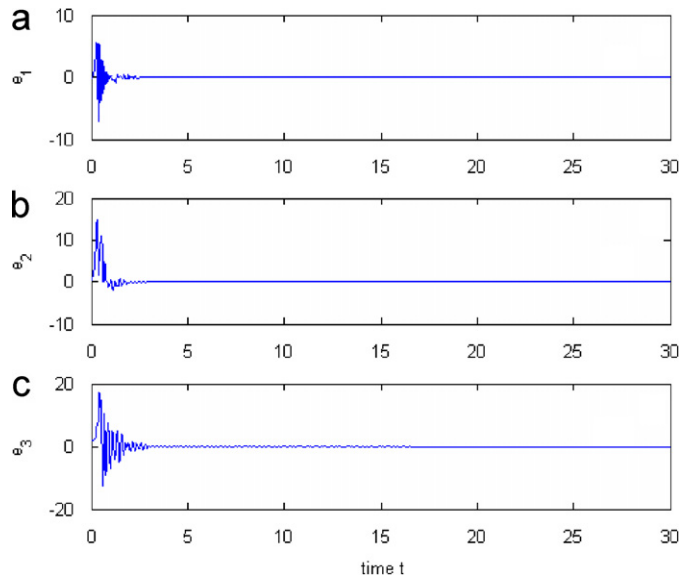


Fig. 2. Time history of synchronization errors.

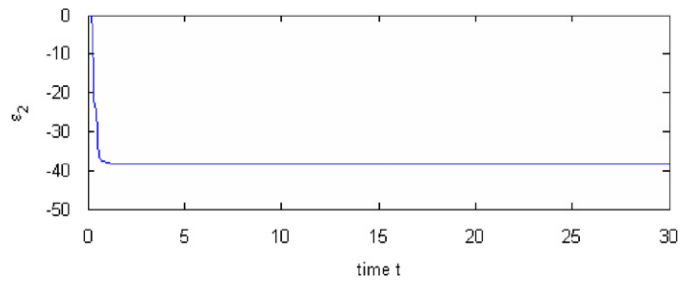


Fig. 3. Time evolution of adaptive parameter ϵ_2 .

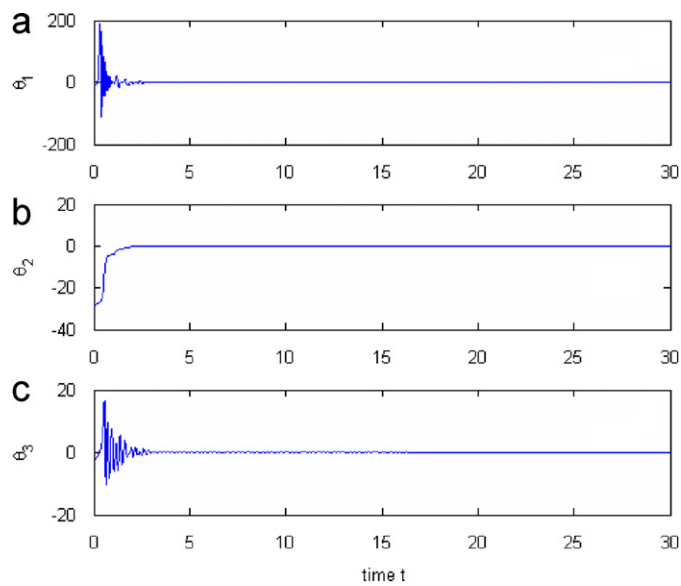


Fig. 4. Estimation errors $\theta = \tilde{\theta} - \hat{\theta}$ for parameters $\hat{\theta}$.

where

$$f(x) = \begin{bmatrix} x_4 \\ -x_1x_3 \\ x_1x_2 \\ x_2x_3 \end{bmatrix}, \quad F(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & 0 & 0 \\ 0 & 0 & 0 & -x_3 & 0 \\ 0 & 0 & 0 & 0 & x_4 \end{bmatrix}, \quad \hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5)^T,$$

is the unknown parameter vector.

$$\dot{y} = g(y) + U, \tag{25}$$

where

$$g(y) = \begin{bmatrix} 35(y_2 - y_1) + y_2y_3 \\ 25y_1 - y_1y_3 - y_2 - y_4 \\ y_1y_2 - \frac{8}{3}y_3 \\ 71.2y_1 + 0.5y_2y_3 + y_4 \end{bmatrix}.$$

We choose the row transformation

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

such that

$$\begin{aligned} \Delta(g(y) - g(x)) &= \begin{bmatrix} -e_2 + 25e_1 - (y_1e_3 + e_1x_3) - e_4 \\ -\frac{8}{3}e_3 + y_1e_2 + e_1x_2 \\ 35(e_2 - e_1) + x_2e_3 + e_2y_3 \\ 71.2e_1 + 0.5(y_2e_3 + x_3e_2) + e_4 \end{bmatrix} \quad \text{and} \\ \Delta F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix} &= \begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 \\ 0 & 0 & 0 & -x_3 & 0 \\ x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_4 \end{bmatrix}. \end{aligned} \tag{26}$$

Comparing Eq. (26) with Eq. (4), one can see that

$$\begin{aligned} M &= [e_2, e_3]^T, \quad N = [e_1, e_4]^T, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & \frac{8}{3} \end{bmatrix}, \\ p(M, N, x, y) &= \begin{bmatrix} 25e_1 - (y_1e_3 + e_1x_3) - e_4 \\ y_1e_2 + e_1x_2 \end{bmatrix}, \quad w(M, N, x, y) = \begin{bmatrix} 35(e_2 - e_1) + x_2e_3 + e_2y_3 \\ 71.2e_1 + 0.5(y_2e_3 + x_3e_2) + e_4 \end{bmatrix}. \end{aligned}$$

Furthermore, choosing $a \geq \max\{|x_2|, |y_2|\}$, $b \geq \max\{|x_3|, |y_3|\}$, $G = H = I$, we have

$$M^T G p(M, N, x, y) + N^T H w(M, N, x, y)$$

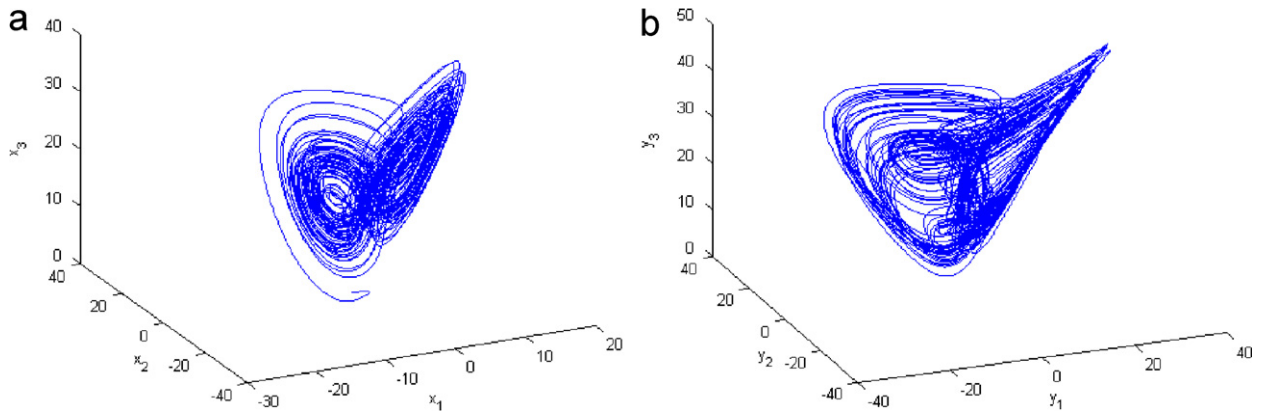


Fig. 5. Phase portraits: (a) hyperchaotic Chen system in (x_1, x_2, x_3) space; and (b) hyperchaotic new system in (y_1, y_2, y_3) space.

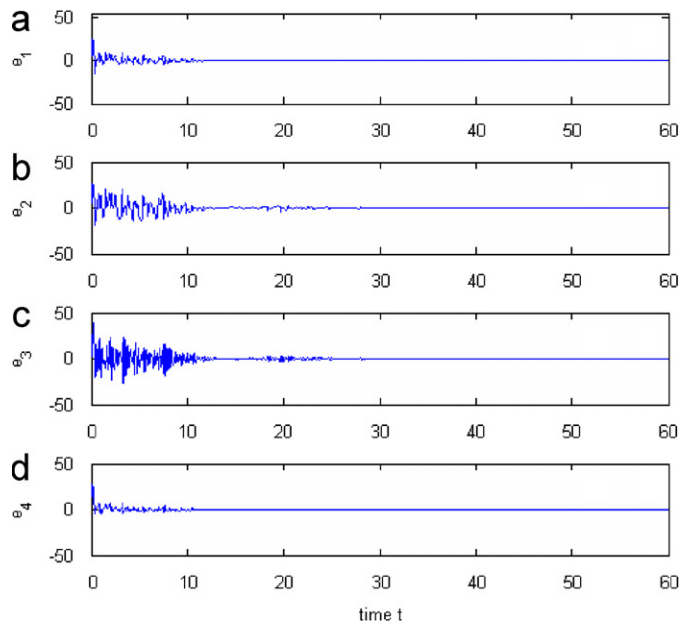


Fig. 6. Time history of synchronization errors.

$$\begin{aligned}
 &= (60 - x_3 + y_3)e_1e_2 + (0.5x_3 - 1)e_2e_4 + 2x_2e_1e_3 \\
 &\quad + 71.2e_1e_4 + 0.5y_2e_3e_4 - 35e_1^2 + e_4^2 \\
 &\leq M^T \phi M + N^T \psi N,
 \end{aligned}$$

where

$$\phi = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad \psi = \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix},$$

with

$$\begin{aligned}
 \psi_1 &= (60 + 2b)^2 + 2a^2 + 0.6, \\
 \psi_2 &= (0.5b + 1)^2 + 0.125a^2 + 36.6.
 \end{aligned}$$

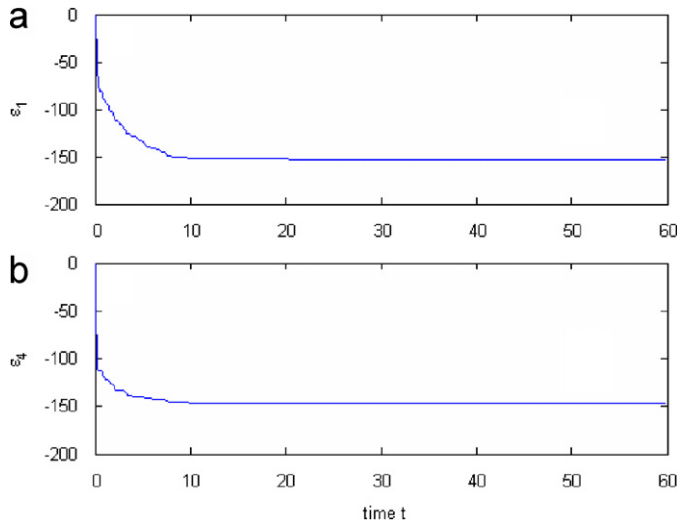


Fig. 7. Time evolution of adaptive parameters $\varepsilon_1, \varepsilon_4$.

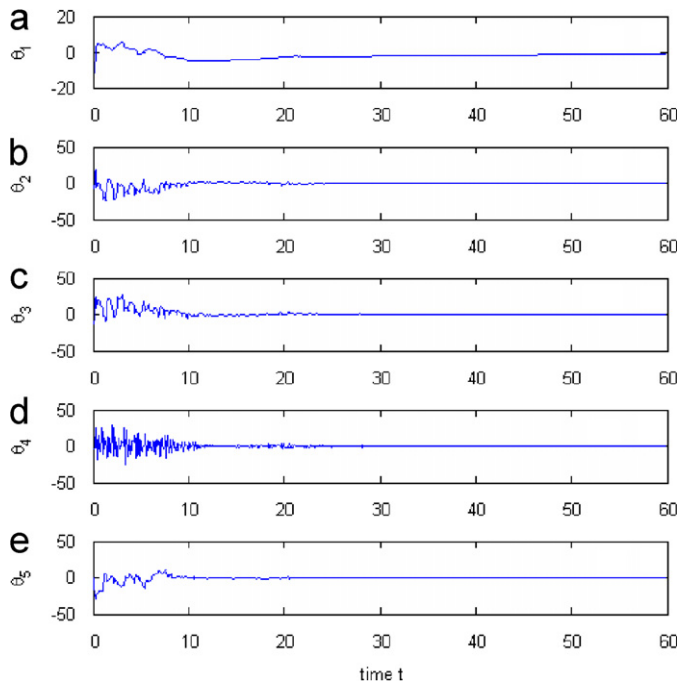


Fig. 8. Estimation errors $\theta = \tilde{\theta} - \hat{\theta}$ for parameters $\hat{\theta}$.

Therefore, Assumptions 1 and 2 are satisfied. According to Theorem 1, the controller is taken as

$$U = f(x) - g(x) + F(x)\tilde{\theta} + \varepsilon e = \begin{bmatrix} (35 - \tilde{\theta}_1)(x_1 - x_2) + x_4 - x_2x_3 + \varepsilon_1e_1 \\ (\tilde{\theta}_2 - 25)x_1 + (\tilde{\theta}_3 + 1)x_2 + x_4 \\ (\frac{8}{3} - \tilde{\theta}_4)x_3 \\ -71.2x_1 + (\tilde{\theta}_5 - 1)x_4 + 0.5x_2x_3 + \varepsilon_4e_4 \end{bmatrix}, \quad (27)$$

and adaptive laws of parameters are chosen as

$$\dot{\varepsilon}_1 = -\beta_1 \varepsilon_1^2, \quad \dot{\varepsilon}_4 = -\beta_4 \varepsilon_4^2 \quad (\beta_1 > 0, \beta_4 > 0), \quad (28)$$

$$\dot{\hat{\theta}} = - \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} GM \\ HN \end{bmatrix} = \begin{bmatrix} (x_1 - x_2)e_1 \\ -x_1 e_2 \\ -x_2 e_2 \\ x_3 e_3 \\ -x_4 e_4 \end{bmatrix}, \quad (29)$$

where the true values of unknown parameters $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5)^T$ can be estimated by Eq. (29).

In the simulations, we choose the unknown parameter vector $\hat{\theta} = (35, 7, 12, 3, 0.082)^T$. Fig. 5(a)–(b) display phase portraits of two chaotic attractors, from which we can take $a = 40$, $b = 50$. Under the initial conditions $x(0) = (-3, 0, 0, 5)^T$, $y(0) = (1, 5, 3, 17)^T$, $\hat{\theta}(0) = (23, 0, 0, 0, 0)^T$, $\varepsilon_1(0) = 0$, $\varepsilon_4(0) = 0$ and the coefficients $\beta_1 = 1$, $\beta_4 = 1$, Figs. 6–8 show time evolution curves of synchronization errors, adaptive parameters ε_1 , ε_4 and the estimation errors of parameters $\hat{\theta}$, respectively. From Figs. 6–8 one can find that, as time increases, synchronization errors converge to zero, parameters ε_1 , ε_4 tend to constants, $\hat{\theta}$ are able to approach the unknown parameters $\hat{\theta}$. These results show that chaos synchronization and parameters estimation can be achieved simultaneously using the proposed method.

4. Conclusion

This paper studies synchronization and parameters estimation between two different chaotic systems. A model concerning the drive system with unknown parameters and the response system with known parameters is considered. Based on the invariance principle of differential equations and the control technique of partial system states, a simple and general form of the controller is designed to realize synchronization. Because in real applications the unknown parameters often need to be identified, the problem of parameters estimation is also discussed. It is shown that parameter estimation and chaos synchronization can be achieved simultaneously by the proposed method. Two illustrative examples are given to demonstrate the validity of this technique, and numerical simulations are also given to show the effectiveness of the method.

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