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# Design of springs with "negative" stiffness to improve vehicle driver vibration isolation

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**Abstract**

Minimization of the fundamental frequencies of a vibratory system by means of springs with "negative" stiffness is probably the only way to attain infra-frequency vibration isolation under gravitation. Traditionally, the design of similar springs for vehicle driver vibration isolation systems was an art, and design decisions were based primarily on the designer experience. This paper presents an approach, based on the consistent theory of thin shells, for designing compact springs in terms of their compatibility with the room available for packaging the vehicle suspensions and simultaneous extension of the height control region where fundamental frequencies are kept minimal. In the approach, a generic model of a simple springing element with "negative" stiffness in the large is proposed. A simple iterative procedure is formulated to solve the geometrically nonlinear problem of large-amplitude post-bucking of springing elements and to represent them in a way that enables an optimal, computable scheme for the design of springs. Validity of the approach is assessed by a comparison of the computation and measurement results. Using the approach, we propose a generic spring module applicable to any vehicle suspension, whether it is a seat suspension, a cab mounting, or a cargotainer platform.


## 1. Introduction

Vibrations in the infra-frequency range, $f = 0.5{-}5\,\mathrm{Hz}$, are the most harmful and dangerous to human health and activity. The effects of these vibrations on humans have been investigated and classified by many researchers, and in particular by Griffin [1]. Reason is the highest sensitivity of a man to vibrations in these frequencies. Infra-frequency vibration isolation is possible if the fundamental frequencies $f_0$ of a vibration isolation system (VIS) are shifted below the external vibration frequencies $f$, i.e. if $f_0 \ll f$. This holds true even if gravity is reduced or if suspension stiffness is minimized. This is clear if one considers the single degree of freedom VIS:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{n_g g}{z_0/c_a}}, \tag{1}$$

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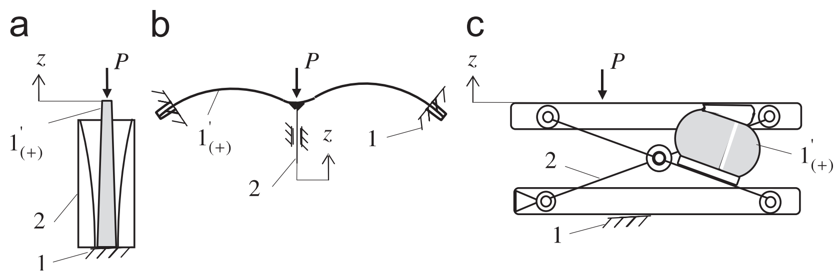


Fig. 1. Conventional seat suspensions with LBSs possessing a certain "negative" stiffness in the small, where 1 is the suspension basis, 2 is the input link of GM, $1'_{(+)}$ is the LBS.

where $k$ is the suspension stiffness, $m$ the sprung mass (including occupant), $n_g \in (0,1]$ the coefficient of gravitation, $g \approx 9810\,\text{mm}\,\text{s}^{-2}$, $z_0$ the suspension stroke, and $c_a$ the asymmetry parameter of the stroke (which depends on the suspension height control with respect to mid-ride).

Formula (1) shows that the minimization of $k$ is the only way to attain infra-frequency vibration isolation under gravitation. Low values of $k$ can be obtained by using a spring with "negative" stiffness. This type of elasticity, under a certain load $P$, is a result of local buckling of load-bearing springs (LBSs) numbered in Fig. 1 as $1'_{(+)}$, e.g., metal elastic rods (see Figs. 1(a),(b)) or rodless air-springs (see Fig. 1(c)). The subscript in $1'_{(+)}$ denotes "positive" stiffness of LBS in operation. This may lead to certain minima of $k$ and $f_0$, and finally to perfect vibration isolation, but under small-amplitude movement in the $z$-direction, as indicated. Even so, the performance of such springs come into conflict with their dimensions with respect to vehicle driver VISs as explored in detail in Refs. [2,3] and in many other publications.

More general and operative designs are the composites consisting of suspensions with "positive" stiffness and springs with "negative" stiffness that might have new suspensions with stiffnesses much smaller than those of their "positive" constituents [4–8]. Since a spring with "negative" stiffness has no load capacity, it can operate while connected in parallel with mechanical, hydraulic, pneumatic, or other LBSs with "positive" stiffness. This raises the following questions: Is there a way to evaluate the spring design in terms of its compatibility with the room available for packaging the new suspension and extension of the height control region where the fundamental frequencies can be minimized? Is there a way to rationalize the design process and make it less dependent on designer experience and chance?

In this paper, we attempted to identify springs with "negative" stiffness as design building blocks and to represent them in a way that enables an optimal, computable scheme for upgrading suspensions for vehicle driver VISs. Based on the results of investigations of a wide variety of devices, we first designed a simple springing element with "negative" stiffness and then packed a compact spring, which was compatible and connectible at will in space with the function-generating mechanism or better to say with the guide mechanism (GM) of a vehicle suspension. Then, we proposed a simple iterative procedure for solving the geometrically nonlinear problem of large-amplitude post-buckling of the springs. Numerical results of the spring analysis and design are verified through measurements. Finally, we demonstrated, along with design samples, the compatibility of the springs with different types of vehicle suspensions.

## 2. Generic model of springing element with variable "negative" stiffness

Rod-shaped elastic structures traditionally were and still remain a benchmark for springs with "negative" stiffness. This phenomenon is outlined e.g. in Ref. [9]; the springs' use for vibration isolation began in the 1960s [2,10–12]. Let us construct a generic model of the structure. Let a slender beam of length $2l$ be simply supported as shown in Fig. 2(a). Considering a state evolution, we bring the beam in a springing element with "negative" stiffness. Each pre- and post-equilibrium state is shown by the dotted and solid lines, respectively. First, under a certain axial load $P_a$, the beam is buckled as shown in Fig. 2(a); here, $\varepsilon_0$, $\varpi_0$, and $\psi_0$ are the axial pre-compression, bending deflection, and end-slope, respectively. Obviously, $\varpi_0$ and $\psi_0$ are uniquely dependent on $\varepsilon_0$, which defines the beam shape in post-buckling. Second, combining the $P_a$ force and $T_2^{(a)}$

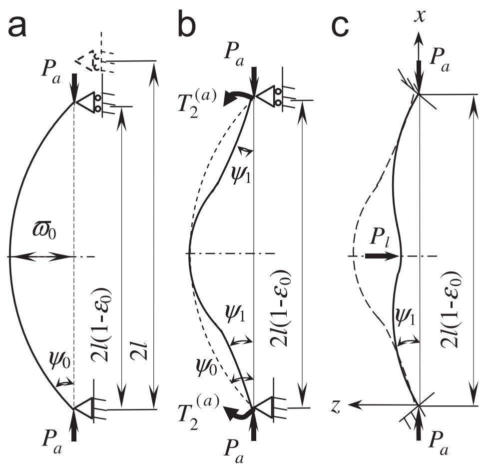


Fig. 2. A sequence of the model construction of a springing element with "negative" stiffness.

moment loading without a change in $\varepsilon_0$ leads to the shape shown in Fig. 2(b). Given $T_2^{(a)}$, one may obtain a certain value of the slope $\psi_1$. Finally, a transverse load $P_l$, using previous values for $\varepsilon_0$ and $\psi_1$, results in the state shown in Fig. 2(c).

A well-known exact equation for transverse-longitudinal bending of a slender beam is

$$M_F'' + (N\varpi')' = 0, \tag{2}$$

where $M_F$ and $N$ are the bending moment and axial force, respectively, $s$ is the coordinate along the beam axis line, and a prime (′) denotes differentiation with respect to $s$. We formulate $M_F$ based on the assumption that the axis line is not stretched, giving $M_F = EJ\varpi''[1-(\varpi')^2]^{-0.5}$. If we also assume (a) $(\varpi')^2 \ll 1$ and (b) $N \approx P_a$, then the $M_F$ can be linearized, and so can Eq. (2):

$$\varpi'''' + c^2\varpi'' = 0, \tag{3}$$

where $c^2 = P_a(EJ)^{-1}$, and $EJ$ is the bending rigidity.

To find the $\varepsilon_0$-function, we use the nonlinear equation

$$\int_0^{2l} \mathrm{d}x = \int_0^{2l} [1 - (\varpi')^2]^{0.5}\, \mathrm{d}s = 2l(1 - \varepsilon_0). \tag{4}$$

Considering also the assumption, $[(\varpi')^2 \ll 1]$, Eq. (4) yields

$$\varepsilon_0 \approx \frac{1}{4l}\int_0^{2l} (\varpi')^2\, \mathrm{d}s. \tag{5}$$

Since Eq. (3) is linear, then the bending amplitude one should define from nonlinear Eq. (5).

Any solution of Eq. (3) is of the form, $\varpi = C_1 \sin cs + C_2 \cos cs + C_3 s + C_4$, where $C_1 - C_4$ are the constants of integration determined from the boundary conditions. Solving the problem by the models taken in Fig. 2 (considering only half the length of the beam by symmetry, and properly changing the boundary conditions), we finally obtain, under condition, $\psi_1 = 0$, the following:

$$\tilde{P} = \frac{P_l l^2}{\pi^3 EJ}\frac{l}{\varpi_0} = \mp \frac{(0.5\tilde{c})^2 \cos 0.25\pi\tilde{c}}{\sqrt{2 + \cos 0.5\pi\tilde{c} - 6(\pi\tilde{c})^{-1} \sin 0.5\pi\tilde{c}}}, \tag{6a}$$

$$\tilde{\varpi} = \frac{\varpi}{\varpi_0} = \pm\left(\frac{2}{\tilde{c}}\right)\frac{0.25\pi\tilde{c} \cos 0.25\pi\tilde{c} - \sin 0.25\pi\tilde{c}}{\sqrt{2 + \cos 0.5\pi\tilde{c} - 6(\pi\tilde{c})^{-1} \sin 0.5\pi\tilde{c}}}, \tag{6b}$$

where $\tilde{c} = 2lc/\pi$ is a key parameter for designing a springing element with "negative" stiffness.

The slope $\psi_1$ is a design parameter which significantly extends the region of ''negative'' stiffness and simultaneously saves suspension room without performance loss [7]. From decision analysis, we find that a certain minimal value for $\psi_1 \neq 0$ should be taken in the range $\tilde{c} > 2$:

$$\psi_{1\ \min} = \left\{8\pi^{-2}\tilde{c}^{-1}[2 + \cos 0.5\pi\tilde{c} - 6(\pi\tilde{c})^{-1} \sin 0.5\pi\tilde{c}]^{0.5}\right\}_{\min}. \tag{7}$$

Eqs. (6) define the parametric dependence between the force $\tilde{P}$ and bending $\tilde{\varpi}$. The $(\tilde{P}, \tilde{\varpi})$-curves have the forms shown in Fig. 3, regardless of the sectional shape of the springing element with ''negative'' stiffness. Also, Eq. (6a) yields a fundamental relation for the element design. For instance, in case of a rectangular cross-section with dimensions $b \times h$ we obtain

$$P_l = \tilde{P}_l \frac{\pi^3 Ebl}{12(1 - v^2)} \left(\frac{\varpi_0}{l}\right) \left(\frac{h}{l}\right)^3, \tag{8}$$

where $b$ and $h$ are the width and thickness (cross-sectional height) of the element, respectively, and $E$ and $v$ are the Young's modulus and Poisson's ratio for spring steels [13], respectively.

Supplementing Eq. (8) with the strength criterion as

$$\tilde{\sigma}_F = \left(\frac{M_F}{EJ}\frac{h}{2}\right)_{\max} = \left[\frac{0.125\pi^2\tilde{c}}{\sqrt{2 + \cos 0.5\pi\tilde{c} - 6(\pi\tilde{c})^{-1} \sin 0.5\pi\tilde{c}}}\right]_{\max} \left(\frac{h}{l}\right)\left(\frac{\varpi_0}{l}\right) \leqslant \frac{[\sigma]_F}{E} \tag{9}$$

yields simultaneous equations for designing the element, here $[\sigma]_F$ is the allowable bending stress.

Analysis of Eq. (8) while considering Eq. (9) leads to some important predictions in designing the springs with ''negative'' stiffness to upgrade a vehicle suspension, whether it is a driver seat, cab mounting or other object. These predictions, in a sense justify, the failure of the evolution in designing the springs that started in 1960s and have been for the most part empirical. So, we take the below as a breakthrough:

(a) The $(h/l)$-ratio means that, considering mechanical data of spring steels, the springing element is to be a thin-walled structure [5,7].
(b) The $(bl)$-product means that the springing element is more likely to be in the form of a thin shallow shell or thin plate than of a slender beam or other rod-shaped design [14]. In this case, one may obtain large-amplitude post-buckling and, respectively, to extend the ''negative'' stiffness region without increasing the suspension room and without loss of the spring strength.
(c) The springs are to be arranged from the $n_{pl}$-number of the elements in order to pack the springs perfectly compactly in the suspension room and to simultaneously minimize the stiffness of vehicle suspensions [4,5,7].

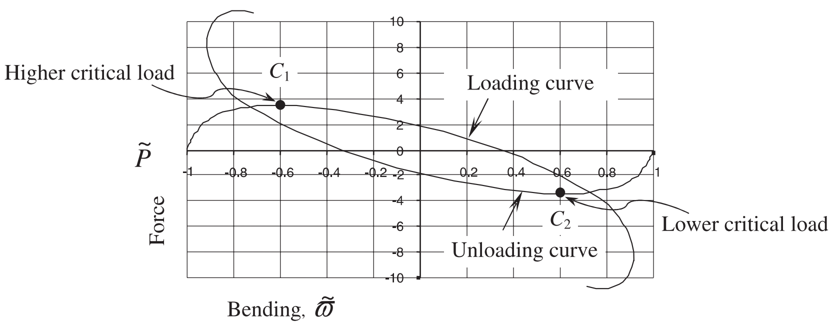


Fig. 3. Boundaries of the region of ''negative'' stiffness in the large.

## 3. Spring design procedure

### 3.1. Formulation

As predicted above, thin shallow spherical and cylindrical shells, thin plates under cylindrical bending, made of elastic materials behave as "off-the-shelf" structures that can be used to design springing elements with "negative" stiffness. The behavior of any of these structures can be evaluated precisely in the framework of the nonlinear theory of elasticity. And there exist many perfect and developing numerical methods to formulate and solve the nonlinear problems, in particular [15–22]. Though in engineering design, one prefers approaches that enable a reduction the geometrically nonlinear problem of large-amplitude post-buckling of thin-walled structures to a sequence of linear boundary value problems, a simple and sufficiently precise numerical algorithm can also give a solution that converges sufficiently quickly.

One of the approaches is based on the following hypothesis [14]: Generally, the nodes of a finite element (FE) model of the structure move along nonlinear paths during deformation. What if the final displacement of each FE is the sum of a "large" displacement of the FE as a rigid body and a "small" displacement under deformation?

Let us consider, in the general case, a shell structure of arbitrary shape and curvature. The initial state of its median surface is described by the vector, $\mathbf{x} = \mathbf{x}(\theta^1,\theta^2)$, where $\theta^1$ and $\theta^2$ are the curvilinear coordinates. Assume that this surface moves into a final state, $\mathbf{x}^+ = \mathbf{x} + \mathbf{u}_{gl}$, so that the $n$th FE undergoes a total displacement of, $\mathbf{u}_{gl}{}^{(n)} = \mathbf{u}_*^{(n)} + \mathbf{u}^{(n)}$, where $\mathbf{u}_*^{(n)}$ is the vector of "large" displacements of the FE as a rigid body and $\mathbf{u}^{(n)}$ is the vector of "small" displacements from the state described by the vector, $\mathbf{x}_*^{(n)} = \mathbf{x}^{(n)} + \mathbf{u}_*^{(n)}$. Hence, the variation of elastic energy of the FE is determined by the displacements $\mathbf{u}^{(n)}$ only. If assume the smallness of $\mathbf{u}^{(n)}$, then one may use linear relations for the strain energy and equations of state. It is clear also that a greater number of FEs gives a more accurate assumption about the smallness of the displacements.

The hypothesis allows for the formulation a simple iterative procedure for solving the above nonlinear problem:

(1) Let a state for $n$th FE be described by the vector

$$\mathbf{x}^{(n,k)} = \mathbf{x}^{(n)} + \mathbf{u}_{gl}^{(n,k)}, \tag{10}$$

where $k$ is the iteration. At the first subsequent iteration $(k+1)$, the vector $\mathbf{u}_*^{(n,k+1)}$ is determined so that the strain energy of each FE becomes minimal, giving displacements of

$$\Delta\mathbf{u}^{(n,k+1)} = \mathbf{u}_{gl}^{(n,k)} - \mathbf{u}_*^{(n,k+1)}. \tag{11}$$

The vector $\mathbf{u}_*^{(n,k+1)}$ is typically a function of six arbitrary constants, variable for each FE and determined at this stage.

Thus, the structure is divided into an arbitrary number of FEs moving as a system of free rigid bodies from an initial state to an intermediate one.

(2) Then, each FE moves to a final state to be conjugated into an entire structure by "small" displacements $\mathbf{u}^{(n,k+1)}$ from the state $\mathbf{x}_*^{(n,k+1)} = \mathbf{x}^{(n)} + \mathbf{u}_*^{(n,k+1)}$. To determine these displacements evaluating the deflected modes of each FE, the strain energy of the structure is minimized.

The strain energy of each FE in a local system of coordinates, determined by the vector $\mathbf{x}_*^{(n,k+1)}$, can be expressed in the following form:

$$E_p^{(n,k+1)} = 0.5\mathbf{V}_{\text{loc}}^{(n,k+1)\text{T}} K_{\text{loc}}^{(n)} \mathbf{V}_{\text{loc}}^{(n,k+1)}, \tag{12}$$

where $\mathbf{V}_{\text{loc}}^{(n,k+1)}$ and $K_{\text{loc}}^{(n)}$ are the vectors of node displacements and the stiffness matrix of the $n$th FE in the local system of coordinates, respectively.

If we find a relation between the vectors $\mathbf{V}_{\text{loc}}^{(n,k+1)}$ and $\mathbf{V}_{gl}^{(n,k+1)}$ which are connected via the matrix $B^{(n,k+1)}$ and the vector $\mathbf{C}^{(n,k+1)}$ as

$$\mathbf{V}_{\text{loc}}^{(n,k+1)} = B^{(n,k+1)}\mathbf{V}_{gl}^{(n,k+1)} + \mathbf{C}^{(n,k+1)}. \tag{13}$$

Then one may formulate the total strain energy of the structure (in the global system of coordinates):

$$E_p^{(k+1)} = 0.5\mathbf{V}_{gl}^{(k+1)\mathrm{T}}K_{gl}^{(k+1)}\mathbf{V}_{gl}^{(k+1)} + \mathbf{P}_{gl}^{(k+1)\mathrm{T}}\mathbf{V}_{gl}^{(k+1)} + E_{p0}^{(k+1)}, \tag{14}$$

where the stiffness matrix, the vector of the imaginary load (it makes allowance for nonlinearities in transformation the local matrix into global one), and the component of the strain energy independent of the vector of global displacements, are, respectively,

$$K_{gl}^{(k+1)} = \sum_n K_{gl}^{(n,k+1)} = \sum_n B^{(n,k+1)\mathrm{T}}K_{\text{loc}}^{(n)}B^{(n,k+1)}, \tag{15a}$$

$$\mathbf{P}_{gl}^{(k+1)} = \sum_n B^{(n,k+1)\mathrm{T}}K_{\text{loc}}^{(n)}\mathbf{C}^{(n,k+1)}, \tag{15b}$$

$$E_{p0}^{(k+1)} = 0.5\sum_n \mathbf{C}^{(n,k+1)\mathrm{T}}K_{\text{loc}}^{(n)}\mathbf{C}^{(n,k+1)}. \tag{15c}$$

Minimizing the $E_p{}^{(k+1)}$ yields a system of linear equations for FE analysis:

$$K_{gl}^{(k+1)}\mathbf{V}_{gl}^{(k+1)} = \mathbf{P}_{gl}^{*} - \mathbf{P}_{gl}^{(k+1)}, \tag{16}$$

where $\mathbf{P}_{gl}^{*}$ is the vector of effective external forces.

The iterative procedure is complete if the differences between vectors $\mathbf{u}_*^{(n,k)}$ and $\mathbf{u}_*^{(n,k+1)}$ of "small" displacements of the FE system are smaller than a specified allowance. The linear elasticity solution can be accepted as an initial estimate $\mathbf{u}_{gl}^{(n,0)}$ for the process.

Thus, the hypothesis does not require the compatibility equations between the FEs, when a system of free rigid bodies in space is considered (instead of an entire structure), at the iteration. The strain energy of an FE is determined for conjugating displacements only. From the assumption about the smallness of these displacements, the relations from linear thin shell theory hold in formulating the strain energy function, constructing the stiffness matrices, etc. The compatibility equations can be expressed as by Novozhylov [23]. At the same time, we modify the state equations. Analysis shows that the modified equations do not result in greater solution accuracy. They do allow, however, the formulation of simultaneous equations to describe the large-amplitude post-buckling of a thin-walled structure. The modified equations differ from the well-known ones [23] by the lowest terms. The equations, however, simplify the formulation of shape functions for the structures [14]; it was confirmed through study the other solutions, in particular by Gallagher [24], Bogner et al. [25], and Levin [26]. The validity of the approximate model is determined by the fineness of the FE mesh and was verified by comparing computations and measurements (see below), suggesting that the procedure effectively accounts for all the systemic nonlinearities.

### 3.2. Results and discussion

Based on the above procedure for the analysis of thin-walled structures, we proposed a multilevel scheme of the design: simple springing element—spring with "negative" stiffness in the large—spring mechanism easy mountable into a vehicle suspension.

In view of the above predictions, the spring can be arranged e.g. with $n_{pl}$ thin plates spaced around a central link. Packaging the $n_{pl}$ springing elements with $n_{pl}/n_{\text{set}}$ thin plates in each lamination gives the configuration shown schematically in Fig. 4(a). In FE analysis sampled below we assume (a) there is no interference by the thin plates, but (b) the plates are in strong order in lamination. Dimensionless length, external radius of central link 2, axial pre-compression, and the end-slope of thin plate $2'_{(-)}$ (the subscript in $2'_{(-)}$ denotes "negative"

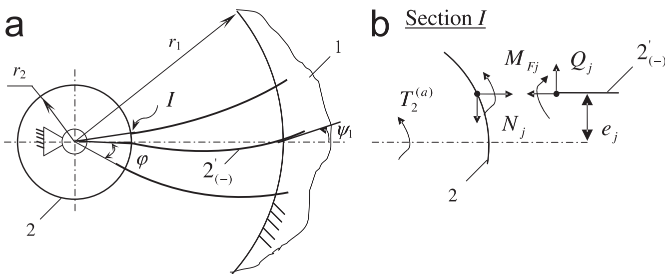


Fig. 4. A model of spring with "negative" stiffness: (a) layout of a mechanism with the spring consisting a set of thin plates; (b) free body diagram formed by cutting jth thin plate at section I.

stiffness in operation), and are the main variables, respectively,

$$\tilde{l} = lh^{-1}\sqrt{12(1-v^2)},\ \tilde{r}_2 = r_2 l^{-1},\ \tilde{\varepsilon}_0 = 1-(r_1-r_2)l^{-1},\ \tilde{\psi}_1 = \psi_1 l^{-1}, \tag{17a–d}$$

where $r_1$ is the inner radius of spring casing 1.

The dimensionless performance of the spring and structural strength of the thin plate are

$$\tilde{T}_2^{(a)} = \frac{T_2^{(a)} n_{pl}}{Ebl^2},\ \tilde{k}_2 = \frac{\mathrm{d}\tilde{T}_2^{(a)}}{\mathrm{d}\varphi},\ \tilde{\sigma}_{F(\max)} = \frac{\sigma_{F(\max)}}{\sigma_e} = \frac{E}{2(1-v^2)\sigma_e}\tilde{l}^{-1}\left(\frac{\mathrm{d}^2\tilde{\varpi}}{\mathrm{d}\tilde{x}^2}\right)_{(\max)}, \tag{18a–c}$$

where central link 2 is loaded with an external torque $T_2^{(a)}$ in order to transform a set of thin plates $2'_{(-)}$ into a spring with "negative" (torsional) stiffness $k_2$, $\varphi$ is the rotation angle of the link, $\sigma_e$ is the elastic limit [13], $\tilde{\varpi} = \varpi/l$, and $\tilde{x} = x/l$ is the moving coordinate for the rectangular cross-section of the thin plate $2'_{(-)}$.

Torque, $T_2^{(a)} = \sum_{k=1}(M_{Fk} - N_k e_k - Q_k r_2)$, is defined by the equilibrium of central link 2; here, $e$ is the eccentricity of the $k$th thin plate relative to central one (see Fig. 4(b)). Solving the equations given by Eq. (16) yields the relations for the bending moment $M_F$, membrane force $N$, and shear force $Q$ in the thin plate. The stress peak-peak is to be $\tilde{\sigma}_{F(\max)}^{\max} < 1$. Otherwise, $\tilde{\varepsilon}_0$ is reduced, and the iterative procedure is repeated. Solution of the problem results in a range of $\varphi = \varphi_0$ where $\tilde{T}_2^{(a)}$ is between critical points $C_1$ and $C_2$, i.e. where the spring has a certain "negative" stiffness, $\tilde{k}_2$.

Solving the problem, one may formulate a few simple relationships for optimal design of the springs. Optimizing $\tilde{\varepsilon}_0$ using the design database for spring steels [13], we formulated the relationships between the design parameters and the performance of the spring mechanism, in particular [7],

$$(r_1 - r_2)h^{-1} = 6.25\varphi_0 - 8\times 10^{-4}\varphi_0^2 - 6.25\times 10^{-4}\varphi_0^3, \tag{19a}$$

$$\psi_1 \approx 1.08375\varphi_0. \tag{19b}$$

The focus of the examples below is to demonstrate, using Eqs. (19), the possibilities for the design of perfectly compact spring mechanisms and simultaneous extendibility of the region of "negative" stiffness to improve vibration isolation of a vehicle object.

Fig. 5 demonstrates validity of the design approach (compare the strain states, computed and operative, of the spring in Figs. 5(a),(b)) with respect to seat suspensions. As can be seen from the example (compare also the performance of the spring mechanism in Fig. 5(c)), $\varphi_0 > 28°$. Hence, if such a mechanism is connected to a GM, then it results in extremely soft suspension within the stroke around $z_0 \geqslant 130$–150 mm that is over the room provided with design of GMs [8].

Principally, the ranges of $\varphi_0$, $T_2^{(a)}$, and $|-k_2|$ depend on $\varepsilon_0$. Optimizing $\varepsilon_0$-value, we design the spring mechanism which dimensions and performance give the best fit with respect to a certain type of vehicle suspensions. Specifically, analysis of Eqs. (19) shows that one can increase the range of $\varphi_0$ with insignificant gain or without gain in the dimensions of spring mechanisms and connect them at will in space to seat suspensions of any type (see samples in Fig. 6). As shown, central link 2 of the spring mechanism and the input link of the GM can be coupled via force closure (see Figs. 6(a),(b)) or kinematically as shown in Fig. 6(c) [6].

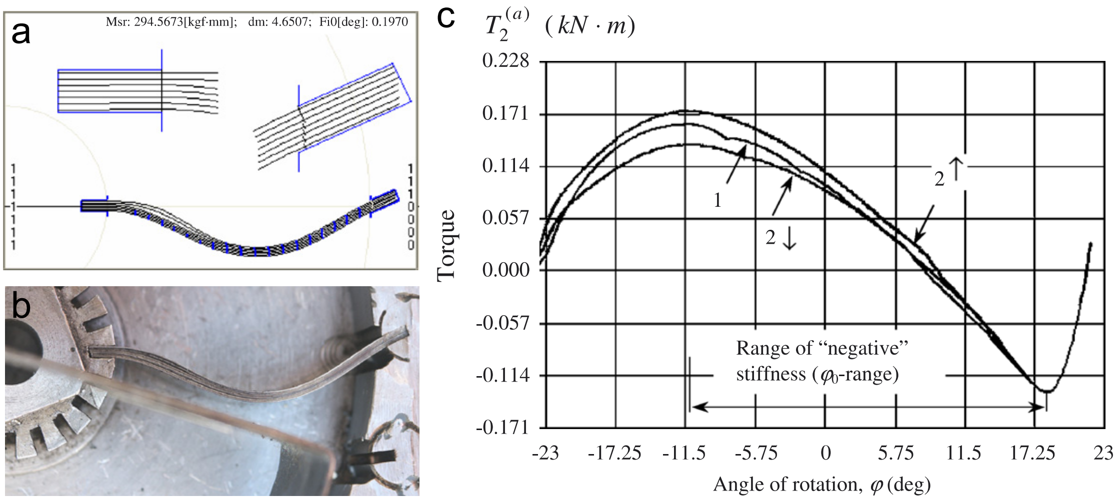


Fig. 5. Comparison of computation and measurement results applied to design of springs with ''negative'' stiffness to a seat suspension: (a) the spring state of strain by FE analysis; (b) identification of same state via digital photography of the full-scale spring prototype; (c) performance of the spring mechanism (1—computation, ignoring structural friction, 2—measurement, where 2↑ and 2↓ are the loading and unloading curves).

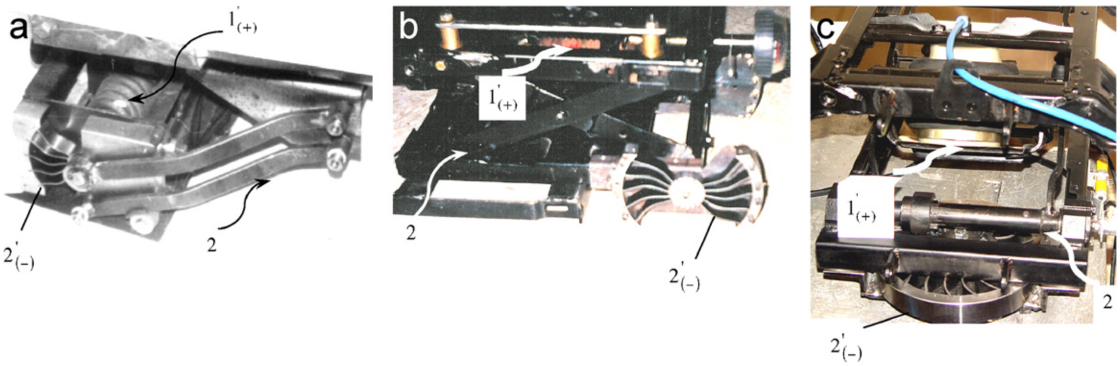


Fig. 6. Seat suspensions upgraded: (a,b) layouts of mechanical and (c) pneumatic suspensions, here, $1'_{(+)}$ are the torsion, tension or compression LBSs, respectively; $2'_{(-)}$ are the mechanisms with springs of ''negative'' stiffness in the large, 2 are the GMs.

By varying the $\psi_1$-angle one can resolve ambiguity due to the nonlinearities and also extend the range of $\varphi_0$ (see Fig. 7). In the example shown in Fig. 7(b), the performance of the spring mechanism is identical in the stroke and reverse, and the range of $\varphi_0$ is increased by 23%, in comparison with the sample in Fig. 7(a). Properly adjusting $\psi_1$ one may decrease the dimensions of the mechanism, in additional. In the example, the extra benefit is 10% [7].

Performance of the spring mechanism is varied multiply, and with insignificant changes of design parameters. Analysis of Eqs. (19) shows also that such a mechanism might be used as a generic module to minimize the stiffness of any of vehicle suspensions, whether it is a seat, cab mounting, or cargotainer platform. Fig. 8 shows that useful variability of the ranges of $\varphi_0$ and $|-k_2|$ can be obtained through the changes of two parameters only.

## 4. Conclusion

This paper presented an approach, based on the consistent theory of thin shells, for designing springs used to effectively minimize the suspension stiffness and thus the fundamental frequencies of vehicle VISs.

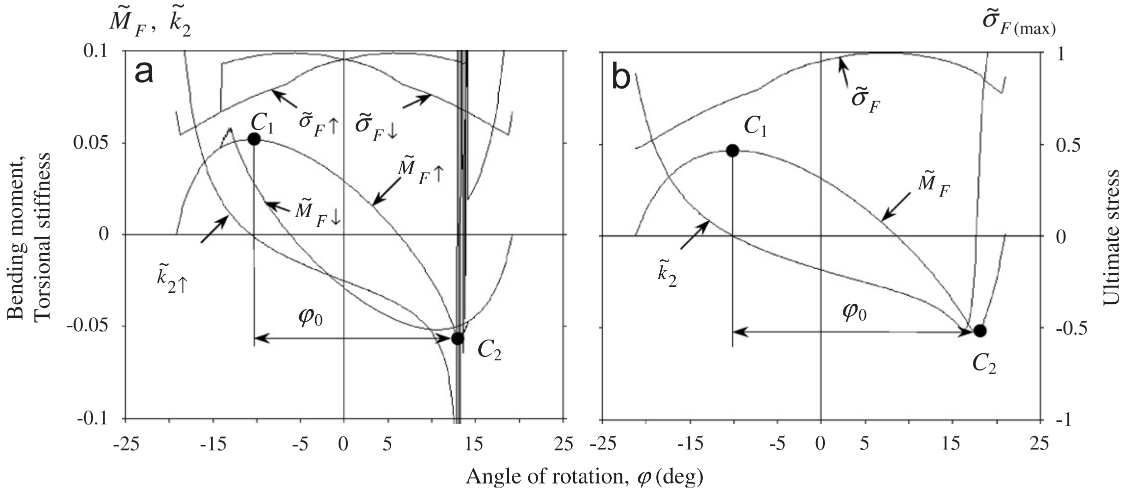


Fig. 7. Performance of the spring mechanism: (a) if $\psi_1 = 0°$ (here, ↑ and ↓ are the stroke and reverse curves, respectively); (b) if $\psi_1 \approx 30°$.

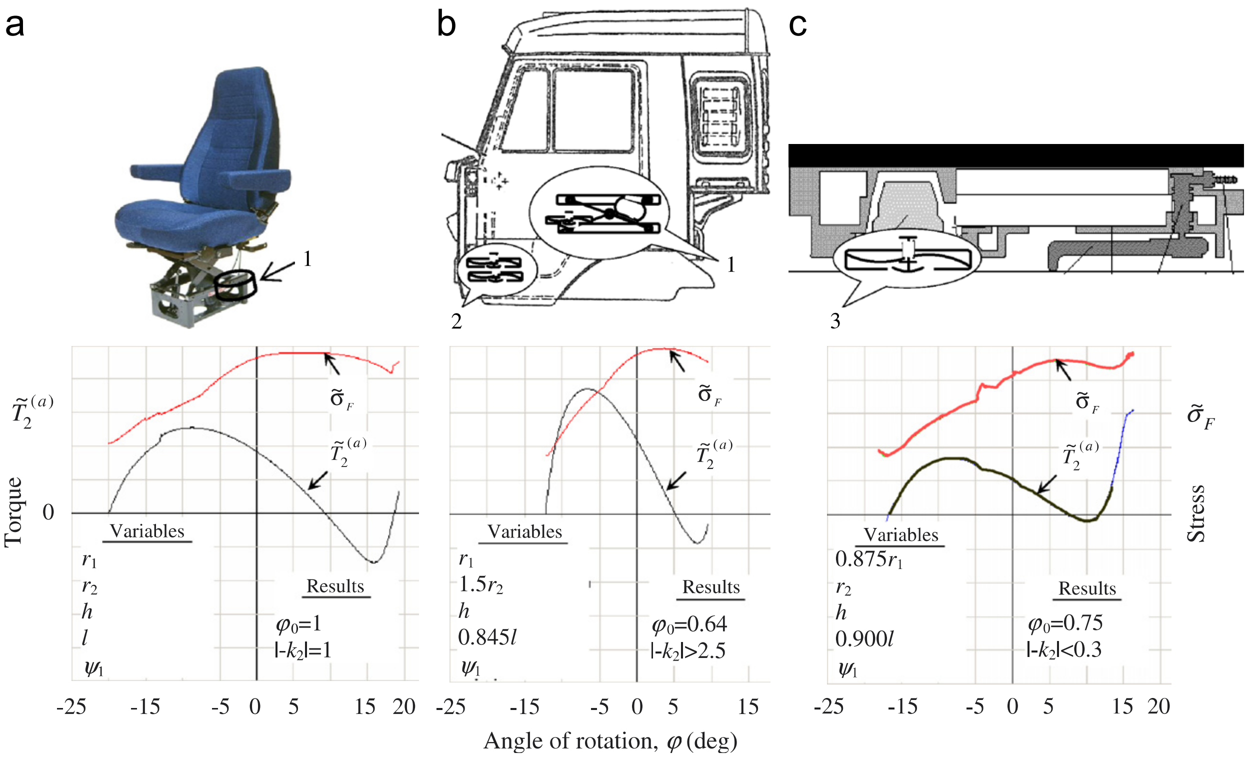


Fig. 8. Layouts of spring mechanisms and their performance relations with respect to: (a) the seat suspension; (b) seat suspension and cab mounting; (c) cargotainer platform, here the mechanisms are scaled for seat suspensions (1), the dumping device (2), and the platform (3).

The minimization, in turn, leads to improving of vibration isolation up to an absolute immobility of an object under infra-frequency (extremely low) vibrations, which are the most harmful and dangerous to a vehicle driver and some other objects must to be protected. In the approach, a generic model of a springing element with variable "negative" stiffness was proposed. Within the scope of the FE method, a simple iterative procedure was formulated to solve the geometrically nonlinear problem of large-amplitude post-bucking of the elements, and to represent them in a manner that enables an optimal, computable scheme for the design of springs with "negative" stiffness in the large and thus the spring mechanisms compact and easy mountable into vehicle suspensions. Samples of the mechanisms have been used in seat suspensions and tested successfully in the land vehicles, construction equipment, and agricultural machines, within parallel testing of conventional seat suspensions [5,8]. They also have been tested in helicopters [8], in what was apparently the first successful application of seats suspended in aircrafts. Using the approach, one may design generic modules and use them for perfect improving of vibration isolation in both large equipment, like a driver seat, cab mounting, cargotainer platform, and in miniature one, like a vehicle-borne mini-device.

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