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A new approach to identification of structural damping ratios

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Abstract

A new approach to identification of modal damping ratios from free vibration response of a linear structure with viscous damping is proposed in this paper. For a single degree of freedom (dof) system, the formulated relations among areas forming from time history of response are used to determine the damping ratio for the free decay vibration. Comparing with traditional logarithmic-decrement method, the approach in this paper has advantages such as stronger anti-noise ability, higher precision, better stability and convenience. For a multi-dof system, the free vibration response measured at one point on the structure is expressed in an analytical form at first, and then multiplied by e^{at}. Giving the initial value, a can be obtained by Newton's dichotomy or golden section method while the product of the response and e^{at} has equal vibration amplitudes after a period of time. Accordingly, the damping ratio for this mode can be determined. Repeating the above-mentioned process, all damping ratios for different modes can be obtained in the same way. Results of digital simulations and field tests for the Dongting Lake Cable-stayed Bridge in Yueyang show that the approach presented in this paper is effective and practical.

1. Introduction

In the advanced research areas of structural dynamics such as fault diagnosis, real-time monitoring of vibration, response prediction, load identification and so on, it is very important to build accurate dynamics models. Because of the complexity of large engineering structures and the existence of measurement error, the dynamic characteristics of systems obtained by the theoretical calculation and practical measurement may be quite different. When carrying out the calculation of structural dynamics, modal damping is often used and considered as proportional damping for the purpose of simplicity. Even though, the selection of proportional coefficient still depends on more engineering experience. Modal damping ratios of a structure can be identified or estimated from tested data by the methods of parameter identification. There are many methods to identify modal damping ratios both in the time domain and in the frequency domain. Methods of the time domain include logarithmic-decrement method, ITD method [1], STD method [2], random decrement technique [3], weighted response-integral method [4], etc. Methods of the frequency domain include half-power bandwidth method, peak picking method, admittance circle method [5], etc. Further possibilities include wavelet

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transform [6,7] and EMD-HT method [8]. In comparison with the other parameters like natural frequencies or mode shape, the damping ratios have lower identification accuracy, especially when the tested data is contaminated by noise. As a result, how to improve the identification accuracy of damping ratio is an active research in structural dynamics.

To improve the identification accuracy of damping ratios, Refs. [9,10] presented the solutions for damping matrix identification to be independent of mass and stiffness matrices, which performed in the frequency domain and needed to test the frequency response function. For a large and complex structure like a bridge, the acquisition of the frequency response function is very difficult because the exciting force is inconvenient to be measured in this case. Therefore, the purpose of this paper is to propose a new approach to identification of modal ratios from free vibration response (displacement, velocity or acceleration) measured at one point on the structure in the time domain. This approach needs not to test the input data and is suitable for single degree of freedom (dof) systems and multi-dof systems. For the measurement noise, ITD method and STD method take the noise modes into consideration and have to characterize the useless noise, consequently lead to the increase of computation time. The approach in this paper determines the damping ratios using the formulated relation among areas forming from time history of response. In practical application, all of areas are obtained by summating trapezia forming from sampling response data. Although noise may have a serious effect on discrete sampling data, it has much less effect on the summation of the total response, especially when the noise is Gaussian white noise with zero mean as the positives and the negatives counteract. So, this approach has stronger anti-noise ability, higher precision and better stability.

For a multi-dof system, the free vibration response $x(t)$ exponentially decays with time and gradually reaches zero. If $x(t)$ is multiplied by $e^{\xi_i\omega_{ni}t}$, $x(t)e^{\xi_i\omega_{ni}t}$ would have equal vibration amplitudes after a period of time. In fact, $\xi_i\omega_{ni}$ is unknown at first, but it can be obtained by search algorithm. Then ω_{ni} can be determined through frequency spectrum analysis on $x(t)e^{\xi_i\omega_{ni}t}$ and ξ_i can be determined accordingly.

To illustrate this idea, digital simulations and field tests are presented in this paper.

2. Theoretical background

2.1. Single dof system

Consider a single dof linear and viscous system

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0, \tag{1}$$

where x, $\dot{x}$ and $\ddot{x}$ are displacement, velocity and acceleration, respectively, ξ is damping ratio, and ω_n is natural frequency. For under damped case, the free decay vibration response of this system can be written as

$$x(t) = Ae^{-\xi\omega_n t}\sin(\omega_d t + \varphi) \tag{2}$$

in which $\omega_d = \omega_n\sqrt{1-\xi^2}$ is damped natural frequency, and A and φ are constants determined by initial conditions. The logarithmic-decrement ratio

$$\delta = \ln\frac{A_1}{A_{n+1}} = \ln\frac{Ae^{-\xi\omega_n t_i}}{Ae^{-\xi\omega_n(t_i+nT_d)}} = \xi\omega_n T_d n = \frac{2n\pi\xi}{\sqrt{1-\xi^2}}, \tag{3}$$

where $T_d = 2\pi/\omega_d$ is the damped period corresponding to the damped natural frequency, and n is the number of selected periods. The damping ratio can be obtained through Eq. (3).

There exists in two problems for the logarithmic-decrement method. The first one is that peak values A_1 and A_n are sampling values, and they are not sure of being equal to the actual maximum values. The second one is that this method is easily contaminated by noise. If $x(t)$ is contaminated, the peak values A_1, A_n might have great local changes which could influence the identification result. Now it is improved as follows.

The time history of response $x(t)$ is shown in Fig. 1. Suppose the response $x(t)$ and time t intersect at points $t_1, t_2, \ldots, t_{2N+1}$, and absolute areas enclosed between $x(t)$ and t are $S_1, S_2, \ldots, S_{2N}$, respectively. If the intersections do not exactly correspond to the sampling points, $t_1, t_2, \ldots, t_{2N+1}$ are determined by linear

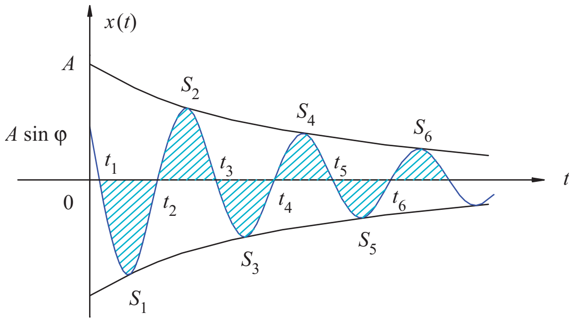

Fig. 1. Free vibration response of the system.

interpolation. Then one gets

$$S_1 = \int_{t_1}^{t_1+\frac{T_d}{2}} |x(t)|\,\mathrm{d}t = \int_0^{\frac{T_d}{2}} |x(t+t_1)|\,\mathrm{d}t = A\mathrm{e}^{-\xi\omega_n t_1} \int_0^{\frac{T_d}{2}} \left|\mathrm{e}^{-\xi\omega_n t} \sin(\omega_d t + \varphi + \omega_d t_1)\right| \mathrm{d}t, \tag{4}$$

$$S_2 = \int_{t_1+\frac{T_d}{2}}^{t_1+T_d} |x(t)|\,\mathrm{d}t = \int_0^{\frac{T_d}{2}} \left|x\left(t+t_1+\frac{T_d}{2}\right)\right| \mathrm{d}t = A\mathrm{e}^{-\xi\omega_n t_1}\mathrm{e}^{-\xi\omega_n\frac{T_d}{2}} \int_0^{\frac{T_d}{2}} \left|\mathrm{e}^{-\xi\omega_n t} \sin(\omega_d t + \varphi + \omega_d t_1)\right| \mathrm{d}t. \tag{5}$$

It is easily obtained

$$S_2 = S_1 \mathrm{e}^{-\xi\omega_n\frac{T_d}{2}}. \tag{6}$$

One can also know in the same way

$$S_{2N} = S_{2N-1} \mathrm{e}^{-\xi\omega_n\frac{T_d}{2}}. \tag{7}$$

So

$$\frac{S_1 + S_3 + \cdots + S_{2N-1}}{S_2 + S_4 + \cdots + S_{2N}} = \frac{S_1 + S_3 + \cdots + S_{2N-1}}{(S_1 + S_3 + \cdots + S_{2N-1})\mathrm{e}^{-\xi\omega_n\frac{T_d}{2}}} = \mathrm{e}^{\xi\omega_n\frac{T_d}{2}} = \mathrm{e}^{\pi\xi/\sqrt{1-\xi^2}}. \tag{8}$$

Taking natural logarithm to the base e to both sides of Eq. (8), the damping ratio is given by

$$\xi = 1\Bigg/\sqrt{1 + \left(\frac{\pi}{E}\right)^2}, \tag{9}$$

where $E = \ln(\sum S_{2k-1}/\sum S_{2k})$, i.e., natural logarithm to the base e to the ratio of area summation of the N odd subscripts to area summation of the N even subscripts. N can be arbitrarily determined. Another form can also be expressed as

$$\frac{S_1 + S_2 + \cdots + S_N}{S_{N+1} + S_{N+2} + \cdots + S_{2N}} = \frac{S_1 + S_2 + \cdots + S_N}{(S_1 + S_2 + \cdots + S_N)\mathrm{e}^{-\xi\omega_n n T_d}} = \mathrm{e}^{\xi\omega_n n T_d} = \mathrm{e}^{2n\pi\xi/\sqrt{1-\xi^2}} \tag{10}$$

the damping ratio is given by

$$\xi = 1\Bigg/\sqrt{1 + \left(2n\frac{\pi}{E}\right)^2}, \tag{11}$$

where $E = \ln(\sum S_k / \sum S_{k+N})$, i.e., natural logarithm to the base e to the ratio of area summation of the former N subscripts to area summation of the latter N subscripts.

2.2. Multi-dof system

An N-dof vibration system is governed by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{0\}, \tag{12}$$

in which $[M],\ [C],\ [K] \in R^{N\times N}$ are mass, damping and stiffness matrices, respectively, and $\{\ddot{x}\}, \{\dot{x}\}, \{x\} \in R^N$ are acceleration, velocity and displacement vectors, respectively.

Response measured at one point on the system of Eq. (12) is expressed as

$$x(t) = \sum_{i=1}^{N} A_i \mathrm{e}^{-\xi_i \omega_{ni} t} \sin(\omega_{di} t + \varphi_i), \tag{13}$$

in which ξ_i is the ith damping ratio, ω_{ni} is the ith natural frequency, ω_{di} is the ith damped natural frequency, and A_i, φ_i are constants determined by initial conditions.

For simplification of statement, suppose $\xi_1\omega_{n1} \leqslant \xi_2\omega_{n2} \leqslant \cdots \leqslant \xi_N\omega_{nN}$. Multiplying Eq. (13) by e^{at} results in

$$y(t) = x(t)\mathrm{e}^{at} = \sum_{i=1}^{N} A_i \mathrm{e}^{-(\xi_i\omega_{ni} - a)t} \sin(\omega_{di} t + \varphi_i). \tag{14}$$

Now three cases will be discussed as follows:

(1) If $a > \xi_1\omega_{n1}$, then $\mathrm{e}^{(a-\xi_1\omega_{n1})t}$ is divergent. The absolute areas forming from t and $y(t)$ will become larger and larger after a period of time, i.e., the latter area is larger than the former area.

(2) If $a = \xi_1\omega_{n1}$, then $\mathrm{e}^{(a-\xi_1\omega_{n1})t} = 1$, $y(t)$ can be expressed as

$$y(t) = \sum_{i=2}^{N} A_i \mathrm{e}^{-(\xi_i\omega_{ni} - a)t} \sin(\omega_{di} t + \varphi_i) + A_1 \sin(\omega_{d1} t + \varphi_1). \tag{15}$$

It is obvious that $y(t)$ will vibrate in equal amplitudes A_1 with frequency ω_{d1} if the time interval is long enough. Determining A_1 and ω_{d1}, the damping ratio ξ_1 is given by

$$\xi_1 = \frac{a}{\omega_{n1}} = \frac{a\sqrt{1-\xi_1^2}}{\omega_{d1}}. \tag{16}$$

So

$$\xi_1 = 1 \Bigg/ \sqrt{1 + \left(\frac{\omega_{d1}}{a}\right)^2}. \tag{17}$$

(3) If $a < \xi_1\omega_{n1}$, then $\mathrm{e}^{(a-\xi_1\omega_{n1})t}$ decays with time, and $y(t)$ decays as well. The absolute areas forming from t and $y(t)$ will become smaller and smaller after a period of time, i.e., the latter area is smaller than the former area.

Normally, a is not directly available to make $y(t)$ vibrate in equal amplitudes, but it can be determined by search algorithm. Giving the initial value, for example, letting $a \in (0, 1)$, calculate the ratio changes of areas at $a = 0, 0.5, 1$ after enough long time interval, $a = \xi_1\omega_{n1}$ can be obtained by Newton's dichotomy or golden section method, and ξ_1, ω_{n1} and A_1 can be determined. Theoretically, the product of component $A_1 \sin(\omega_{d1} t + \varphi_1)$ in Eq. (15) and e^{-at} is the contribution of the mode with damping ratio ξ_1 to response $x(t)$. Subtracting $A_1\mathrm{e}^{-at}\sin(\omega_{d1} t + \varphi_1)$ from $x(t)$ and repeating above-mentioned process, $\xi_2, \omega_{n2}, A_2, \ldots, \xi_N$, ω_{nN}, A_N can be obtained correspondingly.

Single dof system is the special case of multi-dof system. Therefore, its damping ratio ξ can also be gotten in the same way.

3. Digital simulations and field tests

3.1. Simulation of single dof system

The single dof system is given by

$$\ddot{x} + 0.5\dot{x} + 25x = 0. \tag{18}$$

The response is represented by

$$x(t) = \mathrm{e}^{-0.25t}\sin(\omega_d t + \varphi), \tag{19}$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$, $\varphi = \arctan 10$. The free decay response in Eq. (19) and the response contaminated by Gaussian white noise with zero mean are shown in Figs. 2 and3, respectively. Identification results obtained by traditional logarithmic-decrement method and approach in this paper are given in Table 1.

All areas are calculated by summating trapezium. Sampling frequency is 200 Hz. Two cases are considered, i.e., $E = S_1/S_2$ and $E = (S_1+S_2)/(S_3+S_4)$, and identified results are almost the same.

From the results, it can be seen that both logarithmic-decrement method and this approach can obtain satisfactory damping ratios if there is no noise contamination. However, when the free response is contaminated by Gaussian white noise, the relative errors of damping ratio obtained by traditional logarithmic-decrement method increase much more than those obtained by approach in this paper as the noise-to-signal ratios increase. Filtering the contaminated response through low-pass filter, the precision of logarithmic-decrement method can be greatly improved, but it is unsteady.

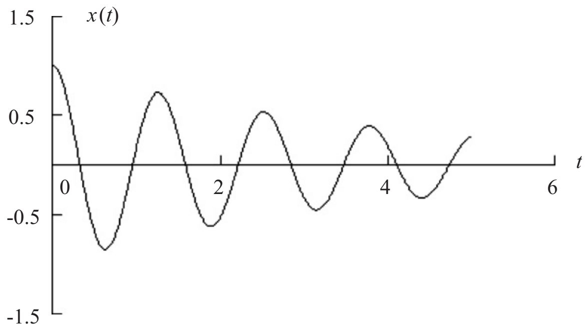

Fig. 2. Free decay response.

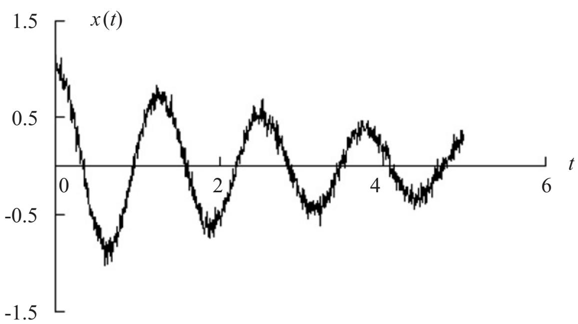

Fig. 3. Free response contaminated by Gaussian white noise with zero mean.

Table 1
Comparison of damping ratio identified by traditional logarithmic-decrement method and approach in this paper

Noise-to-signal ratio	Response without filtering				Response with low-pass filtering			
	Approach in this paper (%)	Relative error (%)	Logarithmic-decrement method (%)	Relative error (%)	Approach in this paper (%)	Relative error (%)	Logarithmic-decrement method (%)	Relative error (%)
0	5.0	0	5.0	0	5.0	0	5.0	0
5%	4.99	0.2	4.31	13.8	4.99	0.1	4.88	2.5
10%	4.98	0.4	2.73	45.4	4.99	0.19	4.66	6.8
20%	4.95	1.0	1.24	75.4	4.98	0.36	4.5	10.1

Table 2
Comparison of each order damping ratio identified by approach in this paper, ITD method and STD method

Noise-to-signal ratio	0			5%		
	ξ_1 (relative error (%))	ξ_2 (relative error (%))	ξ_3 (relative error (%))	ξ_1 (relative error (%))	ξ_2 (relative error (%))	ξ_3 (relative error (%))
Approach in this paper (%)	5.01(0.2)	3.75(0)	1.0(0)	4.97(0.6)	3.77(0.53)	1.02(2)
ITD method (%)	5.02(0.4)	3.72(0.8)	1.05(5)	5.15(3)	3.90(4)	1.30(30)
STD method (%)	5.01(0.2)	3.73(0.53)	1.0(0)	5.10(2)	3.82(1.87)	1.12(12)

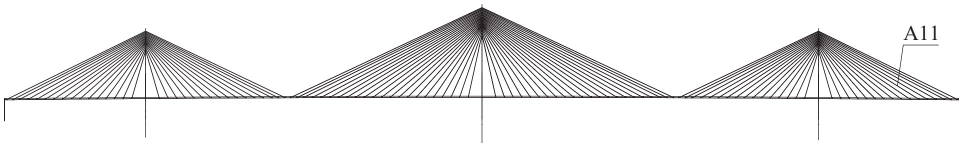

Fig. 4. The elevation view of the Dongting Lake Bridge.

3.2. Simulation of multi-dof system

Response of a three-dof vibration system is assumed as

$$x(t) = \mathrm{e}^{-0.25t}\sin(5t+\varphi_1) + 1.5\mathrm{e}^{-0.3t}\sin 8t + 2.0\mathrm{e}^{-0.1t}\sin 10t, \tag{20}$$

where $\varphi_1 = \arctan 5$, $\xi_1 = 5\%$, $\xi_2 = 3.75\%$, $\xi_3 = 1.0\%$. Identification results of three modal damping ratios obtained by approach in this paper, ITD method and STD method are listed in Table 2.

All areas are calculated by summating trapezia. Sampling frequency is 200 Hz. When calculating areas forming from t and $x(t)\mathrm{e}^{at}$, it starts at moment $t = 40$ s in order to allow the response to decay sufficiently. 5% in Table 2 means noise-to-signal ratio level.

3.3. Field tests and results analysis

The field tests are carried out on the Dongting Lake Cable-stayed Bridge (DLB), which locates in Yueyang city, Hunan province of China. The DLB is a prestressed concrete cable-stayed bridge with three-pylon, the main 310 m span, and four-lane highway deck. It opened to traffic in October 2000. A total of 111 pairs of cables are stayed on the bridge. The middle pylon is 100 m high, and the other two side pylons are equally 75 m high from the deck. The elevation view of the bridge is shown in Fig. 4.

According to actual situation of cable vibrations and theoretical analysis, cable A11 is selected as the experimental cable. Standard of the cable A11 is PES7-163. This cable has following geometric parameters: 114.719 m length, 68.944 m height from the deck, 51.8 kg weight per unit length, $6.272 \times 10^{-3}\,m^2$ cross-sectional area, 3.095×10^6 N dead loading tension and 2.0×10^{11} Pa elastic modulus.

Experimental devices consist of two parts: the supporting system is used to support the cable-MR-damper system on the deck, and the excitation system is used to vibrate the cable. The supporting system is fixed on the deck by eight foundation bolts. In order to verify the effectiveness of cable vibration mitigation, the cable-MR-damper system is designed to be separable. The excitation system is composed of an electric motor, a steel cable, a load cell, a frequency converter, a reduction gear, a trigger and a fastener. Layout of experimental devices is shown in Fig. 5. Other experimental equipment includes two model RD-1005 MR dampers, four model CA-YD piezoelectric accelerometers, one model DH5938 vibration analysis and processing system, one voltage regulator, and a portable computer are used to measure the acceleration responses of the cable A11.

In the system of cables with MR dampers, cables occupy dominate status and have viscous damping. Therefore, it is basically reasonable to assume the cable-MR-damper system is viscous system although MR damper itself has characteristic of non-viscous damping.

Cable A11 was excited in sinusoid by the electric motor before and after MR dampers installed. The excitation frequencies were adjusted to near the first three natural frequencies of cable A11 obtained by ANSYS programs. After stabilization, unloading excitation forces through the trigger enabled the cable or the cable-MR-damper system to vibrate in the form of free decay. The voltage levels applied on the MR dampers would be changed to 0, 0.25, 0.5, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0 V, respectively. The acceleration responses from accelerometers were amplified and sampled by DH5936 vibration analysis and processing system. The sampling frequency was set as 500 Hz. Recording time for each sample was about 400 s. Acceleration responses of cable A11 with and without MR dampers would be contrasted. The first four theoretical and measured natural frequencies of cable A11 are listed in Table 3.

It can be seen from Table 3 that measured natural frequencies have good agreements with theoretical ones. Tests under different excitation frequencies for the cable A11 with MR dampers under different voltage levels and without MR dampers were performed on the DLB. Typically free vibration responses of the cable A11 with and without MR dampers are shown in Figs. 6 and 7, respectively. Obviously, the equivalent modal damping ratio of the cable-MR-damper system is greatly larger than that of the cable itself. Based on the approach to identification of damping ratio presented in this paper, and by

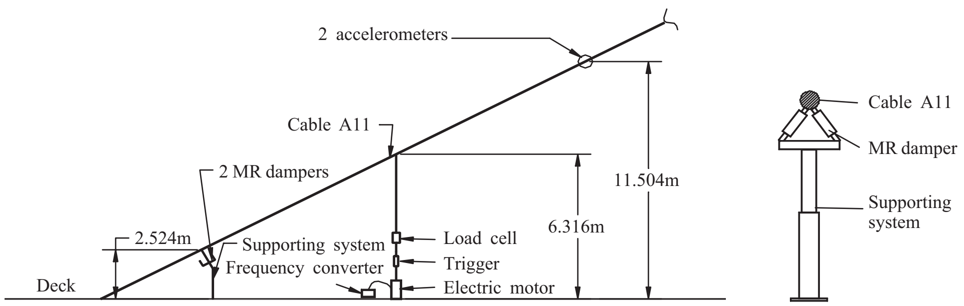

Fig. 5. Layout of experimental devices.

Table 3
The first four theoretical and measured natural frequencies of cable A11 (Hz)

Mode	1	2	3	4
Theoretical values	1.0963	2.1925	3.2888	4.3851
Measured values	1.0986	2.1973	3.2959	4.3945

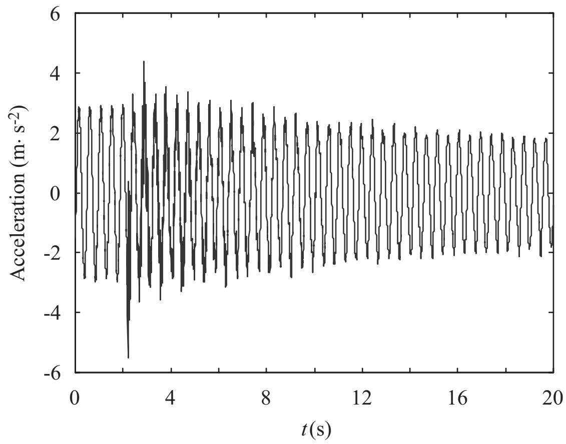

Fig. 6. Free vibration response without dampers.

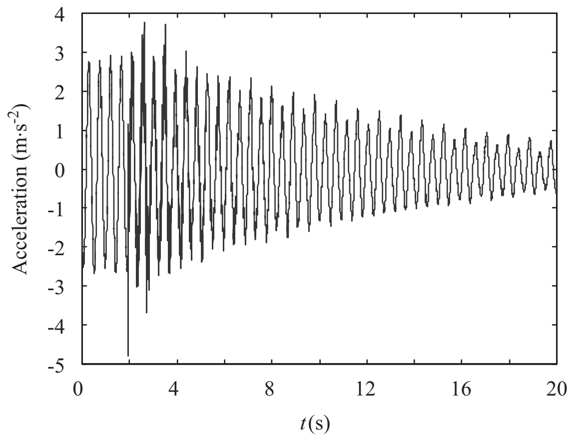

Fig. 7. Free vibration response with dampers.

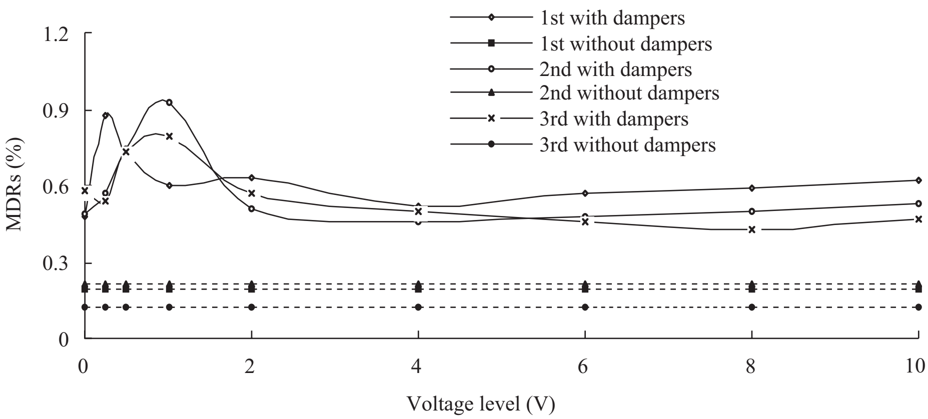

Fig. 8. Equivalent damping ratios under different voltage levels.

using MATLAB as a platform, the programs were developed to analyze measured data. The identified modal damping ratios for the cable itself and cable-MR-damper system under different voltage levels are shown in Fig. 8 and Table 4.

Table 4
The identified modal damping ratios for the cable itself and cable-MR-damper system under different voltage levels by approach in this paper and approach in Ref. [7]

Cable A11 modes	Without dampers	Damping ratios (%)								
		With dampers (voltage levels)								
		0	0.5 V	1 V	2 V	4 V	8 V	12 V	16 V	20 V
1st										
Approach in this paper (%)	0.19	0.48	0.87	0.73	0.60	0.63	0.52	0.57	0.59	0.62
Approach in Ref. [7] (%)	0.19	0.47	0.86	0.75	0.61	0.63	0.50	0.58	0.61	0.61
2nd										
Approach in this paper (%)	0.21	0.49	0.57	0.74	0.93	0.51	0.46	0.48	0.50	0.53
Approach in Ref. [7] (%)	0.21	0.52	0.59	0.77	0.95	0.51	0.47	0.48	0.53	0.50
3rd										
Approach in this paper (%)	0.12	0.58	0.54	0.73	0.79	0.57	0.50	0.46	0.43	0.47
Approach in Ref. [7] (%)	0.11	0.57	0.51	0.73	0.81	0.55	0.49	0.46	0.45	0.44

The first three modal damping ratios of the cable A11 itself are 0.19%, 0.21% and 0.12%, and the first three optimal equivalent modal damping ratios of the cable A11 incorporated with MR dampers are 0.87%, 0.93% and 0.79%, respectively. The latter increases about 4–7 times as comparing with the former. Even no power applied to MR dampers, the first three equivalent modal damping ratios increases about 2–4 times as comparing with the former. In Fig. 8, it is also displayed that the first three modal damping ratios get the optimal values when voltage levels reach 0.25, 1.0, 1.0 V, respectively.

4. Conclusions and discussion

A new approach to identification of modal damping ratios from free vibration response of a structure is presented in this paper. The approach is suitable for both single dof systems and multi-dof systems. Modal damping ratios are obtained by using the formulated relations among areas forming from time history of free response. In practical, all areas are obtained from response by numerical integration, which may cause slight error theoretically as areas forming from time history of response are replaced by summating trapezium. Though noise has a serious effect on discrete sampled data in local part, it has much less effect on areas because integral calculations can cancel the positive and the negative out for the response signals countermined by noise, especially by zero-mean Gaussian white noise. Therefore, compared with traditional logarithmic-decrement method, the approach in this paper has stronger anti-noise ability, higher precision better stability, much convenience and practicality. Results of digital simulations and field tests demonstrate that it is successful and effective.

Compared with the Wavelet based methods in Refs. [6,7], the presented method is simpler, more convenient and more practical.

It should be pointed out that the measured responses of the DLB include the component contributed by random wind, earth pulse and deck vibration. For limitation of the space and complexity of the problem, it is not further discussed in the paper.

When calculating areas, if there is no noise contamination, sampling frequency has no other requirement except satisfying the Sampling theory, and if there is noise, sampling frequency must be increased because low sampling frequency is disadvantageous to eliminate the effect of noise. Generally, sampling frequency is selected as 10 times that of the maximal frequency.

Damping ratios are identified from responses measured at one point on the structure in this paper. If there are enough responses measured at more points, then all modal parameters such as damping ratios, natural frequencies and modal shapes could be identified.

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