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Application of stochastic resonance in target detection in shallow-water reverberation

Bohou Xu[a,*], Lingzao Zeng[a], Jianlong Li[b]

[a]*Department of Mechanics, the State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, PR China*

[b]*Institute of Information and Communication Engineering, Zhejiang University, Hangzhou 310027, PR China*

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Abstract

In the processing of sea-bottom echo signal, the total received noise is approximated to Lorentzian form according to its vertical coherence. Then a method based on parameter-induced stochastic resonance (PSR) is presented. By means of tuning system parameters, numerical simulation shows that PSR method can effectively recover the spatial signal interfered by Gaussian noise. It also shows that PSR method has well applicability when processing spatial signal interfered by K-distributed envelope noise.

1. Introduction

When detecting buried target in shallow sea bottom, the echo signal is interfered by background noise and reverberation. In shallow water, reverberation mostly arises from the sea bottom reverberation. The roughness of the sea bottom is dominant for reverberation [1]. Processing echo signal embedded in reverberation is an attractive problem in acoustic signal processing. In the echo signal processing, both the ability to detect the presence of a target echo and the accuracy of measurements (range, angle, etc.) are limited by signal-to-noise ratio (SNR) [2].

Stochastic resonance has been developed rapidly in a variety of fields both in theory and application since 1981, when it was proposed by Benzi et al. [3–7] to explain the periodicity of ice ages. As a counterintuitive phenomenon in the nonlinear system, stochastic resonance is the cooperation between the stochastic excited nonlinear system and the external deterministic force. Under certain conditions, noise plays an active role in the system output. Getting the maximum SNR to realize stochastic resonance by tuning system parameters, this is the basis of parameter-induced stochastic resonance (PSR) [7–9].

In this paper, a method based on PSR is developed for processing spatial echo signal. This paper is organized as follows: in Section 2, the spatial echo signal model is presented. In Section 3, the spatial signal is processed by the PSR method, which is extended from the PSR signal processing in time domain [7–9,16]. The

*Corresponding author. Tel.: +86 571 8795 1769.
E-mail address: xubohou@zju.edu.cn (B. Xu).

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numerical simulation is carried out in Section 4. Section 5 is devoted to simply explore the applicability of PSR method to process signal interfered by non-Gaussian noise. The conclusions are presented in the Section 6.

2. Spatial echo signal model

In the detection of the buried target in shallow sea bottom, active sonar emits non-directive sine wave with frequency f_c (Hz), duration τ_t (s), and the depth of sea is h (m). The receiving uniform line array (ULA) with interval Δz is shown in Fig. 1. At the time t ($t \gg \tau_t$), take the coordinates (z, r) as shown in Fig. 2, the interfering reverberation is the sum of echo scattered by the scatterers distributed at the cirque on the sea bottom [10,11]. At the nth receiving element, the echo signal $S(n)$ is interfered by the background noise $W(n)$ and reverberation noise $R(n)$. So we obtain

$$y(n) = S(n) + R(n) + W(n), \tag{1}$$

where R is statistically independent with W, both R and W are assumed to be Gaussian.

Without losing generality, assume the signal received by the first element at time t is $s(1) = \sin(2\pi f_c t + \phi_0)$, where ϕ_0 is the arbitrary phase. Then the received signals can be written in vector form

$$\begin{aligned}[S(1)\;\; S(2)\cdots S(N)]^{\mathrm{T}} \\ = [\sin\phi_t \;\; \sin(2\pi\Delta z f_c \sin\theta/c + \phi_t) \;\; \cdots \;\; \sin[2\pi(N-1)\Delta z f_c \sin\theta/c + \phi_t]^{\mathrm{T}},\end{aligned} \tag{2}$$

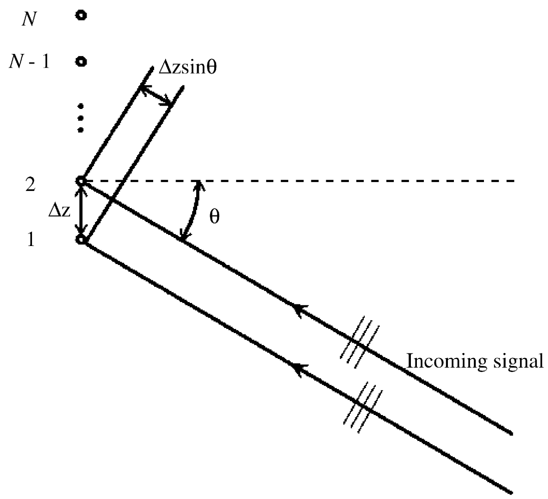

Fig. 1. Uniform line array.

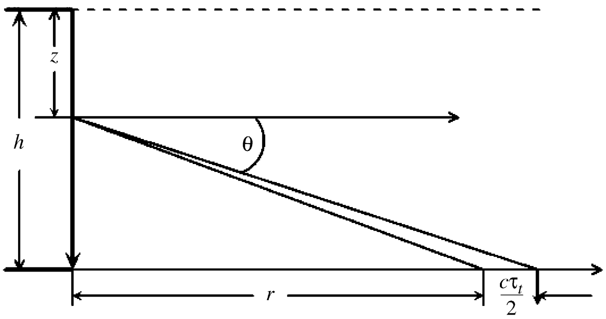

Fig. 2. Reverberation spatial sketch map.

where c is acoustic velocity in sea, $\phi_t = 2\pi f_c t + \phi_0$. Therefore, the received signal series $S(n),\ n = 1, 2, \ldots, N$ are the spatial samples along the coordinate z with sample interval Δz. Since $r \gg h$, the spatial wavenumber of echo with direct path is $2\pi f_c \sin\theta/c \approx 2\pi f_c(h - z)/cr$, which varies linearly with coordinate z.

According to the normal mode reverberation theory, the normalized vertical spatial correlation of reverberation R can be described as [12]

$$C_R(z) = \left(1 + \frac{k^2 z^2 h}{2Qr}\right)^{-(l+2)/4}, \tag{3}$$

where z is the spatial interval between any two receiving elements, $k = 2\pi f_c/c$ is the wavenumber, Q is the sea bottom reflection coefficient, and l is reverberation model parameter (for Lambert reverberation model, $l = 2$).

3. Spatial signal processing by parameter-induced stochastic resonance

Take the spatial coordinate z as the variable, and the received signal $S(z) + R(z) + W(z)$ as the input of the bistable system. When the bistable system gets stable quickly enough, the varying signal in a short interval can be viewed as a constant value H, which can be determined by the weighed average [8]. Therefore, we obtain the following:

$$\frac{\mathrm{d}x}{\mathrm{d}z} = ax - \mu x^3 + H + R(z) + W(z), \tag{4}$$

where x is the system stochastic output, a and μ are the adjustable system parameters.

The total received noise $R(z) + W(z)$ can be approximated to Lorentzian noise $\xi(z)$, which has the following stationary correlation form:

$$\langle \xi(z)\xi(0)\rangle = \frac{D}{\tau_z}\exp\left(-\frac{|z|}{\tau_z}\right), \tag{5}$$

where D is the noise intensity, and τ_z is the correlation length of noise. The Lorentzian noise has the spectrum form

$$S(\omega) = \frac{2D}{1 + \omega^2\tau_z^2}, \tag{6}$$

where ω denotes the variable in the wavenumber domain. The parameters τ_z can be estimated from the total noise data by the method of least-squares data fitting. The parameter D can be derived by $D = \tau_z\sigma^2$, where σ^2 is the noise variance.

Assume the correlation length τ_z is small, and then the corresponding approximating Fokker–Planck equation of Eq. (4) is [13]

$$\frac{\partial P(x,z)}{\partial z} = -\frac{\partial}{\partial x}[f(x)P(x,z)] + D\,\frac{\partial^2}{\partial x^2}\left[\frac{P(x,z)}{1 - \tau_z f'(x)}\right], \tag{7}$$

where $P(x, z)$ is the probability density function (PDF) of the stochastic output x at spatial coordinate z, $f(x) = ax - \mu x^3 + H$. The stable solution of Eq. (7) is given by [14]

$$P_S(x) = N|1 - \tau_z f'(x)|\exp\left[\int_0^x \frac{f(\xi)}{D}\,\mathrm{d}\xi - \frac{1}{2D}\,\tau_z f^2(x)\right], \tag{8}$$

where N is the normalizing factor.

The solution of Eq. (7) can be numerically obtained by a variational method based on eigenfunction expansion [8,15]. The minimum non-zero eigenvalue λ_1 is regarded as the system response speed, which dominates how fast the output reaches the stable state. The SNR of the stable output x is

$$\mathrm{SNR} = \frac{E[x]}{\sqrt{E[x^2] - (E[x])^2}}, \tag{9}$$

where

$$E[x] = \int_{-\infty}^{\infty} x P_S(x)\,\mathrm{d}x,$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 P_S(x)\,\mathrm{d}x. \tag{10}$$

After selecting a certain response speed λ_1, the maximum SNR can be obtained by tuning the system parameters a and μ.

Since the system is nonlinear and the signal is varying, the output signal is distorted. A formula to obtain the recover signal $\hat{H}$ from the stochastic output of the system with optimal parameters a and μ is given as follows [16]:

$$\hat{H} \approx \mu\bar{x}^3 - a\bar{x}, \tag{11}$$

where $\bar{x} = E[x]$. Using the spline matching method to the results $\hat{H}$, a good signal curve is recovered.

There is an optimal system response speed, which is related to the wavenumber and the amplitude of input signal as well as the properties of interfering noise. Here a simple method is provided to choose the optimal system response speed. Let $\hat{H}(z)$ be the recovery signal, and $H(z)$ be the input reference signal. The cross-correlation between $H(z)$ and $\hat{H}(z)$ is

$$\mathrm{Cor}(H(z), \hat{H}(z)) = \frac{\int \hat{H}(z)H(z-\varphi)\,\mathrm{d}z}{\int \hat{H}^2(z-\varphi)\,\mathrm{d}z}, \tag{12}$$

where φ is the phase lag (there is a phase lag between the recovery signal and input signal). Hence, the recovery signal $\hat{H}(z)$ is passed through a correlation receiver with a varying delay. According to Eq. (12), with each given system response speed, each maximum cross-correlation is derived by varying the delay of the receiver. Then the system response speed corresponding to the maximum cross-correlation is optimal.

4. Numerical simulation

Consider the emitted signal is sine wave with frequency $f_c = 20\,\text{kHz}$ and pulse duration $\tau_t = 1\,\text{ms}$, the depth of sea $h = 100\,\text{m}$, and the sea bottom reflection coefficient $Q = 0.3$. Choose Lambert scattering model $l = 2$ and spherical spreading model, take the absorption coefficient of sea as $\beta = 3.8\,\text{dB/km}$, then the transfer loss is $\text{TL} = 20\lg r + \beta r \times 10^{-3}\,\text{dB}$. Hence, the level of received signal is $(\text{SL} - 2\text{TL} + \text{TS})\,\text{dB}$. The level of received reverberation noise is $(\text{SL} - 2\text{TL} + S_b + 10\lg c\tau_t\pi r)\,\text{dB}$, where S_b is the sea bottom scattering strength [17]. Then the ratio of signal to reverberation is $(\text{TS} - S_b - 10\lg c\tau_t\pi r)\,\text{dB}$, and the ratio of signal-to-background noise is $(\text{SL} - 2\text{TL} + \text{TS} - \text{NL})\,\text{dB}$, where NL is background noise level. Assume the source level is $\text{SL} = 210\,\text{dB}$, the target strength is $\text{TS} = 5\,\text{dB}$, the sea bottom scattering strength is $S_b = -35\,\text{dB}$, the background noise is $\text{NL} = 45\,\text{dB}$. Choose spatial sample interval $\Delta z = 1\,\text{cm}$. Without losing generality, assume the amplitude of received target echo signal is normalized to 1. According to the ratio of signal to reverberation and background noise, the variance of normalized background noise and reverberation at different range is obtained. Then the parameters D and τ_z of the normalized total noise varying with range r can be obtained with the forgoing approximation method.

As shown in Fig. 3, the Lorentzian parameters D, τ_z vary with the increase of range. Hence the optimal system parameters a and μ vary with range r, where $r = ct/2$.

The numerical simulation is carried out by Simulink of MATLAB. The system is shown in Fig. 4. In order to simulate the reverberation, the white noise is filtered by the designed filter, which makes the correlation of the output fulfill the foregoing reverberation vertical correlation Eq. (3). The simulating signal is chirp signal, which simulates the varying wavenumber spatial signal.

At the distance $r = 4\,\text{km}$, with the given parameters, the normalized variances of reverberation and background white noise are $\sigma_R^2 = 0.94$, $\sigma_W^2 = 1.4$. The corresponding Lorentzian noise parameters $D = 5.8 \times 10^{-2}$, $\tau_z = 2.5 \times 10^{-2}$. Considering the highest wavenumber is 0.6π, we take λ_1 as 3. According to PSR theory, the system parameters can be chosen as $a = 0.35$, $\mu = 74$. The input interfered signal

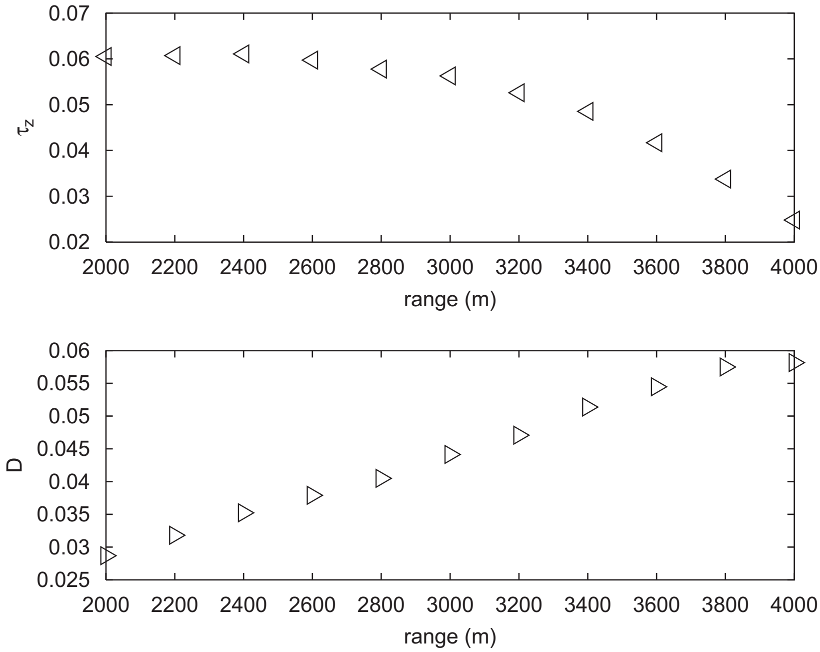

Fig. 3. The Lorentzian parameters against range.

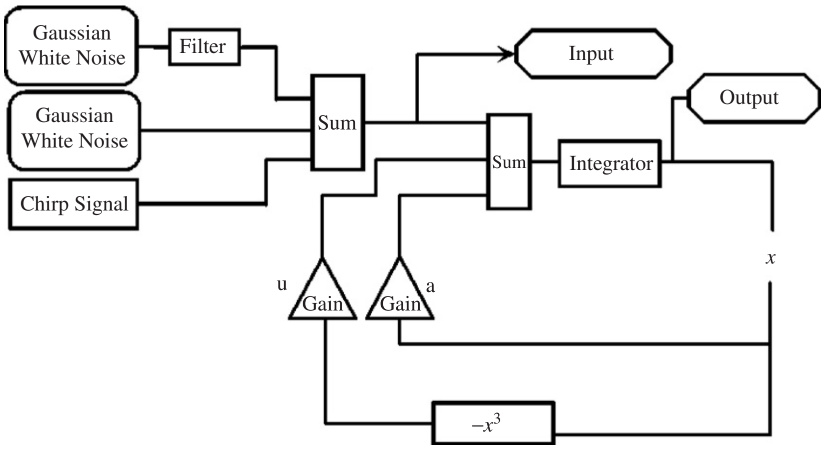

Fig. 4. The block scheme for simulating bistable system.

and the output of the system are shown in Fig. 5(a) and (b). The input signal and recovery signal are shown in Fig. 5(c).

It is shown that the stochastic resonance system can recover interfered signal effectively, therefore PSR provides a potential applicability in sonar echo signal processing. In different shallow sea environment, the vertical correlation of the interfering noise is different, hence the approximated Lorentzian parameter varies. It has been shown that the effects of the correlation length on the numerical calculation is not great [8]. However, in some sense, the mechanism of SR can be seen as the transferring of noise energy from high-frequency band to the signal frequency domain, therefore the shorter the correlation length of interfering noise is, the more superiority of PSR method to the linear filter will exhibit. The goal of the PSR system is to improve the reception of echo signal with direct path, the reception ability of sequent echo signals with multipath will be

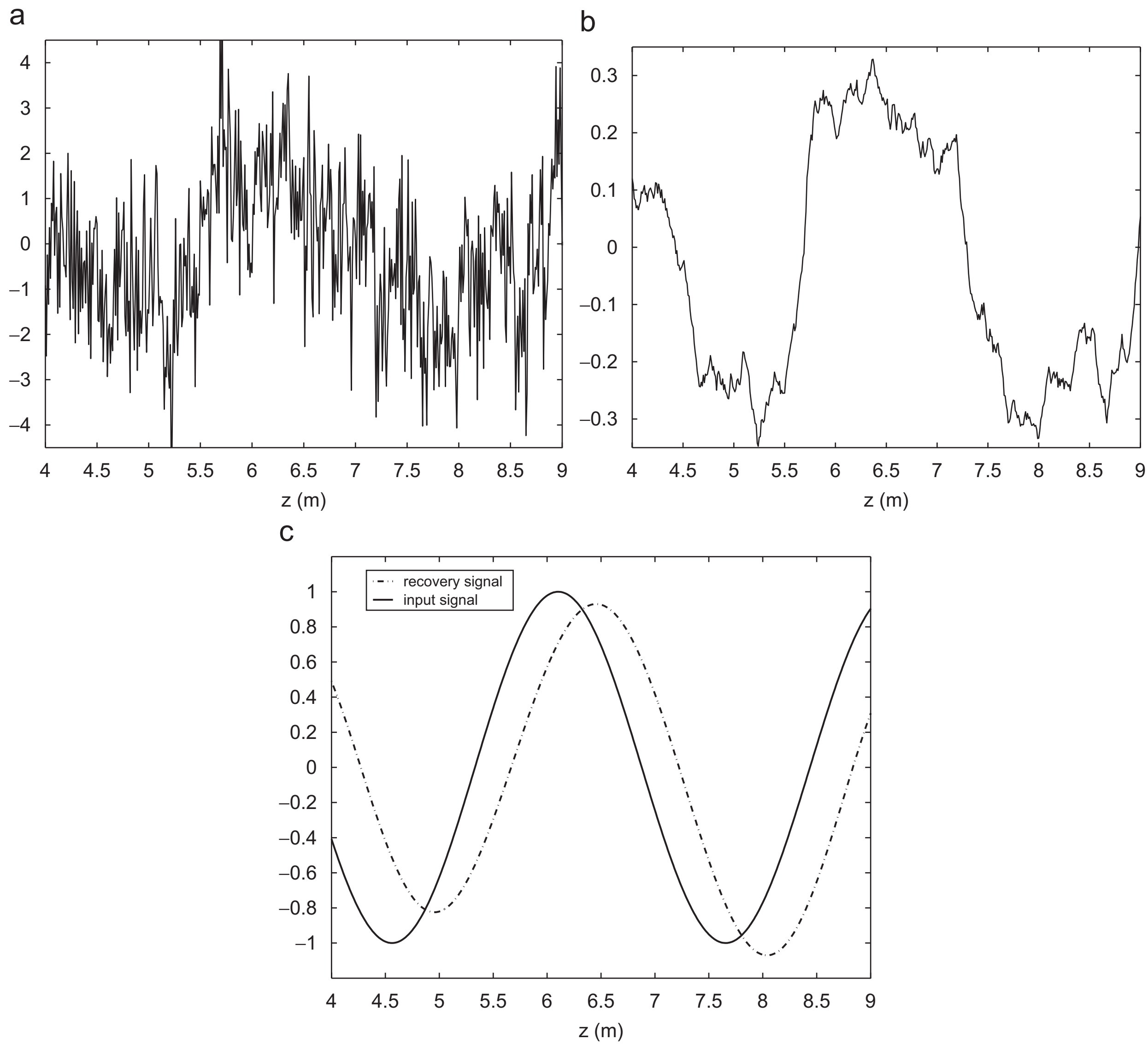

Fig. 5. (a) The signal interfered by Gaussian noise with correlation length $\tau_z = 2.5 \times 10^{-2}$, (b) the output of the system and (c) the input signal and recovery signal.

reduced. The reason is along the vertical coordinate z, the spatial wavenumber of the echo with multipath is different to that of the echo with direct path, and the PSR system is optimized for a certain wavenumber.

5. The effect of non-Gaussian noise

The Gaussian distribution of reverberation is derived by the central limit theorem under the assumption that there are a large number of scatters in a range-bearing resolution cell. However, with the advent of high-resolution sonar, the conditions of central limit theory are violated, so reverberation is no longer Gaussian [18,19]. In the preceding method, Fokker–Planck equation is derived by assuming that the noise is Gaussian. The departure caused by the probability mismatch is simply explored in the following section.

The K-distributed reverberation-envelope is a standard model to describe the PDF of non-Gaussian reverberation. The noise with shape parameter α and scale parameter λ is generated using the

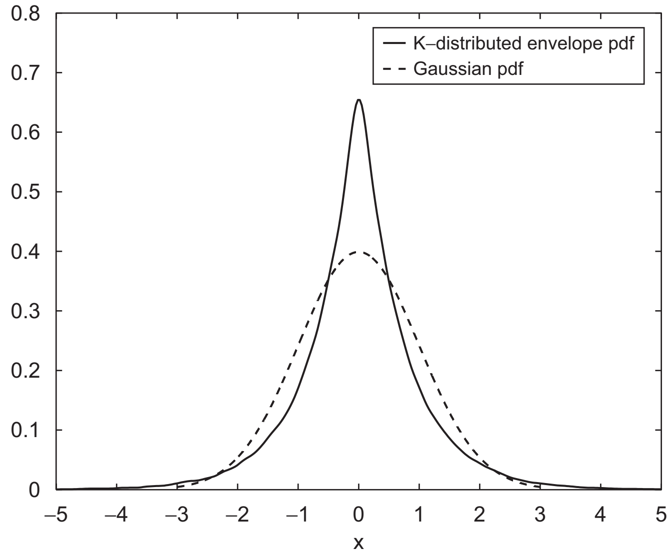

Fig. 6. The K-distributed envelope PDF and Gaussian PDF.

compound method [19]

$$X = \sqrt{V}Z, \tag{13}$$

where V is the Gamma random variable with parameters α and 1, Z is the Gaussian random variable with zero mean, variance λ, and V is statistically independent with Z. When $\alpha \to \infty$, the distribution of X tends to be Gaussian.

Consider an input signal $s(z) = \sin(0.6\pi z)$ is interfered by a K-distributed envelope white noise with intensity $D = 0.025$, shape $\alpha = 1$ and scale $\lambda = 1$. The probability density of the noise is shown in Fig. 6 with comparison to Gaussian probability density with unit variance. Choose $\Delta z = 0.01$ m, hence the noise variance is $\sigma^2 = 5$, according to PSR theory, choose system parameters as $a = 0.4$, $\mu = 80$. The interfered signal, system output and the recovery signal is shown in Fig. 7(a)–(c), respectively. It can be shown that although there is a probability density mismatch in the noise distribution, the method based on Gaussian precondition shows well applicability.

6. Conclusion

In this paper, the spatial echo signal is processed by the method based on parameter-induced stochastic resonance. Numerical simulation shows that the PSR method can effectively recover echo spatial signal interfered by reverberation in shallow sea. The method can be summarized as follows:

(1) Approximate the total noise to Lorentzian form according to environmental and sonar system parameters.
(2) According to the highest wavenumber of spatial signal, select a certain system response speed λ_1. Tune the system parameters a and μ to get the maximal SNR gain.
(3) Recover the output of the system according to the recovery formula.

Notice that the properties of both noise and signal vary with range r, therefore, the optimal PSR system is time-varying. As shown in the above numerical simulation, the sample interval is taken as $\Delta z = 1$ cm, which is not easy to be realized for the minimum separation of hydrophones. To get small spatial sample interval, investigation involved with moving array is now being undertaken. Also, the effects of non-Gaussian noise in the Fokker–Planck equation need to be studied in further work.

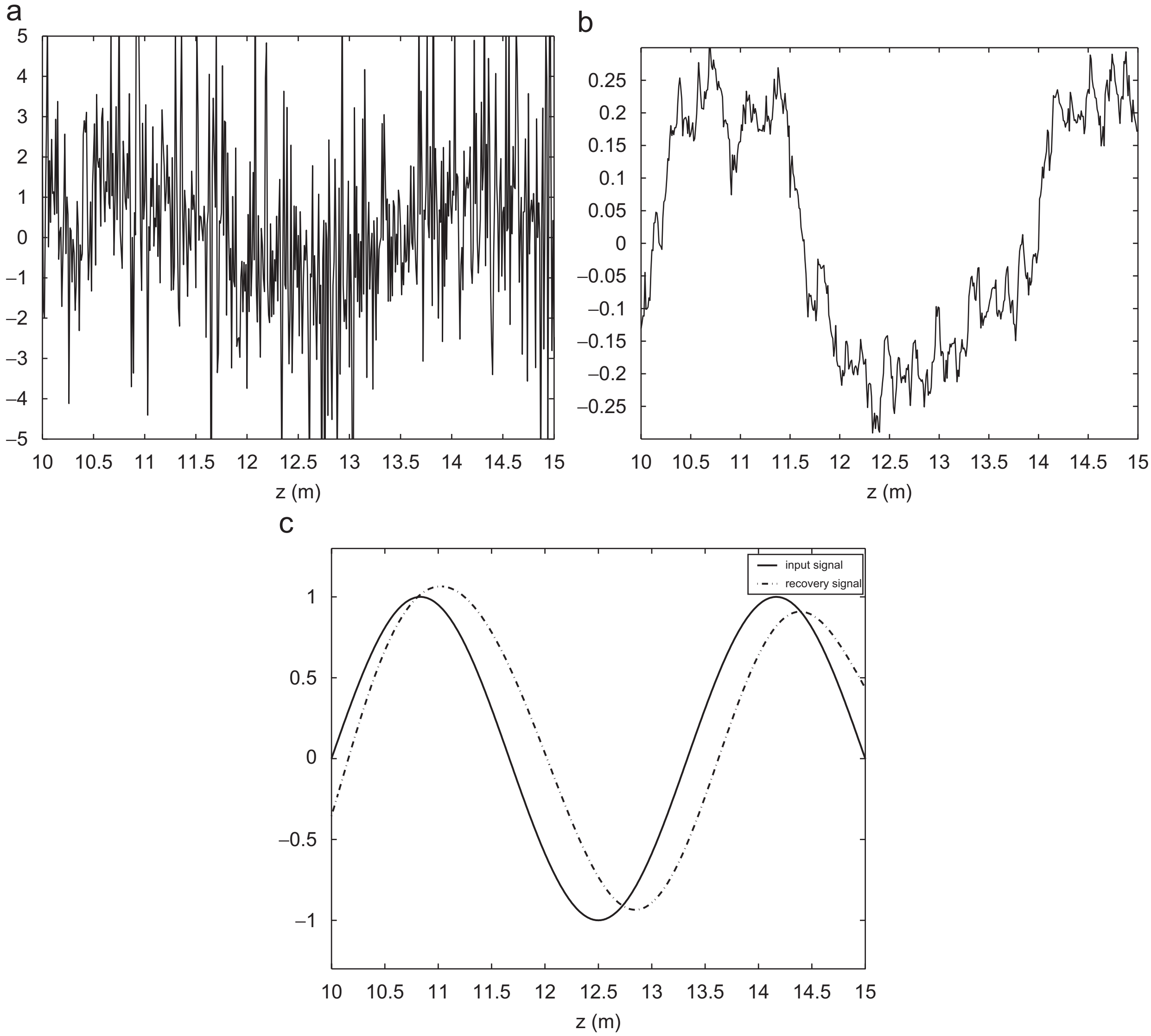

Fig. 7. (a) The signal interfered by K-distributed envelope noise, (b) the output of system interfered by K-distributed envelope noise and (c) the input signal and recovery signal.

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References

[1] P.C. Etter, *Underwater Acoustic Modeling*, second ed., Spon, London, 1996.
[2] M.I. Skolnik, *Introduction to Radar Systems*, first ed., McGraw-Hill, New York, 1962.
[3] R. Benzi, A. Sutera, A. Vulpiani, The mechanism of stochastic resonance, *Journal of Physics A: Mathematical and General* 14 (1981) 453–457.

[4] F. Moss, D. Pierson, D.O. Gorman, Stochastic resonance: tutorial and update, *International Journal of Bifurcation and Chaos* 4 (6) (1994) 1383–1397.
[5] L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Stochastic resonance, *Reviews of Modern Physics* 70 (1) (1998) 223–287.
[6] T. Wellens, V. Shatokhin, A. Buchleitner, Stochastic resonance, *Reports on Progress in Physics* 67 (2004) 45–105.
[7] B. Xu, F. Duan, R. Bao, J. Li, Stochastic resonance with tuning system parameters: the application of bistable systems in signal processing, *Chaos, Solitons and Fractals* 13 (4) (2002) 633–644.
[8] B. Xu, J. Li, F. Duan, J. Zheng, Effects of colored noise on multi-frequency signal processing via stochastic resonance with tuning system parameters, *Chaos, Solitons and Fractals* 16 (2003) 93–106.
[9] B. Xu, J. Li, J. Zheng, How to tune the system parameters to realize stochastic resonance, *Journal of Physics A: Mathematical and General* 36 (2003) 11969–11980.
[10] R. Zhang, G. Jin, Normal-mode theory of average reverberation intensities in shallow water, *Journal of Sound and Vibration* 119 (1987) 215–223.
[11] D.D. Ellis, A shallow-water normal-mode reverberation model, *Journal of the Acoustical Society of America* 97 (5) (1995) 2804–2814.
[12] F. Li, J. Liu, The vertical coherence of shallow water reverberation, *Acta Acustica* 28 (6) (2003) 494–503.
[13] J.M. Sancho, M.S. Miguel, S.L. Katz, J.D. Gunton, Analytical and numerical studies of multiplicative noise, *Physical Review A* 26 (3) (1982) 1589–1609.
[14] P. Hänggi, P. Jung, C. Zerbe, F. Moss, Can colored noise improve stochastic resonance, *Journal of Statistical Physics* 70 (1–2) (1993) 25–47.
[15] H. Risken, *The Fokker– Planck Equation*, Springer, New York, 1984.
[16] H. Li, B. Xu, R. Bao, Post-treatments of multi-frequency analog signal processing via QDW systems, *Communications in Nonlinear Science and Numerical Simulation* 11 (2006) 452–462.
[17] A.D. Waite, *Sonar for Practising Engineers*, third ed., Wiley, New York, 2002.
[18] N.P. Chotiros, H. Boehme, T.G. Goldsberry, S.P. Pitt, R.A. Lamb, A.L. Garcia, R.A. Altenburg, Acoustic backscattering at low grazing angles from the ocean bottom, part II: statistical characteristics of bottom backscatter at a shallow water site, *Journal of the Acoustical Society of America* 77 (3) (1985) 975–982.
[19] D.A. Abraham, A.P. Lyons, Simulation of non-Rayleigh reverberation and clutter, *Journal of Oceanic Engineering* 29 (2) (2004) 347–362.