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Finite element model updating using vibration test data under base excitation

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Abstract

Most model updating methods use measured frequency response function (FRF) data to update analytical models whereas only response functions under base excitation can be obtained in practical vibration test due to difficulties and constraints which prevent conventional FRFs from being measured accurately. This paper presents a new model updating method, which can employ measured response function data under base excitation directly for updating. Mathematical formulations using measured response function data under base excitation to identify mass and stiffness modeling errors, have been established. Through simulated numerical case studies based on a cantilever beam as well as a practical GARTEUR structure, it has been proved that the proposed method is feasible and effective when applied to the identification of mass and stiffness modeling errors. It is also shown that this method has considerable noise-resisting ability in the case where the measured response function data are contaminated by certain level measurement noise.

1. Introduction

In engineering practice, accurate mathematical models representing the dynamic characteristics of various engineering structures have been required for structural design and analysis. However, current finite element (FE) analysis cannot provide sufficiently accurate FE models, which are in good agreement with measured results. As a way to improve FE models, model updating procedure has been introduced and widely used to correct analytical FE models by using experimental test data.

In the past 30 years, a large number of model updating methods have been developed as discussed in the literature surveys carried out by Mottershead and Friswell [1,2]. In 1980s, direct modal based methods had been developed, such as the Lagrange multiplier methods introduced by Baruch [3] and Berman [4], matrix mixing methods developed by Caesar [5] and Link et al. [6] and the error matrix methods [7]. Though these early methods were computationally efficient, modal analysis was needed in these methods in order to obtain modal data before model updating can be carried out. Eigensensitivity-based iterative methods [8,9] had become dominant since 1990s due to the fact that these methods can preserve physical connectivity of an original FE model. These methods used the measured modal data as targets for updating FE models. As a

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result, modal analysis errors and incompleteness of measured modal data are inevitable during the updating process, which can bear significant effect on the accuracies of updated models.

The FRF-based methods [10–12] are the most promising model updating methods, which can produce accurate updated analytical models. These methods used measured FRF data directly to optimize a penalty function, which is defined in terms of the different types of error functions. Lin and Ewins [12] presented an iterative frequency response function (FRF) method in which the physical difference between the measured and analytical receptances was written as a linear function of the parameters to be updated. This method is able to produce highly accurate updated system matrices through iteration since the FRF sensitivity matrix involved is formulated exactly when measured coordinate data are complete and accurately when measured coordinate data are incomplete. It has been believed that the FRF-based method is more suitable for updating FE models since lots of measured FRF data are available and damping matrix of a system can be identified if the proper algorithm is applied. However, it needs to be further investigated and developed due to its convergence problem.

Modal testing is widely and successfully used for the determination of vibration properties of structural systems in engineering practice. In many situations, in order to simulate operational loads, base motion excitation is involved in simulated operational tests. Due to the size limitation of test structural systems such as micro-systems, base excitation technique is also applied in modal testing. Although base excitation test by itself is a technique of vibration measurement, the measured response functions, which are defined by displacement output and acceleration input, do represent the actual dynamic properties of structural systems just as traditional FRFs do. To date, most of the modal testing techniques developed do not specifically address the identification of modal parameters using this type of test data acquired using base excitation. Beliveau et al. [13] considered the relative motion of a structural system with respect to its base and presented a procedure to obtain modal information directly from the measured frequency response of the acceleration for the case of base excitation. In Ref. [14], Thomas and David developed a method which can be used to convert the motion-to-motion FRFs under base excitation test into motion-to-force FRFs using the equation of relative motion. Then, the modified FRF data could be analyzed directly using algorithms of modal parameter estimation.

It is believed that in the current state of practice, only a few of the model updating methods are capable of incorporating base excitation test data directly. This may be because most of updating methods developed assume the availability of measured FRF data or modal data, which can be acquired relatively easily using most existing modal testing techniques. Mark [15] presented a model updating procedure, in which a large mass was introduced to convert the driving motions (base excitations) into equivalent external excitation forces so that test data under base excitation can be used in FRF based updating methods indirectly. However, some approximation about the applied forces has been made during the modeling procedure. Moreover, the value of the large mass can not be determined accurately, which is usually chosen based on experiences and can affect the accuracy of the updated results. In fact, base excitation test data can be adopted to update FE models directly since the measured response function data naturally represent the vibration properties of structural systems. And model updating using base excitation test data is likely to be more appropriate than using modal data. This is because modal analysis may introduce additional analysis errors and the obtained modal data are usually incomplete due to the limitation of test techniques.

In the response-based model updating method the response function measured under base excitation can be considered as correlation targets directly. In this case, the method of updating would seem particularly similar to the FRF-based updating methods since the properties of the response functions measured in case of base excitation test are similar to FRFs obtained from force input. Moreover, an updating method based on test data under base excitations is sometimes more appropriate for updating FE models of both macro/large and micro/small structural systems. This is because a base excitation technique using shakers is usually more preferred in the vibration testing of structures with very small feature size where conventional testing techniques such as attaching an exciting shaker or using an impulse hammer becomes difficult to apply.

In the present paper, a new model updating method is presented which seeks to update analytical FE models of a group of structures on which only vibration tests under base excitations can be made to measure response functions. Compared with other existing FRF-based updating methods, the proposed new method has the advantage that it can be applied to updating erroneous FE models accurately using measured response function data under base excitation directly. In order to demonstrate the practical applicability of the proposed method, extensive numerical simulations have been carried out based on a cantilever beam and a truss structure.

2. Base excitation

Typically modal testing involves exciting a structural system with either an impact hammer or a shaker. The location and magnitude of the loads are selected such that enough energy can be imparted to the system to excite the modes of interest. In many applications, the information needed from a test is not only for the identification of the modes, but also for the verification of the response to operational loads. For this reason, simulated operational tests are performed, many of which involve the applications of base motion excitations. In this type of test, a test structure is fixed onto a shake table or displacement driven actuators subject to controlled motions. Usually, to evaluate the response to road input, automotive systems are tested based on known road/base excitation inputs. Many aerospace components are also tested in this manner to evaluate their dynamic responses to launch, or flight vibrations. Due to the limitation of input techniques when applied to test structures with small feature size, vibration tests of micro-systems such as hard disk drive are also performed using base excitation test to investigate the dynamic characteristics of these systems.

In engineering practice, base-excitation model is widely used for studying buildings subjected to earthquakes, packaging during transportations, vehicle responses, and even the design of accelerometers. To demonstrate the basic concept of base excitation, a simple mass–spring system with three degrees-of-freedom (dof) shown in Fig. 1(a) is considered. Here, all dofs of the system are relative to ground. Hence, the measured responses of the system would also be relative to ground (in the later formulas and examples, if not specifically mentioned, all the responses would be relative to ground). The system is only subjected to a displacement input from the moving base. The governing equations of motion of this mass–spring system with base excitation are written as

$$\begin{cases} m_3\ddot{y}_3 + k_3(y_3 - y_2) = 0, \\ m_2\ddot{y}_2 + k_2(y_2 - y_1) + k_3(y_2 - y_3) = 0 \\ m_1\ddot{y}_1 + k_1(y_1 - y_0) + k_2(y_1 - y_2) = 0, \end{cases}, \tag{1}$$

where y_0 is the displacement input of the base. Assume that the displacement responses are expressed as $y_j = Y_j e^{i\omega t}$ ($j = 1,2,3$) due to a base excitation input $y_0 = Y_0 e^{i\omega t}$. Then (1) can be re-written in matrix form in frequency domain as

$$\left(\begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_1 \end{bmatrix} - \omega^2 \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \right) \begin{Bmatrix} y_3 \\ y_2 \\ y_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ k_1 y_0 \end{Bmatrix}. \tag{2}$$

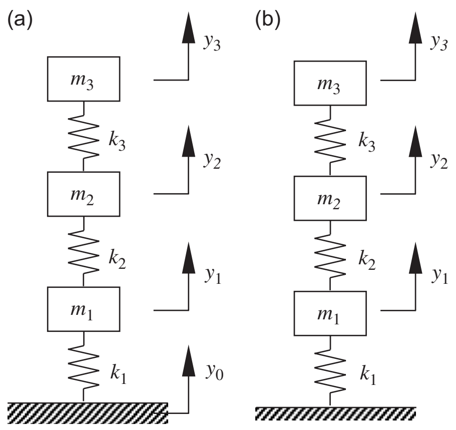

Fig. 1. A three dofs mass–spring system with a (a) moving base (b) fixed base.

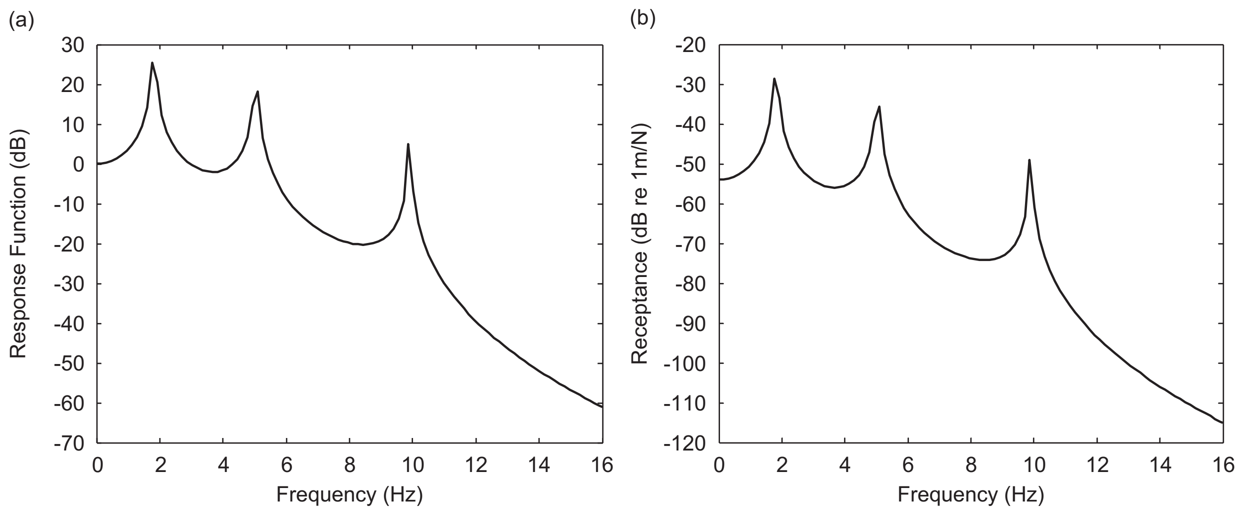

Fig. 2. Response functions of a system with moving and fixed bases.

Now, define new response functions with base excitation input as $H_j \equiv y_j/y_0$ ($j = 1, 2, 3$), after some manipulation, Eq. (2) becomes

$$\left(\begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_1 \end{bmatrix} - \omega^2 \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}\right) \begin{Bmatrix} H_3 \\ H_2 \\ H_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ k_1 \end{Bmatrix}. \tag{3}$$

Upon solving Eq. (3), the response functions of the system with base excitation input can be obtained as

$$\begin{Bmatrix} H_3 \\ H_2 \\ H_3 \end{Bmatrix} = \left[\begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_1 \end{bmatrix} - \omega^2 \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}\right]^{-1} \begin{Bmatrix} 0 \\ 0 \\ k_1 \end{Bmatrix}. \tag{4}$$

Based on Eq. (4), a typical response function, which is the transfer function between response output y_3 and the base excitation input y_0, has been calculated and is shown in Fig. 2(a). Compared with conventional FRF of the same mass–spring system fixed to the ground with the input excitation force being applied at coordinate y_1, the curve of the new response function under base excitation has similar shape to that of the conventional FRF, although the actual values of amplitudes of the two curves are different. The frequency locations of the resonance peaks of the two different types of response functions are the same. This means that the natural frequencies of the system measured using base excitation test are the same as those measured using input excitation force. If appropriate modal test techniques are used, modal data can also be obtained from the measured response functions under base excitation.

3. Model updating using vibration test data under base excitation

3.1. Theory

To demonstrate how a model updating method using vibration test data under base excitation can be developed, the mass–spring system shown in Fig. 1(a) with analytical modeling errors is first considered. The equations (3) of motion of the experimental model and the analytical model of the system can be written, respectively, as

$$([K_X] - \omega^2[M_X])\{H_X\} = \{F_X\}, \tag{5}$$

$$([K_A] - \omega^2[M_A])\{H_A\} = \{F_A\}, \tag{6}$$

where $\{H_X\}$ and $\{H_A\}$ are the experimental and analytical response functions under base excitation and, $\{F_A\} = \{0 \quad 0 \quad k_1\}^{\mathrm{T}}$ and $\{F_X\} = \{0 \quad 0 \quad (k_1 + \Delta k_1)\}^{\mathrm{T}}$, respectively. Assume that $[K_X] = [K_A] + [\Delta K]$, $[M_X] = [M_A] + [\Delta M]$ and $\{F_X\} = \{F_A\} + \{\Delta F\}$, upon subtracting (6) from Eq. (5) and rearranging, one has,

$$([\Delta K] - \omega^2[\Delta M])\{H_X\} = -([K_A] - \omega^2[M_A])\{\Delta H\} + \{\Delta F\}. \tag{7}$$

A model updating formula can then be developed based on Eq. (7) to identify stiffness and mass modeling errors $[\Delta K]$ and $[\Delta M]$, assuming $[K_A]$ and $[M_A]$ are given and $\{H_X\}$ has been measured. The formulation of such an updating procedure is presented as follows. Upon substituting the actual parameters of the mass–spring system into Eq. (7), one has,

$$\begin{bmatrix} \Delta k_3 - \omega^2\Delta m_3 & -\Delta k_3 & 0 \\ -\Delta k_3 & \Delta k_3 + \Delta k_2 - \omega^2\Delta m_2 & -\Delta k_2 \\ 0 & -\Delta k_2 & \Delta k_2 + \Delta k_1 - \omega^2\Delta m_1 \end{bmatrix} \begin{Bmatrix} H_{3X} \\ H_{2X} \\ H_{1X} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \Delta k_1 \end{Bmatrix}$$
$$- \begin{bmatrix} k_{3A} - \omega^2 m_{3A} & -k_{3A} & 0 \\ -k_{3A} & k_{3A} + k_{2A} - \omega^2 m_{2A} & -k_{2A} \\ 0 & -k_{2A} & k_{2A} + k_{1A} - \omega^2 m_{1A} \end{bmatrix} \begin{Bmatrix} H_{3X} - H_{3A} \\ H_{2X} - H_{2A} \\ H_{1X} - H_{1A} \end{Bmatrix}, \tag{8}$$

where $\{H_X\} \equiv \{H_{3X} \quad H_{2X} \quad H_{1X}\}$ and $\{H_A\} \equiv \{H_{3A} \quad H_{2A} \quad H_{1A}\}$. After some further mathematical manipulations, Eq. (8) can be turned into the following:

$$\begin{bmatrix} -\omega^2 H_{3X} & 0 & 0 & H_{3X} - H_{2X} & 0 & 0 \\ 0 & -\omega^2 H_{2X} & 0 & -H_{3X} + H_{2X} & H_{2X} - H_{1X} & 0 \\ 0 & 0 & -\omega^2 H_{1X} & 0 & -H_{2X} + H_{1X} & H_{1X} - 1 \end{bmatrix} \begin{Bmatrix} \Delta m_3 \\ \Delta m_2 \\ \Delta m_1 \\ \Delta k_3 \\ \Delta k_2 \\ \Delta k_1 \end{Bmatrix}$$
$$= - \begin{bmatrix} k_{3A} - \omega^2 m_{3A} & -k_{3A} & 0 \\ -k_{3A} & k_{3A} + k_{2A} - \omega^2 m_{2A} & -k_{2A} \\ 0 & -k_{2A} & k_{2A} + k_{1A} - \omega^2 m_{1A} \end{bmatrix} \begin{Bmatrix} H_{3X} - H_{3A} \\ H_{2X} - H_{2A} \\ H_{1X} - H_{1A} \end{Bmatrix}. \tag{9}$$

Eq. (9) is formulated using response functions under base excitation measured at one single frequency ω. If sufficient number of frequency data points are used, Eq. (9) will become a set of over-determined equations. And general inverse of the coefficient matrix can be used to solve the equation sets for the unknown mass and stiffness modeling errors. Subsequently, an updated analytical model can be reconstructed after the modeling errors have been identified.

Having established the formulation of the updating method using measured response functions under base excitation for the simple mass–spring system, application of the method to more general case of continuous systems needs to be developed. For a continuous structural system with moving base, the equations of motion of the system in frequency domain can be written as

$$([K] - \omega^2[M])\{u\} = \{f\}. \tag{10}$$

Here, it is assumed that the structural system under consideration is undamped. For simplicity, assume further that the first node in the FE formulation of the structural system is fixed at the base whose motion is specified, then this node will have the same motion as that of the base. Cases other node(s) being fixed to the base can be similarly treated by rearranging the mass and stiffness matrices. Under the base excitation in this case, the only possible external unknown forces applied to the system are applied at the fixed node. Suppose that the displacement vector of the fixed node is $\{\hat{u}_0\}$, Eq. (10) can then be written as

$$([K]-\omega^2[M])\left\{\begin{array}{c}\{\hat{u}_0\}\\ \{\hat{u}\}\end{array}\right\}=\left\{\begin{array}{c}\{\hat{f}\}\\ \{0\}\end{array}\right\}, \tag{11}$$

where $\{\hat{u}\}$ is the displacement vector corresponding to those unfixed nodes and $\{\hat{f}\}$ is the unknown external force vector applied at the fixed node. Assume that the stiffness and the mass matrices of the structural system can be partitioned into sub-matrices according to the dofs of the fixed nodes and the dofs of the unfixed nodes as

$$[K]=\begin{bmatrix}[K_{11}] & [K_{12}]\\ [K_{21}] & [K_{22}]\end{bmatrix} \quad \text{and} \quad [M]=\begin{bmatrix}[M_{11}] & [M_{12}]\\ [M_{21}] & [M_{22}]\end{bmatrix}, \tag{12}$$

where subscript 1 corresponds to the dofs of the fixed node and subscript 2 corresponds the dofs of the unfixed nodes, respectively. Separating $\{\hat{u}_0\}$ and $\{u\}$ in the displacement vector, we can rewrite Eq. (11) as

$$([K]-\omega^2[M])\left\{\begin{array}{c}\{0\}\\ \{\hat{u}\}\end{array}\right\}=\left\{\begin{array}{c}\{\hat{f}\}\\ \{0\}\end{array}\right\}-([K]-\omega^2[M])\left\{\begin{array}{c}\{\hat{u}_0\}\\ \{0\}\end{array}\right\}. \tag{13}$$

Substituting the sub-matrices of $[M]$ and $[K]$ into Eq. (13), one can obtain the following two equations:

$$([K_{12}]-\omega^2[M_{12}])\{\hat{u}\}=\{f\}-([K_{11}]-\omega^2[M_{11}])\{\hat{u}_0\}, \tag{14}$$

$$([K_{22}]-\omega^2[M_{22}])\{\hat{u}\}=-([K_{21}]-\omega^2[M_{21}])\{\hat{u}_0\}. \tag{15}$$

Suppose that the fixed node has motion only in one direction as it is usually the case in base excitation where base motion is specified in one specific direction, the displacement vector of this node $\{\hat{u}_0\}$ can be written as $\{\hat{u}_0\}=\{u_0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathrm{T}}$ (considering one node has six dofs). Then, upon dividing both sides by u_0, (14) and (15) become

$$([K_{12}]-\omega^2[M_{12}])(\{\hat{u}\}/u_0)=(\{\hat{f}\}/u_0)-([K_{11}]-\omega^2[M_{11}])\{e\}, \tag{16}$$

$$([K_{22}]-\omega^2[M_{22}])(\{\hat{u}\}/u_0)=-([K_{21}]-\omega^2[M_{21}])\{e\}, \tag{17}$$

where $\{e\}\equiv\{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathrm{T}}$. From Eq. (17), we can obtain the response functions of the system under base excitation input as

$$\{H(\omega)\}\equiv(\{\hat{u}\}/u_0)=-([K_{22}]-\omega^2[M_{22}])^{-1}([K_{21}]-\omega^2[M_{21}])\{e\}. \tag{18}$$

Here, it should be noted that the response function is obtained in the case of arbitrary excitation input. No matter what kind of base excitation (sinusoidal excitation or random excitation) is applied, the response function is same since it is the natural property of the mechanical system. Now we have established the relationship between the response functions under base excitation and system matrices. Based on Eq. (18), a model updating method can be developed assuming that the response functions data under base excitation $\{H(\omega)\}$ have been measured. From Eq. (18), one can have the following for the analytical and experimental models, respectively,

$$([K_{22}]_A-\omega^2[M_{22}]_A)\{H\}_A=-([K_{21}]_A-\omega^2[M_{21}]_A)\{e\}, \tag{19}$$

$$([K_{22}]_X-\omega^2[M_{22}]_X)\{H\}_X=-([K_{21}]_X-\omega^2[M_{21}]_X)\{e\}. \tag{20}$$

Let the sub-matrices of the system matrices and the error matrices be related as

$$\begin{cases}[K_{22}]_X=[K_{22}]_A+[\Delta K_{22}], & [M_{22}]_X=[M_{22}]_A+[\Delta M_{22}],\\ [K_{21}]_X=[K_{21}]_A+[\Delta K_{21}], & [M_{21}]_X=[M_{21}]_A+[\Delta M_{21}]\end{cases} \tag{21}$$

and the analytical and experimental response functions under base excitation of the system as

$$\{H\}_X=\{H\}_A+\{\Delta H\}. \tag{22}$$

Upon substitution of (21) and (22) into (20), one has,

$$[[K_{22}]_A + [\Delta K_{22}] - \omega^2([M_{22}]_A + [\Delta M_{22}])][\{H)_A + \{\Delta H\}]$$
$$= -[[K_{21}]_A + [\Delta K_{21}] - \omega^2([M_{21}]_A + [\Delta M_{21}])]\{e\}. \tag{23}$$

Subtracting (19) from Eq. (23) and rearranging, following equation can be established:

$$[[K_{22}]_A - \omega^2[M_{22}]_A]\{\Delta H\} + [[\Delta K_{22}] - \omega^2[\Delta M_{22}]]\{H\}_X$$
$$= -[[\Delta K_{21}] - \omega^2[\Delta M_{21}]]\{e\}. \tag{24}$$

After some rearranging, (24) becomes

$$[[\Delta K_{22}] - \omega^2[\Delta M_{22}]]\{H\}_X + [[\Delta K_{21}] - \omega^2[\Delta M_{21}]]\{e\}$$
$$= -[[K_{22}]_A - \omega^2[M_{22}]_A]\{\Delta H\}. \tag{25}$$

Eq. (25) can be employed to solve for the unknown modeling errors. However, before this can be achieved, some parameterization is needed. To parameterize the modeling errors, it is assumed here in this paper without much loss of generality that the error mass and stiffness matrices can be expressed as linear combinations of element mass and stiffness matrices, respectively,

$$[\Delta M] = \sum_{i=1}^{N} a_i[M^e]_i \quad \text{and} \quad [\Delta K] = \sum_{i=1}^{N} b_i[K^e]_i, \tag{26}$$

where $[M^e]_i$ and $[K^e]_i$ are the ith element mass and stiffness matrices, respectively, and a_i and b_i are the design parameter changes associated with the ith element. And the $\sum$ sign denotes matrix building and not straight summation. From Eq. (26), one can also derive the following:

$$\begin{cases} [\Delta M_{22}] = \sum_{i=1}^{N} a_i[M^e_{22}]_i, \\ [\Delta M_{21}] = \sum_{i=1}^{N} a_i[M^e_{21}]_i, \\ [\Delta K_{22}] = \sum_{i=1}^{N} b_i[K^e_{22}]_i, \\ [\Delta K_{21}] = \sum_{i=1}^{N} b_i[K^e_{21}]_i, \end{cases} \tag{27}$$

where $[M^e_{22}]_i$ and $[M^e_{21}]_i$ are the sub-matrices of the ith element mass matrix and $[K^e_{22}]_i$ and $[K^e_{21}]_i$ are the sub-matrices of the ith element stiffness matrix, which are accordingly expanded and partitioned. Upon substituting Eq. (27) into Eq. (25), one can obtain

$$\left[\sum_{i=1}^{N} b_i[K^e_{22}]_i - \omega^2 \sum_{i=1}^{N} a_i[M^e_{22}]_i\right]\{H\}_X + \left[\sum_{i=1}^{N} b_i[K^e_{21}]_i - \omega^2 \sum_{i=1}^{N} a_i[M^e_{21}]_i\right]\{e\}$$
$$= -\left[[K_{22}]_A - \omega^2[M_{22}]_A\right]\{\Delta H\}. \tag{28}$$

Eq. (28) can be transformed into a set of linear algebraic equations in terms of unknown design parameter changes a_i ($i = 1, 2,\dots,N$) and b_i ($i = 1, 2,\dots,N$) as

$$\begin{bmatrix} s^a_1 & s^a_2 & \cdots & s^a_N & s^b_1 & s^b_2 & \cdots & s^b_N \end{bmatrix} \begin{Bmatrix} \{a\} \\ \{b\} \end{Bmatrix} = -\left[[K_{22}]_A - \omega^2[M_{22}]_A\right]\{\Delta H\}, \tag{29}$$

where s_i^a, s_i^b, $\{a\}$ and $\{b\}$ are

$$\begin{cases} s_i^a = -\omega^2[M_{22}^e]_i\{H\}_X - \omega^2[M_{21}^e]_i\{e,\}, \\ s_i^b = [K_{22}^e]_i\{H\}_X + [K_{21}^e]_i\{e\}, \\ \{a\} = (\,a_1 \quad a_2 \quad \cdots \quad a_N\,)^{\mathrm{T}}, \\ \{b\} = (\,b_1 \quad b_2 \quad \cdots \quad b_N\,)^{\mathrm{T}}. \end{cases} \tag{30}$$

Eq. (29) is established based on measured response function data under base excitation at one measurement frequency. In practical vibration test under base excitation, response function data are measured at many different measurement frequencies. When response function data at sufficient number (n) of measurement frequencies are used, Eq. (29) can be turned into a set of over-determined algebraic equations, which can be simply written as

$$[S]\{p\} = \{q\}, \tag{31}$$

where

$$[S] = \begin{bmatrix} s_1^a(\omega_1) & s_2^a(\omega_1) & \cdots & s_N^a(\omega_1) & s_1^b(\omega_1) & s_2^b(\omega_1) & \cdots & s_N^b(\omega_1) \\ s_1^a(\omega_2) & s_2^a(\omega_2) & \cdots & s_N^a(\omega_2) & s_1^b(\omega_2) & s_2^b(\omega_2) & \cdots & s_N^b(\omega_2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ s_1^a(\omega_n) & s_2^a(\omega_n) & \cdots & s_N^a(\omega_n) & s_1^b(\omega_n) & s_2^b(\omega_n) & \cdots & s_N^b(\omega_n) \end{bmatrix},$$

$$\{p\} = \begin{Bmatrix} \{a\} \\ \{b\} \end{Bmatrix} \quad \text{and} \quad \{q\} = \begin{Bmatrix} -[[K_{22}]_A - \omega_1^2[M_{22}]_A]\{\Delta H(\omega_1)\} \\ -[[K_{22}]_A - \omega_2^2[M_{22}]_A]\{\Delta H(\omega_2)\} \\ \vdots \\ -[[K_{22}]_A - \omega_n^2[M_{22}]_A]\{\Delta H(\omega_n)\} \end{Bmatrix}.$$

Here, $[S]$ is a known coefficient matrix which is formed using the given analytical model and the measured response function data under base excitation. Eq. (31) can be solved for $\{p\}$ using linear least squares methods and then the solution $\{p\}$ is used to reconstruct the updated analytical model together with the original analytical model itself. Following above updating procedures, an accurate updated model can be obtained.

It has to be pointed out that during the development of the new model updating method under base excitation, it has been assumed that only one node is assumed to be fixed to the base where excitation is generated. In practice, however, there might be several nodes specified in an analytical model which are fixed to the base. In the case where multiple nodes are fixed, only the sub-matrices of the system matrices associated with the dofs of the fixed nodes and unfixed nodes will need to be changed through proper partition. And the vector $\{e\}$ which is associated with the dofs of the fixed nodes will also be changed accordingly.

3.2. Practical considerations

During the derivation of the formulas of the updating method, displacement response functions are applied while, in practical vibration testing, the measured responses of the system would typically be accelerations and not displacements. Since the displacement response function is defined as $H(\omega) = u_i(\omega)/u_0(\omega)$, $(i = 1, 2, \ldots, n)$, where $u_0(\omega)$ and $u_i(\omega)$ are the Fourier transforms of excitation and response time signals, respectively, then the acceleration response function would be $I(\omega) = \ddot{u}_i(\omega)/u_0(\omega)$, $(i = 1,\ 2, \ldots, n)$. Similar to the relationship between receptance and accelerance, there would be a certain relationship between these two kinds of response functions. In the general case, where an arbitrary excitation is applied and the response is measured, the relationship can be expressed as $H(\omega) = -I(\omega)/\omega^2$, which may be derived from the non-sinusoidal vibration and FRF properties theory in Ref. [16]. Therefore, if the acceleration response function is measured, the displacement response function would also be available based on the relationship. During the practical updating procedure, $H(\omega) = -I(\omega)/\omega^2$ should be employed to replace displacement response function in the formulas when measured acceleration responses data are available.

Eq. (31) is obtained using the analytical and measured response function data under base excitation in the case where damping matrix of the experimental model is not considered. In engineering practice, experimentally derived responses function data contain damping information and are always complex in nature. Since the measured response function data of practical structures are complex while this updating method requires real response function data, some kind of numerical pre-treatment of measured complex response function data becomes necessary before the suggested updating method can be performed. Here, are two complex-to-real conversion methods are proposed and developed. One is choosing the real parts of the measured complex response function data as the 'experimental' response function data and these real data are used for updating for frequency points off resonances. The alternative is to take the modulus of the complex response function data as the modulus of the real response function data. The signs of the converted real response function data are the signs of cosines of the phase angles associated with these response function data points. In general, these two conversion methods are approximate methods in which the imaginary part of measured response function data are is ignored. Since it is believed that most practical structures are lightly damped, these methods do provide real experimental response function data for model updating without introducing larger errors in the procedure.

In practice, it is not realistic to assume that all the coordinates, which are specified in the analytical model have been measured since some coordinates are physically inaccessible such as internal dofs and some others are very difficult to measure such as rotational dofs. When the measured coordinates are incomplete, direct solution of the updating problem is generally not possible and some approximation has to be introduced. During the calculation of the coefficient matrix $[S(\omega)]$ and the difference vector $\{q(\omega)\}$, those unmeasured elements of the response function vector are replaced by their analytical counterparts. Then, it will lead to the same linear algebraic equations as (31), which constitute a first-order approximation due to the incompleteness of measured coordinates. Again, when data at several frequency points are used for the equations, linear least squares methods can be use to solve for $\{p\}$. Of course, the obtained $\{p\}$ in this way is only a first-order approximation and an iteration scheme is required in the process in order to accurately obtain the exact solution. During the iteration process, the objective function to be minimized should be chosen in order to determine whether the convergent p-values is obtained. Here, the Euclindean norm of p-values, $\|\{p\}\|$ is define

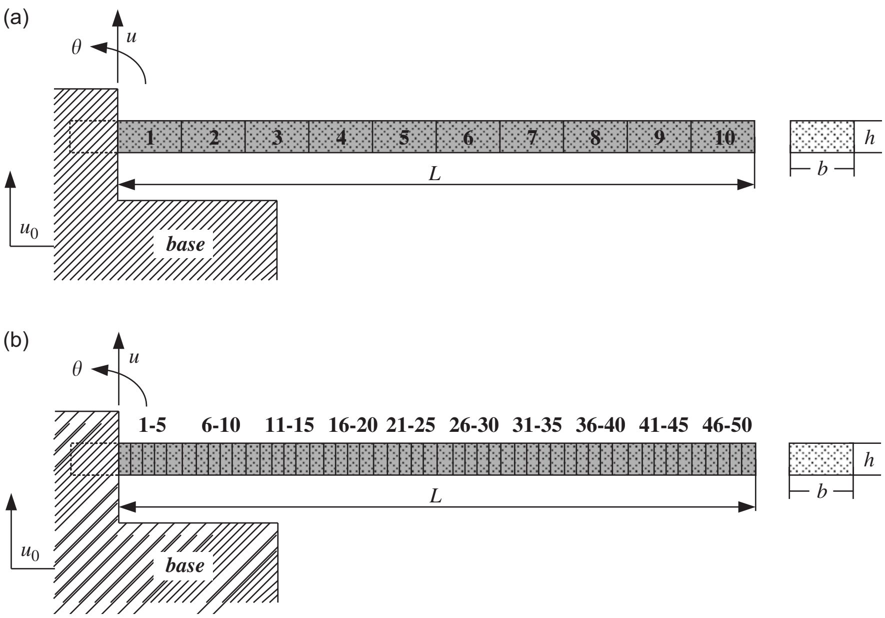

Fig. 3. A cantilever beam attached to a base: (a) the analytical FE model, (b) refined FE mesh used for 'experimental' model.

as the objective function for convergence criterion. Generally, it is believed that convergence of the p-values is achieved when the value of $\|\{p\}\|$ is less than 10^{-5}.

It should also be noted that convergence of updated parameters in iteration process is an important measure of the success of the updating procedure. Mathematically, the updating formulation in Eq. (31) is based on a particular form of matrix perturbation analysis in the case where the measured coordinates are incomplete. According to matrix perturbation theory, in order to guarantee convergence of the iteration process, some restriction should be made on the extent the difference between the analytical and experimental models. The restriction mentioned in Ref. [12] may be considered here. The Euclindean norms of the errors matrices should be of second-order when compared with those of the analytical mass and stiffness matrices themselves, namely,

$$\frac{||[\Delta M]||_F}{||[M_A]||_F} \leqslant \varepsilon, \frac{||[\Delta K]||_F}{||[K_A]||_F} \leqslant \varepsilon \quad \text{where } ||D||_F = \sqrt{\sum_{i=1}^{n} \sum_{i=1}^{n} d_{ij}^2}. \tag{32}$$

Although the value of ε varies for different systems, computational experience shows that for structural dynamic systems, the maximum value of ε may reach 0.3. Since the Frobenius norm is used and modeling errors are generally localized, the relative amplitudes of changes for individual design parameters can easily be more than 100% and still satisfy this requirement, as shown in the numerical case studies below.

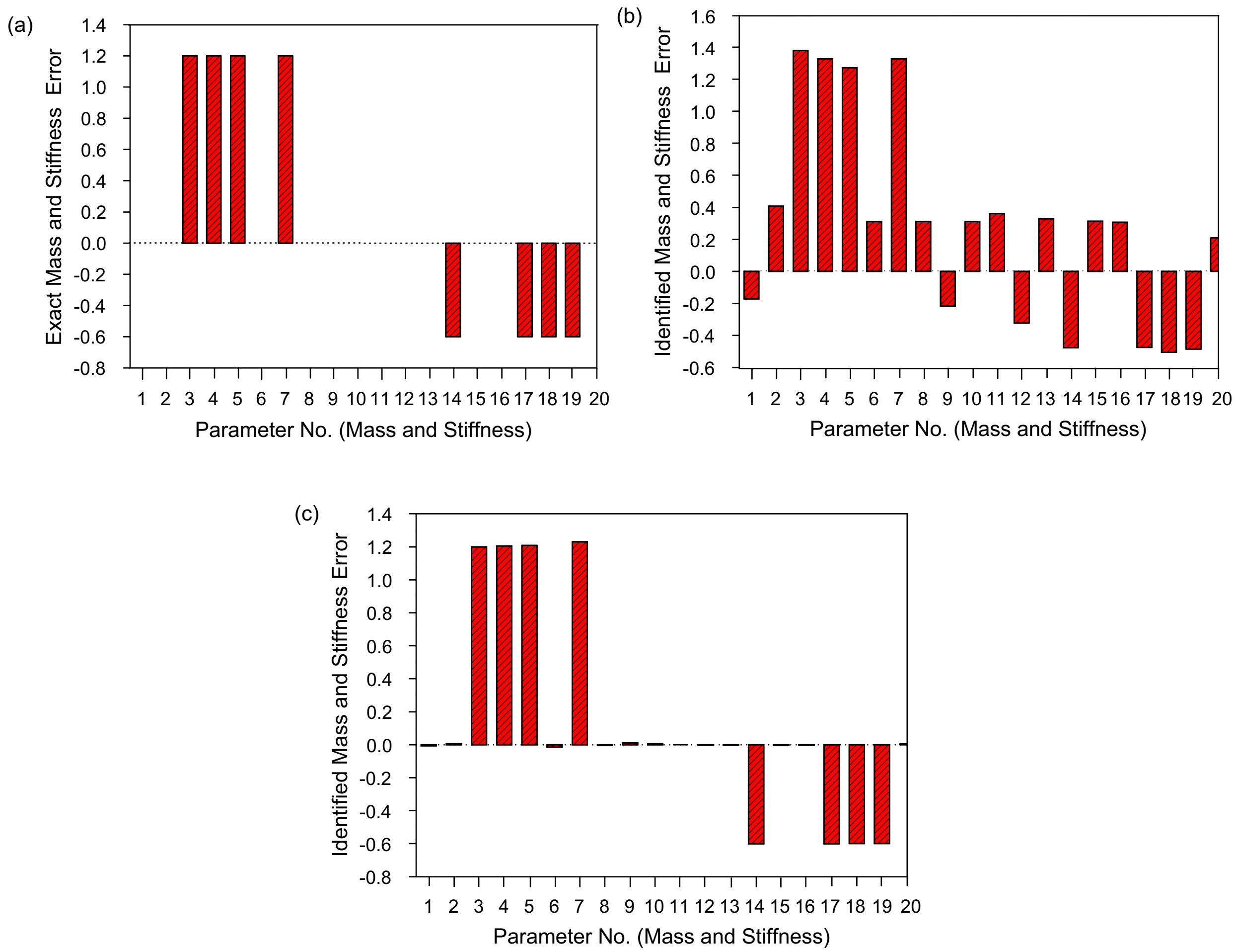

Fig. 4. Comparison of introduced and identified modeling errors (case 1).

4. Numerical case studies

4.1. Case 1—cantilever beam

In order to demonstrate its practical application, the proposed model updating method using vibration test data under base excitation has been applied to a cantilever beam shown in Fig. 3. The left end of this cantilever beam is fixed to the base, which has the same displacement as that of base motion in vertical direction.

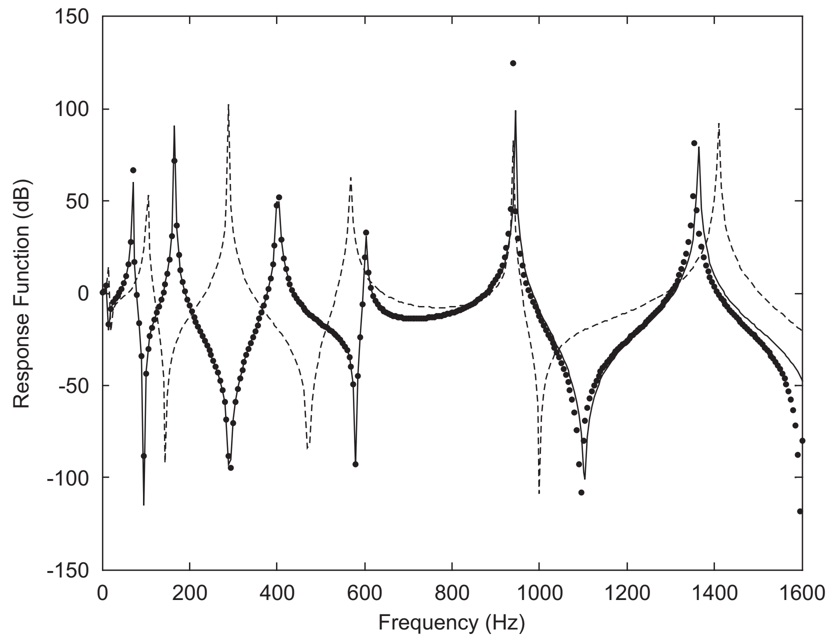

Fig. 5. Comparison of analytical, 'experimental' and updated response function curves (case 1) (—— updated, ● experimental, -------- analytical).

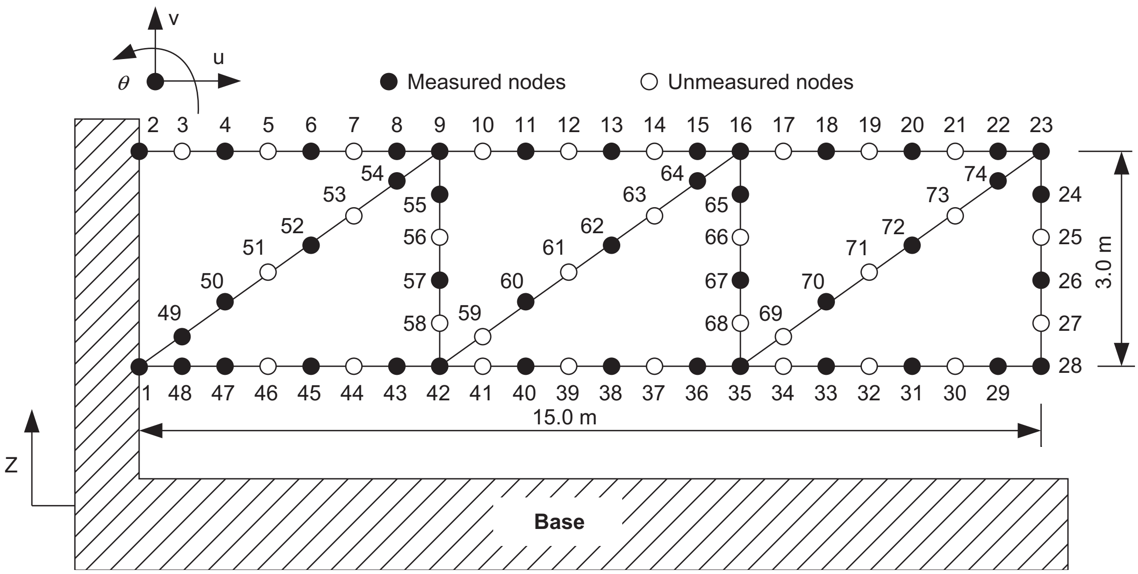

Fig. 6. The **GARTEUR** structure with excitation base.

Following parameters are used for this beam: Young's modulus of elasticity $E = 2.06E\ 11\,\mathrm{N/m^2}$, cross sectional area $A = b \times h = 0.02 \times 0.006\,\mathrm{m^2}$; length of the beam $L = 1.0\,\mathrm{m}$; material density $\rho = 7895.0\,\mathrm{kg/m^3}$. The analytical FE model of the beam, which is to be updated, is formulated using 10 bending beam elements as shown in Fig. 3(a). In order to simulate the practical structure more realistically, the 'experimental' model in this case is generated using a much refined FE mesh as shown in Fig. 3(b). And this model consists of 50 beam elements, which is five times that of the coarse analytical FE model and may be considered to be close to the practical structure. To generate the 'experimental' response function data, it is assumed that the cross-section areas of the 11–15th, the 16–20th, the 21–25th and the 31–35th elements are increased by 120% and the second moments of area of the 16–20th, the 31–35th, the 36–40th, and the 41–45th elements are reduced by 60%, respectively.

In this case study, only the translational dofs of the bending beam are assumed to have been measured since the measured dofs are usually incomplete in vibration test. We suppose that the response function data of the 'experimental' model have been measured over a frequency range covering just the first 6 resonances, with base

Table 1
Mass and stiffness modeling errors location (case 2)

Element no.	34				66			72			73	
A Error (%)	100				100			100			100	
Element no.	1	2	28	29	43	44	45	46	47	48	49	50
I Error (%)	−90	−90	100	100	100	100	100	−90	−90	−90	−90	−90

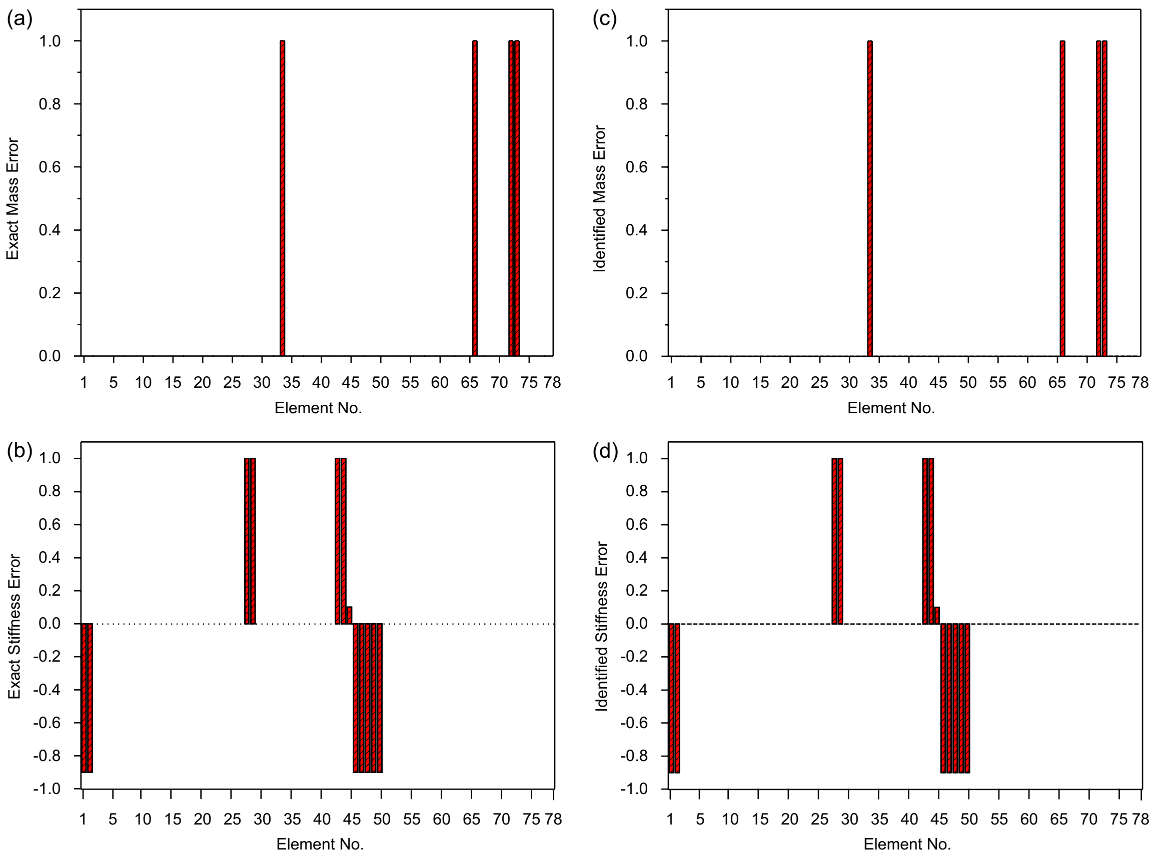

Fig. 7. Comparison of the exact and identified modeling errors (case 2).

excitation applied at the fixed node in u direction. Based on Eq. (31), the response function data at 10 measurement frequency points are chosen in the measured frequency range to construct the coefficient matrices $[S(\omega)]$ and $\{q(\omega)\}$. Since approximation is made during the derivation of the updating formulation of this proposed method due to the incomplete measured dofs in this case and the refined 'experimental' model is used, iteration process is required during updating. The iteration results for identification of element modeling errors are shown in Fig. 4. It can be seen that all the introduced modeling errors are well identified after 9 iterations. The response function curves of the 'experimental', analytical and updated models are shown in Fig. 5. From this figure, one can expect that the regenerated response function data from the updated model overlay those of the 'experimental' response function data. There are a few differences between the regenerated

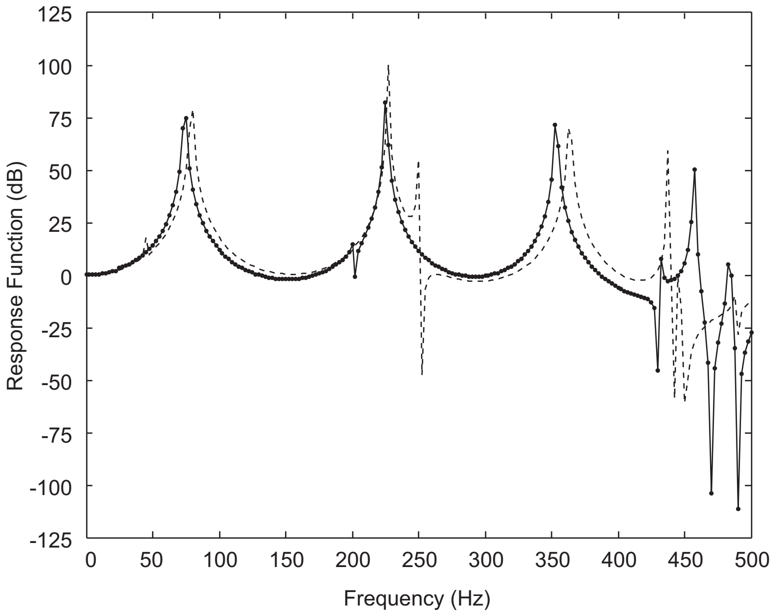

Fig. 8. Comparison of the analytical, the 'experimental' and the updated response function curves (case 2) (—— updated, ● experimental, -------- analytical).

Table 2
Mass and stiffness modeling errors location (case 3)

Element no.	34				66			67	
A Error (%)	200				200			200	
Element no.	7	8	26	27	40	41	47	48	54
I Error (%)	−90	−90	200	200	200	200	20	−90	−90

Table 3
The selected elements to be updated and their corresponding parameter number (case 3)

Parameter no.	1	2	3	4	5	6	7	8	9	10	11	12	13
Element no.	1	2	7	8	14	15	21	22	26	27	33	34	40
Parameter no.	14	15	16	17	18	19	20	21	22	23	24	25	
Element no.	41	47	48	54	55	59	60	66	67	71	72	78	

and 'experimental' response function data only at the resonant and anti-resonant regions, which is also the phenomenon in the response function data of two incompatible FE models.

4.2. Case 2—truss structure with complete coordinates

In this case, a simulation example based on a GARTEUR structure [12] shown in Fig. 6 is presented. The GARTEUR structure has two points fixed to the base. The FE model of the GARTEUR structure consists of 78 2-D beam elements. Each beam segment is a superposition of an axial bar element and a bending beam element. Each node of the beam element has three dofs (two translations and one rotation) and hence, the total number of dofs in the FE model is 222. Following material properties are used during FE modeling:

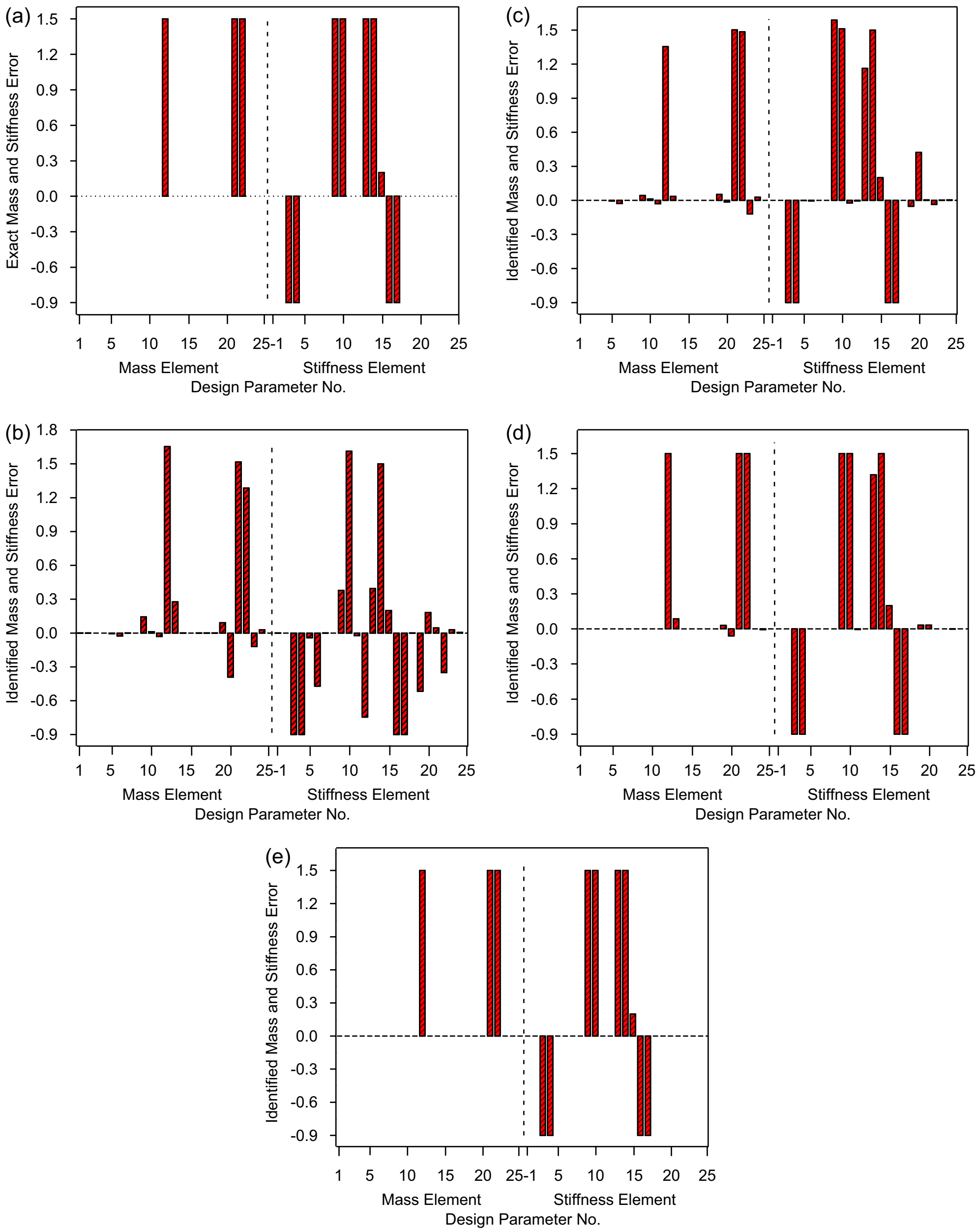

Fig. 9. Comparison of the exact and identified modeling errors (case 3).

Young's modulus is assumed to be $E = 0.75 \times 10^{11}\,\mathrm{N/m^2}$ and density $\rho = 2800\,\mathrm{kg/m^3}$. For the bar element, the cross-sectional areas are $S_h = 0.004\,\mathrm{m^2}$ (horizontal), $S_v = 0.006\,\mathrm{m^2}$ (vertical), and $S_d = 0.003\,\mathrm{m^2}$ (diagonal). For the bending beam elements, the second moment of area is assumed to be the same for all the elements and is assumed to be $I = 0.0756\,\mathrm{m^4}$. In order to generate the 'experimental' data, mass and stiffness modeling errors are introduced in the elements of the analytical model of the structure by changing the cross-section area A and the second moment of area I of some of elements as shown in Table 1.

We suppose that the response functions of all dofs have been measured over a frequency range covering just the first 8 resonances, with base excitation applied at the fixed nodes in the v direction. Based on Eq. (31), response function data at 20 frequency points are chosen in the measured frequency range to construct the coefficient matrices $[S(\omega)]$ and $\{q(\omega)\}$. The results for the identification of element modeling errors are shown in Fig. 7. The response function curves of the 'experimental', analytical and updated models are shown in Fig. 8. From this figure, it is found that the regenerated response function data from the updated model overlay perfectly with those of the 'experimental' model.

4.3. Case 3—truss structure with incomplete coordinates

In vibration test, it is not realistic that all the coordinates, which are specified in the analytical model have been measured. Therefore, the effect of coordinate incompleteness upon the updating procedure should be assessed. Due to the incomplete measured dofs, those unmeasured elements of the response function vector are replaced by their analytical counterparts when the coefficient matrix $[S(\omega)]$ and the difference vector $\{q(\omega)\}$ are calculated. Since some approximation has been made during the formulation of the coefficient matrix and the difference vector, iterations are required in the updating process in order to accurately identify the modeling errors.

In this numerical example, the same GARTEUR structure fixed to the base shown in Fig. 6 was considered. And the response function data of the 'experimental' model are supposed to be incomplete in the sense that only the translational dofs (u and v) of those hatched nodes are measured. In order to simulate the 'experimental' data, mass and stiffness modeling errors are introduced in the analytical model of the structure

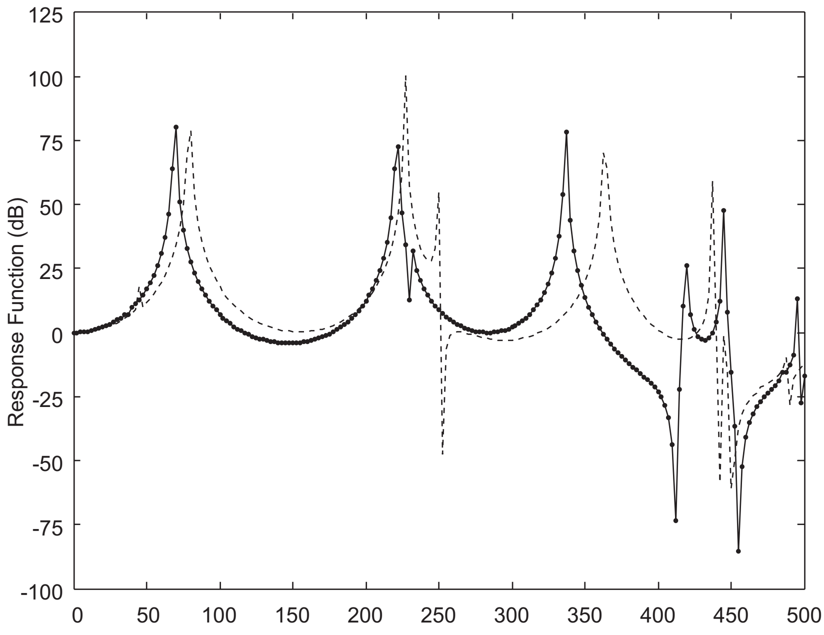

Fig. 10. Comparison of the analytical, the 'experimental' and the updated response function curves (case 3) (—— updated, ● experimental, -------- analytical).

by changing the cross-section area A and the second moment of area I of some elements as shown in Table 2. Since in practical modeling of structures, modeling errors generally occur in the modeling of joint elements, localization of modeling errors can be considered first before the application of the updating method in order to reduce computational time and improve the condition. Therefore, in this case we only choose those elements, which are located at joint positions to be updated as shown in Table 3. The mass and stiffness modeling errors of the selected 25 elements are shown in Fig. 9(a). The response function data have been measured over a frequency range covering just the first 8 resonances, with base excitation at the fixed nodes in the v direction. Based on Eq. (31), response function data at 20 frequency points are randomly chosen for each iteration in the measured frequency range to construct the coefficient matrices $[S(\omega)]$ and $\{q(\omega)\}$. After 20 iterations, convergence of the solution is obtained. The iteration results for the identification of element modeling errors are shown in Fig. 9. The response function curves of the 'experimental', analytical and updated models are shown in Fig. 10. From this figure, it is found that the agreement between the updated and 'experimental' response function data is excellent.

4.4. Case 4—noise problem (cantilever beam)

In engineering practice, the measured response function data from typical vibration test are usually contaminated by measurement noise. Therefore, in order to investigate the effect of measurement noise on the updating process, based on the same cantilever beam model, 6% uniform distributed random noise was added to the simulated measured response function data. To generate the 'experimental' response function data, the analytical FE model shown in Fig. 3(a) having both mass and stiffness modeling errors is assumed as the 'experimental' model. The modeling errors are introduced due to a 100% increase of cross-section area, A, in the 3rd, 4th, 5th 7th elements and a 60% reduction of second moment of area, I, in the 4th, 7th, 8th, 9th

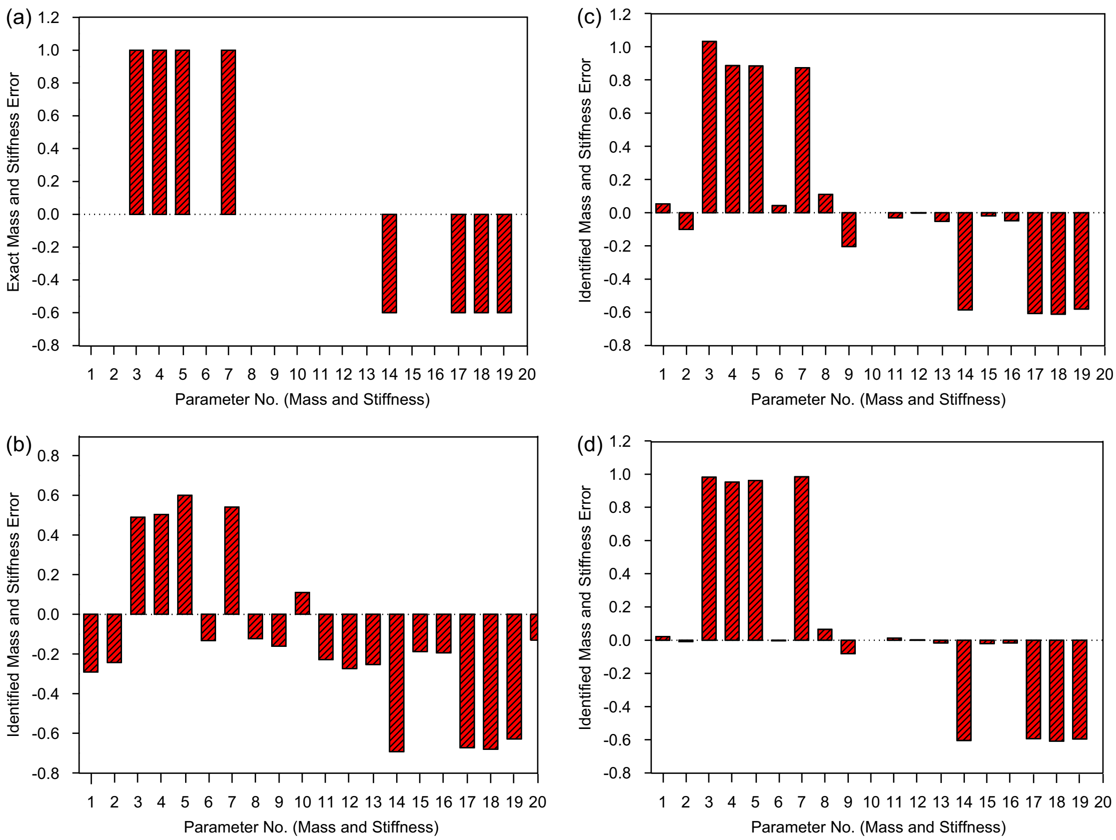

Fig. 11. Iteration results of identification modeling errors (case 4).

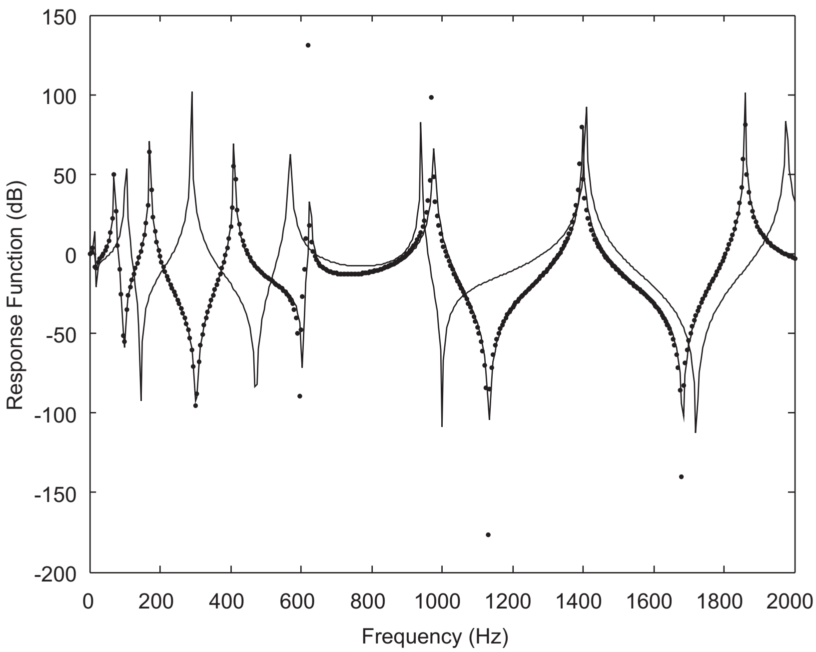

Fig. 12. Comparison of the analytical, the 'experimental' and the updated response function curves (case 4) (—— updated, ● experimental, -------- analytical).

elements, respectively. The introduced modeling errors in the 'experimental' model for this case are shown in Fig. 11(a). Considering that not all dofs can be measured in practical test, we assume that only the translational dofs are measured with base excitation applied at the fixed node in the *u* direction. The response function data have been selected in a frequency range covering just the first 7 resonances. Since the contaminated and incomplete response function data are used, iterations in the updating procedure are needed in order to average the effect of measurement noise. In each iteration, 30 different frequency points are randomly selected from total 250 frequency points to calculate the coefficient matrix. The iteration results of identified element modeling errors are shown in Fig. 11. From this figure, it can be seen that convergence of the solution is obtained after 20 iterations. The response function curves of the 'experimental', analytical and updated models are shown in Fig. 12. From this figure, it is observed that a good agreement between the regenerated response function data from the updated model and those of the 'experimental' model is reached even in the presence of measurement noise.

5. Concluding remarks

In many practical cases of vibration test, only response functions under base excitation can be obtained due to difficulties and constraints which prevent conventional FRFs from being measured accurately, such as in the cases of civil structural systems and micro-electro-mechanical systems whereas analytical models need to be updated/validated using measured vibration test data. In this paper, a new model updating method has been developed, which employs response function data measured under base excitation directly. Instead of using conventional FRF data or the derived modal data which are not readily available in the case of base excitation test, mathematical formulations of a model updating method using measured response function data under base excitation directly to identify mass and stiffness modeling errors, have been successfully established. Simulated numerical case studies based on a cantilever beam as well as a more realistic practical GARTEUR structure with complete and incomplete coordinates have been carried out to assess the applicability of the proposed model updating method. The success of these cases has proved the feasibility and practicality of the proposed method when applied to the identification of mass and stiffness modeling errors. Very promising results have been obtained even for the case where the measured response function data are contaminated by 3% measurement noise. Although in these cases the structural models assumed are relatively simple, the method has demonstrated its potential to be applied to model updating of more complex practical structures using base excitation test data.

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