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On the use of transmissibility measurements for finite element model updating

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Abstract

Conventional finite element updating techniques make use of measured frequency response functions and modal parameter estimates to update the FE model parameters. This paper presents a new technique to update the FE model from output-only transmissibility measurements assuming the location of the excitation force is known. A transmissibility function is defined as the ratio between two (measured) responses. In general, the poles that are identified from transmissibility measurements do not correspond with the system's poles. It can be proven that the peaks of the transmissibility function coincide with the modal frequencies of the system with the excitation degree of freedom constrained. The effectiveness of the proposed procedure will be illustrated by means of a case study.

1. Introduction

Finite element model updating is an inverse problem used to correct uncertain modelling parameters leading to a better prediction of the dynamic behavior of structures [1]. The procedure of updating a finite element model uses measurement data as reference data. The estimated resonance frequencies and mode shapes are usually considered and the modelling parameters are updated to minimize the differences between experimental and numerically calculated resonance frequencies and mode shapes.

In the process of the structural integrity and operational survivability assessment of mechanical structures, dynamic mathematical models are used for response prediction. These analytical models need to be test-verified. Therefore, it is of vital importance to correlate the finite element model with experimental vibration data and to further fine-tune and update the model. Classically, the modal test is performed in the laboratory. Frequency response functions are measured and used as input for modal analysis software in order to estimate eigenfrequencies, damping ratios, mode shapes and participation factors. Once the analytical model is test verified and the component loads have been estimated, the structure is often subjected to shaker tests to ensure their structural integrity. This issue is complicated by the fact that during qualification tests only response data are measured while the actual loading conditions are unknown.

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Peeters et al. [2] investigate the possibility to integrate both operational modal analysis (OMA) and vibration qualification testing of satellite structures and the results obtained with OMA are compared with the results of a traditional input–output modal analysis. Füllekrug et al. [3] develop the theoretical background for the identification of modal parameters, the generalized and effective masses from multi-axial base-driven tests. The identification of modal parameters from uni-axial translational base excitation tests can be considered as a special case of the general multi-axial base excitation. Although modal identification by base-driven tests is unable to deliver a complete set of modal parameters due to the limited controllability of mode shapes, the investigated test method excellently identifies all those modes which are excited in real operation of a clamped and base-accelerated structure.

In Ref. [4] the identification of free and fixed interface normal modes by base excitation is investigated. Base acceleration can be used to identify the modal parameters of the unbound structure as well. The introduction of a device for measuring the interface forces enables the identification of the structure in unbound conditions without changing the test setup. In Ref. [5], an approach to identify modal parameters from output-only transmissibility measurements is introduced. In general, the poles that are identified from transmissibility measurements do not correspond with the system poles. However, by combining transmissibility measurements under different loading conditions, it is shown that model parameters can be identified. The corresponding theory will be briefly discussed in Section 2.

This paper presents a new technique to update the FE model from output-only transmissibility measurements assuming the location of the excitation force is known. The force is presumed to be unmeasurable, but can be any persistent excitation (swept sine, random noise,...). The experimental results are compared with the results of a finite element model and the finite element model will be updated, based on the information gained from the transmissibility measurements. The output-only results are compared with the results of a traditional input–output modal analysis.

The proposed technique can be described as follows:

- Measurement of at least two accelerations on the structure with one acceleration measured at the excitation point in the excitation direction.
- Calculation of the transmissibility function with the driving point or control acceleration as reference.
- Estimation of the transmissibility poles.
- Calculation of FE resonance frequencies with the excitation degree of freedom constrained.
- Making use of transmissibility poles to update the FE model with fixed excitation degree of freedom.
- Recalculation of FE resonance frequencies with constraint on the excitation degree of freedom removed.

The key added value of this technique is that it can be used for FE model updating when only output measurements (accelerations) can be performed instead of input–output FRF measurements. In contradiction to the OMA approach, there is no need to make assumptions on the nature of the input excitation signal compared with the assumption of white noise excitation in the OMA approach [6]. Another reason for not using cross spectra as is done in OMA, is that the interpretation of the modes is generally ambiguous. When the excitation spectrum is not flat, one cannot guarantee that the identified modes are the free-interface modes. The assumption of a flat excitation spectrum is not always satisfied. This will be the case in the measurements to be investigated. Also from the discussed application, it will be seen that it is not always beneficial to use cross spectra in a subsequent OMA as they look much noisier than transmissibilities. An explanation for this beneficial behavior will be given in Section 2.

To summarize, in this work of research the usage of transmissibility measurements is investigated for FE model updating because:

- The estimation of system poles is possible from output-only transmissibility measurements.
- Transmissibility functions are generally less noisy than crosspower spectra.
- The assumption of a flat excitation spectrum is not required in order to identify the modes correctly as the transmissibility functions are independent from the nature of the excitation signal. Only the excitation location must be known.

- Taking into account the constraint on the force degree of freedom, output-only transmissibilities can be used for FE updating purposes.

2. Transmissibility functions

In this paper attention will be paid to the use of transmissibility functions to update the finite element model. In general, it is not possible to identify modal parameters from transmissibility measurements [7]. Transmissibilities are obtained by taking the ratio of two response spectra:

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)}, \tag{1}$$

with $T_{ij}(\omega)$ the transmissibility function defined as the ratio of response spectra $X_i(\omega)$ and $X_j(\omega)$.

By assuming a single force that is located in, say, the input degree of freedom k, it is readily verified that the transmissibility reduces to

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{H_{ik}(\omega)F_k(\omega)}{H_{jk}(\omega)F_k(\omega)} = \frac{H_{ik}(\omega)}{H_{jk}(\omega)}, \tag{2}$$

with $H_{ik}(\omega)$ and $H_{jk}(\omega)$ the measured frequency response functions between output degree of freedom i, j, respectively, and input degree of freedom k.

From Eq. (2) it can be seen that in the case of a single excitation force the transmissibility function only depends on the location of the force, but not the nature of the force signal nor its amplitude. The influence of the input signal (excitation force) is eliminated. This way, transmissibility functions are generally less noisy compared with measured crosspower spectra which are function of the excitation force.

In Ref. [5] it is shown that by combining transmissibility measurements under different loading conditions it is possible to identify the modal parameters (resonance frequencies, damping ratios and mode shape vectors). In general, the poles that are identified from transmissibility measurements do not correspond with the system's poles. However, by combining transmissibility measurements under different loading conditions, it is shown that model parameters can be identified.

Making use of the modal model between input dof k and output dof i:

$$H_{ik}(\omega) = \sum_{m=1}^{N_m} \frac{\phi_{im} L_{km}}{i\omega - \lambda_m} + \frac{\phi_{im}^* L_{km}^*}{i\omega - \lambda_m^*}. \tag{3}$$

The limit of the transmissibility function (2) for $i\omega$ going to the system's poles, λ_m, converges to

$$\lim_{i\omega \to \lambda_m} T_{ij}^k(\omega) = \frac{\phi_{im} L_{km}}{\phi_{jm} L_{km}} = \frac{\phi_{im}}{\phi_{jm}} \tag{4}$$

and is independent of the (unknown) force at input dof k. Consequently, the subtraction of two transmissibility functions with the same output dofs (i, j) but with different input dofs (k, l) satisfies

$$\lim_{i\omega \to \lambda_m} (T_{ij}^k(\omega) - T_{ij}^l(\omega)) = \frac{\phi_{im}}{\phi_{jm}} - \frac{\phi_{im}}{\phi_{jm}} = 0. \tag{5}$$

To sum up, the system's poles, λ_m, are the zeroes of the rational function $\nabla^2 T_{ij}^{kl}(\omega) \triangleq T_{ij}^k(\omega) - T_{ij}^l(\omega)$ and consequently the poles of its inverse.

$$(\nabla^2)^{-1} T_{ij}^{kl}(\omega) \triangleq \frac{1}{\nabla^2 T_{ij}^{kl}(\omega)} = \frac{1}{T_{ij}^k(\omega) - T_{ij}^l(\omega)}. \tag{6}$$

In general, only a subset of the poles of $(\nabla^2)^{-1} T_{ij}^{kl}(\omega)$ will correspond to the real system's poles.

Ewins and Liu [8] discuss the physical interpretation and application of the transmissibility properties of mdof chain-like mass–spring systems. A theorem dealing with the relationship between FRFs is proposed and a transmissibility function is defined to describe the correlation characteristics between FRFs. Ribeiro and Maia [9,10] propose an approach that aims at obtaining the transmissibility matrix from the FRF matrices.

This transmissibility matrix allows for the determination of the responses at an arbitrary set of the structure's coordinates from another set of coordinates where the responses are known. The number of coordinates in this last set must be, at least, equal to the number of coordinates where applied forces may exist. The FRFs among the various coordinates involved in the process must be known, either from laboratory tests or from numerical or analytical models. Varoto and McConnell [11] discuss motion transmissibility concepts and their application to test environments. The single point transmissibility FRF concept is extended to the case where the test object inputs are given in terms of a set of multiple interface motions that are simultaneously applied to the test object. An expression relating the test object unknown external accelerations to the input interface acceleration is obtained.

3. Theoretical equations

In this section, a brief summary of the used theoretical equations will be described.

The dynamic equilibrium can be expressed as

$$[\mathbf{M}]\{\ddot{x}(t)\} + [\mathbf{C}]\{\dot{x}(t)\} + [\mathbf{K}]\{x(t)\} = \{f(t)\}, \tag{7}$$

where $[\mathbf{M}], [\mathbf{C}], [\mathbf{K}]$ are the mass, damping and stiffness matrices; $\{x(t)\}, \{\dot{x}(t)\}, \{\ddot{x}(t)\}$ are the structural displacements, velocities and accelerations at continuous time t. The vector $\{f(t)\}$ contains the external forces.

In the Laplace domain, Eq. (7) can be written as

$$[Z(s)]\{a(s)\} = \{f(s)\}, \tag{8}$$

where s is the Laplace variable, $\{a(s)\}$ is the Laplace transform of the acceleration vector $\{\ddot{x}(t)\}$ and $[Z(s)]$ is defined as

$$[Z(s)] = [\mathbf{M}] + \frac{1}{s}[\mathbf{C}] + \frac{1}{s^2}[\mathbf{K}]. \tag{9}$$

The FRF matrix $H(s)$ is defined as

$$[H(s)] = [Z(s)]^{-1} = \frac{1}{|Z(s)|}\,\mathrm{adj}(Z(s)), \tag{10}$$

where $|Z(s)|$ denotes the determinant and adj$(Z(s))$ the adjoint of matrix $[Z(s)]$. The roots of the denominator $|Z|$ are the resonances (poles) of the structure and the denominator is common to all elements of the FRF matrix.

In the case of base or shaker excitation, the only external force is applied at the base interface. If it is assumed that the control (driving point) accelerometer is at the same base location, Eq. (8) can be partitioned as

$$\begin{bmatrix} z_{cc} & \lfloor \mathbf{Z_{cm}} \rfloor \\ \{\mathbf{Z_{mc}}\} & [\mathbf{Z_{mm}}] \end{bmatrix} \begin{bmatrix} a_c \\ \{\mathbf{a_m}\} \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}, \tag{11}$$

where a_c is the control or driving point acceleration and $\{\mathbf{a_m}\}$ is the vector of remaining accelerations that have been measured.

In case of an unknown input force f, Eq. (11) remains valid but the lower part of the matrix equation can be rewritten as

$$\{\mathbf{Z_{mc}}\}a_c + [\mathbf{Z_{mm}}]\{\mathbf{a_m}\} = 0 \tag{12}$$

or

$$\{\mathbf{a_m}\} = -[\mathbf{Z_{mm}}]^{-1}\{\mathbf{Z_{mc}}\}a_c, \tag{13}$$

$$\{\mathbf{a_m}\} = \{\mathbf{T_{mc}}\}a_c, \tag{14}$$

with

$$\{\mathbf{T_{mc}}\} = -[\mathbf{Z_{mm}}]^{-1}\{\mathbf{Z_{mc}}\} = -\frac{1}{|\mathbf{Z_{mm}}|}\,\mathrm{adj}(\mathbf{Z_{mm}})\{\mathbf{Z_{mc}}\}, \tag{15}$$

where the transmissibility function $\{\mathbf{T_{mc}}\}$ is defined as the ratio of accelerations $\{\mathbf{a_m}\}$ over a_c. The roots of the denominator $|\mathbf{Z_{mm}}|$ are the resonances of the transmissibility functions. The physical interpretation of the resonance modes of the transmissibilities follows from the input–output relations of the structure that is fixed at the interface or excitation point. The dynamic equilibrium of such a system is obtained by eliminating the row and the column involving the driving point from the equilibrium equations of the free structure.

By consequence, the FRF matrix of the fixed structure $[\mathbf{H}]_{\text{fixed}}$ equals:

$$[\mathbf{H}]_{\text{fixed}} = [\mathbf{Z}]_{\text{mm}}^{-1} = \frac{1}{|\mathbf{Z_{mm}}|}\,\text{adj}(\mathbf{Z_{mm}}). \tag{16}$$

By comparing Eq. (16) with Eq. (15), it is observed that the same poles are found in both expressions as the roots of the denominator $|\mathbf{Z_{mm}}|$.

4. Updating the finite element model of a mobile substation support structure

4.1. Introduction on the procedure

The procedure to update a finite element model by making use of transmissibility functions can be described as follows:

- Measurement of at least two accelerations on the structure with one acceleration measured at the excitation point in the excitation direction.
- Calculation of the transmissibility function with the driving point or control acceleration as reference.
- Estimation of the transmissibility poles.
- Calculation of FE resonance frequencies with the excitation degree of freedom constrained.
- Making use of transmissibility poles to update the FE model with fixed excitation degree of freedom.
- Recalculation of FE resonance frequencies with constraint on the excitation degree of freedom removed.

In the following sections, each step in this list will be examined and discussed more in detail. To follow a clear line of thought, from initial measurement data set and initial FE model to updated FE model, the structure of this section is organized as follows:

- Description of measurement setup and modal results from measured FRFs.
- Modal parameters from transmissibility functions.
- Comparison and updating of constrained FE model.
- Recalculation to unconstrained FE modal parameters and final validation with reference modal results.

4.2. Description of measurement setup and modal results from measured FRFs

The finite element model to be updated will represent a part of a mobile substation support structure. Mobile substations (Fig. 1) can be defined as completely equipped electrical substations mounted on a trailer. Improper dimensioning of this structure can result in an important degree of mechanical failure during transport [7]. Lightly damped structures that have one or more natural modes of oscillation within the frequency band of transport excitation can experience considerable amplification of operational deflections. Thus, mobile substation components whose natural frequencies lie in the normal frequency range of transport motion are particularly vulnerable to damage and fatigue. Therefore, one is particularly interested in analyzing the natural frequencies, damping ratios and level of accelerations of those components.

The support beam (Fig. 2) was excited with an electromagnetic shaker at a height of 28 cm along the vertical z-axis with excitation direction in the horizontal x direction. A total number of 9 measurement points were used, spread across the structure, and accelerations were measured in three directions in order to identify the modes of the structure. A driving point accelerometer was used as a reference and will play an important role in the described transmissibility updating approach. Frequency response function measurements are

Fig. 1. A mobile substation.

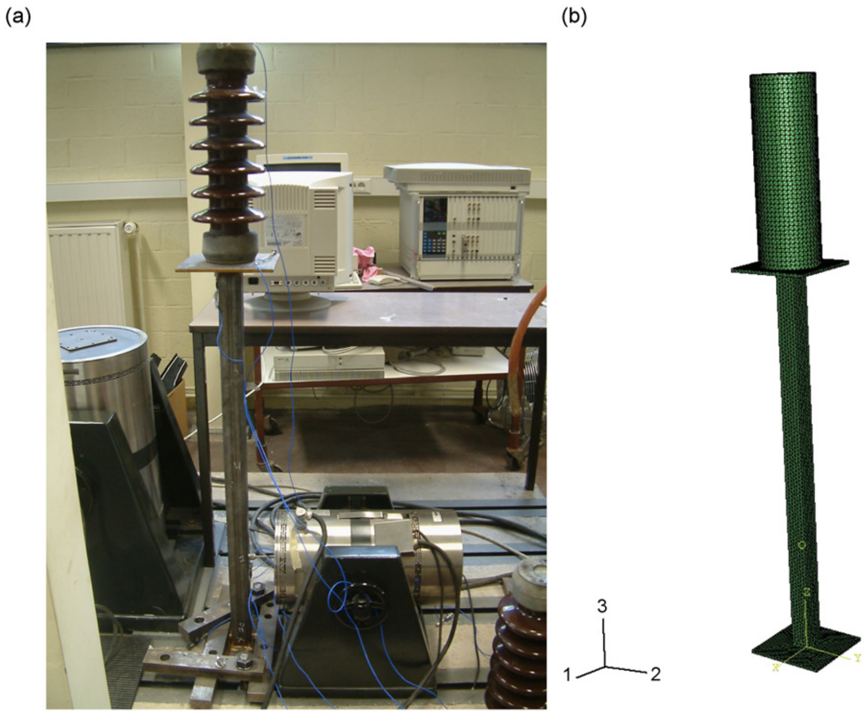

Fig. 2. Measurement setup (a) and finite element model (b) of the support structure.

performed in clamped boundary condition with the structure clamped at its lower end to the ground, with a frequency resolution of 0.125 Hz using a burst random excitation signal to avoid leakage errors. The modal parameters are estimated, using the PolyMAX frequency-domain estimator [12]. To make the unconstrained and constrained behavior more similar, the current setup treats a clamped structure, driven by a regular shaker. The input for the FRFs is the force at the excitation point.

The created FE model (Fig. 2) consists out of 4 parts: the steel beam, 2 steel plates rigidly connected to the beam and the isolator with the material properties of porcelain. Initially, the porcelain isolator was rigidly connected to the upper plate. During the FE updating process, the rigid connection was replaced by 4 bolt

Table 1
Comparison between estimated test and calculated FE modes

Test mode (Hz)	FE mode (Hz)	Description
8.5	9.7	Clamped rigid beam mode x direction
8.5	9.7	Clamped rigid beam mode y direction
88.5	101.1	First bending x direction
88.5	101.1	First bending y direction
126.0	108.1	Torsion mode
188.0	377.4	Longitudinal z direction mode
266.5	367.8	Beam-isolator mode x direction (anti-phase)
266.5	367.8	Beam-isolator mode y direction (anti-phase)
588.6	862.2	Second bending x direction
588.6	862.2	Second bending y direction

connections with the bolt connection stiffness as updating parameter (Fig. 7). These springs influence the stiffness of the coupling between the porcelain isolator and the steel beam. Mesh convergence was checked, using 18 734 linear tetrahedral elements.

As a result of the experimental model analysis, measuring frequency response functions in 9 structure points using shaker excitation and the PolyMAX estimation of the modal parameters, one finds the following comparison between estimated test and calculated FE modes in Table 1. The finite element model is not yet updated.

4.3. Modal parameters from transmissibility functions

Once the measurements are performed, one can calculate the transmissibility functions between two measurement points. According to the theory described in Section 3, one has to take the driving point acceleration as a reference for the calculation of the transmissibilities. A measurement point (‘beam:8:+y’) along the support beam is chosen with horizontal measurement y direction, perpendicular to the horizontal excitation direction (x direction). Calculation of the transmissibility function between the driving point accelerometer and measurement point 8 in y direction leads to the following transmissibility, shown in Fig. 3. Estimation of the transmissibility poles [13] will lead to the following results, listed in Table 2.

Based on the equations, described in Section 3, one can now use these measurement results to perform an updating of the FE model with fixed excitation degree of freedom. Once this constrained model is updated, one can recalculate and check the FE resonance frequencies of the FE model with the excitation degree of freedom kept free.

The transmissibility functions ‘T:8:+x’(dotted line) and ‘T:8:+y’(full line) between response point ‘beam:8’ in x and y direction and driving point acceleration are calculated and shown in Fig. 4. The vertical lines represent the frequency lines where stable poles are estimated. Comparing transmissibility functions in different directions of the same measurement point, one can draw the following conclusions.

When looking at the important modes, one will notice amplitude differences in transmissibilities at the corresponding peaks. Poles that represent modes with a modal displacement mainly in one direction are more clear (higher amplitude) in the transmissibility function of the corresponding direction. This can be clearly seen for the poles at frequencies 88.5 Hz (bending mode in y direction) and 91 Hz (bending mode in x direction). Due to the fact that these bending modes are very close in frequency, both corresponding peaks in the transmissibility function will “drown” in the peak of the mode in the dominating direction. Although only one peak in the transmissibility function is clearly visible, the used estimator will detect both closely spaced modes. This is also the case for the bending modes at 9.7 and 11.2, 365.2 and 393.9 Hz. For the torsional deflection modes (pole at 127.7 Hz), the transmissibility poles are coinciding in both directions. There is still an amplitude difference that is explained by the fact that the structure is excited in the x direction. Thus, the

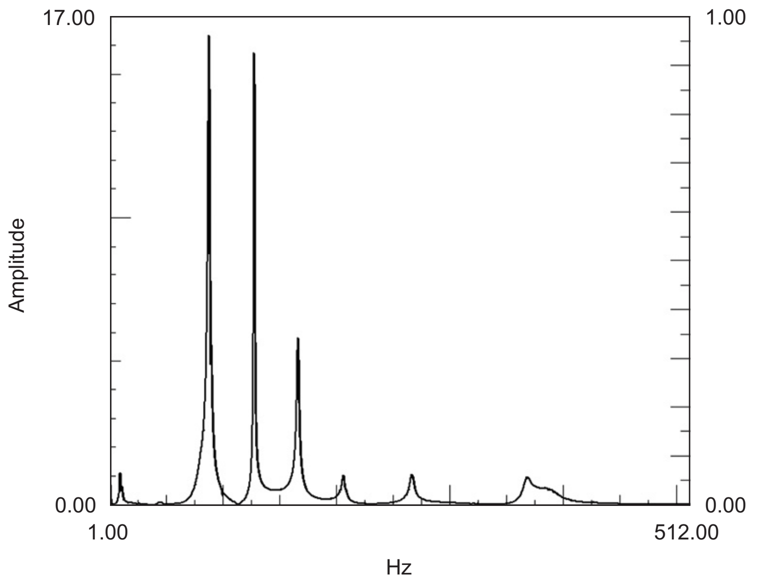

Fig. 3. Transmissibility function ('T:8:+y') between response acceleration ('beam:8:+y') and driving point acceleration.

Table 2
Estimated poles of the support structure transmissibility function ('T:8:+y')

Pole	Transmissibility frequency (Hz)
1	9.7
2	11.2
3	88.5
4	91.0
5	127.7
6	167.4
7	206.7
8	267.0
9	365.2
10	393.9

structure will have higher displacement amplitudes in x direction and as a consequence also resulting in higher transmissibility function amplitudes corresponding with the excitation direction.

4.4. Comparison and updating of constrained FE model

In order to update the finite element model of the support structure using output-only transmissibility functions, it is necessary to constrain (clamp) the excitation degree of freedom (x direction) in the corresponding shaker excitation point. In the finite element model, the FE keypoint or node that geometrically corresponds best with the shaker excitation point needs to be constrained in the x direction in accordance with the excitation direction (Fig. 5).

This will lead to an altered set of finite element modes with respect to the FE modes without the excitation degree of freedom clamped. These modes will now be used in the finite element updating process and their respective resonance frequencies will be compared with the frequencies of the transmissibility poles found in Table 2. The frequencies will be compared up to 500 Hz and the results are listed in Table 3. The updated parameters are the cross sections and thus the stiffness properties of the 4 bolt connections that are used to

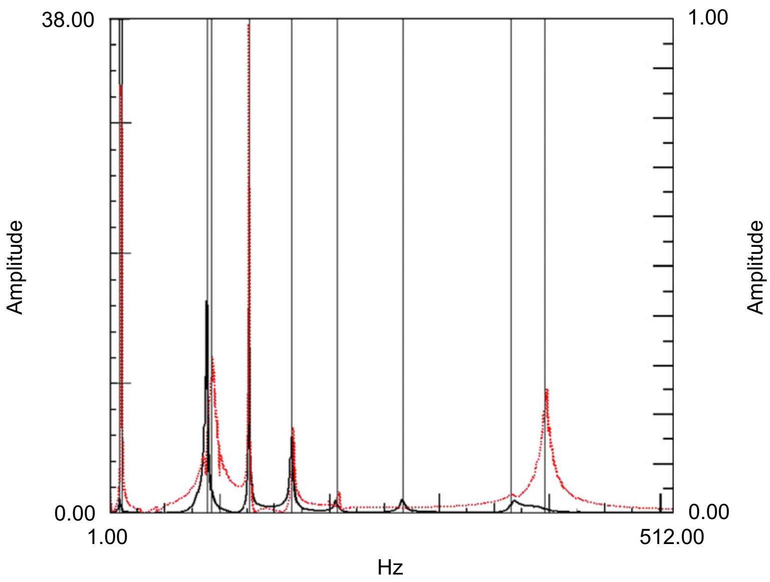

Fig. 4. Transmissibility functions 'T:8:+x' (dotted line) and 'T:8:+y' (full line) between response point 'beam:8' in x and y direction and driving point acceleration.

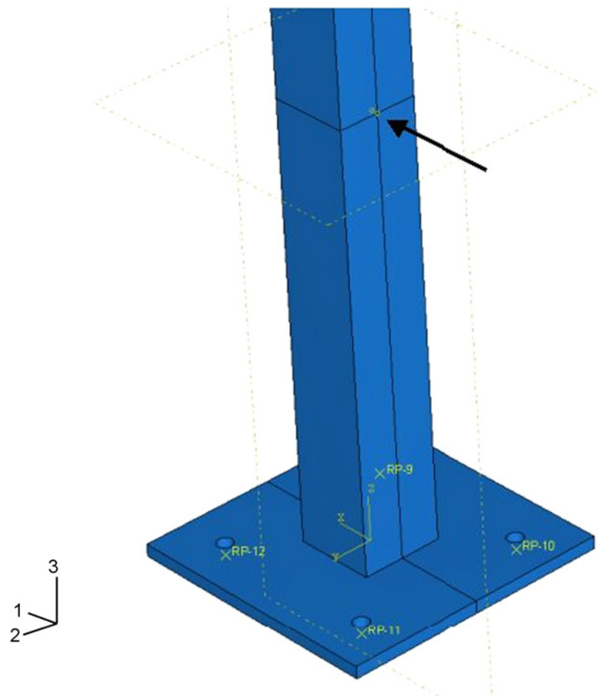

Fig. 5. FE model with constrained degree of freedom (x direction) in shaker excitation point.

connect the rigid isolator mass with the steel beam part of the structure. Overall, a good FE correlation is achieved between the FE resonance frequencies and the transmissibility poles.

In Fig. 6, the most important measured mode shapes with constrained excitation degree of freedom are presented. This is noticeable in the second measurement point from the bottom of the beam which is constrained in x direction.

Table 3
Updating results for the FE model with constrained force degree of freedom

Transmissibility pole (Hz)	Initial FE mode (Hz)	Updated FE mode (Hz)
9.7	9.8	8.3
11.2	12.7	11.2
88.5	101.1	88.5
91.0	106.9	93.0
127.7	108.1	127.1
167.4	143.0	180.2
206.7	336.4	200.3
267.0	370.9	271.5
365.2	377.4	340.5
393.9	556.5	415.0

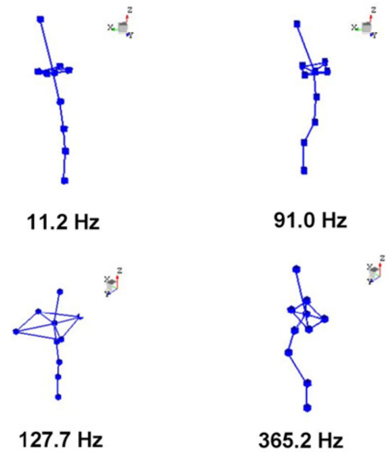

Fig. 6. Most important measured mode shapes with constrained excitation degree of freedom.

4.5. Final validation with reference modal results

After the finite element updating with the estimated transmissibility poles as a reference, one can now compare the FE resonance frequencies with the measured and estimated test frequencies. In order to do this, the fixed excitation force degree of freedom is eliminated and the FE resonance frequencies are recalculated. The finite element model is already updated on the basis of the transmissibility functions. The results are presented in Table 4. Overall, a good correlation is achieved between the estimated test resonance frequencies and the calculated FE resonance frequencies.

5. Automated updating using two updating parameters

5.1. Selection of updating parameters

The original FE model is updated on the level of the bolt connections between the isolator mass and the steel beam. The physical bolt connections are remodelled and exist of 4 spring connections with spring stiffness values as design variables. Due to symmetry reasons, only one stiffness value k_1 is used as updating parameter for all four bolts connecting the steel beam with the isolator mass. Also only one stiffness value k_2 is used as updating parameter for the connections of the lower steel plate with the ground. In both cases, using simple rigid connection for both leads to an overestimation of the actual stiffness resulting in higher FE resonance

Table 4
Mode comparison for the FE model with unconstrained force degree of freedom

Test mode (Hz)	Initial FE mode (Hz)	Updated FE mode (Hz)
8.5	9.7	8.3
8.5	9.7	8.3
88.5	101.1	88.5
88.5	101.1	88.5
126.0	108.1	127.1
188.0	377.4	180.2
266.5	367.8	271.5
266.5	367.8	271.5
588.6	862.2	578.2
588.6	862.2	578.2

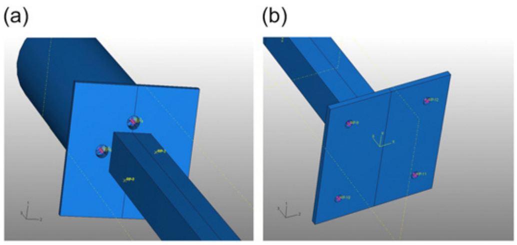

Fig. 7. Spring stiffness values k_1 (a) and k_2 (b) as FE updating parameters.

frequencies (Table 1). The outer endings of the springs are rigidly coupled in a uniform way to the holes of the respective rectangular plates (Fig. 7).

5.2. Detailed procedure and objective function

In this section, the updating method presented in Ref. [14] will be briefly explained in order to update the finite element frequencies based on two updating parameters, using the transmissibility poles as reference data. The FE model is exported as an ascii model file where one can write the model file as a function of one or more parameters. The different spring stiffness values of the springs connecting the steel beam to the isolator mass and the ground will be considered as two updating parameters. A nonlinear least squares problem is solved in Matlab in order to minimize an objective function consisting of the sum of the squares of the frequency differences between estimated transmissibility poles and calculated FE resonance frequencies:

$$\min_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \sum_i r_i^2(\boldsymbol{\theta}), \tag{17}$$

with

$$r_i(\boldsymbol{\theta}) = f_i^{\mathtt{trans}} - f_i^{\mathrm{FE}}(\boldsymbol{\theta}), \tag{18}$$

where $\boldsymbol{\theta}$ is a vector, $r(\boldsymbol{\theta})$ is a function that returns a vector value, $\ell(\boldsymbol{\theta})$ is the objective function to be minimized with $f_i^{\mathtt{trans}}$ the ith transmissibility pole and $f_i^{\mathrm{FE}}(\boldsymbol{\theta})$ the ith calculated FE frequency.

If the uncertainties on the estimated $f_i^{\mathtt{trans}}$ are known, one can use the variances $\sigma_{f_i}^{\mathtt{trans}}$ as weighting factors:

$$r_i(\boldsymbol{\theta}) = \frac{f_i^{\mathtt{trans}} - f_i^{\mathrm{FE}}(\boldsymbol{\theta})}{\sigma_{f_i}^{\mathtt{trans}}}. \tag{19}$$

If all variances are equal or have the same order of magnitude, it does not make a difference when using individual weighting factors.

For solving the nonlinear least squares problem, one needs to specify an upper and lower boundary value for the plate length [15]. The least squares problem is solved with the used boundary values as starting values. A regression algorithm with increasing regression order is used to calculate the regression coefficients of the interpolation polynomial in order to fit an interpolation curve through the already known (numerically calculated) modal frequencies. For the fitting of two updating parameters, one needs three starting values in order to start from a linear regression fit for the updating parameter. In a first step, the objective function will be evaluated based on the initially calculated parameter value eigenfrequencies and the measured transmissibility poles. By calculating a higher order fit through the already known values of the updating parameters and the calculated FE frequencies, respectively, the FE model is replaced by an interpolation polynomial and needs to be calculated only a few times. The updating algorithm will stop when the difference in FE eigenfrequency after a parameter update becomes smaller than a user-defined threshold.

To summarize: the updating algorithm consists of three steps:

(1) Calculate interpolation polynomial between calculated resonance frequencies and their respective updating parameter values (k_1, k_2).
(2) Minimize objective function based on measured and interpolated resonance frequencies.
(3) Solve nonlinear least squares problem to estimate new parameter values (k_1, k_2), based on the defined objective function, measured (transmissibilities) and interpolated eigenfrequencies and two parameter starting value combinations.

More information and details on the updating algorithm can be found in Ref. [14].

5.3. Physical explanation of changes

The original FE model is updated on the level of the bolt connections between the isolator mass and the steel beam. The physical bolt connections are remodelled and consist of 4 spring connections with spring stiffness values as design variables. In the initial FE model, a pure rigid connection was used to connect the steel beam with isolator mass. This rigid connection was made over the whole contact surface between isolator mass and steel beam. In reality steel bolts are used, that are of course not completely rigid but contain a certain stiffness. Defining four springs on these four locations with a certain stiffness value as updating parameter will yield a more accurate modelled connection. Furthermore, the rigid (clamped) connection of the lower steel plate with the ground is replaced by 4 spring connections with updated spring stiffness values. As a consequence of these modifications that make the FE model a more accurate representation of reality, the final updating results for the eigenfrequencies will improve with respect to the values in the original FE model (Fig. 8).

5.4. Updating using eigenfrequencies from FRF data

Together with the measured accelerations, also frequency response functions were measured that can be used to compare updating results. Instead of transmissibility functions, FRFs are now used for modal parameter estimation and the estimated resonance frequencies can now be used to perform a new model updating on the FE model. These new updating results are presented in Table 5. The updating results are compared on the level of resulting sum of squared errors between the test and FE frequencies. The transmissibility updating gives slightly better results, although both updating models have the same order of magnitude on the level of squared frequency differences. Table 6 lists the updated spring parameter values for the FE updating based on transmissibilities and the FE updating based on the estimated FRF poles. These stiffness values are in accordance with the FE bolt stiffness calculations found in the work of McCarthy et al. [16].

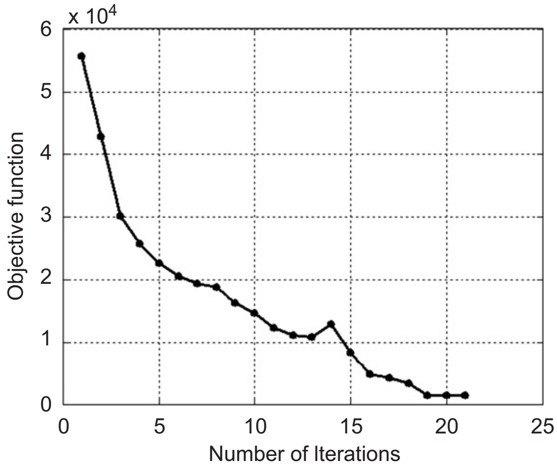

Fig. 8. Convergence of the objective function during the optimization routine.

Table 5
Comparison of updating results with unconstrained force degree of freedom

Test mode (Hz)	FE transmissibility updating (Hz)	FE FRF updating (Hz)
8.5	8.3	8.4
8.5	8.3	8.4
88.5	88.5	88.7
88.5	88.5	88.7
126.0	127.1	115.4
188.0	180.2	190.5
266.5	271.5	274.5
266.5	271.5	274.2
588.6	578.2	598.5
588.6	578.2	598.5
Sum of squared errors	3.28E + 02	4.36E + 02

Table 6
Comparison of optimized spring parameter values k_1 and k_2

Optimized spring parameter	FE transmissibility updating (N/m)	FE FRF updating (N/m)
k_1	1.47E + 08	1.50E + 08
k_2	1.75E + 07	1.75E + 07

6. Comparison with operational modal analysis

Classical modal analysis methods require FRF or impulse response data whereas OMA methods deal with measured cross spectra [6]. In case of an environmental test, also the use of transmissibilities as input data for parameter estimation methods lead to sensible results. Only when using FRFs or transmissibilities, it is clear which modes will be identified: the modes of the free structure in case of FRFs and the modes of the fixed interface structure in case of transmissibilities. If cross spectra are used, some information about the input excitation is needed in order to interpret the modes correctly. If the excitation force has a flat spectrum, it can

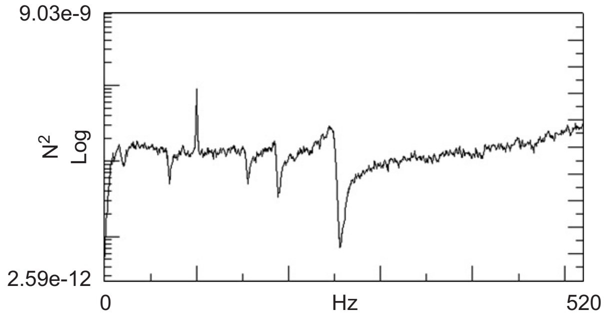

Fig. 9. Autopower spectrum of the shaker excitation force.

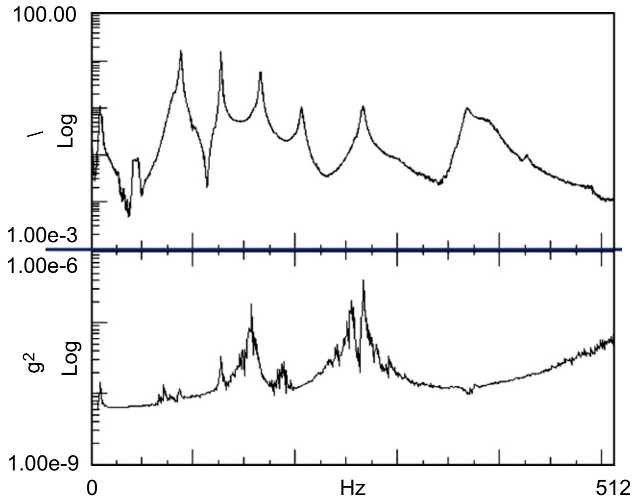

Fig. 10. Comparison of measured transmissibility function (upper) with crosspower spectrum (lower) between Beam:8:+Y and Force:1:+X.

be seen that the cross spectra resonances coincide with the resonances of the FRFs [2]. This is the not completely the case as the excitation signal autopower spectrum contains some peaks and dips, caused by the structural resonances and stinger resonances (Fig. 9). Hence, one of the main advantages of working with transmissibilities again surfaces.

During the FRF measurements, also the crosspower spectra were measured and can be used for modal parameter estimation. Because of the constrained degree of freedom, the poles from the OMA cannot be directly compared with the transmissibility poles. As a consequence, the OMA poles will be compared with the resonance frequencies of the updated FE model (Table 4) and also with the estimated poles from the measured FRFs (Table 1). By visually comparing the cross spectra with transmissibilities (Fig. 10), it is seen that it is not always beneficial to use cross spectra in an operational modal analysis as they look much ‘‘noisier’’ than transmissibilities. Another reason for not using cross spectra is that the interpretation of the modes is generally ambiguous. Only if the excitation spectrum is flat, the free interface modes are found. However, in environmental conditions this assumption is not always the case.

Based on the crosspower spectra, a modal analysis is performed and the OMA modes are estimated. A comparison between the FRF modes and the OMA modes based on crosspower spectra is presented in Table 7. Although there are some differences in frequency, probably caused by fluctuations due to noise on the input data, the estimated frequencies are comparable. Using one of both frequency sets will yield comparable FE frequencies after updating the model by using this data.

Table 7
Comparison of estimated EMA modes and OMA modes

Modes EMA (Hz)	Modes OMA (Hz)
8.5	8.5
8.5	8.5
88.5	88.3
88.5	88.3
126.0	125.5
188.0	188.6
266.5	263.8
266.5	263.8
588.6	575.2
588.6	575.2

7. Conclusions

This paper presents a new technique to update a finite element model by making use of output-only transmissibility measurements. It is not necessary that the force is measurable and it can be any persistent excitation (swept sine, random noise,...). It is shown that the resonances (poles) of the transmissibilities determine the structural modes of the structure when it is constrained at the excitation point. The output-only results are compared with the results of a traditional input–output modal analysis. The effectiveness of the suggested procedure is illustrated by means of a case study. It is shown that when updating the finite element model, an good correlation is achieved between the FE resonance frequencies and the transmissibility poles. These findings may lead to improved model identification if one is unsure about the nature of the excitation signal. When using cross spectra, the interpretation of the modes is generally ambiguous. Only if the excitation spectrum is flat the free interface modes are found. However, in environmental conditions this assumption is not always the case. Another reason is the fact that in general the quality of the transmissibility data is less noisy compared to measured crosspower spectra. A comparison between the classical EMA, OMA and transmissibility OMA is made on a structure and comparable updating results are generated concerning both techniques. Another potential application consists in the fact that an operational updating can be performed on a substructure as part of a system, based on transmissibility measurements only.

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