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Short Communication

Identification of nonlinear viscous damping and Coulomb friction from the free response data

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Abstract

In this paper, a single degree of freedom system with nonlinear viscous damping and Coulomb friction is considered. A procedure for simultaneous identification of the nonlinear damping and Coulomb friction from the free response signals of the system is described. The procedure is based on the moving rectangle window method. The free vibration differential equation of the single degree of freedom system is established, and from its free response curve, the nonlinear damping characteristics and the Coulomb friction can be obtained by moving the rectangle window with fixed length along the time axis. In simulations, different kinds of the nonlinear damping models contained in the single degree of freedom system are respectively investigated, and the Coulomb friction value keeps constant. The validity and accuracy of the proposed method are illustrated by the good simulation results. In addition, the computing accuracy of the nonlinear viscous damping is higher than that of the Coulomb friction.

1. Introduction

Nonlinear behavior is present in systems ranging from piping network to mechanical systems. On the one hand, with the development of the damping technique, material with high damping has been widely used in the vibration and noise reduction of systems. Generally speaking, the viscous damping of the mechanical system is nonlinear. And the nonlinear characteristics of damping are related with many factors, such as temperature, frequency, strain, and strain time rate [1]. On the other hand, one of the common non-linearities in the system is friction [2]. And it is a main factor of power dissipation in these systems. Coulomb friction is one of the common friction models. And it exists in most system where the relative motion exists between bodies. In order to evaluate the nonlinear damping performance of these systems, it is necessary to have an accurate and reliable method for the identification of nonlinear damping mechanisms.

Problems of identification of system parameters are very important in engineering. There exist two types of identification methods, i.e. online and offline. Online identification implies permanent monitoring of the system performance. It can detect undesirable failures timely and apply control force. Offline identification implies the identification of system parameters from the measured response. And it does not apply control

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force timely. Many identification methods have been put forward. Smith [3] put forward two methods for identifying damping from transient response of a single degree-of-freedom system having either nonlinear Coulomb or quadratic damping based on a periodic Fourier series decomposition and a Hilbert transform technique. And Smith classified nonlinear damping in a transient reponse based on the envelope signal, as well as quantified the nonlinear damping level. Frederick and Inderjit [4] presented the moving block technique and the sparse time domain method to estimate equivalent damping characteristics from transient response data involving nonlinear damping. Iourtchenko [5] put forward in-service identification of damping characteristic from measured stationary response based on the stochastic averaging method. And the explicit analytical solution is obtained for the integral equation which relates the desired damping characteristics. Trueba [6] assumed that the general nonlinear damping terms are proportional to the power of velocity in the oscillators. The effect of nonlinear dissipation is equivalent to a linearly damped nonlinear oscillator with a modified damping coefficient by using the idea of Melnikov equivalence. Feeny [7] put forward a decrement method for the estimation of Coulomb and viscous friction, but the viscous damping is considered only as a constant.

According to the characteristic of nonlinear damping, the rectangle window is introduced in this paper. The paper presents a simultaneous identification of nonlinear viscous damping and Coulomb friction based on the moving rectangle window technique.

The paper is organized as follows. In Section 2, the differential equation of a system with nonlinear viscous damping and Coulomb friction is established, and the analytic solution of the equation is obtained through the differential equation theory. In Section 3, the identification theory and algorithm of the nonlinear viscous damping and Coulomb friction are described. In Section 4, the simulation examples are presented to demonstrate the validity of the proposed method. Conclusions are drawn in Section 5.

2. Description of a system with nonlinear viscous damping and Coulomb friction

A single degree of freedom system with nonlinear viscous damping and Coulomb friction is considered. The differential equation of motion is

$$m\ddot{x} + c(x)\dot{x} + kx + f_c\,\mathrm{sgn}(\dot{x}) = 0, \tag{1}$$

where, m, $c(x)$ and k are the mass, nonlinear viscous damping and stiffness respectively. Damping parameter $c(x)$ is seemed as a function of vibration amplitude x, and f_c is the Coulomb friction level. The symbolic function $\mathrm{sgn}(\dot{x})$ is defined as

$$\mathrm{sgn}(\dot{x}) = \begin{cases} -1, & \dot{x} < 0, \\ 0, & \dot{x} = 0, \\ 1, & \dot{x} > 0. \end{cases} \tag{2}$$

Let x_k be a locus of equilibrium, and then $f_c = kx_k$. So Eq. (1) can be rewritten as

$$m\ddot{x} + c(x)\dot{x} + kx + kx_k = 0, \quad \dot{x} > 0 \tag{3}$$

and

$$m\ddot{x} + c(x)\dot{x} + kx - kx_k = 0, \quad \dot{x} < 0. \tag{4}$$

Eqs. (3) and (4) can be further written into

$$\ddot{x} + 2\xi(x)\omega_n\dot{x} + \omega_n^2 x + \omega_n^2 x_k = 0, \quad \dot{x} > 0, \tag{5}$$

$$\ddot{x} + 2\xi(x)\omega_n\dot{x} + \omega_n^2 x - \omega_n^2 x_k = 0, \quad \dot{x} < 0, \tag{6}$$

where, $\omega_n^2 = k/m$, $2\xi\omega_n = c/m$.

3. Identification of nonlinear viscous damping and Coulomb friction

3.1. Constant viscous damping computation

The free response of Eq. (1) is obtained by the Runge–Kutta method as shown in Fig. 1. The rectangle window is adopted to extract several waves of the free response curve in Fig. 1. And assume that the damping in the extracted waves is constant. Now, the waves in the rectangle window satisfy the following equations:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x + \omega_n^2 x_k = 0, \quad \dot{x} > 0, \tag{7}$$

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x - \omega_n^2 x_k = 0, \quad \dot{x} < 0. \tag{8}$$

It is assumed that the initial displacement of the free vibration at time t_0 is $x(t_0) = X_0$, and the initial speed is $\dot{x}(t_0) = 0$. At the first speed interval from $\dot{x} = 0$ to <0 , i.e. at the first time interval from t_0 to $t_1 = t_0 + \pi/\omega_d$, the analytic solution of Eq. (8) is

$$x(t) = (X_0 - x_k)e^{-\xi\omega_n(t-t_0)}(\cos(\omega_d(t - t_0) + \beta \sin \omega_d(t - t_0)) + x_k, \tag{9}$$

where, $\omega_d = \omega_n\sqrt{1 - \xi^2}$ and $\beta = \xi/\sqrt{1 - \xi^2}$. At time t_1, the amplitude is $X_1 = X(t_1) = -e^{-\beta\pi}X_0 + (e^{-\beta\pi} + 1)x_k$. At the second speed interval from $\dot{x} = 0$ to >0, i.e. at the second time interval from t_1 to $t_2 = t_1 + \pi/\omega_d$, the analytic solution of Eq. (7) is

$$x(t) = (X_1 + x_k)e^{-\xi\omega_n(t-t_1)}(\cos(\omega_d(t - t_1) + \beta \sin \omega_d(t - t_1)) - x_k. \tag{10}$$

At time t_2, the free response amplitude is $X_2 = X(t_2) = -e^{-\beta\pi}X_1 + (e^{-\beta\pi} + 1)x_k$. Then a series of X_i ($i = 1, 2, \ldots, n$) are obtained until the vibration stops. According to the data X_i ($i = 1, 2, \ldots, n$), the recursive formula of the successive wave crests and wave hollows is obtained

$$X_i = -e^{-\beta\pi}X_{i-1} + (-1)^{i-1}(e^{-\beta\pi} + 1)x_k. \tag{11}$$

According to paper [7], the parameter β can be computed by

$$\hat{\beta} = -\frac{1}{\pi}\log\frac{X_{i-1} - X_{i+1}}{X_i - X_{i-2}}. \tag{12}$$

Because that $\beta = \xi/\sqrt{1 - \xi^2}$, the damping ratio ξ is got as

$$\hat{\xi} = \hat{\beta}/\sqrt{1 + \hat{\beta}^2}. \tag{13}$$

So the damping ratio ξ are achieved by Eqs. (12) and (13).

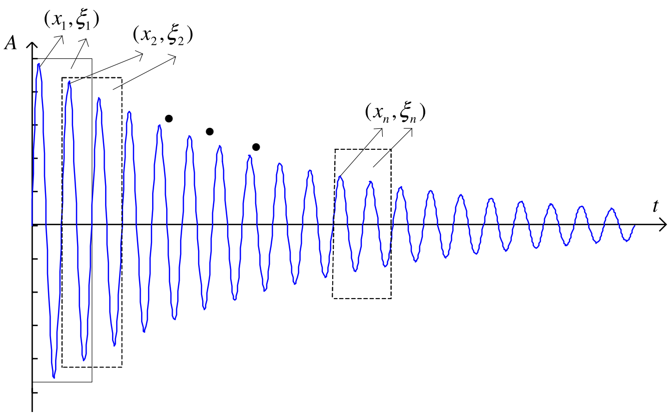

Fig. 1. Free response signal of Eq. (1).

According to Eq. (11), x_k is expressed as

$$\hat{x}_k = \frac{X_i + e^{-\hat{\beta}\pi} X_{i-1}}{(-1)^{i-1}(e^{-\hat{\beta}\pi} + 1)}. \tag{14}$$

Then the Coulomb friction can be got by the following equation:

$$\hat{f}_c = k\hat{x}_k. \tag{15}$$

Seeing from Eqs. (12) and (13), the damping ratio ξ can be obtained from at least four continuous extreme displacements X_i, i.e. two peaks and two valleys. The length of the rectangle window is equal to two periods of the response curve, as shown in Fig. 1.

3.2. Nonlinear damping computation

The basic idea for identifying the nonlinear viscous damping is to move the rectangle window along the time axis, as shown in Fig. 1. First, according to the waves in the first rectangle window. The damping ratio ξ_1 of the extracted waves is calculated according to Eqs. (12) and (13). At the same time, the amplitude value of the former peak x_1 can be got. Then ascribing the value of the damping ratio ξ_1 to the amplitude x_1. So a set of data of amplitude and damping ratio (x_1, ξ_1) can be obtained. Next, moving the rectangle window for one period along the time axis, the differential equation of the waves in the second rectangle window is established. Similarly, another set of data of amplitude and damping ratio (x_2, ξ_2) can be also obtained. Repeatedly, a series of data of amplitude and damping ratio (x_i, ξ_i) can be got by moving the rectangle window along the time axis continuously. Therefore, the nonlinear relationship curve of damping ratio versus amplitude is achieved.

3.3. Nonlinear damping models

According to paper [8], there are several nonlinear relationships between damping ratio and vibration amplitude: (1) quadratic type $\xi(x) = ax^2 + bx + c$; (2) inverse hyperbolic type $1/\xi(x) = a/x + b$; (3) exponent type $\xi(x) = ae^{bx}$; (4) inverse exponent type $\xi(x) = ae^{-b/x}$, where, a, b and c are constant coefficients, ξ is the damping ratio, and x is the vibration amplitude.

4. Simulations

In this section, a SDOF system with nonlinear viscous damping and Coulomb friction is adopted to verify the validity of the proposed identification method. The physical parameters in Eq. (1) are: $m = 1$ kg, $k = 10$ N/m, and $f_c = 0.01$ N. Six kinds of nonlinear damping types are verified, respectively.

Type 1: Constant damping is independent of amplitude: $c(x) = d$, $\xi(x) = c(x)/2m\omega_n = 0.047434$, where $d = 0.3$.
Type 2: Damping ratio is a linear function of amplitude: $\xi(x) = bx + d$, where $b = 0.1$ and $d = 0.3$.
Type 3: Damping ratio is a quadratic function of amplitude: $\xi(x) = ax^2 + bx + d$, where $a = 0.2$, $b = 0.1$, and $d = 0.3$.
Type 4: Damping ratio is an inverse hyperbolic function of amplitude: $1/\xi(x) = a/x + b$, where $a = 0.6$ and $b = 1$.
Type 5: Damping ratio is an exponent function of amplitude: $\xi(x) = ae^{bx}$, where $a = 0.2$ and $b = 1$.
Type 6: Damping ratio is inverse exponent curve function of amplitude: $\xi(x) = ae^{-b/x}$, where $a = 0.8$ and $b = 0.2$.

The simulation results are shown in Figs. 2–7, respectively. The relationship curves in the figures are the nonlinear relationship descriptions of damping ratio ξ and amplitude x.

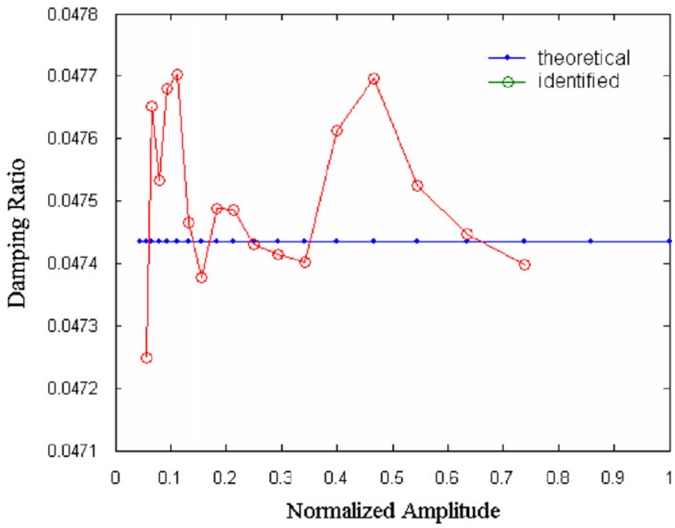

Fig. 2. Relationship curve of constant damping versus amplitude.

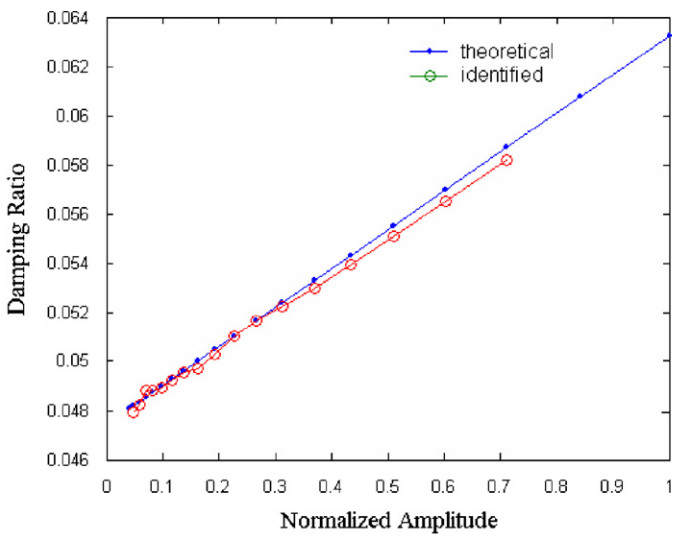

Fig. 3. Linear relationship curve of damping ratio versus amplitude.

Here, the maximum relative error (MRE) is given to check the estimation accuracy of ξ and f_c. The maximum relative error is respectively defined as

$$\xi_{R\,\text{error}} = \max\left(\left|\xi_{\text{theory},i} - \xi_{\text{obtain},i}\right|/\xi_{\text{theory},i}\right) \times 100\% \quad (i = 1, 2, \ldots, n),$$
$$f_{c\,R\,\text{error}} = \max\left(\left|f_{c\,\text{theory},i} - f_{c\,\text{obtain},i}\right|/f_{c\,\text{theory},i}\right) \times 100\% \quad (i = 1, 2, \ldots, n).$$

In order to express obviously and conveniently, the vibration amplitudes are normalized for simplicity, i.e.

$$\tilde{A} = \frac{X_i}{X_1} \quad (i = 1, 2, \ldots, n), \tag{16}$$

where $\tilde{A}$ is the normalized amplitude, X_i $(i = 1, 2, \ldots, n)$ are the peaks of the free response signal, and A_1 is the first peak.

The maximum relative errors of the estimated viscous damping are shown in Table 1. The identified values of Coulomb friction force and the maximum relative error are shown in Table 2.

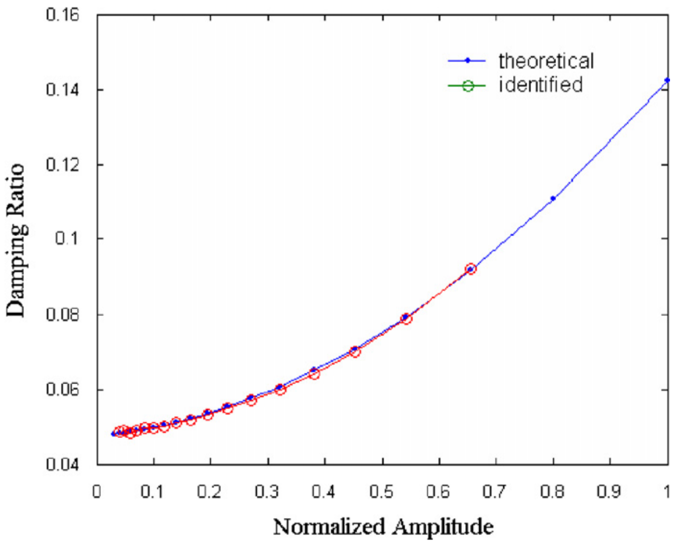

Fig. 4. Quadratic relationship curve of damping ratio versus amplitude.

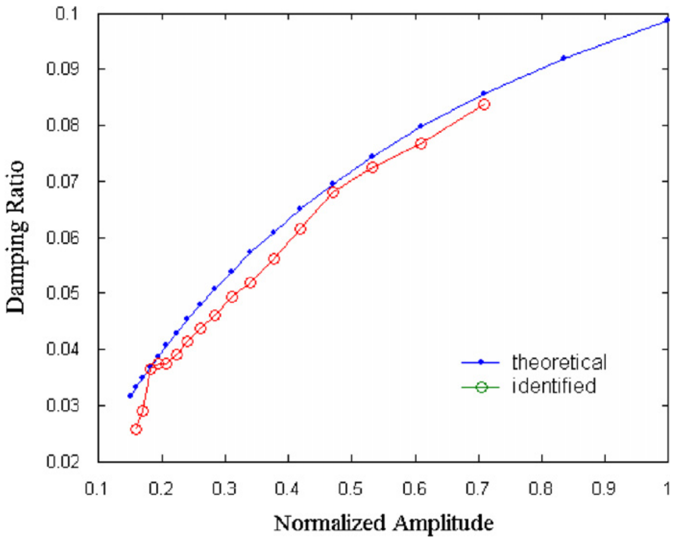

Fig. 5. Inverse hyperbolic relationship curve of damping ratio versus amplitude.

It can be concluded from Figs. 2 to 7 and Table 1 that: (1) For the case of constant damping, the identified values of the constant damping ratio fluctuate on the theoretical values, and the amplitude of fluctuation is very small. The mean value is almost equal to the theoretical value. (2) When the damping ratio is a linear function, a quadratic function, an inverse hyperbolic function, and an exponent curve function of amplitude, the identified values are very closed to the theoretical ones. The identification accuracy of the quadratic damping is the highest among the six kinds of nonlinear viscous damping. Table 2 shows that the identification accuracy of the Coulomb friction in the second type is higher than the others.

In accordance with the above figures, some observations can be made. (1) The damping identification method presented is not only applicable to the constant damping but also to the nonlinear damping. (2) The identification accuracy of the nonlinear damping is less than 1% except the inverse exponent type. (3) The identification accuracy of the Coulomb friction force is lower than that of the nonlinear damping.

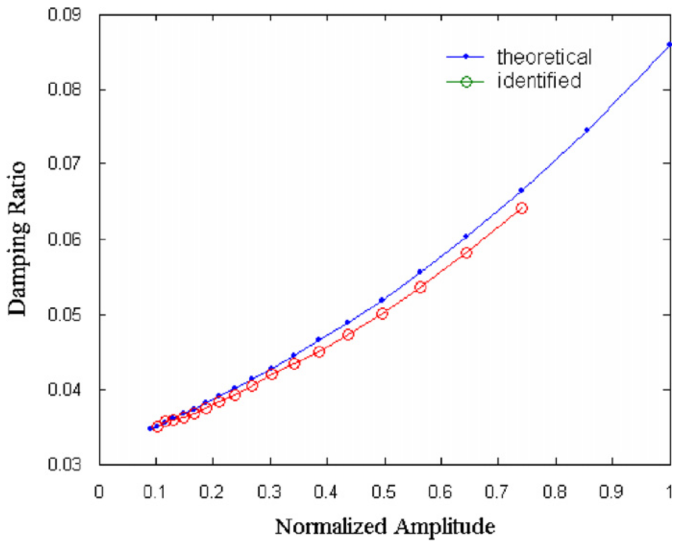

Fig. 6. Exponent relationship curve of damping ratio versus amplitude.

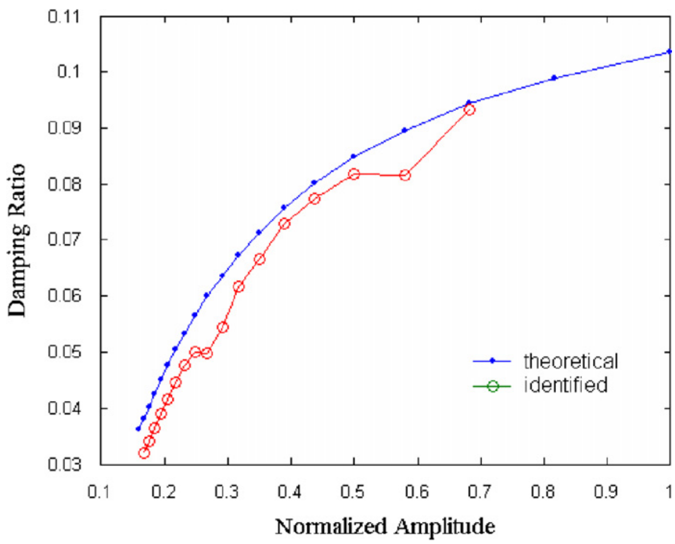

Fig. 7. Inverse exponent relationship curve of damping ratio versus amplitude.

Table 1
Maximum relative errors of viscous damping

Type	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
MRE	0.5569%	05820%	0.3165%	0.3331%	0.3587%	1.2346%

Table 2
Identified values of Coulomb friction and the maximum relative errors

Type	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Theoretical value f_c	0.01 N	0.01 N	0.01 N	0.01 N	0.01 N	0.01 N
Identified value $\hat{f}_c$	0.0101 N	0.01005 N	0.0098 N	0.01048 N	0.008 N	0.0109 N
MRE	1%	0.5%	2%	4.8%	2%	9%

5. Conclusions

A simultaneous identification method of the nonlinear damping and Coulomb friction in a mechanical system is presented. The differential equation of the system with Coulomb friction and nonlinear damping is established. The analytic solution of differential equation is obtained by means of the differential equation theory. The moving rectangle window method is introduced to identify the nonlinear damping and Coulomb friction. Simulations of several types of nonlinear damping model show that the proposed identification method is valid and applicable, and the identification accuracy of the nonlinear damping is higher than that of the Coulomb friction force. The method can be extended to multi-degree-of-freedom systems with nonlinear damping.

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