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An active–passive absorber by using hierarchical fuzzy methodology for vibration control ☆

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Abstract

It has been shown that piezoelectric materials are highly promising as passive electromechanical vibration absorbers when shunted with electrical networks. However, these passive devices have limitations that restrict their practical applications. The main goal of this study is to develop an innovative approach for achieving a high performance adaptive piezoelectric absorber—an active–passive hybrid configuration. This investigation addresses the first application of the concept of hierarchy for controlling fuzzy systems in such an active–passive absorber. It attempts to demonstrate the general methodology by decomposing a large-scale system into smaller subsystems in a parallel structure so that the method developed here can be applied for studying complex systems. The design of the lower-level controllers takes into account each subsystem ignoring the interactions among them, while a higher-level controller handles subsystem interactions. One of the main advantages of using a hierarchical fuzzy system is to minimize the size of the rule base by eliminating ''the curse of dimensionality''. Therefore, the computational complexity in the process can be reduced as a consequence of the rule-base size reduction. Although the performance of the optimal passive absorber is already much better than the original system (no absorber), the intelligent active–passive absorber can still significantly outperform the passive system.

1. Introduction

Reducing the vibrations of structures using piezoelectric damping materials has long been a subject of study in the fields of aeronautical, mechanical, and civil engineering. As a consequence, interest in the use of piezoelectric materials as actuators and sensors for controlling vibrations in flexible structures has also increased. Piezoelectric materials provide inexpensive, reliable, and nonintrusive means of actuating and sensing vibrations in flexible structures.

The French scientists Pierre and Paul-Jacques Curie discovered piezoelectricity in 1880. An electric voltage or change in electric voltage is generated when a mechanical force is applied to a piezoelectric material. However, when an electric field is applied to such a material, a mechanical force is induced by the converse

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Nomenclature

A	system matrix
A_b	cross-sectional area of the beam
B_2	control matrix
b_s	width of the beam and PZT
C	damping matrix of the system
C_p^s	capacitance of the piezoelectric under constant strain
E	Young's modulus
$\hat{f}$	external disturbance vector
h_b	distance from the beam neutral axis to the outside surface of the beam
h_s	distance from beam neutral axis to the outside surface of the PZT
h_{31}	piezoelectric constant
I	moment of inertia
K	stiffness matrix of the system
K_c	coupling vector (conversion from mechanical energy to electrical energy)
l	length of beam
L_p	passive inductance of the shunt circuit
L_T	total number of levels in the hierarchy
M	mass matrix of the system
$q_i(t)$	modal displacement
Q_p	charge on the piezoelectric
R_p	passive resistance of the shunt circuit
u_F	final control action
u_i	control action obtained by consulting the ith level rule set
V_c	control voltage
β_{33}	dielectric constant of PZT
ρ	linear mass density of the beam
$\phi_i(x)$	normalized mode shapes function
$\mu_{A_i}(x)$	membership functions
v	Poisson's coefficient

piezoelectric effect. Recent advances in piezoelectric actuators, based on the converse piezoelectric effect, have great potential for the active control of vibrations, especially for suppressing or isolating vibrations [1–6].

Numerous applications exist in which the addition of passive vibration damping to a structural system can significantly improve system performance or stability. Piezoelectric materials have been shown to have potential as passive electromechanical vibration absorbers when shunted with electrical networks. Piezoelectric transducers (PZTs), in conjunction with appropriate circuitry, can serve as a mechanical energy dissipation device. By placing electrical impedance across the terminals of the PZT, the passive network is capable of damping structural vibrations. If a simple resistor is placed across the terminals of the PZT, the PZT will act as a viscoelastic damper. If the network consists of a series inductor-resistor *R–L* circuit, the passive network combined with the inherent capacitance of the PZT creates a damped electrical resonance. The resonance can be tuned so that the PZT element acts as a tuned vibrational energy absorber [7]. The damping methodology is commonly referred to as *passive shunt damping*. Passive shunt damping is regarded as a simple, low cost, lightweight, and easily implemented method of controlling structural vibrations. A desirable property of passive shunt damping is that the controlled system is guaranteed to be stable in the presence of structural uncertainties.

Flexible mechanical structures have an infinite number of resonant frequencies (or structural modes). If the tuned energy absorber were used to minimize a number of modes, one would need an equal number of PZT patches and shunting circuits. This is clearly impractical.

In order to alleviate problems associated with single mode damping, multimode shunt damping has been introduced; specifically, the use of a piezoelectric patch to damp several structural modes. Therefore, Ref. [8] reports a method of damping multiple vibration modes using a single PZT. Furthermore, [9] proposes a new approach for the optimization and implementation of multimode piezoelectric shunt damping systems. A synthetic impedance, consisting of a voltage-controlled current source and a digital signal processor system, is used to synthesize the terminal impedance of a shunt network. By modeling the compound system, an optimization problem has been formulated that minimizes the H_2 norm of the resulting system.

Recently, the concept of semi-active piezoelectric absorbers to suppress harmonic excitations with time-varying frequencies has also been proposed. Owing to their active and passive damping features, piezoelectric materials have been explored to determine their active–passive hybrid control abilities, advantageous to both passive and active systems. Hence, Ref. [10] outlines new insights derived from analyzing the active–passive hybrid piezoelectric network (APPN) concept. The integrated APPN design is more effective than a system with separated active and passive elements.

Although of considerable potential, these semi-active devices have limitations that restrict their practical application. For instance, the variable capacitor method [11] limits the tuning of the piezoelectric absorber to a relatively small frequency range. Accordingly, an approach proposing a high-performance active–passive alternative to semi-active absorbers is shown in Ref. [5]. Furthermore, the effectiveness of this new absorber design is first demonstrated through experimental investigations, as indicated in Ref. [6].

The studies noted above fail to provide any examples of the application of the fuzzy control theory in active–passive vibration absorption using a hierarchical concept. Consequently, this study develops an innovative approach for achieving a high-performance adaptive piezoelectric absorber—an active–passive hybrid configuration. In the following discussion, the rules are structured hierarchically to ensure that the total number of rules is a linear function of the number of system variables. A methodology for designing fuzzy controllers is examined, and system performance is measured and expressed using fuzzy variables. In fuzzy control, the hierarchy is also effective in structuring the rules to make the fuzzy controller appropriate for a relatively large system.

2. Modeling the compound system

This section sketches how the dynamics of a piezoelectric laminate beam, as illustrated in the following figure, can be derived.

Fig. 1 schematically illustrates the proposed system, which consists of a piezoelectric actuator integrated with an active voltage source in series with an RL circuit. The elastic deflection of a beam is described by the one-dimensional Bernoulli–Euler beam equations, described below:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y(x,t)}{\partial x^2} - C_a v_a(x,t)\right] + \rho A_b \frac{\partial^2 y(x,t)}{\partial t^2} = 0, \tag{1}$$

where E, I, A_b, and ρ represent the Young's modulus, moment of inertia, cross-sectional area, and linear mass density of the beam, respectively. The additional term is due to the moment applied to the neutral axis of the beam by the actuator piezoelectric layer, i.e., $M_a = C_a v_a(x,t)$, where C_a is a constant dependent on the actuator properties [9].

This study assumes that each piezoelectric patch is very thin and the beam deflects only in the y-axis. Using the modal analysis techniques, the position function $y(x,t)$, can be expanded as an infinite series

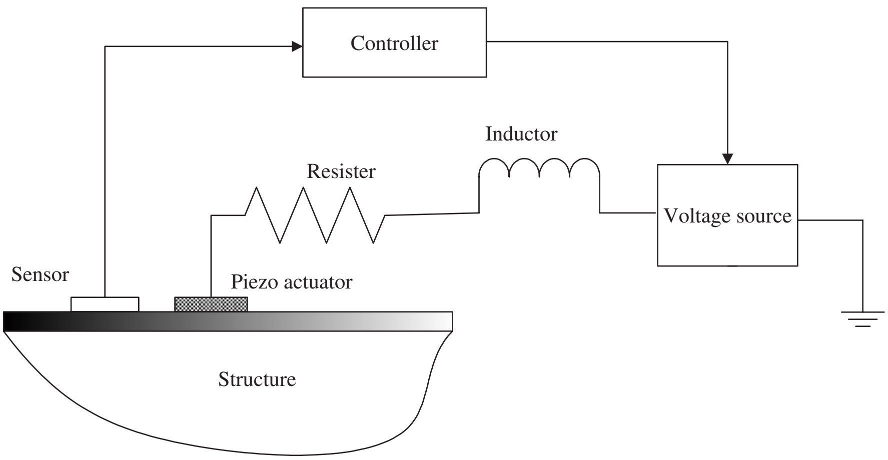

Fig. 1. Adaptive structure with active–passive hybrid piezoelectric networks.

of the form [12–15]

$$y(x,t)=\sum_{i=1}^{\infty}\phi_i(x)q_i(t), \tag{2}$$

where $\phi_i(x)$ denote the normalized mode shapes function and $q_i(t)$ represent the modal displacements.

In this investigation, the system considered is a general mechanical structure that is integrated with a piezoelectric actuator. Additionally, it will also be assumed that the local vibration in the structure is detected using a piezoelectric sensor. The actuator is connected to an RL circuit as well as an active voltage source. This study assumes that the model of the structure and the piezoelectric absorber can be obtained, either analytically or experimentally, in the form displayed in Eq. (3):

$$\begin{aligned} &M\ddot{q}+C\dot{q}+Kq+K_cQ=\hat{F}\cdot f(t),\\ &L_p\ddot{Q}_p+R_p\dot{Q}_p+\tfrac{1}{C_p^s}Q_p+K_c^{\mathrm{T}}q=V_c, \end{aligned} \tag{3}$$

where q, $\dot{q}$ and $\ddot{q}$, are vectors of generalized displacement, velocity, and acceleration [5].

Moreover, the matrices M, C, and K are the mass, damping, and open-circuit stiffness matrices of the system; L_p and R_p are the passive inductance and resistance of the shunt circuit, Q_p denotes the charge on the piezoelectric, C_p^s represents the capacitance of the piezoelectric under constant strain, and V_c is the control voltage. The coupling vector K_c represents the conversion from mechanical energy to electrical energy and vice versa.

Furthermore, C_p^s and K_c^T can be defined separately as [10]

$$C_p^s=\frac{b_s(x_2-x_1)}{\beta_{33}(h_s-h_b)}, \tag{4}$$

$$K_c^{\mathrm{T}}=\frac{h_{31}(h_s^2-h_b^2)}{2(x_2-x_1)}[\phi'(x_2)-\phi'(x_1)], \tag{5}$$

where b_s denotes the width of the beam and PZT, h_b represents the distance from the beam neutral axis to the outside surface of the beam, and h_s is the distance from the beam neutral axis to the outside surface of the PZT. Additionally (x_2-x_1) is the length of the PZT, h_{31} denotes the piezoelectric constant and β_{33} represents the dielectric constant of PZT.

Hence, the system can be expressed in a standard state-space form

$$\dot{x}=A(R_p,L_p)x+B_1\hat{f}+B_2(R_p,L_p)u, \tag{6}$$

where x denotes the state vector, u is the control input, $\hat{f}$ represents the external disturbance vector. The system matrix, A, and the control matrix, B_2, are functions of the passive resistance and inductance.

This study defined,

$$x=[q \quad Q \quad \dot{q} \quad \dot{Q}]^{\mathrm{T}},$$

$$A=\begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -M^{-1}K & -M^{-1}K_c & -M^{-1}C & 0\\ -K_c^{\mathrm{T}}/L_p & -C_p^s/L_p & 0 & -R_p/L_p \end{bmatrix},$$

$$B_1=[0 \quad 0 \quad M^{-1}\hat{F} \quad 0]^{\mathrm{T}},$$

$$B_2=[0 \quad 0 \quad 0 \quad 1/L_p]^{\mathrm{T}}.$$

The system described above has $(n+1)$ modes. The $(n+1)$th mode is due to the passive circuit. Significantly, the addition of an active absorber of this type to a principal system results in a combined system with an added degree of freedom. Because the comparison functions in the expansion are chosen to be the eigenfunctions of a cantilever beam, the ith generalized coordinate closely resembles the ith structural modal coordinate ($i = 1, 2, 3,\ldots,n$). Moreover, for obvious reasons the infinite-order model produced by the model analysis techniques is not suitable for use in the optimization. Consequently, to perform the optimization, an accurate model of the system is required. The following section proposes an optimization approach for determining appropriate values for the shunt circuit.

3. Determining the shunting circuit via optimization

For easy comparison with the active–passive hybrid system, instead of using the classical procedure to find the optimal passive resistance and inductance, a different method is used. However, the concept and results are quite similar to those proposed in Refs. [7,10]. Hence, the state space form for the passive damping can express the system equations:

$$\dot{x} = A(R_p, L_p)x + B_1\hat{f}. \tag{7}$$

Here, the control action u is not included since this system is a passive system. This investigation mathematically represents the disturbances as a stochastic process, which is modeled as the output of a linear system driven by white noise. Thus, this study assumes that $\hat{f}(t)$ is given by

$$\hat{f}(t) = D_d(t)x_d(t). \tag{8}$$

Here, $x_d(t)$ is the solution of

$$\dot{x}_d(t) = A_d(t)x_d(t) + B_d w(t), \tag{9}$$

where $w(t)$ is white noise. This work further assumes that both $x(t_0)$ and $x_d(t_0)$ is stochastic variables. Moreover, the mean and spectral density of $w(t)$ are given by $E[w(t)] = 0$ and $E[w(t)w^{\mathrm{T}}(\tau)] = V(t)\delta(t-\tau)$; $E[\bullet]$ is the expectation operator.

This study combines the description of the system and the disturbances by defining an augmented state vector $\tilde{x}(t) = [\,x(t) \quad x_d(t)\,]^{\mathrm{T}}$, and the overall system state equations become

$$\dot{\tilde{x}} = \begin{bmatrix} A(t) & D_d(t) \\ 0 & A_d(t) \end{bmatrix}\tilde{x} + \begin{bmatrix} 0 \\ B_d \end{bmatrix} w(t) = A_a\tilde{x} + B_a w(t). \tag{10}$$

This investigation now turns its attention to the optimization criteria. The deterministic regulator problem considered the cost function

$$J_t = \lim_{t\to\infty} E[\tilde{x}^{\mathrm{T}}Q\tilde{x}], \tag{11}$$

where Q represents a nonnegative definite weighting matrix. Here, $\tilde{x}^{\mathrm{T}}Q\tilde{x}$ represents the overall structure energy.

Moreover, the system response consists of a state vector with zero mean and a variance given by the solution ($\tilde{P}$) to the Lyapunov function

$$A_a^{\mathrm{T}}\tilde{P} + \tilde{P}A_a + Q = 0. \tag{12}$$

This equation leads to the following constrained optimization problem:

$$R^* = \arg \min_{s.t.g=0} \; J, \tag{13}$$

where $g = A_a^{\mathrm{T}}\tilde{P} + \tilde{P}A_a + Q$.

With a given set of passive parameters (R and L), the cost function L is

$$L = tr(B_a D_d B_a^{\mathrm{T}}\tilde{P}). \tag{14}$$

Notably, for each set of the passive control parameters R and L, there exists an optimal control with a corresponding minimized cost function. A sequential quadratic programming algorithm [17] can be used to determine the resistance and inductance that further minimizes J.

Notably, shunting the piezoelectric does not preclude using the shunted element as an actuator in an active control system, but rather modifies the passive characteristics of the actuator. Modifying the passive stiffness of the piezoelectric to include material damping can introduce perfectly collocated damping into the system. This passive damping can be useful in stabilizing controlled structures in which a mechanical actuator is passively damped.

4. Active–passive control law design

Since the resistor R_p and inductor L_p values are chosen here to optimize the passive system, it is not obvious that they will be best for maximizing the active action. For example, while the resistor is designed to dissipate the structural vibration energy, it can simultaneously dissipate the control power from the active element. That is, the resistor reduces the active authority of the actuator. However, passive control is economical but can only control the vibration up to a certain limit. On the other hand, an active control operates with external energy continuously supplied. Stated differently, this study is applying active control on an optimized (tuned circuit) passive system.

Accordingly, a scheme is synthesized to concurrently incorporate the passive elements and the active control law. The proposed approach ensures that the active and passive elements are configured in a systematic and integrated manner.

4.1. Analysis of hierarchical fuzzy system

Traditional linear quadratic regulator (LQR) synthesis methods are known to guarantee stability margins. Unfortunately, these characteristics only hold for large-scale systems under some limited conditions. One of the most important deficiencies of such large-dimensional systems is the computational impractically of the direct application of LQR methodology. This impracticality is due to the presence of several complexities of the system, such as its large dimensions, nonlinearities, coupling, time delays, and the physical separation of its components. Moreover, this controller requires a mathematical model and assumes that all of the system parameters are known.

Eq. (2) shows that the beam vibration system is an infinite series form. However, it must be approximated by a lower-order model and controlled by a finite-order controller because of limitations of the onboard computer, the inaccuracy of sensors and noise of the system. Additionally, the large-scale system has been limited to a reduced-order truncated system. Through this process, by virtue of linearization, delay approximation, decomposition, and model reduction, each step and/or assumption has introduced a degree of uncertainty into the system, moving the model away from the true physical situation. The above discussion brings up another point, namely that frequent, simplifying assumptions make the problem at hand too uncertain to be of practical use. The design and analysis of a large-scale system should be based on the best available knowledge instead of the simplest available model to treat system uncertainties. Therefore, a large-scale system is better treated via knowledge-based methods such as fuzzy logic, neural networks, etc. [18].

Fuzzy control has become a very popular approach to controller design because it enables human skills to be transferred into linguistic rules. Consequently, fuzzy control has frequently been applied to poorly defined systems or systems without mathematical models. Moreover, fuzzy controllers afford a simple and robust framework for specific nonlinear control laws that accommodate uncertainty and imprecision.

The design of fuzzy controllers is often a time-consuming activity that depends on knowledge acquisition, the definition of the controller structure, and the definition of rules and other parameters. Currently, an important issue related to fuzzy logic systems is the reduction of the total number of rules and their corresponding computational demands. This section addresses the concept of hierarchies in fuzzy control systems [14].

One of the main purposes of using a hierarchical fuzzy system is to minimize the size of rule base by eliminating "the curse of dimensionality". Furthermore, the computational complexity in the process can be reduced as a consequence of the rule-base size reduction, which has become one of the main concerns among system designers. As the application domain of fuzzy control expands from simple systems to more complex systems, a serious limit on the standard fuzzy controller arises as the number of rules in a standard fuzzy controller increases exponentially with the number of variables involved. Given n variables and m fuzzy sets defined for each variable, m^n rules are required to generate a complete fuzzy controller. As n increases, the rule base quickly overloads the memory and makes the fuzzy controller difficult to implement. The concept of the "hierarchical rule set" is elucidated to overcome this problem. Given this hierarchical structure, the number of rules increases linearly (not exponentially) with the number of system variables. Therefore, when considering a complex system with more state variables, employing hierarchical techniques to study fuzzy logic control can significantly reduce the complexity of the design of the rule base [14–22].

Hierarchical fuzzy control contains several level rule sets. The first-level rule set gives a basic control action, while the higher-level rule sets initiate fine tuning control action based on the base (gross) control action. Generally, the first-level rule set depends upon only a few important system variables, while the higher-level rule sets rely on a larger number of system variables. Each controller aims at the global behavior of the reference FLC, regardless of the missing information from the other inputs. To avoid initiating an undesirable control action, the final control action should mainly depend on the first-level rule set in the event of system parameter perturbation. Furthermore, for such a fuzzy system illustrated in Fig. 2, with four variables and seven fuzzy sets (labels), the number of rules is reduced from $7^4 = 2401$ to $7^2 + 7^2 + 7^2 = 147$, indicating a 93.88% reduction. Clearly, depending on how many flexible modes can be fused and in what order when these modes are put into a hierarchical structure, the size of the rule base is reduced differently.

4.2. Fuzzy control structure of the system

The ultimate goal of controller design for a structure is to regulate structural vibration to a desired level by providing a proper driving actuator. The complexity increases when the example is extended to a larger system and when it involves more complicated modes and many more sensors and actuators. The strategy developed here is to combine the hierarchical fuzzy control theory with the passive system parameter values.

Typical hierarchical techniques divide the complex system into several subsystems, which may interact with one another. This study attempts to demonstrate the general methodology by dividing a large-scale system into smaller subsystems in a parallel structure so that the method developed here can be applied for studying complex systems [16].

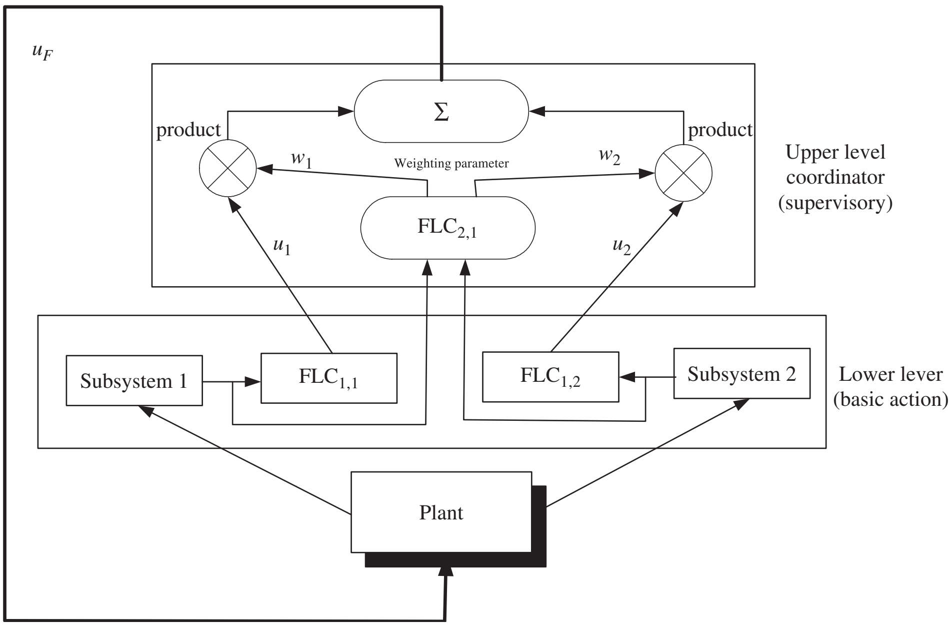

Fig. 2. Implementation of hierarchical fuzzy controller.

The design of the local controllers takes into account each subsystem, ignoring the interactions among them, while a higher-level controller handles subsystem interactions. Fig. 2 illustrates the employment of the hierarchical technique for the system.

For such a case, the hierarchical fuzzy controller suppresses vibrations in a flexible structure using piezoelectric actuators. Typically, the response of a beam is dominated by the lower (1st, 2nd, 3rd…) modes. Consequently, few flexible retained modes are selected for the system in an approximate dynamic model.

To implement the proposed technique, the system is decomposed into two subsystems: the first subsystem takes q_{f1} and $\dot{q}_{f1}$ as local variables, while the second subsystem takes q_{f2} and $\dot{q}_{f2}$ as local variables. Hence, in the proposed hierarchical fuzzy control structure, the first subsystem rules are those associated with the first flexible mode, and its derivatives are used to generate the first-level hierarchy. The second most dominant mode and its derivative are selected as inputs to another fuzzy controller at the same level, and so on.

The fuzzy logic controller FLC_1 takes q_{f1} and $\dot{q}_{f1}$ as inputs to generate the local control action u_1 while fuzzy logic controller FLC_2 takes q_{f2} and $\dot{q}_{f2}$ to generate another control action u_2. Thus, at the local level, each subsystem is designed separately. The fuzzy logic rule base for each subsystem is designed based on the dynamic response of each mode when a control force is activated on the flexible system.

At the upper level, the information from each subsystem is taken as an input to the coordinator. The coordinator, which is based linguistic syntax variables of system performance adjusts the weighting factors of the hierarchical fuzzy controller to achieve a better performance in case of changes in system parameters. To coordinate the local subsystems, the upper level FLC takes q_{f1} and $q_{f1} - q_{f2}$ and $\dot{q}_{f1} - \dot{q}_{f2}$ as inputs to generate the weighting functions w_1 and w_2. Hence, the upper level controller monitors these differences and causes the supervisory decision to be fed to the lower level. The weight factors w_1 and w_2 generated by the supervisory fuzzy logic controller are multiplied with the local controls u_1 and u_2. These are then summed to form the total control u_F for feeding back to the flexible beam. Furthermore, the weighting factor (output scaling factor) is self-regulated during the control process, and can optimize the gain for the hierarchical fuzzy controller [23].

In particular, it is assumed that higher levels in the hierarchy, that is planning and supervision deal with a more abstract view of the control problem and to do so in less precise terms. The proposed control structure constructs the upper-level coordinator (supervisory) to deal with the model reduction error and makes the supervisory decision to the lower level. In the design of the hierarchical fuzzy control structure, the lower level controllers take into account each subsystem ignoring the interactions among them, while the higher-level supervisory fuzzy rule set is used to adjust the weighting factors of the hierarchical fuzzy controller to achieve better performance even in the case of unexpected changes in system parameters.

However, fuzzy logic may be best used to implement high level or supervisory functions in a hierarchically structured intelligent control system. As depicted in Fig. 2, fuzzy logic control can appear at the higher levels, and acts as the supervisory control or coordinator. Fuzzy logic offers a twofold advantage in this setting. First, fuzzy logic can facilitate the synthesis of supervisory control strategies due partly to its ability to better represent the semantics of linguistic terms and constructs often used in supervisory control strategies.

Second, because fuzzy logic-based control strategies can be viewed as nonlinear control strategies, analytical study of the resulting hierarchical system can be facilitated [24].

4.3. Fuzzy logic controller design

4.3.1. Define input and output variables, fuzzy partition, and building the membership functions

Error and error change are two commonly used variables in fuzzy control. This study uses the vibration states and their rate variables as inputs, with the voltage applied to the voltage amplifier as the output. Since the fuzzy inference system handles smooth membership functions better than trapezoidal ones in vibration control, bell-shaped functions are employed to convert these inputs and output variables into linguistic control variables. Generally, $\mu_{A_i}(x)$ is chosen as bell-shaped, with a maximum of 1 and a minimum of 0, such as

$$\mu_{A_i}(x) = \exp\left\{-\left[\left(\frac{x - c_i}{a_i}\right)^2\right]\right\}^{b_i}, \tag{15}$$

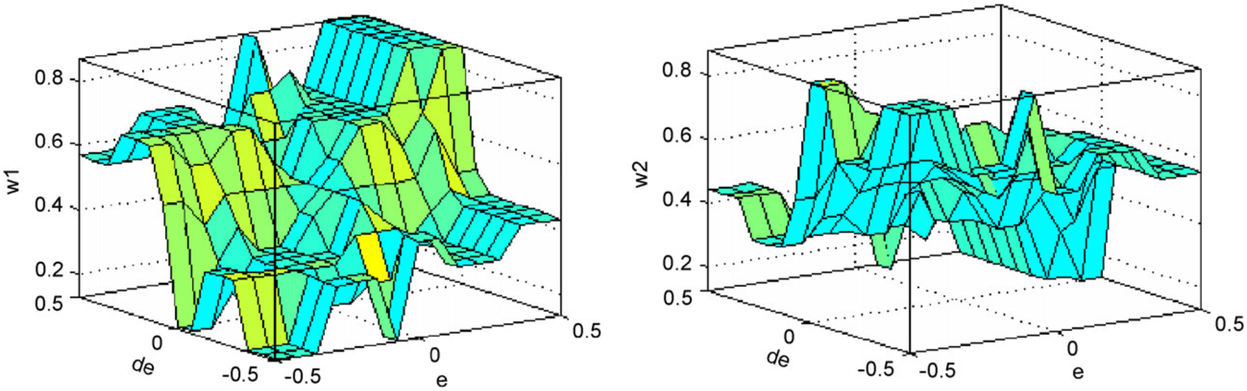

Fig. 3. The viewer surface for the weighting factor w_1 and w_2.

where $\{a_i, b_i, c_i\}$ denotes the parameter set. As the values of the parameters change, the bell-shaped functions vary accordingly, thus exhibiting various forms of membership functions on linguistic label A_i. The fuzzy partition of universes of discourse and the creation of the rule base were drawn from the criteria of skilled operators.

The rule-base design of two subsystems is based on pre-simulation investigations. Fuzzy quantities such as *large negative* (LN), *medium negative* (MN), *small negative* (SN), *zero* (ZE), *small positive* (SP), *medium positive* (MP), *large positive* (LP), and so on, are used in the statements, and the corresponding membership functions thus are needed. Moreover, in building the supervisory rule set, the knowledge of the on tuning weighting factor, gained from the experience (or knowledge) of tuning the fuzzy logic controller, is used and stated as a set of linguistic statements. For the present research, the same kind of fuzzy logic may also be applied for the supervisory fuzzy tuning set to initiate a particular tuning action. The FLC coordinator has two inputs and two outputs. Furthermore, the inference process of the coordinator is illustrated in the **rule view** of the window of the **fuzzy logic toolbox**. The viewer surface for the weighting factor is presented in Fig. 3. Moreover, Fig. 4 displays the rule view of the upper level coordinator.

4.3.2. *Composite control of the system*

A hierarchical fuzzy approach is pursued which allows the adaptation of a composite control strategy. The total (final) control action of the hierarchical fuzzy controller is composed of the control actions due to different level rule sets; that is

$$u_F = \sum_{i=1}^{L_T} w_i u_i, \tag{16}$$

where L_T denotes the total number of levels in the hierarchy and u_F represents the final control action; u_i is the control action obtained by consulting the ith level rule set, and w_i is the corresponding weighting factor.

The overall hierarchical fuzzy controller is implemented according to the block diagram in Fig. 5. The main advantages of the hierarchical structure in Fig. 5 are a significant reduction of memory demand in the implementation.

5. Results and discussion

This section applies the system model to examine the open-loop effects of the circuit parameters (resistance and inductance) on the passive damping ability and active hierarchical fuzzy control authority of the active–passive absorber. The example used in this section is identical to the cantilever beam system identified in Section 2. A test apparatus was constructed, constituted of a flexible cantilever aluminum beam type

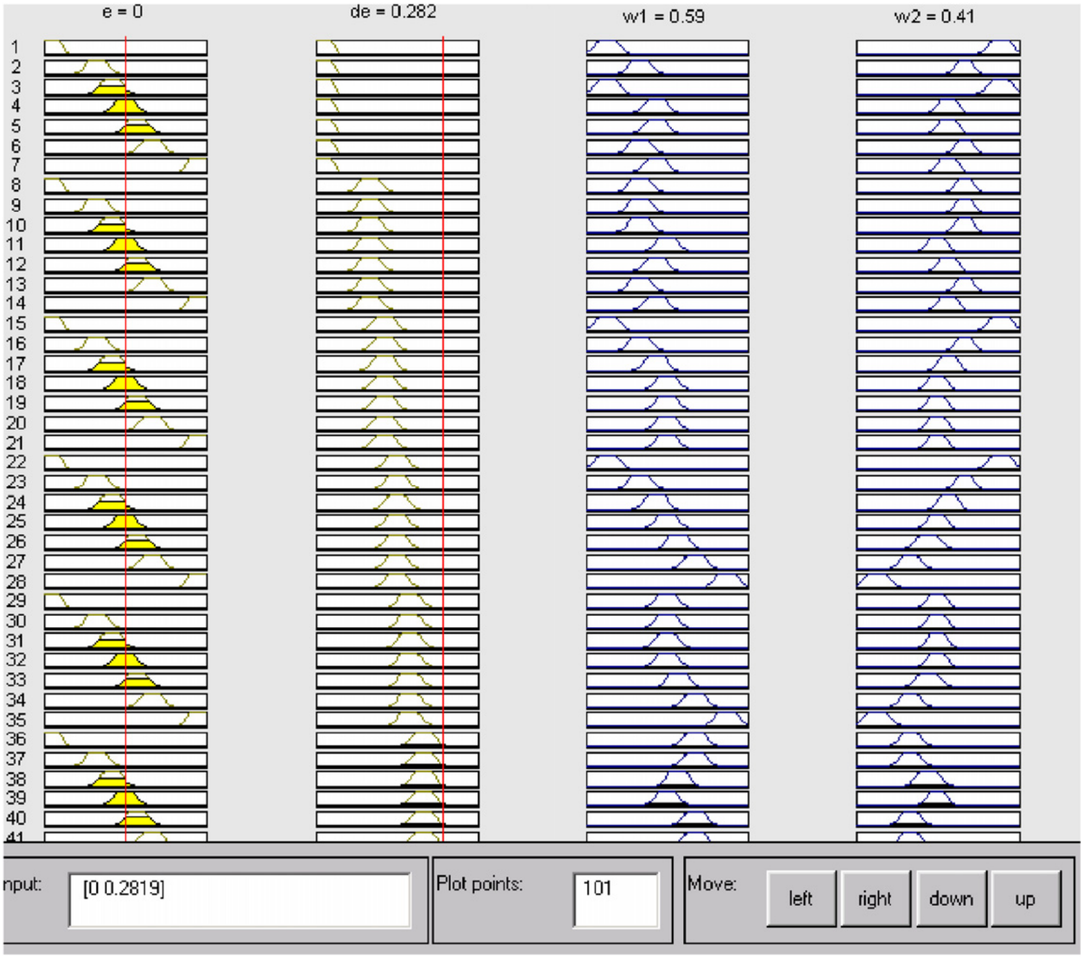

Fig. 4. Rule view of the upper-level coordinator.

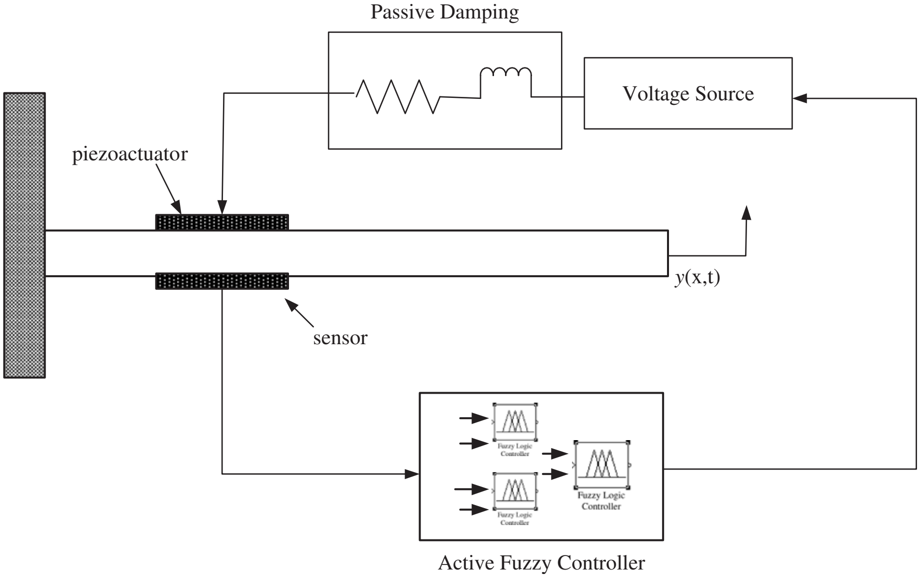

Fig. 5. Schematic diagram of the control experiment.

structure with piezoelectric patches symmetrically bonded on both sides to provide structural bending. Numerical results are obtained by the aforementioned methods for the following system parameters.

The tested beam is a uniform aluminum beam with a rectangular cross-section and experimentally fixed–free boundary conditions. The structure consists of a 15-cm (l) long uniform and a rectangular cross-section $19\,\text{mm} \times 3\,\text{mm}$ $b_s \times h_s$. The common properties of the system parameters are

$$\begin{aligned}
&\text{Young's modulus} = 7.1 \times 10^{10}\,\text{N/m}^2,\\
&\text{Density} = 2700\,\text{kg/m},\\
&\text{Poisson's coefficient} = 0.36,\\
&\text{Piezoelectric constant of PZT} = 7.664 \times 10^{8}\,\text{N/C},\\
&\text{Dielectric constant of PZT} = 7.331 \times 10^{7}\,\text{V m/C}.
\end{aligned}$$

5.1. Case 1. Frequency responses

For the system described in the previous section, the overall structural response is a sum of the response contributed from the excitation force $\hat{f}$ and that contributed from the control voltage V_c. Fig. 6 plots the measured midpoint deflection of the beam and the step excitation frequency responses. It compares the peak magnitude with and without the shunt circuit. The resonant responses of the first two modes were reduced considerably once the shunt circuit was introduced. That is, the RL circuit enhanced the passive damping ability around the first resonant frequency. One may argue that such a circuit only can be shunted to one mode to achieve damping. However, Fig. 6 appears to suggest that two modes have been damped and another resonance has been introduced just before the second resonance. In order to explain this issue, some of the viewpoints should be addressed in the following. It is important to realize that the addition of the RL circuit of a principal system (Eq. (1)) results in a combined system (Eq. (3)) having an added degree of freedom. Therefore, the passive absorber can be used to virtually eliminate vibration in systems in which it is particularly undesirable, and to reduce excessive amplitudes of vibration in others. Nevertheless, the passive damping capability of the second mode is slightly modify while applied the RL circuit. It means that the passive damping ability of the second mode is not significant as the first mode in such case. Consequently, while applied in one RL circuit only, it could be damped multimodes. However, only the first mode will be damped significantly, then the second mode, and the other modes damping effect are trivially. Moreover, another resonance was introduced before the second resonance is owing to the nonlinear effect and measurement inaccuracy while the RL circuit applied (Fig. 7).

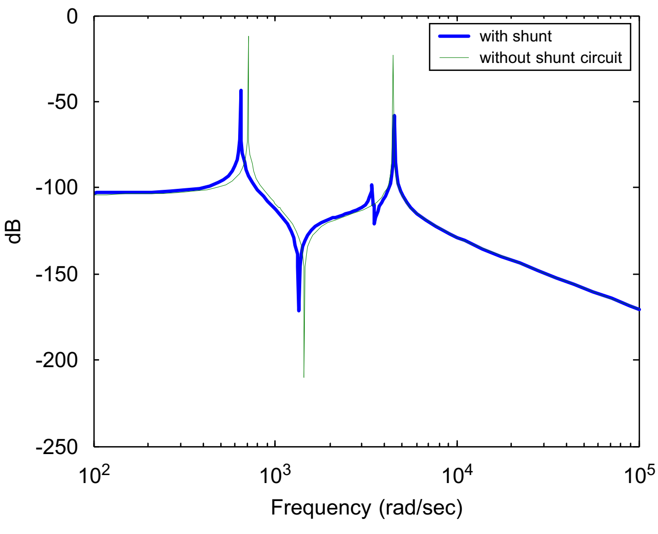

Fig. 6. Frequency response with shunt circuit ($2500\,\Omega \times 79\,\text{H}$).

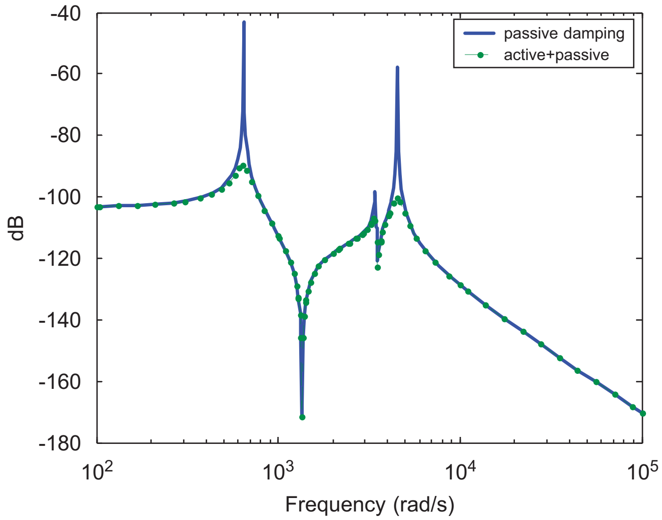

Fig. 7. Frequency response of the controller ($5000\,\Omega \times 79\,H$).

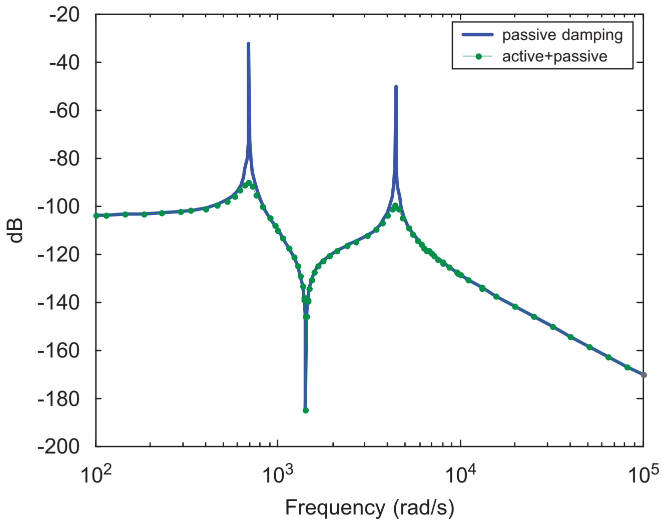

Fig. 8. Frequency response of the controller ($5000\,\Omega \times 22\,H$).

The frequency response of the controller is illustrated in Figs. 8 and 9. The frequency response under passive damping (open loop) and the control of the hierarchical fuzzy logic (closed loop) are presented. The controller is observed to have a resonant structure, as expected. Moreover, the resonant response of the first two modes was reduced over the entire beam due to the controller action. Table 1 lists the magnitude reduction at the first resonance frequency for each specified *RL* circuit of the beam vibration. The table also lists the magnitude reduction, such as the shift in the first resonant frequencies of the beam vibration. The simulation demonstrates that the resonant responses (711.5 rad/s) of the first mode were reduced by around 11.26–30.85 dB when the shunt circuit was applied. Furthermore, compared to passive damping, the first modal resonant magnitudes were reduced by up to 66.63 dB while the active controller was engaged. Notably, the reduction in peak vibration amplitude was greater for the active–passive absorber than for passive damping such as the shift in resonant frequencies. Hence, the controller reduced the resonant responses of the structure by increasing the system damping at resonant frequencies. Moreover, as can be seen, the system sensitivities to the active controller exceeded those for passive damping. As the resonant frequency reduced,

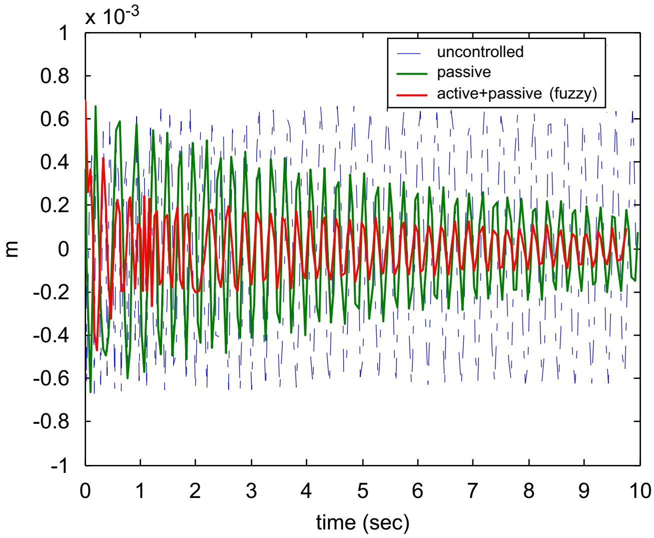

Fig. 9. Vibration at the midpoint of the beam (2500 Ω × 79 H).

Table 1
Magnitude reduction at the first resonant frequency

Type/*RL*	Reduction 1 (711.5 rad/s) (dB)	Reduction 2 (711.5 rad/s) (dB)	Reduction 1 (259.4 rad/s) (dB)	Reduction 2 (259.4 rad/s) (dB)	Reduction 1 (850.4 rad/s) (dB)	Reduction 2 (850.4 rad/s) (dB)
1500 Ω × 22 H	11.26	66.63	11.18	74.76	11.33	65.07
2500 Ω × 22 H	14.7	63.2	14.62	71.32	14.75	61.63
5000 Ω × 22 H	19.89	58.01	19.80	66.14	19.96	56.44
1500 Ω × 79 H	20.94	56.48	20.17	63.78	21.20	54.82
2500 Ω × 79 H	25.07	52.36	24.29	59.65	25.33	50.68
5000 Ω × 79 H	30.85	46.85	30.07	53.87	31.12	44.91

Note: Reduction 1: (uncontrolled)–(passive); Reduction 2: (passive)–(active + passive); first resonant frequency $\omega_{n1} = \frac{1.875}{l}\sqrt{\frac{EI}{\rho}}$ (for cantilever beam [25]).

the magnitude reduction increased In contrast, the magnitude reduction decreased with increasing resonant frequency. Therefore, the proposed fuzzy controllers afford a simple and robust framework for resonant frequency shift.

5.2. *Case 2. Time responses*

Figs. 9 and 10 plot the time response for vibrational displacement at the midpoint of a beam under an initial excitation step for various resistance and inductance values. These indicate the effectiveness of the hierarchical fuzzy controller in minimizing structural vibration in the time domain. The settling time of the position response is considerably reduced by the fuzzy control action. It also demonstrates that the convergence rate is faster than for passive damping when using the hierarchical fuzzy control techniques. Additionally, Figs. 9 and 10 also show the effectiveness of the controller effectiveness in minimizing beam vibration in the time domain.

Furthermore, Table 2 lists the normalized root-mean-square (rms) vibrational displacement at the midpoint of a beam under uncontrolled, passive, and active–passive absorber for various resistance and inductance values. Fig. 11 demonstrates the flexible modes response for q_{f1} and q_{f2} of the elastic beam with LQR and the proposed hierarchical fuzzy controller. Evidently, a vibration reduction (%) for passive absorber is highly dependent upon the resistance and inductance values with the shunt circuit. Moreover, a hierarchical fuzzy

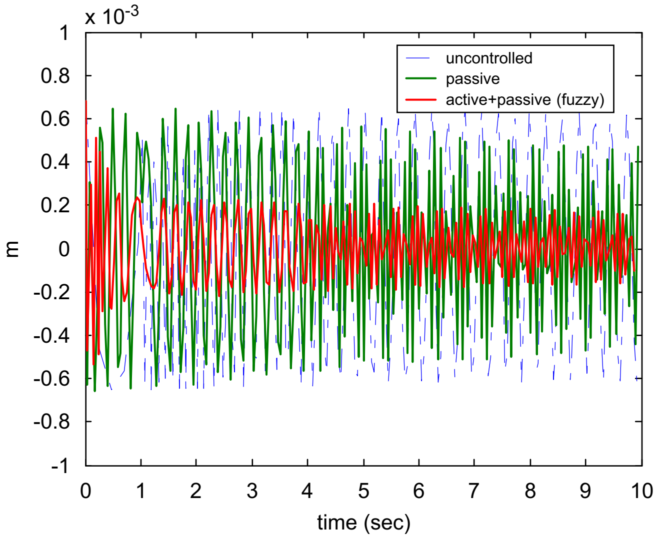

Fig. 10. Vibration at the midpoint of the beam (2500 Ω × 22 H).

Table 2
Normalized rms vibrational displacements at the midpoint of a beam

Type/*RL*	Uncontrolled (I)	Passive (II)	Active + passive (III) (LQR)	Active + passive (IV) (fuzzy)	Reduction (I–II)/I (%)	Reduction (I–IV)/I (%)	Reduction (II–IV)/II (%)	Reduction (III–IV)/III (%)
1500 Ω × 22 H	4.6e−4	4.2e−4	3.0e−4	1.7e−4	8.7	63.0	59.5	43.3
2500 Ω × 22 H	4.6e−4	4.0e−4	2.8e−4	1.6e−4	13	65.2	60.0	42.9
5000 Ω × 22 H	4.6e−4	3.5e−4	2.7e−4	1.5e−4	23.9	67.4	57.1	44.5
1500 Ω × 79 H	4.6e−4	3.3e−4	2.5e−4	1.5e−4	28.3	67.4	54.5	40
2500 Ω × 79 H	4.6e−4	2.8e−4	2.2e−4	1.4e−4	39.1	69.6	50.0	36.4
5000 Ω × 79 H	4.6e−4	2.1e−4	2.0e−4	1.2e−4	54.3	73.9	42.9	40

controller yields a more significant improvement in displacement reduction over that obtained by LQR techniques which design techniques is shown in Appendix A.

One of the most important deficiencies for such a large-dimensional system is computational impractically of the direct application of LQR methodology [18]. Therefore, it is very difficult to choose the weighting matrix Q and R, and to solve a suitable feedback gain K to damp out the vibration modes in such a case. That is why the LQR methodology only controls the vibration up to a certain limit just as shown in Table 2. As a result, the proposed hierarchical fuzzy controller can damp out the vibration more quickly. In practice, dynamical models of a flexible structure, as represented in Eq. (2) have to be truncated to represent a system by a finite dimensional model. However, if the complex system were decomposed into several subsystems, the model reduction error will introduce into the system. Traditional LQR has no way to cope with this point. It needs to develop another modified design techniques while keeping the system model as realistic as possible [18]. Fortunately, the proposed control structure constructs the upper-level coordinator (supervisory) to deal with the model reduction error and makes the supervisory decision to the lower level. In the design of the hierarchical fuzzy control structure, the lower-level controllers take into account each subsystem ignoring the interactions among them, while the higher-level controller handles subsystem interactions. The upper-level supervisory fuzzy rule set is used to adjust the weighting factors of the hierarchical fuzzy controller to achieve better performance even in the case of unexpected changes in system parameters. In particular, it is assumed that the higher levels in the hierarchy, that is planning and supervision deal with a more abstract view of the control problem and do so in less precise terms.

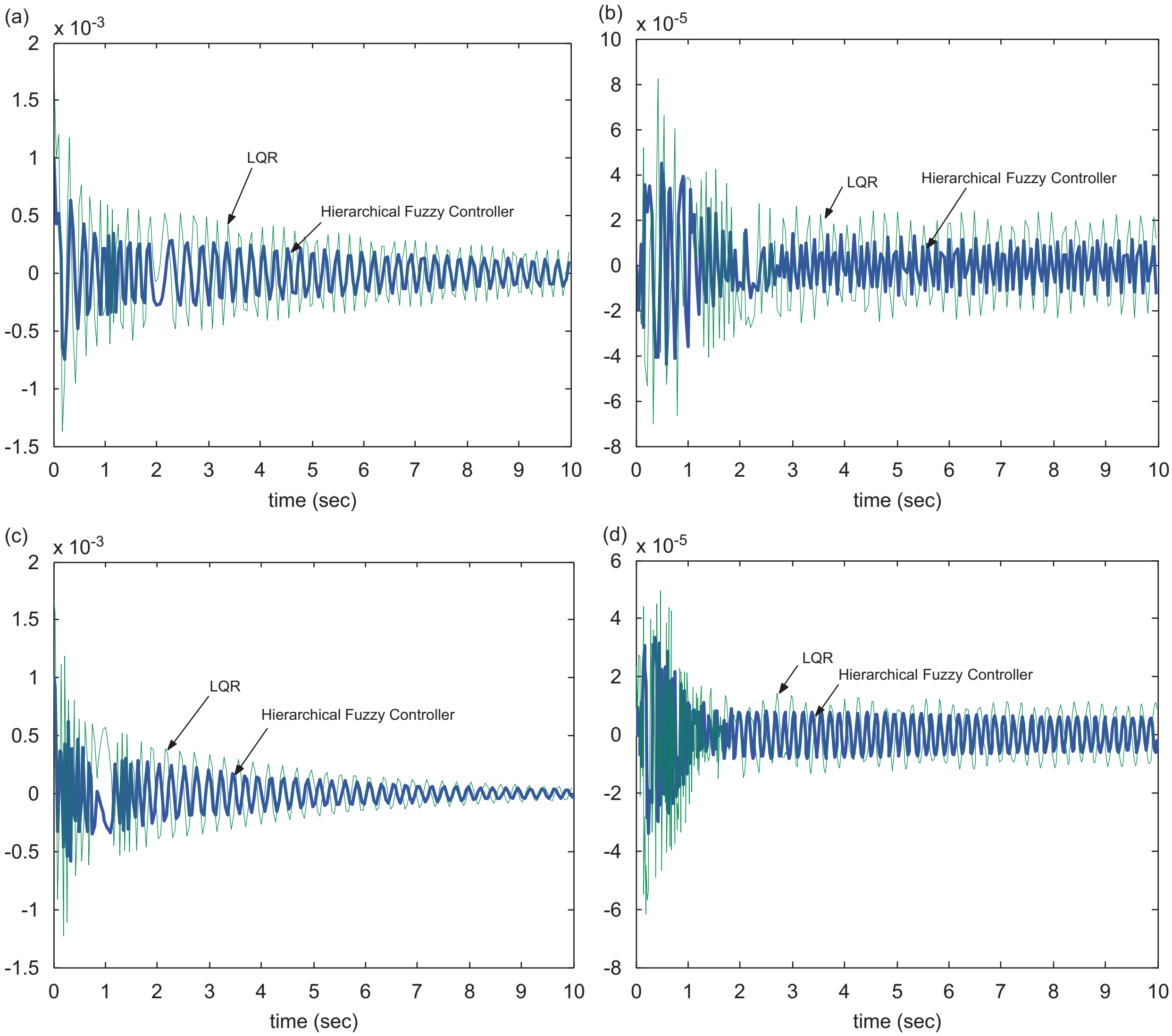

Fig. 11. (a) Vibration mode response for q_{f1} ($2500\,\Omega \times 22\,\mathrm{H}$). (b) Vibration mode response for q_{f2} ($2500\,\Omega \times 22\,\mathrm{H}$). (c) Vibration mode response for q_{f1} ($5000\,\Omega \times 79\,\mathrm{H}$). (d) Vibration mode response for q_{f2} ($5000\,\Omega \times 79\,\mathrm{H}$).

Table 2 also reveals that the proposed passive absorber reduces the displacement due to vibration of an uncontrolled by approximately 8.7–54.37% at each specified *RL* circuit. Moreover, once again the performance of the intelligent active–passive absorber is relatively unaffected by the change in the shunt circuit parameters. The normalized RMS is around 1.2e−4 to 1.7e−4 when implementing the intelligent active–passive absorber. Such a controller is observed to result in suppression of the transverse deflection of the structure. Consequently, the active gain is updated with variation in passive parameters. That is, simultaneously varying the values of the fuzzy control gains and passive parameters can obtain the "optimized" optimal control. Compared with the system with no absorber, the performance of the intelligent active–passive absorber is considerably better than that of the passive damping in vibration reduction. The active–passive piezoelectric absorber presented here may have the performance and robustness required for such a case. Moreover, hierarchical control technique is a powerful and efficient means to cope with complex systems. A complex system with more state variables, the employment of hierarchical techniques in the study of fuzzy logic control can greatly reduce the complexity of the design of the rule base. Also, as provide in the large-scale systems theory, the parallel structure will reduce the computation time.

6. Conclusions

It has been shown that piezoelectric materials can be used as passive electromechanical vibration absorbers by shunting them with electrical networks. The active gain is updated with variation in the passive parameters. That is, simultaneously varying the values of the control gains and passive parameters can obtain the "optimized" optimal control. This study addresses the first application of the concept of hierarchy for controlling fuzzy systems in such active–passive absorbers. This research also demonstrates the feasibility of using hierarchical fuzzy logic control in dealing with many mode dynamic system problems. Furthermore, the main contribution of the hierarchical structure is a significant reduction in the amount of memory required for implementation. Consequently, it is shown that the active–passive absorber not only can provide passive damping, but can also enhance the active action authority. The proposed active–passive absorber is significant compared with the passive baseline systems. Such an investigation could provide insight and design guidelines for a new absorber system. The investigation results also demonstrate that the proposed design can outperform one passive and one (LQR) active–passive vibration control methods while requiring less control effort. The proposed fuzzy control method is quite useful in terms of reliability and robustness.

Acknowledgments

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Appendix A. Reduction of vibration using the linear quadratic regulator (LQR)

Principles of optimization are applied to the smart panel. An LQR is applied [12]. The aggregation procedure is addressed from the perspective of optimal control. Consider the system equation (15) $\dot{x} = Ax + B\tilde{V}$, with a quadratic cost function

$$J = \frac{1}{2}\int_0^{\infty} (x^{\mathrm{T}}(t)Qx(t) + \tilde{V}^{\mathrm{T}}(t)R\tilde{V}(t))\,\mathrm{d}t, \tag{A.1}$$

where A, B and x, are the system matrix, control matrix and state vector, respectively, and Q and R are nonnegative and positive-definite matrices. The optimal control problem is to determine a control input $\tilde{V}$ such that Eq. (15) is satisfied while the cost function (A.1) is minimized.

The solution to this so-called "state regulator" is well known:

$$\tilde{V}(t) = -R^{-1}B^{\mathrm{T}}Kx(t), \tag{A.2}$$

where K is the symmetric positive-definite matrix solution to the following algebraic matrix and Riccati equation

$$KA + A^{\mathrm{T}}K - KSK + Q = 0, \tag{A.3}$$

where

$$S = BR^{-1}B^{\mathrm{T}}.$$

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