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Short Communication

Implications of time-delayed feedback control on limit cycle oscillation of a two-dimensional supersonic lifting surface ☆, ☆☆

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Abstract

In this paper, the problem of implications of time delay feedback control of a two-dimensional supersonic lifting surface on the flutter boundary and on its character, that is, benign or catastrophic, is addressed. In this context, the structural and aerodynamic nonlinearities are included in the aeroelastic governing equations. The model and the associated theory are developed for linear and nonlinear plunging and pitching full-state proportional and velocity feedback controls. Center manifold reduction and normal form theory are applied to investigate the stability in the post-flutter flight speed regimes. Numerical simulations are carried out to determine the implications of time delay in the considered controls, but are restricted to the cases of proportional feedback control and no structural damping.

1. Introduction

The study of the aeroelastic behavior of flight vehicles in the pre- and post-flutter regimes is of crucial importance towards increasing their operational life and the avoidance of catastrophic failures. A nonlinear model of a wing section of high-speed aircraft incorporating active control has been proposed in Ref. [1] and further studied recently using linear and nonlinear feedback controls [2]. In Ref. [1], the dynamic behavior of the system without time delay in the control was studied in the vicinity of an Hopf bifurcation critical point. Depending on the characteristics of the aeroelastic system, it was shown that with the increase of the flight speed, stable (or unstable) equilibria and stable (or unstable) limit cycles might exist. Also, the effect of structural nonlinearities [3,4] on the character of the flutter boundary has been considered. Furthermore, in Ref. [2], the effect of the time-delayed proportional feedback control on the flutter instability boundary and its

☆We would like to honor and dedicate this work to Professor Liviu Librescu of Virginia Tech, for his scientific achievements in life and for sacrifying his life to save his students.

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character (benign/catastrophic) was discussed. Bifurcations into limit cycles (Hopf bifurcation) were investigated with respect to system parameters as well as the time delay. Numerical simulations were employed to verify the analytical predictions. It has been shown that incorporation of a linear feedback control is always beneficial in controlling both the initiation of Hopf bifurcation and the stability of motions, regardless whether the time delay is added or not. Introducing a time delay into the feedback control could have a profound effect on the stability of the bifurcating motions. However, it has been found that larger time delay is not beneficial in delaying the Hopf bifurcation. When nonlinear feedback control is applied and the nonlinear control is combined with a larger time delay, the situation becomes even more complicated in the sense that it may destabilize the bifurcating motions. Therefore, based on the study in Ref. [2], it was suggested that both linear and nonlinear controls with small time delay should be applied in order to obtain the best control design. However, further studies are necessary to get better understanding of dynamic behavior of the model with other controls.

In this paper, we consider the effect of the time delay on proportional and velocity, linear and nonlinear force and moment feedback controls. The main attention will be focused on Hopf bifurcation with no structural damping in the system. Hopf bifurcation has been extensively studied using various methods [5,6], for example, via Lyapunov's first quantity, see e.g. Refs. [2,7–11]. Also nonlinear systems involving time delay have been studied by many authors (e.g. see Refs. [12–14]). In the past two decades, there has been rapidly growing interest in bifurcation control and chaos control (e.g. see Refs. [15,16]), which has a wide variety of promising potential applications. In general, the aim of bifurcation control is to design a controller such that the bifurcation characteristics of a nonlinear system undergoing bifurcations can be modified to achieve some desirable dynamical behaviors, such as changing a subcritical Hopf bifurcation to supercritical, eliminating chaotic motions, etc.

As it clearly appears, within the problem addressed in this paper, two principal issues deserve special attention: (i) increase, without weight penalties, of the flutter speed, and (ii), possibilities to convert unstable limit cycles into stable ones. While the achievement of (i) can result in the expansion, of the flight envelope, that related with (ii) would result in the possibility to operate in close proximity of the flutter boundary without the danger of encountering the *catastrophic flutter* instability, but in the worst possible scenario, crossing the flutter boundary that features a *benign* character. In contrast to the catastrophic flutter boundary in which case, the amplitude of oscillations increase exponentially, in the case of benign flutter boundary, monotonic increase of the oscillation amplitude occurs, and as a result, the failure can occur only by fatigue. It clearly appears that both issues (i) and (ii) are related to controlling Hopf bifurcations. In particular, issue (i) implies increase of the stability of an equilibrium and delay of the occurrence of Hopf bifurcations; while issues (ii) is related to controlling Hopf bifurcations once a periodic vibration has been initiated. Recently, a new control method for Hopf bifurcation has been proposed, and both issues are discussed [16,17].

The present study primarily deals with the determination and control of the flutter instability and of the character of the flutter boundary of supersonic/hypersonic lifting surfaces when including time delay in the control. In contrast to the issue of the determination of the flutter boundary that requires a linearized analysis, the problem of the determination of the character of the flutter boundary, requires a nonlinear approach. As has been shown (see e.g. Refs. [1,7–9]) at hypersonic speeds, the aerodynamic nonlinearities [18,19] play a detrimental role, in the sense that they contribute to conversion of the benign flutter boundary to a catastrophic one. Therefore, an active control capability enabling one to prevent conversion of the flutter boundary into a catastrophic one should be implemented.

2. Aeroelastic model

This investigation is based on a geometrical and aerodynamic nonlinear model of a wing section of high-speed aircraft incorporating an active control capability. The geometry of the model is shown in Fig. 1.

The nonlinear aerodynamic theory considered in this study is based on the third-order approximation of the Piston Theory Aerodynamics (PTA) given by

$$p(x,t) = p_\infty\left[1 + \kappa\frac{v_z}{a_\infty}\gamma + \frac{\kappa(\kappa+1)}{4}\left(\frac{v_z}{a_\infty}\gamma\right)^2 + \frac{\kappa(\kappa+1)}{12}\left(\frac{v_z}{a_\infty}\gamma\right)^3\right], \tag{1}$$

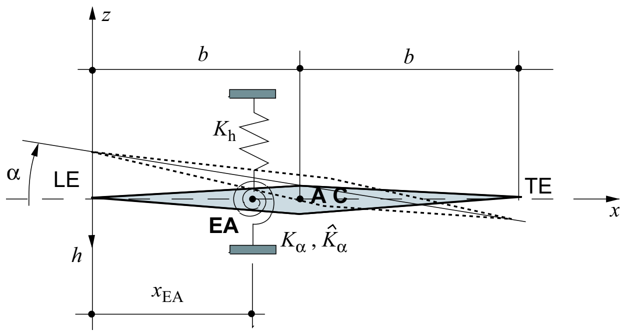

Fig. 1. Geometry of the cross-section of lifting surface.

where $v_z = -((\partial w/\partial t) + U_\infty(\partial w/\partial x))\mathrm{sgn}(z)$ denotes the downward velocity normal to the lifting surface, $a_\infty^2 = \kappa p_\infty/\rho_\infty$, where sgn($z$) is a signum function, assuming the value 1 or -1 for $z>0$ and $z<0$, respectively. In addition, $w(t) = h(t)+\alpha(t)\ (x-b_{x0})$ denotes the transversal displacement, $x_0(\equiv x_{ea}/b)$ is the dimensionless streamwise position of the pitch axis measured from the leading edge, b is the half-chord length of the airfoil. $h(t)$ and $\alpha(t)$ are, respectively, the plunging and pitching about the elastic axis. p_∞, ρ_∞, U_∞ and a_∞ are the pressure, the air density, the airflow speed and the speed of sound of the undisturbed flow, respectively. κ is the polytropic gas coefficient, while $\gamma = M_\infty/\sqrt{M_\infty^2 - 1}$ is an aerodynamic correction factor that enables one to extend the validity of the piston theory aerodynamics to the low supersonic speed range.

The dimensionless aeroelastic equations of a typical cross-section featuring plunging and pitching degrees of freedom, accounting for structural nonlinearities in pitching, aerodynamic nonlinearities and linear and nonlinear time-delayed control, can be written as

$$\begin{aligned} m\ddot{h}(\hat{t}) + S_\alpha\ddot{\alpha}(\hat{t}) + c_h\dot{h}(\hat{t}) + K_h h(\hat{t}) &= L(\hat{t}) - L_c(\hat{t}-\hat{\tau}), \\ S_\alpha\ddot{h}(\hat{t}) + I_\alpha\ddot{\alpha}(\hat{t}) + c_\alpha\dot{\alpha}(\hat{t}) + M_\alpha(\hat{t}) &= M(\hat{t}) - M_c(\hat{t}-\hat{\tau}), \end{aligned} \tag{2}$$

where the dot denotes the differentiation with respect to time $\hat{t}$, $\hat{\tau}$ stands for the time delay. $L(\hat{t})$ and $M(\hat{t})$ denote the aerodynamic force and moment, respectively. $L_c(\hat{t}-\hat{\tau})$ and $M_c(\hat{t}-\hat{\tau})$ represent the delayed nonlinear feedback control force and moment, respectively, while, c/h and c/α denote the structural damping coefficients in plunging and pitching, respectively. The latter ones will be discarded in the numerical simulations.

The dimensionless counterpart of Eq. (2) is as follows [2]:

$$\ddot{\xi}(t) + \chi_\alpha\ddot{\alpha}(t) + \frac{2\bar{\omega}}{V}\varsigma_h\dot{\xi}(t) + \left(\frac{\bar{\omega}}{V}\right)^2\xi(t) = l_a(t) - l_c(t-\tau), \tag{3}$$

$$\frac{\chi_\alpha}{r_\alpha^2}\ddot{\xi}(t) + \ddot{\alpha}(t) + \frac{2\varsigma_\alpha}{V}\dot{\alpha}(t) + \frac{1}{V^2}\alpha(t) + \frac{1}{V^2}\delta_s B\alpha^3(t) = m_a(t) - m_c(t-\tau), \tag{4}$$

where

$$\begin{aligned} l_c &= \left(\frac{\bar{\omega}}{V}\right)^2\Theta_1\xi + \left(\frac{\bar{\omega}}{V}\right)^2\Theta_2\xi^3 + \frac{2\bar{\omega}}{V}\Theta_3\dot{\xi} + \frac{2\bar{\omega}}{V}\Theta_3\dot{\xi}^3, \\ m_c &= \left(\frac{1}{V^2}\right)\Psi_1\alpha + \left(\frac{1}{V^2}\right)\Psi_2\alpha^3 + \frac{2}{V}\Psi_3\dot{\alpha} + \frac{2}{V}\Psi_4\dot{\alpha}^3, \end{aligned} \tag{5}$$

$$
\begin{aligned}
l_a = & -\frac{\gamma}{12\mu M_\infty}\left\{12\alpha + \delta_A M_\infty^2(1+\kappa)\gamma^2\alpha^3 + 12\left[\dot{\xi} + \dot{\alpha}(b - x_{ea})/b\right]\right\}, \\
m_a = & -\frac{\gamma}{12\mu M_\infty}\frac{1}{r_\alpha^2 b}\left\{12(b - x_{ea})\alpha + \delta_A M_\infty^2(b - x_{ea})(1+\kappa)\gamma^2\alpha^3\right. \\
& \left. + 4\left[3(b - x_{ea})\dot{\xi} + \dot{\alpha}(4b^2 - 6bx_{ea} + 3x_{ea}^2)/b\right]\right\}.
\end{aligned} \tag{6}
$$

Here, we still use the overdot to denote the differentiation with respect to the normalized time $t = U_\infty \hat{t}/b$. τ is a dimensionless time delay $\tau = \hat{\tau}\omega_\alpha$, and $\xi = h/b$. The meaning of other parameters can be found in Ref. [2].

In the above equations, the tracers δ_s and δ_A identify the structural and aerodynamic nonlinearities considered in this paper. These assume the values 0 or 1 depending on whether the indicated nonlinearities are accounted for, or discarded, respectively.

3. Formulations

In this paper, we consider an undamped structural model, that is, $\zeta_h = \zeta_\alpha = 0$, and focus on the proportional feedback control only, implying $\Theta_i = \Psi_i = 0$, $i = 3,4$. In order to capture the effect of the time delay, τ, related to the various feedback gains Θ_i and $\Psi_i (i = 1,2,3,4)$, let $\xi = x_1$, $\alpha = x_2$, $\dot{\xi} = x_3$, $\dot{\alpha} = x_4$, and $x_{it} = x_i(t-\tau)$, for $i = 1,2,3,4$. Then, one can convert Eqs. (3) and (4) to the following system in vector form:

$$
\dot{\mathbf{x}}(t) = A_1\mathbf{x}(t) + A_2\mathbf{x}(t-\tau) + \mathbf{F}(x(t),\ \mathbf{x}(t-\tau)), \tag{7}
$$

where $\mathbf{x}$, $\mathbf{F} \in R^4$, A_1 and A_2 are 4×4 matrices. A_1, A_2 and $\mathbf{F}$ are given by

$$
A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_5 & a_6 & a_7 & a_8 \\ b_5 & b_6 & b_7 & b_8 \end{bmatrix}
$$

and

$$
\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ a_9 x_2^3 + a_{10}x_{1t}^3 + a_{11}x_{2t}^3 + a_{12}\dot{x}_{3t} + a_{13}\dot{x}_{4t} \\ b_9 x_2^3 + b_{10}x_{1t}^3 + b_{11}x_{2t}^3 + b_{12}\dot{x}_{3t} + b_{13}\dot{x}_{4t} \end{pmatrix}, \tag{8}
$$

respectively. The explicit expressions of the coefficients a_i and b_i can be found in Ref. [2].

As the first step, we analyze the stability of the trivial solution of the linearized system counterpart of Eq. (7). Its characteristic function can be obtained as: $\det(\lambda I - A_1 - A_2 e^{-\lambda\gamma})$. It can be shown that when $a_1b_2 - a_2b_1 + a_2b_5 - a_5b_2 + a_5b_6 - a_6b_5 + a_6b_1 - a_1b_6 \neq 0$, none of the roots of $D(\lambda)$ are zero. Thus, the trivial equilibrium $\mathbf{x} = \mathbf{0}$ becomes unstable only when the linearized system has at least one pair of purely imaginary roots [2]. The critical values for a Hopf bifurcation to occur can be found by setting the real and imaginary parts of $D(\mathrm{i}\omega)$ to equal to zero. Usually, one can only use numerical approach to determine the relations among the system parameters at the critical point.

In order to obtain the explicit analytical expressions for the stability conditions of the Hopf bifurcation solution, we need to reduce the system given in Eq. (7) to its center manifold. To achieve this, we transform the infinite dimensional problem, described by the delay equation (Eq. (7)), to an abstract evolution equation on Banach space H, consisting of continuously differentiable functions $u:[-\tau, 0] \to R^2$ which results in

$$
\dot{x} = Ax_t + F(t, x_t), \tag{9}
$$

where $x_t(\theta) = x(t+\theta)$ for $-\tau \leqslant \theta \leqslant 0$, and A is a linear operator for the critical case, expressed by

$$
Au(\theta) = \begin{cases} \dfrac{\mathrm{d}u(\theta)}{\mathrm{d}\theta} & \text{for} \quad \theta \in [-\tau, 0), \\ A_1u(0) + A_2u(-\tau) & \text{for} \quad \theta = 0. \end{cases} \tag{10}
$$

The nonlinear operator F is in the form of

$$F = (u)(\theta) = \begin{cases} 0 & \text{for} \quad \theta \in [-\tau, 0), \\ F[u(0), u(-\tau)] & \text{for} \quad \theta = 0. \end{cases} \tag{11}$$

Similarly, we can define the dual/adjoint space H^* of continuously differentiable function determine the stability of solutions, are v: $[0, \tau] \to R^2$ with the dual operator

$$A^* v(\sigma) = \begin{cases} -\dfrac{\mathrm{d}v(\sigma)}{\mathrm{d}\sigma} & \text{for} \quad \sigma \in [-\tau, 0), \\ A_1^* v(0) + A_2^* v(\tau) & \text{for} \quad \sigma = 0. \end{cases} \tag{12}$$

H can be split into two subspaces as $H = P_\Lambda \oplus Q_\Lambda$, where P_Λ is a two-dimensional space spanned by the eigenvectors of the operator A associated with the eigenvalues Λ, while Q_Λ is the complementary space of P_Λ. Then for $u \in H$ and $v \in H^*$, we can define a bilinear operator:

$$\begin{aligned} \langle v, u \rangle &= \bar{v}^{\mathrm{T}}(0)u(0) - \int_{-\tau}^{0} \int_{0}^{\theta} \bar{v}^{\mathrm{T}}(\xi - \theta)[\mathrm{d}\eta(\theta)]u(\xi)\,\mathrm{d}\xi \\ &= \bar{v}^{\mathrm{T}}(0)u(0) + \int_{-\tau}^{0} \bar{v}^{\mathrm{T}}(\xi + \theta)A_2(\xi)u(\xi)\,\mathrm{d}\xi. \end{aligned}$$

Corresponding to the critical characteristic root $\mathrm{i}\omega$, the complex eigenvector $q(\theta) \in H$ satisfies

$$\frac{\mathrm{d}q(\theta)}{\mathrm{d}\theta} = \mathrm{i}\omega q(\theta) \quad \text{for} \quad \theta \in [-\tau, 0), \tag{13}$$

$$A_1 q(0) + A_2 q(-\tau) = \mathrm{i}wq(0) \quad \text{for} \quad \theta = 0. \tag{14}$$

The general solution of Eqs. (13) and (14) is $q(\theta) = C\mathrm{e}^{\mathrm{i}w\theta}$.

From the boundary conditions, given by Eq. (14), we can easily find a real basis for P_Λ denoted as $\Phi(\theta) = (\varphi_1, \varphi_2) = (\mathrm{Re}[q(\theta)], \mathrm{Im}[(q(\theta)])$. Similarly, one can find a real basis for the dual space Q_Λ as $\Psi(\sigma) = (\psi_1, \psi_2) = (\mathrm{Re}[q^*(\sigma)], \mathrm{Im}[q^*(\sigma)])$. Then, by defining $w \equiv (w_1, w_2)^{\mathrm{T}} = \langle \Psi, x_t \rangle$, one can decompose x_t into two parts to obtain

$$x_t = x_t^{PA} + x_t^{QA} = \Phi < \Psi, \quad x_t > + x_t^{QA} = \Phi\omega + x_t^{QA}, \tag{15}$$

which implies that the projection of x_t on the center manifold is Φw. Then, applying Eqs. (9) and (15) results in the center manifold:

$$\dot{w} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} + N(w), \tag{16}$$

where $N(w)$ represents the nonlinear terms steming from the original system contributing to the center manifold. The lowest-order nonlinear terms of the center manifold, needed to determine the stability of solutions, are

$$N_3(w) = \Psi^{\mathrm{T}}(0)F(\Phi w) = \begin{pmatrix} C_{30}^1 w_1^3 + C_{21}^1 w_1^2 w_2 + C_{12}^1 w_1 w_2^2 + C_{03}^1 w_2^3 \\ C_{30}^2 w_1^3 + C_{21}^2 w_1^2 w_2 + C_{12}^2 w_1 w_2^2 + C_{03}^2 w_2^3 \end{pmatrix}.$$

Therefore, we obtain the normal form up to third order, $\dot{r} = Lr^3, \dot{\theta} = \omega + br^2$, where L is a Lyapunov coefficient, referred also to as the Lyapunov first quantity (LFQ), given by $L = \frac{1}{8}(3C_{30}^1 + C_{12}^1 + C_{21}^2 + 3C_{03}^2)$. When $L < 0 (> 0)$, the bifurcating limit cycle is stable (unstable).

4. Results

For a consistent comparison with the findings of Ref. [2], some numerical results are presented to investigate the stability with respect to the variations of the time delay τ and the linear and nonlinear feedback gains Ψ_1, Ψ_2, Θ_1, and Θ_2. For ease of comparison, the same parameter values used in Ref. [1,2] are considered in the numerical simulation. The main parameters chosen to vary are Ψ_1, Ψ_2, Θ_1, Θ_2, V and τ. The values of the parameters given in Eqs. (4) and (5) are selected as

$$\begin{array}{lllll} b=1.5, & \mu=50, & \varpi=1, & \gamma_\alpha=0.5, & \chi_\alpha=0.25, \\ \gamma=1, & \kappa=1.4, & \delta_A=1, & B=1, & x_0=0.5, \\ \omega_\alpha=60\,\mathrm{rad/s}, & \varsigma_h=0, & \varsigma_\alpha=0. & & \end{array}$$

Here, we will analyze the stability of the aeroelastic system in the vicinity of the flutter boundary; our goal is to show the implications of the time-delayed feedback control on the stability of flutter boundary. Similar to the study given in Ref. [2], we consider three typical cases: the system with no delay, a fixed delay, and varied delays.

4.1. Case 1: no delay ($\tau = 0$)

(a) *Vary* Ψ_1, *and set* $\Theta_1 = \Theta_1 = 0$ *and* $\Psi_2 = 0$: This case was considered in both Refs. [1,2], where it was shown that the flutter speed increases monotonically with the increase of flight Mach number M_∞ and/or the control gain Ψ_1. In Ref. [2], it was shown that the slopes of the curves are slightly decreasing as Ψ_1 is increasing, and M_{TR}, denoting the Mach number at which the transition from the benign to catastrophic flutter boundary occurs, is smaller for larger values of Ψ_1.

(b) *Vary* Ψ_1, *and set* $\Psi_2 = 0$, $\Theta_2 = 0.45$: In this case, the results of the flutter velocity with respect to flight Mach number for different values of Ψ_1 and the corresponding Lyapunov coefficients are shown in Figs. 2 and 3, respectively. Two cases: $\Theta_1 = 0.05$ and 0.5 are given in each of the two figures. The trend of the flutter speed is similar to that in case (a), in the sense that it increases monotonically with the increase of flight Mach number M_∞ and/or the control gain Ψ_1. For the same value of the flight Mach number and Ψ_1, the flutter speed is smaller than that obtained in case (a), which indicates that feedback control applied in plunging displacement is not beneficial in increasing the flutter speed (see Fig. 2), and this is especially the case when Θ_1 is increasing. Fig. 3 shows that the critical value M_{TR} decreases with the increase of Ψ. The results also reveal that for the same value of Ψ, the critical value M_{TR} in Fig. 3 again indicates that incorporation of larger plunging displacement control, Θ_1, is not beneficial in stabilizing the Hopf-bifurcation, i.e., it does not make the flutter benign. In particular, larger values of Θ_1 give rise to worse situations of instability. In such cases, M_{TR} decreases.

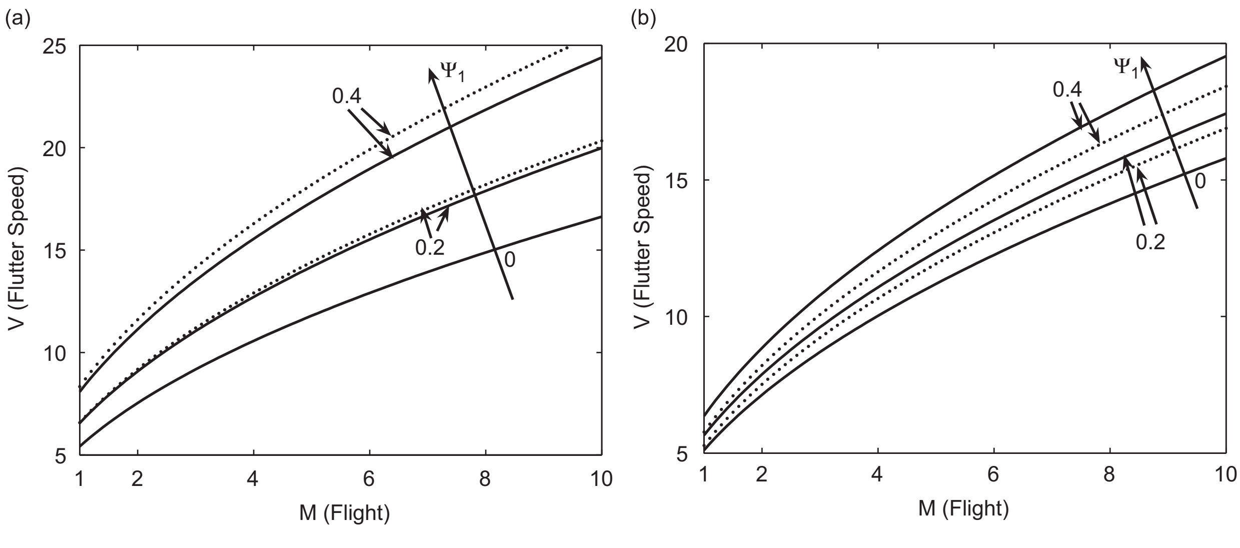

Fig. 2. Effects of the linear control with or without time delay on the flutter boundary for (a) $\Theta_1 = 0.05$ and (b) $\Theta_1 = 0.5$:—— for $\tau = 0$, $\cdots\cdots$ for $\tau \neq 0$.

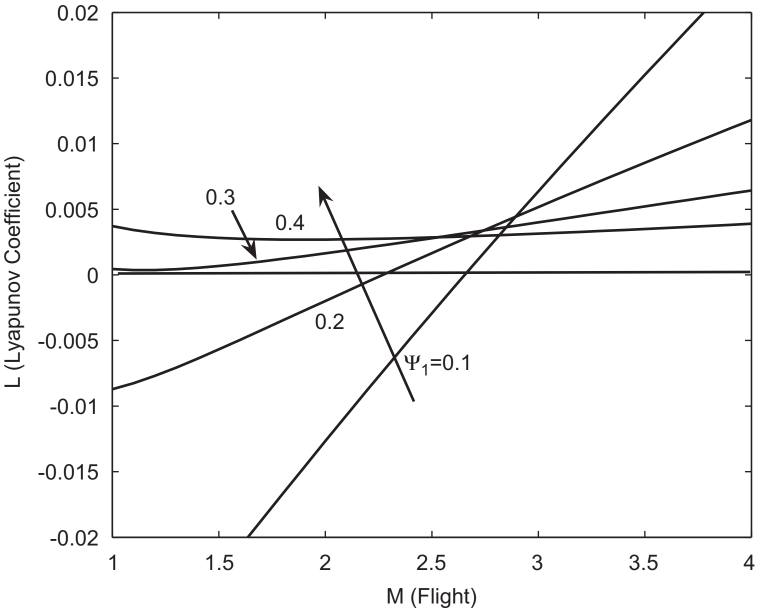

Fig. 3. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 0$, $\tau = 0$ when $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_1 = 0.05$, $\Theta_2 = 0.45$.

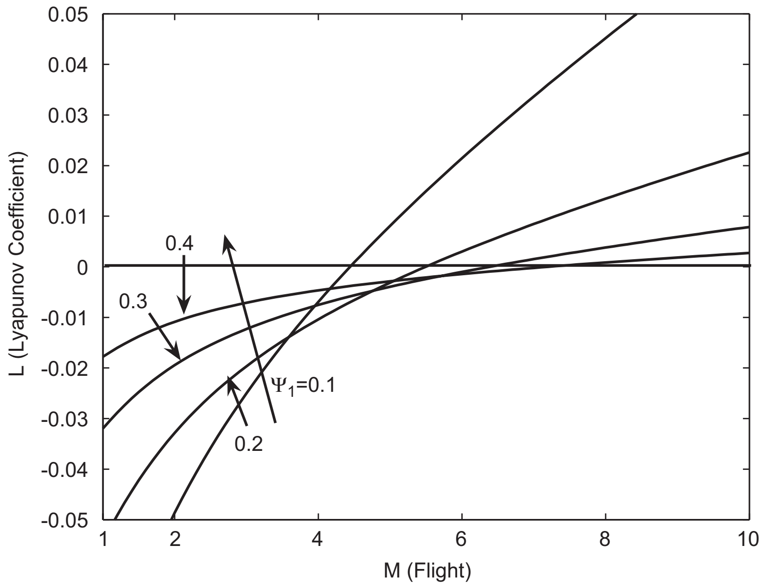

Fig. 4. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 10\Psi_1$, $\tau = 0$ for $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_1 = \Theta_2 = 0$.

(c) $\Psi_2 = 10\Psi_1$: In this case, the presence of Ψ_2 does not change the relation between the flutter velocity and the Mach number because the flutter speed is only determined by linear terms. The Lyapunov coefficients for this case, with and without the presence of plunge displacement control, are depicted in Figs. 4 and 5, respectively. It clearly shows that M_{TR} with plunging displacement control is smaller than that without the plunging displacement control. In both situations, M_{TR} increases with the increase of Ψ_1, and the nonlinear feedback control is more effective than the linear feedback control in rendering the flutter boundary a benign one. Note that Fig. 6 is the same as Fig. 2 in Ref. [2].

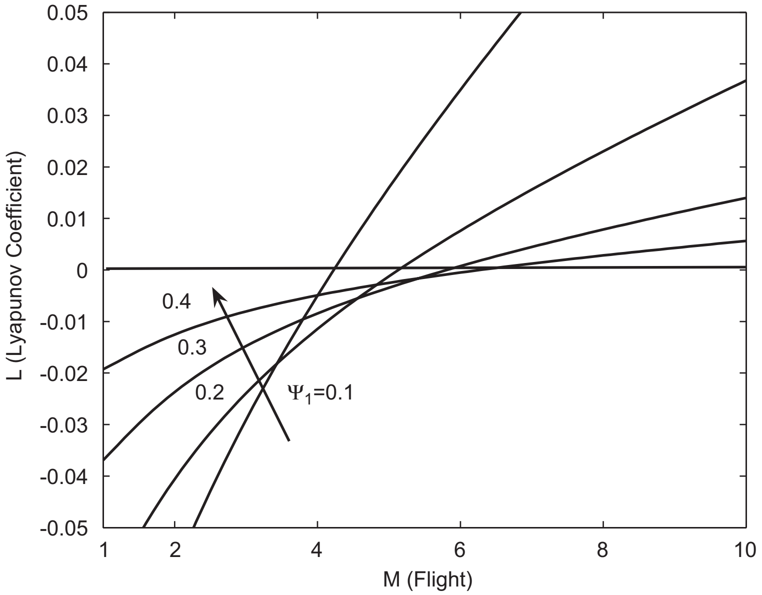

Fig. 5. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 10\Psi_1$, $\tau = 0$ when $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_1 = 0.05$, $\Theta_2 = 0.45$.

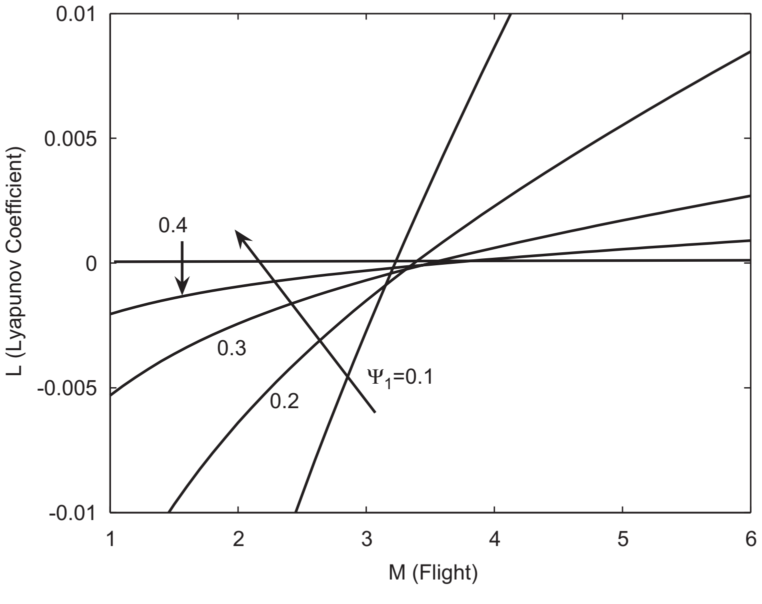

Fig. 6. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 0$, $\tau = 1$ for $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_1 = \Theta_2 = 0$.

4.2. Case 2: fixed delay

The time delay τ is fixed, but Ψ_1 is varied. Note that the time delay given in Eq. (9) is dimensionless, the real time delay is $\hat{\tau} = \tau\omega_\alpha$. We fix $\tau = 1$ and investigate the effects of Ψ_1 and Ψ_2 with and without plunging controls on the flutter stability boundary.

$\Psi_2 = 0$: The changes in flutter speed V_F with variation of Ψ_1 are shown in Fig. 2. They are similar to the case without time delay. V_F is a monotonically increasing function of Ψ_1 and flight Mach number M_∞. In addition, when only delayed pitching displacement control is considered, the value of V_F for any (Ψ_1, M_∞)

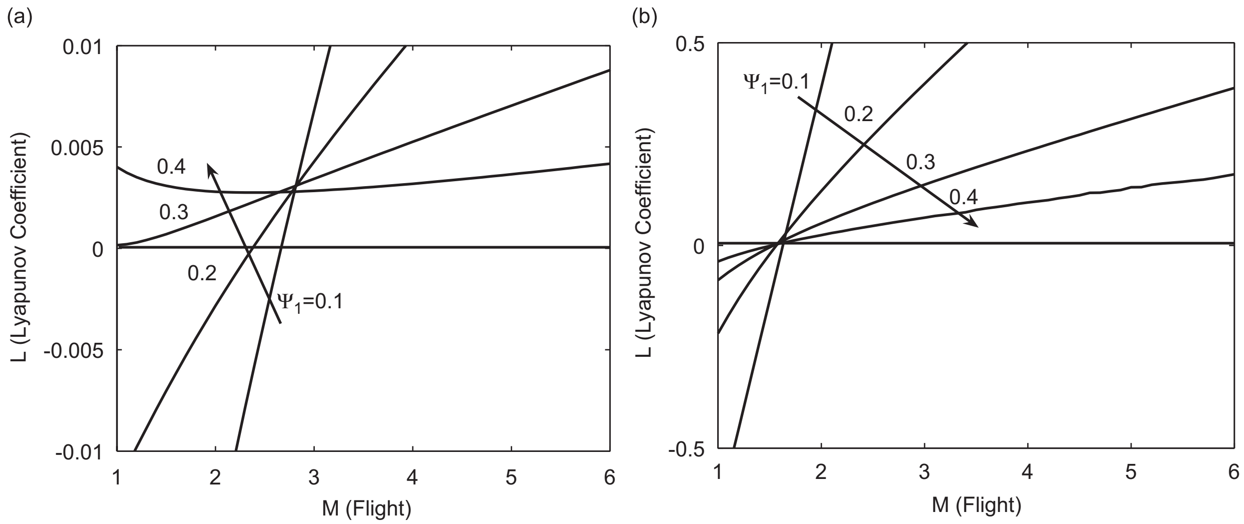

Fig. 7. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 0$, $\tau = 1$ when $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_2 = 0.45$ for (a) $\Theta_1 = 0.05$; and (b) $\Theta_1 = 0.5$.

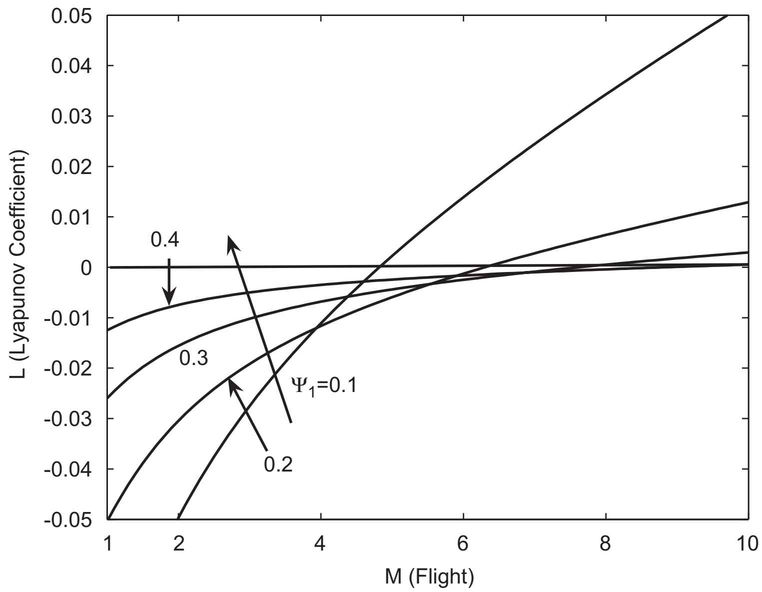

Fig. 8. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 10\Psi_1$, $\tau = 1$ for $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_1 = \Theta_2 = 0$.

experiences an increase, as compared to the case when the time delay is absent. On the other hand, as shown in Fig. 2, when there is a plunging displacement control in the presence of time delay, V_F is smaller than that in the case of no time delay. This indicates that the time delay is not beneficial in delaying the occurrence of flutter (Hopf bifurcation) if a plunging displacement control is applied. It can also be observed from Fig. 2 that the combination of using time delay with the plunging displacement control may be acceptable (see Fig. 4) for a small value of Θ_1, but becomes worse for a large value of Θ_1 (Fig. 4), compared with the case of no time delay ($\tau = 0$). Further, it is seen from Fig. 2 that the effect of τ becomes more prominent for larger values of Φ_1. The Lyapunov coefficients with and without plunging displacement in this case are shown in Figs. 6 and 7, respectively. From Fig. 6 it is easily seen that in the absence of the plunging displacement control, the M_{TR}

increases when the time delay is present. When a plunging displacement control is applied, the situation is much more complex, in the sense that the curve of the Lyapunov coefficient crosses the zero line ($L = 0$) much earlier when Ψ_1 is larger, in particular, if a small plunging displacement control is applied (e.g. $\Theta_1 = 0.05$, as shown in Fig. 7(a)). This indicates that time delay is (is not) beneficial to stabilize Hopf bifurcation for small (large) Ψ_1 for this case.

$\Psi_2 = 10\Psi_1$: The results obtained in this case are shown in Figs. 8 and 9. The effect of the nonlinear control combined with the time delay can be clearly observed from these two figures. Again, it is confirmed that the nonlinear control (Ψ_2) is helpful to stabilize the vibrating motion. By comparing results in Figs. 6–9, respectively, it can be seen that the time delay helps to enhance the character of the flutter boundary, when it is combined with nonlinear control (Ψ_2), if no plunging displacement control exists. Again, it is shown that the time delay is not beneficial when a large plunging displacement control is used. It is noted that results shown in Fig. 8 are similar to results in Fig. 5 of Ref. [2].

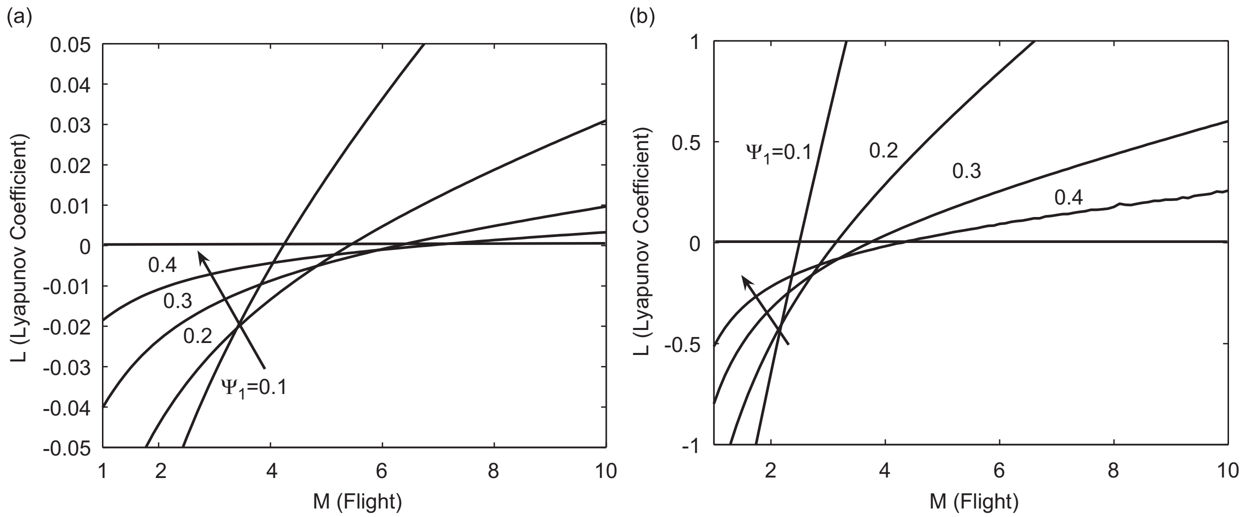

Fig. 9. Benign and catastrophic flutter boundary, Lyapunov first quantity, corresponding to $\Psi_2 = 10\Psi_1$, $\tau = 1$ when $\Psi_1 = 0.1, 0.2, 0.3, 0.4$ and $\Theta_2 = 0.45$ for (a) $\Theta_1 = 0.05$; and (b) $\Theta_1 = 0.5$.

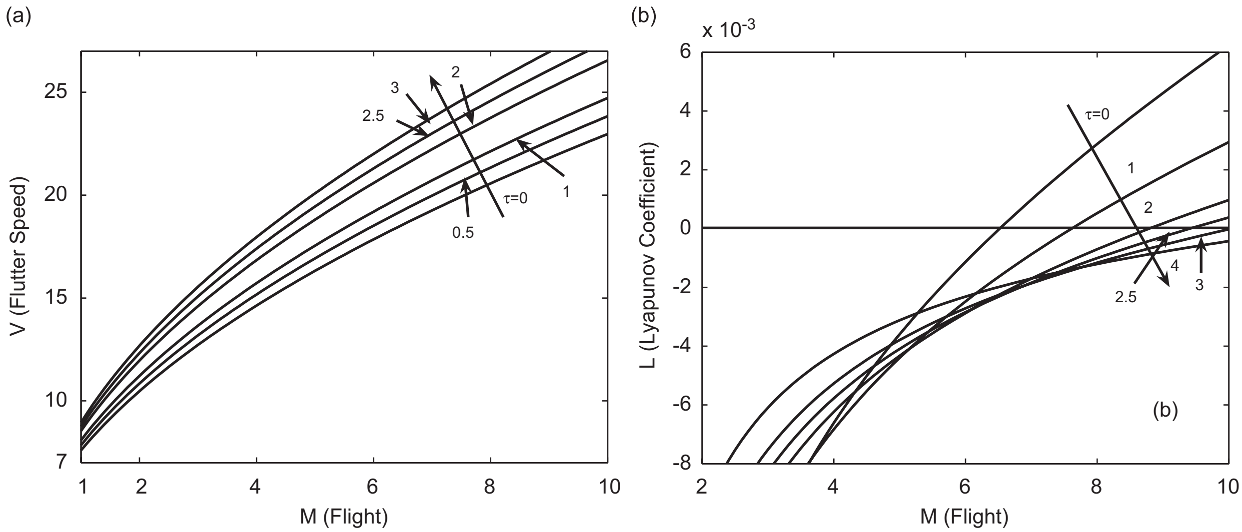

Fig. 10. Effect of the time delay for $\Psi_1 = 0.3$ and $\Theta_1 = \Theta_2 = 0$: (a) on the flutter boundary; and (b) on the Lyapunov coefficient for $\Psi_2 = 3$.

4.3. Case 3: varied delay

Ψ_1 is fixed, but $\tau \neq 0$ is varied. We consider the variation of the time delay τ from 0 to 4, and fix $\Psi_1 = 0.3$. For simplicity, we set $\Psi_2 = 10\Psi_1$. First, in the absence of the plunging displacement control, the effect of time delay on the flutter speed with respect to the variation of the flight Mach number is shown in Fig. 10(a). V_F increases monotonically with the increase of M_∞ and τ. It has been reported in Ref. [2] that a jumping in V_F occurs, for $\tau \in (2.33, 2.73)$. The flutter speed increases when the flight Mach number increases until a critical value, where the flutter speed decreases suddenly to a small value (see Figs. 7–9 in Ref. [2]). The Lyapunov coefficient presented in Fig. 10(b) reveals that the time delay combined with nonlinear control in Ψ_2 is also beneficial in stabilizing the flutter motion when no plunging displacement control is applied. On the other hand, when the plunging displacement control is applied, the effect of time delay on the flutter speed with respect to the variation of the flight Mach number and the Lyapunov coefficient are much more complex, as shown in Figs. 11 and 12, respectively. It can be seen from Fig. 11(a) and (b) that for a small value of Θ_1 the trend is still consistent, which is similar to that observed in Fig. 10(a). However, for a larger value of Θ_1, from Figs. 11(c) and (d) it can be seen that with the increase of τ, V_F decreases for the same flight Mach number when $0 < \tau < 1.5$, and V_F increases with the increase of τ when $\tau > 1.5$. For the stability of the Hopf bifurcation,

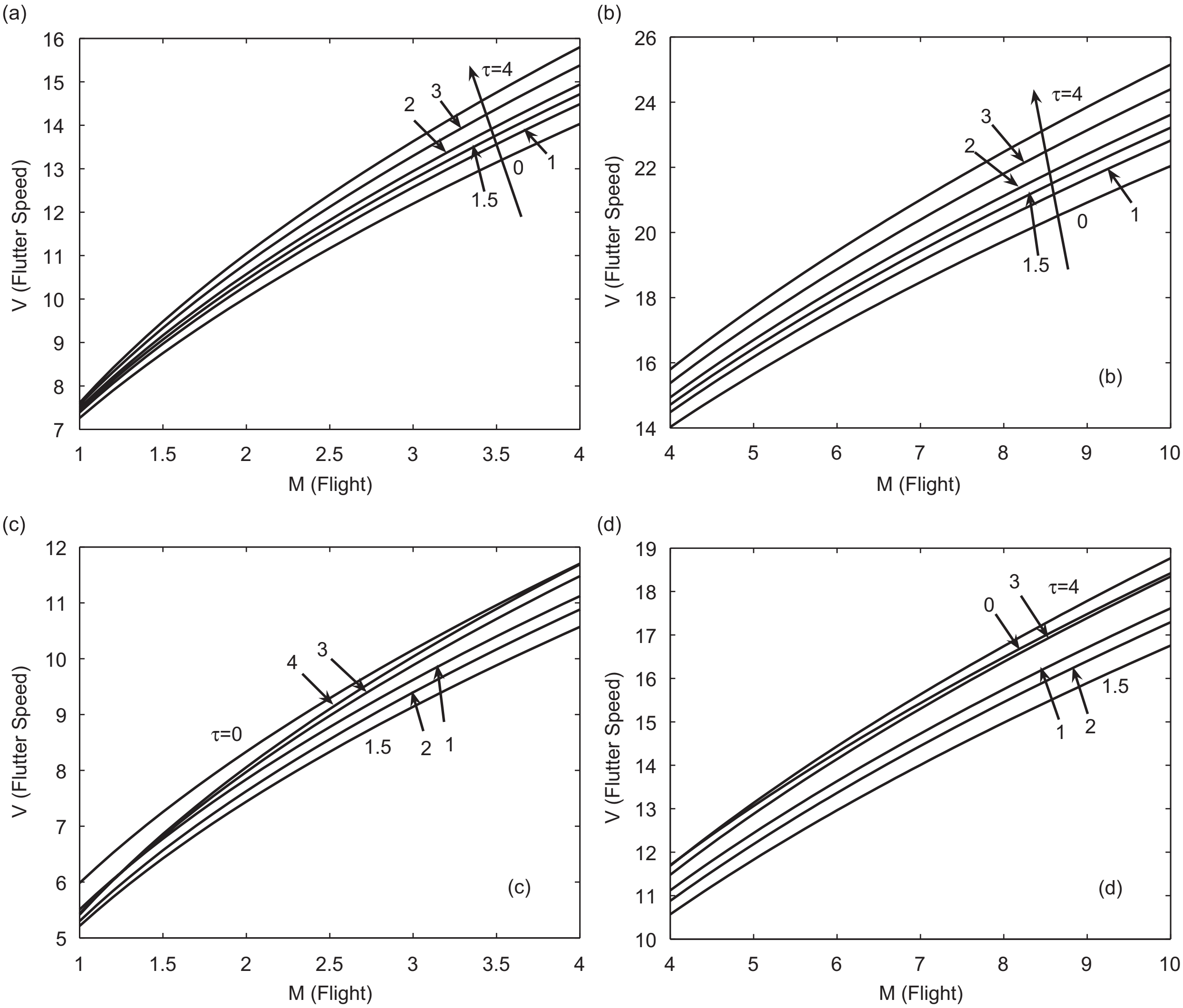

Fig. 11. Effects of the time delay on the flutter boundary when $\Psi_1 = 0.3$ for (a) and (b) $\Theta_1 = 0.05$; and (c) and (d) $\Theta_1 = 0.5$.

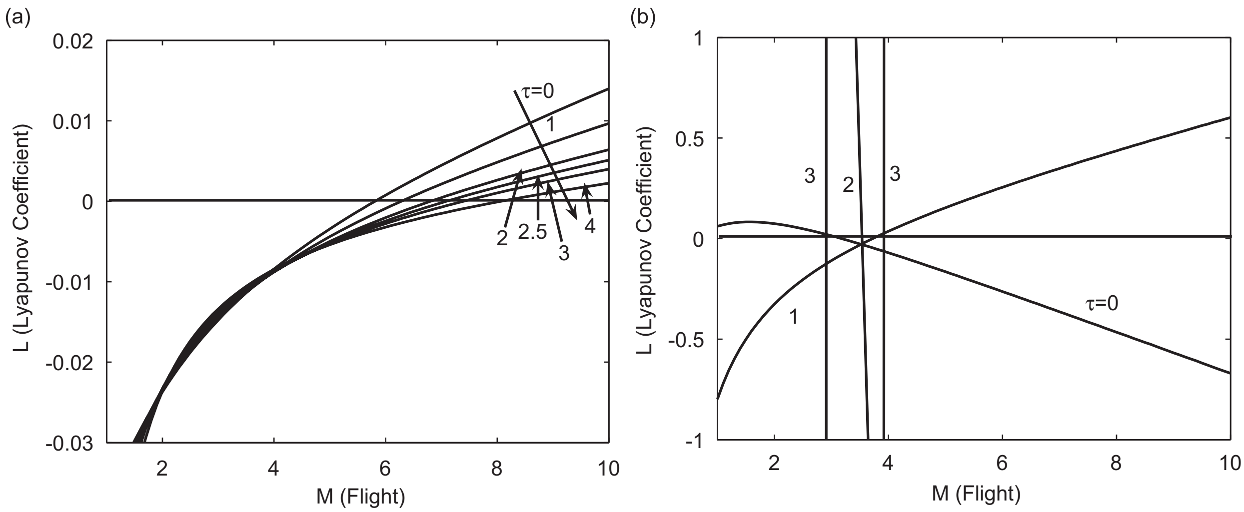

Fig. 12. Effects of the time delay on the Lyapunov coefficient when $\Psi_1 = 0.3$, $\Psi_2 = 3$ and $\Theta_2 = 0.45$ for (a) $\Theta_1 = 0.05$ and (b) $\Theta_1 = 0.5$.

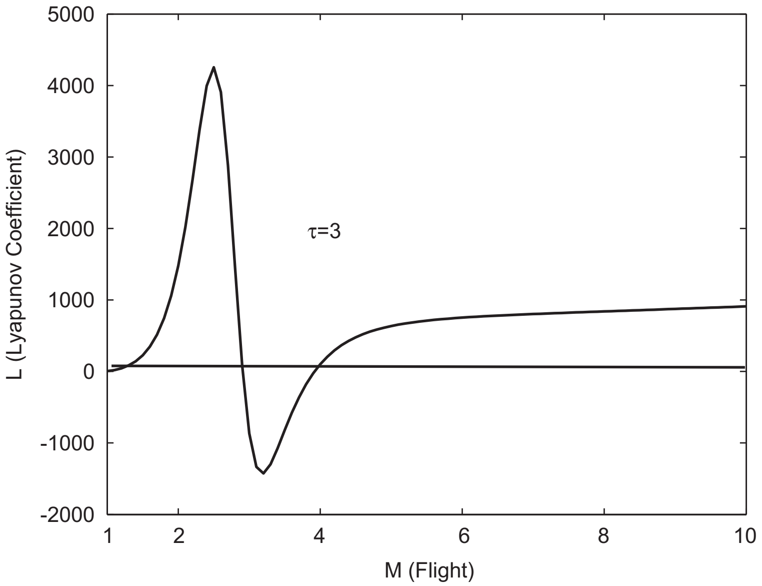

Fig. 13. Lyapunov coefficient for $\tau = 3$, $\Psi_1 = 0.3$, $\Psi_2 = 3$, $\Theta_1 = 0.5$ and $\Theta_2 = 0.45$.

it is seen from Fig. 12(a) that even for a small value of Θ_1, the behavior of the system is quite complex in the sense that the order of the trend with respect to the time delay τ changes twice, around the flight Mach number 2 and 4, respectively. The situation becomes even worse when a large plunging displacement ($\Theta_1 = 0.5$) is applied, as shown in Fig. 12(b), from which we observe that the change of the Lyapunov coefficient is much more complex. For moderate values of τ (e.g. $\tau = 1.5$), the Lyapunov coefficient is negative for any flight Mach number, which suggests that a moderate time delay is beneficial, i.e., it helps to stabilize limit cycle oscillations. A large time delay, usually results in a positive Lyapunov coefficient implying an unstable limit cycle oscillation. The Lyapunov coefficient is negative only in a small range of the flight Mach number (see Fig. 13), which shows that a large time delay may change a benign vibration in to a catastrophic one. Thus, when the aim is to control both the initiation of the Hopf bifurcation and the stability of bifurcation motion, a large time delay is not beneficial.

5. Conclusion

In this paper, the aeroelastic instability in the vicinity of the flutter boundary for a two-dimensional supersonic lifting surface with proportional feedback control was addressed. The effect of time-delayed feedback control on the flutter instability boundary and its character was investigated. It was shown that when pitching control is involved, time delay is beneficial in postponing the initiation of a Hopf bifurcation. However, with plunging displacement control, the effect of time becomes more intricate, in the sense that it can advance the initiation of the Hopf bifurcation. In both cases, the effect of time delay becomes more prominent for large linear control gains. Also, when plunging displacement control is applied, time delays may destabilize bifurcation motions. Thus, based on the study given in this paper, in order to control both the initiation of Hopf bifurcation and the stability of bifurcation motion, effectively, one should apply both linear and nonlinear controls with small time delay, and avoid using plunging displacement controls.

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