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Misalignment in rigidly coupled rotors

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Abstract

Misalignment of multibearing rotor systems is one of the most common fault conditions yet it is still not fully understood. There are numerous (and sometimes confusing) accounts in the literature asserting the presence of harmonics in the vibration signal, but no quantitative descriptions are offered. Harmonics may arise, of course, from the nonlinearities in fluid film journal bearings or from the kinematics of flexible couplings, but in this paper only rigidly coupled rotors mounted on idealised linear bearings are considered. It is shown that even for this case, excitation at twice synchronous speed is developed and an expression for the magnitude and phase of the response is derived. Several examples are then studied to give some insight into the magnitude of these harmonic terms which can arise. It is argued that it is precisely because the harmonic terms can arise from diverse sources, that a full description of the phenomena has proved somewhat elusive.

1. Introduction

Rotor misalignment is one of the most common difficulties in the operation of rotating machinery and is at the heart of mechanical engineering, yet it remains incompletely understood. Despite the rapid increase in understanding of rotor dynamics, no satisfactory analysis explains the range of observed phenomena. There are, in the literature, reports of vibration signals at twice rotational speed and higher harmonics, yet other authors who report only synchronous excitation. Even recent discussions fail to reach consensus on the true nature of the phenomena. At a time when rotor dynamics may be regarded as a mature technology, misalignment remains as an outstanding area, where basic understanding is somewhat lacking.

Part of the difficulty may arise because the single label of 'misalignment' covers a range of situations and the resulting behaviour arises from a combination of physical processes. Some authors have resorted to detailed nonlinear studies in an attempt to understand the nature of the vibrations exhibited by a misaligned machine, but this is not considered an appropriate method. To gain an understanding an attempt is required to understand the basic phenomena. Clearly, harmonics of shaft speed can be generated by any nonlinearity of the system, which may be in a coupling or bearing. Before examining this, however, attention is focused on a simple system with no nonlinear components.

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Nomenclature

j	bolt reference index
I_1, I_2	polar moments of inertia
N	number of coupling bolts
K_b	effective stiffness of each bolt
$\mathbf{K}, \mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3$	component of stiffness matrix
$\mathbf{K}_s$	stiffness matrix of shaft
$\mathbf{K}_c$	constant stiffness contribution of coupling
$\Delta\mathbf{K}(t)$	time-dependent stiffness term
k_x, k_y	shaft stiffness in the 2 orthogonal directions (referred to each rotor if added suffix)
$\mathbf{M}$	mass matrix
r	radius of coupling bolt positions on reference shaft
R_j	radius position of bolt j on second shaft
X_1, Y_1	vibration levels of shaft 1
X_2, Y_2	vibration levels of shaft 2
X, Y	vibration differentials
U_{tor}	stored energy for torsion only case
U	total stored energy
α_j	angular position of bolt j on first shaft
φ_j	angular position of bolt j on second shaft
δ	vertical misalignment at the coupling
ε	torsional displacement
Ω	reference rotor speed
ϕ	total rotation of shaft 1
θ	total rotation of shaft 2

Most authors have sought to explain the harmonic excitation in terms of the nonlinearity of either the bearings and/or flexible couplings in the system, and indeed it is to be expected that these elements will play a part in the dynamics of a real system. Dewell and Mitchell [1] gave a discussion of the harmonics arising in flexible couplings. Al-Hussain and Redmond [2] developed a set of nonlinear equations describing the motion of a misaligned system. However, they report no twice per rev component of vibration. Ref. [3] also reports a full nonlinear analysis which does show nonsynchronous excitation under the appropriate conditions. But the source of these harmonics is the nonlinearities of the system. It is shown in the current paper that a misaligned system can give rise to harmonic terms even when mounted in a purely linear way.

In a pair of papers, Xu and Maragona [4,5] have given an analysis of a system including a flexible coupling and have backed up their predictions with laboratory experiments on a rig. Here again, however, the source of the super harmonic components in the vibration signal emanates from the nonlinear behaviour of the flexible coupling.

In the following section, we develop the simplest possible model for two shafts which are rigidly coupled together. The basic model will be developed in Section 2, whilst the resulting motion for several different scenarios is evaluated in Section 3. Section 4 gives an overview of factors influencing the dynamics of a misaligned system some indication of the experimental work proposed as a next step to the ideas discussed in the present paper.

2. Model development

The case considered comprised two rotors, which are not co-axial but have a relative displacement δ. The two rotors are connected by a series of N bolts, which on the first shaft are distributed around the perimeter at some radius r from the axis of the shaft. On the second shaft, however, if the bolts are to fit easily, the location holes must be positioned on a circle whose centre is offset by δ from the axis of the second rotor, this parameter δ being the extent of the parallel offset or misalignment. Initially, it is assumed that the two portions of the assembly rotate at the same speed, but simple geometry dictates that the two pairs of holes cannot remain aligned since they rotate about different axes.

Fig. 1 illustrates the situation for the case of a coupling with three bolts. The two shafts are assumed to rotate at the same rate but about axes which are displaced. The location points attached to the larger coupling are denoted by diamonds whilst those on the opposing shaft are shown as circles. The figure shows the positions of the bolt locations on both shafts at a number of instants during one full cycle. It can be seen that points move relative to each other generating both forces and moments which vary in time.

On the flange of rotor 1, the coupling bolts will be arranged about the centre. Let the number of bolts be N, equally positioned around the circumference of the first rotor. Then at time zero, the position of

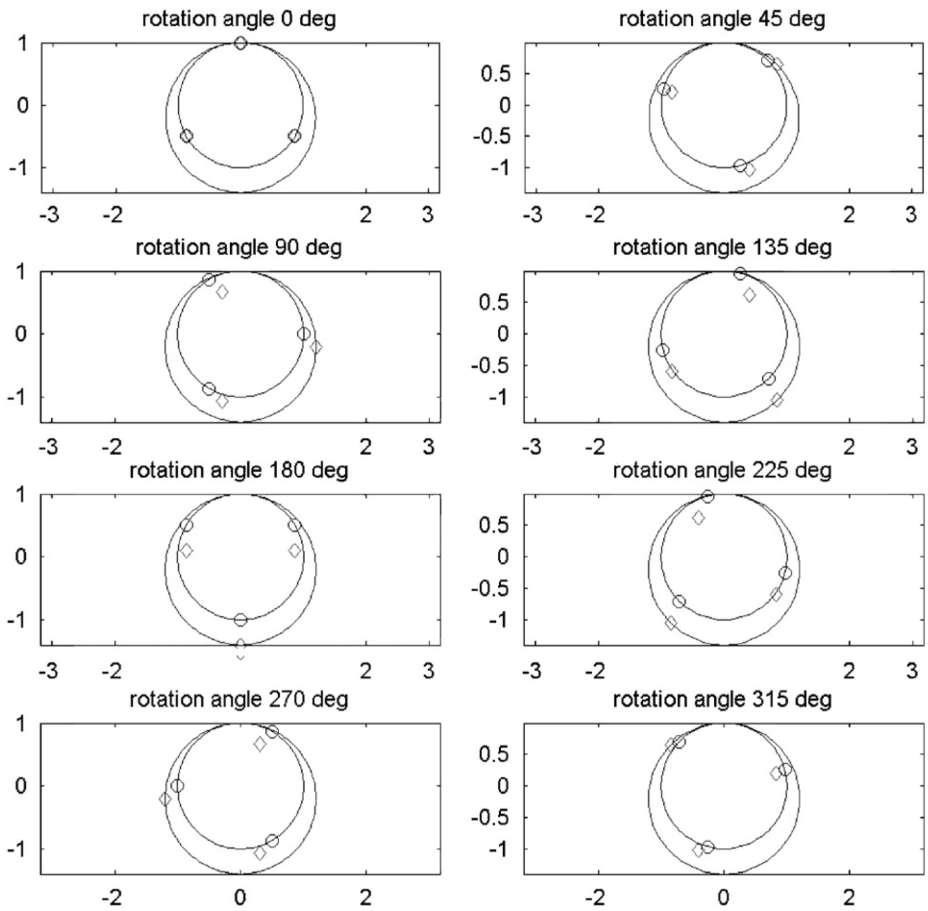

Fig. 1. Orientation of coupling bolts.

bolt j is given by

$$\begin{matrix} x_j = r\cos((j-1)\alpha) \\ y_j = r\sin((j-1)\alpha) \end{matrix} \quad \text{where } \alpha = \frac{2\pi}{N} \quad \text{or more simply} \quad \begin{matrix} x_j = r\cos\alpha_j, \\ y_j = r\sin\alpha_j \end{matrix} \tag{1}$$

(A full list of the notation use is given for reference in the nomenclature). On the other rotor, the holes are not distributed around the centre, but offset. Simple geometry shows that the position of the jth bolt relative to the centre of this rotor is

$$X_j = R_j\cos\varphi_j \begin{Bmatrix} X_j \\ Y_j \end{Bmatrix} = R_j \begin{Bmatrix} \cos\bar{\varphi}_j \\ \sin\varphi_j \end{Bmatrix} + \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} 0 \\ \delta \end{Bmatrix}, \tag{2}$$

where

$$R_j = r\sqrt{(1+\delta^2+2\delta\cos((j-1)\alpha)}, \quad \varphi_j = \tan^{-1}\left(\frac{\delta + r\sin((j-1)\alpha)}{r\cos((j-1)\alpha)}\right). \tag{3}$$

To start with an initially simply model, assume that the two rotors rotate at speeds $\Omega, \dot{\phi}$, respectively, where Ω is taken to be constant (but this restriction can be easily relaxed). Then the kinetic energy is

$$T = \tfrac{1}{2}I_1\Omega^2 + \tfrac{1}{2}I_2\dot{\phi}^2 \tag{4}$$

and the potential energy is

$$U_{\mathrm{tor}} = \frac{K_b}{2}\sum_{j=1}^{N}[(r\cos(\alpha_j + \Omega t) - R_j\cos(\varphi_j + \phi) - \delta\sin\phi)^2] + \frac{K_b}{2}\sum_{j=1}^{N}(r\sin(\alpha_j + \Omega t) - R_j\sin(\varphi_j + \phi) - \delta\cos\phi)^2. \quad (5)$$

Note the locus of the coupling bolts: those on rotor 1 simply follow a single circle, whereas those on rotor 2 each follow a circle of different diameters as shown by Eq. (3). The first shaft is considered to be rigid and has a very large torsional inertia. The location for the single coupling bolt is perfectly positioned at radius r on rotor 1. The second rotor has torsional inertia I_2 and stiffness values k_x,k_y and this rotor has displacements u,v,φ. The arrangement is shown in Fig. 2. This model, considerably simplifies the algebra, but retains the essential physics of the situation. Clearly, the degree of misalignment is substantially exaggerated for the sake of clarity.

The model of the system considered here is shown in Fig. 3. Two rigid rotors, mounted on flexible bearings, are connected by means of a set of N bolts, which have finite stiffness. The axes of the two rotors are separated by δ vertically, and it is the purpose of this analysis to examine the effect of this misalignment (taken to be in the vertical direction) on the dynamics of the system. The system is assumed to be perfectly balanced and hence the only exciting forces arise from the misalignment: more specifically they arise from varying forces in the coupling bolts. It is assumed that in the frequency range of interest, only the first lateral modes of the two rotors, together with the torsional modes, play a part in the motion.

It is assumed that the bolt holes on the flange of the first rotor are arranged around a circle centred on the centre of the shaft cross section, but on the second rotor the bolts holes are again positioned on a circle although the centre of this circle is displaced by δ from the centre of the rotor. Hence, at zero angle, the bolts

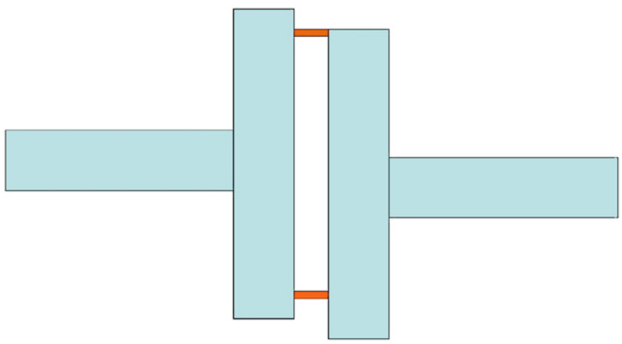

Fig. 2. Idealised coupling.

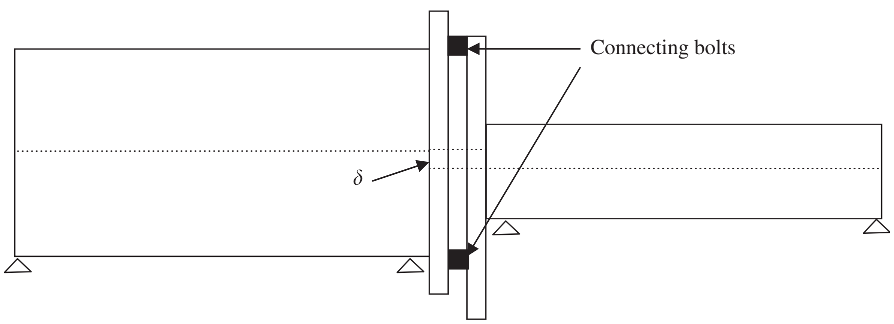

Fig. 3. Model arrangement.

joining the two flanges have no strain, but at all other angles there will be some strain energy in the connecting bolts.

The analysis of the motion commences with an evaluation of the energy stored within the coupling bolts. Recalling that the first shaft rotates at constant speed Ω, whilst that of the second shaft can vary, the potential energy of the bolts is given by

$$U = \frac{K_b}{2}\sum_{j=1}^{N}(R_j\cos(\varphi_j+\theta)+X_2-r\cos(\alpha_j+\phi)-X_1)^2 + \frac{K_b}{2}\sum_{j=1}^{N}(R_j\sin(\varphi_j+\theta)+Y_2-\delta-r\sin(\alpha_j+\phi)-Y_1)^2. \tag{6}$$

This may be re-written as

$$\begin{aligned} 2U/K_b = &\sum_{j=1}^{N}[R_j^2\cos^2(\varphi_j+\theta)+(X_2-X_1)^2+r^2\cos^2(\alpha_j+\phi)+2(X_2-X_1)R_j\cos(\varphi_j+\theta)] \\ &-\sum_{j=1}^{N}2R_jr\cos(\alpha_j+\phi)\cos(\varphi_j+\theta) \\ &-\sum_{j=1}^{N}[2(X_2-X_1)r\cos(\alpha_j+\phi)-R_j^2\sin^2(\varphi_j+\theta)-(Y_2-Y_1-\delta)^2-r^2\sin^2(\alpha_j+\phi)] \\ &+\sum_{j=1}^{N}\Big[2(Y_2-Y_1-\delta)R_j\sin(\varphi_j+\theta)-2R_jr\sin(\alpha_j+\phi)\sin(\varphi_j+\theta)-2(Y_2-Y_1-\delta)r\sin(\alpha_j+\phi)\Big] \end{aligned} \tag{7}$$

This rather clumsy expression can now be simplified to give

$$U=\frac{K_b}{2}\sum_{j=1}^{N}[R_j^2+X^2+r^2+(Y-\delta)^2+2X\delta\sin\theta-2(Y-\delta)\delta\cos\theta-2R_jr\cos(\varphi_j-\alpha_j+\theta-\phi)], \tag{8}$$

where $X=X_2-X_1$ and $Y=Y_2-Y_1$

We can now progress by recalling that

$$\theta=\Omega t+\varepsilon, \tag{9}$$

where ε is the angle of torsional deflection so that, taking first-order small quantities

$$\sin\theta=\varepsilon\cos\Omega t+\sin\Omega t,$$
$$\cos\theta=\cos\Omega t-\varepsilon\sin\Omega t.$$

Since the bolt positioning error is on rotor 1, it is clear that

$$\sum_{j=1}^{N}R_j\cos\varphi_j=0,\quad \sum_{j=1}^{N}R_j\sin\varphi_j=\frac{N\delta}{2}. \tag{10}$$

We now apply Lagrange's equation to the six degrees of freedom and after a little manipulation, the equations of motion can be expressed in the form

$$\mathbf{M}\ddot{z}+\mathbf{K}_s z+\mathbf{K}_c z+\Delta\mathbf{K}(t)z=F(t). \tag{11}$$

The displacement vector takes the form $z = \{X_1 \quad Y_1 \quad \theta \quad X_2 \quad Y_2 \quad \phi\}^{\mathrm{T}}$ and the mass matrix is given by

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_2 \end{bmatrix}. \tag{12}$$

But the stiffness matrices are in three components. The shaft stiffness matrix for this simple case is given by

$$\mathbf{K}_s = \begin{bmatrix} k_{x1} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{y1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{x2} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{y2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{13}$$

Whilst the steady contribution of the coupling is described by

$$\mathbf{K}_c = \begin{bmatrix} NK_b & 0 & 0 & -NK_b & 0 & 0 \\ 0 & NK_b & 0 & 0 & -NK_b & 0 \\ 0 & 0 & NK_b r^2 & 0 & 0 & -NK_b r^2 \\ -NK_b & 0 & 0 & NK_b & 0 & 0 \\ 0 & -NK_b & 0 & 0 & NK_b & 0 \\ 0 & 0 & -NK_b r^2 & 0 & 0 & NK_b r^2 \end{bmatrix}. \tag{14}$$

There is also, however, a fluctuating component arising from the coupling which is given by

$$\Delta\mathbf{K}(t) = \frac{NK_b\delta}{2}\begin{bmatrix} 0 & 0 & \sin\Omega t & 0 & 0 & -\sin\Omega t \\ 0 & 0 & \cos\Omega t & 0 & 0 & -\cos\Omega t \\ \sin\Omega t & \cos\Omega t & 0 & -\sin\Omega t & -\cos\Omega t & 0 \\ 0 & 0 & -\sin\Omega t & 0 & 0 & \sin\Omega t \\ 0 & 0 & -\cos\Omega t & 0 & 0 & \cos\Omega t \\ -\sin\Omega t & -\cos\Omega t & 0 & \sin\Omega t & \cos\Omega t & 0 \end{bmatrix}. \tag{15}$$

The forcing term in Eq. (11) comprises both internal and external components. For the case in which there are no external forces and the excitation arises solely from the geometry of the coupling

$$F = \frac{NK_b\delta}{2}\begin{Bmatrix} \cos\Omega t \\ \sin\Omega t \\ \delta \\ -\cos\Omega t \\ -\sin\Omega t \\ -\delta \end{Bmatrix}. \tag{16}$$

Recognising that for the complete rotor (i.e. coupled), the dynamic behaviour is described by the equation of motion

$$\mathbf{M}\ddot{z} + \mathbf{K}z + \Delta\mathbf{K}(t)z = F(t), \tag{17}$$

where $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_c$.

Having established the equations of motion for the system, a number of scenarios can be identified. The first point to note is that the equations are linear but have some time varying coefficients (arising from the rotary motion). The other major feature to note is that the torsional and flexural motions become coupled. The torsional excitation arising in the coupling will be of order δ^2 and this will feed back into the flexural motion. The three cases to be considered as

(a) A system with an ideal coupling with the torsional excitation from the coupling.
(b) A machine having other torsional fluctuations at synchronous frequency.
(c) A system with a faulty coupling in which terms in the equations of motion change very substantially.

3. Case studies

Some specific cases are now presented to gain some understanding of the behaviour of the model derived above. To illustrate performance features a simplified model with only 3 degrees of freedom has been considered. This has been achieved by considering the first rotor to have infinite mass and inertia. The second rotor was taken to have a mass of 100 kg and a radius of gyration of 0.3 m. The damping of the lateral mode was 1% whilst that of the torsion mode was 0.1% reflecting the observation that many torsional modes are very lightly damped in real machines. This example rotates at 1200 rev/min, which is also its torsional natural frequency, and the lateral resonances occurs at 40 Hz. The degree of misalignment in these studies is 0.5 mm.

Fig. 4 shows the vibrational response with the rotor running at 1200 rev/min. Note that the forcing arises purely from the eccentricity of machining on one of the coupling faces of 0.1 mm. No mass unbalance has been applied to the rotor (although this would be an interesting simulation to examine at a later stage). Although the figure is on a logarithmic scale, the 2 × component of excitation is significant, being of order 2 μm.

The spectrum changes somewhat if there is some torsional excitation (not emanating from the coupling). Fig. 5 shows the situation with a synchronous torsional moment added. Such a torque fluctuation at 1/rev may well arise in many turbo-machines. In the case studied here, the fluctuation is 390 N m at synchronous speed.

The situation changes substantially if the coupling is faulty, in addition to the alignment error. If, for instance one of the bolts fails to take load, this changes the effective terms in the equations of motion. In

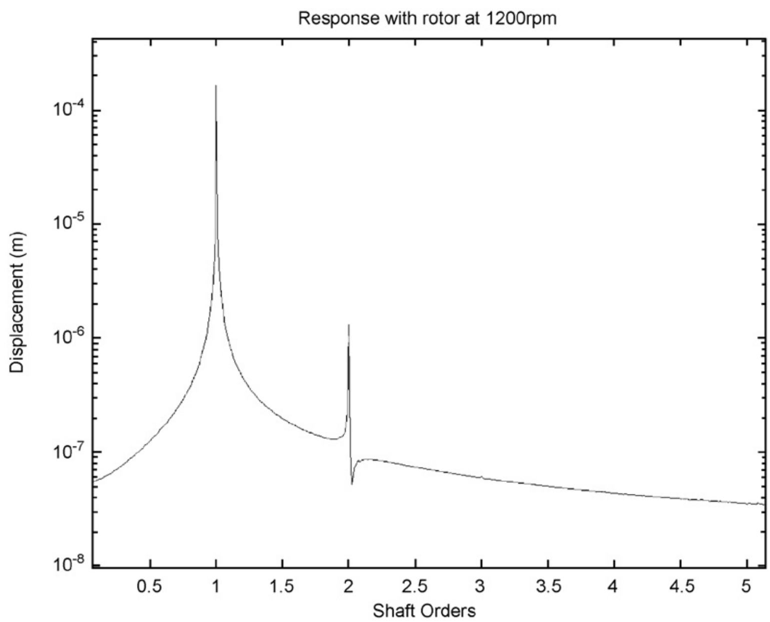

Fig. 4. Response with rotor at 1200 rev/min.

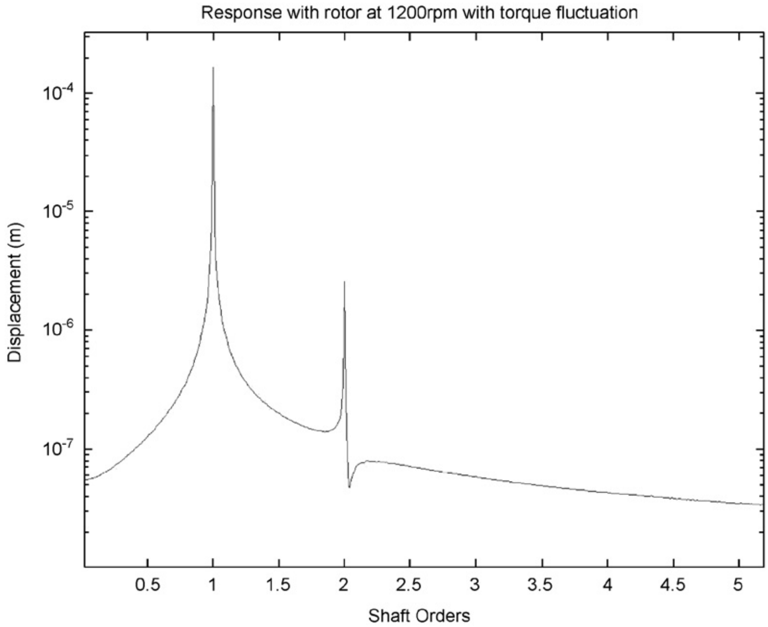

Fig. 5. Response with rotor at 1200 rev/min with torque fluctuation.

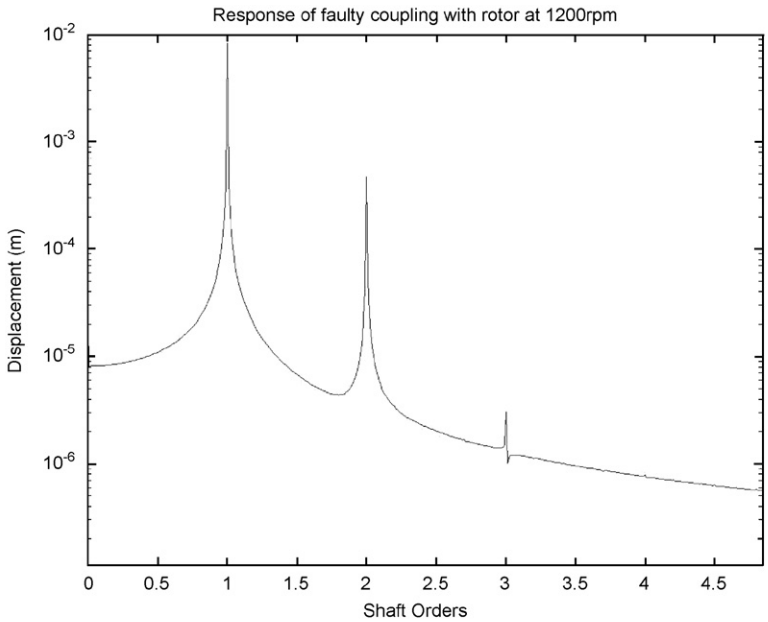

Fig. 6. Response of faulty coupling with rotor at 1200 rev/min.

particular, with one failed bolt equation (10) may be replaced by

$$\sum_{j=1}^{N} R_j \cos\varphi_j = 0, \quad \sum_{j=1}^{N} R_j \sin\varphi_j = \frac{N\delta}{2} - R_1. \tag{18}$$

Inserting this into the model yields the spectrum shown in Fig. 6. Again the various harmonic terms are clear. In this case, no external excitation was given torsionally. The excitation arises solely from the geometry

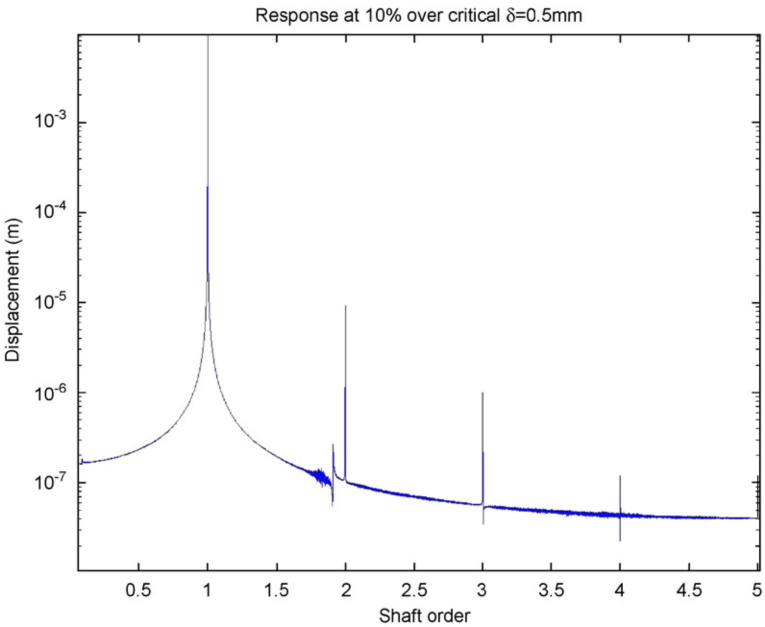

Fig. 7. Response at 10% over critical $\delta = 0.5\,\text{mm}$.

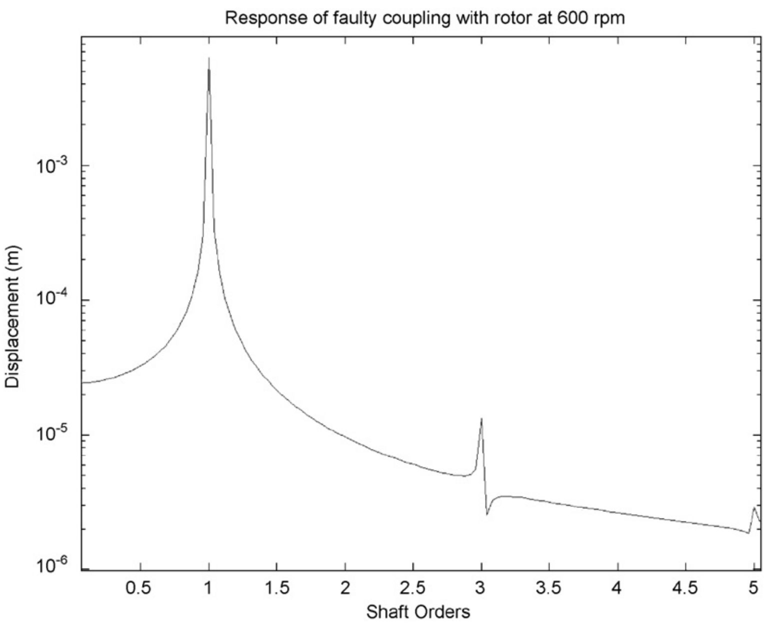

Fig. 8. Response of faulty coupling with rotor at 600 rev/min.

of the coupling. The coupling is considered faulty, and one of the bolts was, in effect, eliminated from the model. Note that whilst this might be considered to be a fairly extreme fault, a coefficient could be conveniently introduced to represent a weakened bolt.

The situation changes if the machine speed is now increased by 10% for this same case with a faulty coupling. The result is shown in Fig. 7. A series of harmonics now arise. The presence of a peak at the flexural natural frequency is just a transient, which would disappear in a longer simulation.

Finally, Fig. 8 shows the situation where the rotor is running at half the torsion critical speed (and quarter of the flexural). There is now significant three times per revolution excitation, of order 10 μm, but nothing observable at twice shaft speed.

4. Discussion

This paper reports some early studies on rotor misalignment and clearly the work has concentrated on a single form of misalignment. As remarked earlier, misalignment is a major fault condition in rotating machinery, yet there is no widely accepted theoretical basis for its analysis. In particular whilst most authors report the generation of vibration response at harmonics of rotor speed (and in particular $2\times$) there is nothing in the literature (to the best of the author's knowledge) showing how these harmonics arise. Al-Hussain and Redmond [2] were unable to detect harmonics in their nonlinear model. It is believed the present paper may offer a clue to this confused state of affairs.

The important feature shown in the analysis of Section 2 is a cross-coupling between torsional and flexural vibration. Furthermore this cross-coupling applies equally to torque oscillations generated by the coupling itself or from elsewhere in the machine. Not surprisingly the consequences in terms of vibration are significantly more onerous when the coupling is faulty. Note that the degree of excitation and twice rotational speed is increased markedly. Whilst the synchronous term also increases, this will normally be eliminated by trim balancing. Hence, it is the absolute value of the twice pre revolution term, which is important rather than its relationship to the synchronous excitation.

It has commonly been assumed that harmonics of shaft speed arise from nonlinearities of the system, in most cases emanating from the bearings, but in reality it appears the situation is somewhat more complex. In the analysis offered here, it is shown that a purely linear model can generate harmonics. The harmonic terms in the response arise from geometric nonlinearities of the motion. In a real system there will, in addition, be nonlinear components which may either enhance or diminish the excitation of response at multiples of shaft speed. Perhaps it is this multiplicity of sources, which imposes some variability on the nature of responses of misaligned machines, and may help to explain why this prominent machine fault is still not fully understood.

The models presented in this paper are clearly highly idealised and merely chosen in an attempt to understand the basic mechanisms. Work is currently in hand to formulate these concepts into a form for inclusion into an FE model of a real machine. This will be reported in the near future. In a real machine the situation is somewhat more complicated and torque will be transmitted partially by interfacial friction. Only a detailed model of the coupling can help resolve this issue. Such a detailed model will also be required to establish the appropriate 'bolt stiffness' term required to yield a realistic model of a given coupling.

There are numerous sources of harmonic response in a real machine. The purpose of this paper, albeit with simplified models, is to illustrate one source of excitation in misaligned rotors with rigid couplings.

5. Conclusions

(a) Using a purely linear model, the equations of motion of a machine with a coupling alignment fault have been derived.
(b) Solutions for some simple cases have been given both analytically and numerically.
(c) It has been shown that the linear model generates responses at harmonics of shaft speed.
(d) The harmonics are caused by an interaction of torsional and flexural effects.
(e) The combination of these mechanisms with the nonlinearities of a real system may explain the confusion in the literature.

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