Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 305 (2007) 298–307
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Structural damage identification based on residual force vector

Q.W. Yang, J.K. Liu*

Department of Mechanics, Zhongshan University, Guangzhou 510275, PR China

Received 17 June 2005; received in revised form 26 October 2005; accepted 17 March 2007

Abstract

Structural damage identification methods based on the residual force vector are studied in this paper. Using the residual force vector, the node residual force vector is defined to locate the suspected damaged elements preliminarily. Then three damage quantification techniques are studied to identify damages more precisely. The first is the algebraic solution of the residual force equation, the second is the MREU technique and the third is the natural frequency sensitivity method. A mode shape expansion technique based on the best achievable eigenvector concept is presented to solve the incomplete measurement problem. These damage detection methods are demonstrated on a numerical example and the measurement noises are discussed.

1. Introduction

Techniques based on dynamic parameters for detecting damages in a structure have attracted much attention in recent years. Modal frequencies and mode shapes are the most popular parameters used in the identification. The basic idea of these techniques is that modal parameters are functions of the physical properties of the structure (mass, damping and stiffness). Therefore, changes in the physical properties will cause changes in the modal properties. Many methods were developed recently using modal parameters as damage indicators.

An important class of damage identification methods is based on the updating or modification of structural matrices. The residual force vector is widely used in many damage detection methods using optimal matrix modification [1–5]. Chen and Garba [1] put forward a theory for assessing the occurrence, location and extent of potential damage using on-orbit response measurements. This method detects damages by using the minimum norm solution of the residual force equation. Zimmerman and Kaouk [2] made use of a minimum rank update theory to detect structural damages. The damage sites are located firstly by the residual force vector and the damage extents are assessed by the minimum rank update theory. Doebling improved this method and presented a new technique termed the minimum-rank elemental update (MREU) by computing the minimum rank updates directly to the elemental stiffness parameters [3]. Chiang and Lai [4] presented a two-stage structural damage detection method. The residual force vector is used to localize damages

*Corresponding author.
E-mail address: jikeliu@hotmail.com (J.K. Liu).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.03.033

preliminarily and the simulated evolution method is employed to determine damage extents. Mares and Surace proposed a genetic algorithm to identify damage in elastic structures. The location and quantification of the extent of the damage is performed with genetic techniques implemented by using the residual force method, which is based on conventional modal analysis theory [5]. In short, these above methods all begin with the residual force vector but use different techniques to obtain damage extents, so the accuracy of the residual force vector is very important to those methods. The minimum norm method is shown to be unfeasible in damage identification in practice because the residual force equation is ill conditioned with the measurement noises, while the minimum rank update techniques can obtain better results only when the number of modes used in calculation equals the rank of perturbed matrix [2,3]. However, the rank of the perturbed matrix is unknown after damages occurrence; in other words, the number of damaged elements is unknown, which limits the use of the minimum rank update method. The genetic algorithms [4,5] need much expense of the computation and the results of damage quantification sometimes are not ideal because the mode shapes used in calculation often have large measurement noises.

In this study, the residual force method is developed in order to detect structural damages successfully when the measured modal parameters are incomplete and have noises. A mode shape expansion algorithm based on the best achievable technique is proposed to solve the problem of the test/analysis degree of freedom (dof) mismatch. Damage locations are determined firstly by the node residual force vector, which is defined by the residual force vector. Three techniques for damage quantification are studied to obtain damage extents after the suspected damaged elements are determined, the first is the algebraic solution of the residual force equation, the second is the MREU technique and the third is the natural frequency sensitivity method. These damage detection methods are demonstrated on a numerical example with the measurement noises.

2. The residual force equation

Without loss of generality, by assuming that the mass matrix is unchanged as damage occurs, the eigenvalue equation for an n dofs finite element model of a damaged structure is

$$(K_d - \lambda_{dj} M)\phi_{dj} = 0, \tag{1}$$

$$K_d = K_u - \Delta K, \tag{2}$$

where M is the mass matrix, K_u and K_d are the stiffness matrices associated with the undamaged and damaged structural models, respectively, ΔK is the corresponding changes, λ_{dj} and ϕ_{dj} are the jth eigenvalue and eigenvector of the damaged structure, respectively.

Substituting Eq. (2) into Eq. (1) yields

$$(K_u - \lambda_{dj} M)\phi_{dj} = \Delta K \phi_{dj}. \tag{3}$$

Letting $b_j = (K_u - \lambda_{dj} M)\phi_{dj}$, Eq. (3) can be rewritten as

$$\Delta K \phi_{dj} = b_j, \tag{4}$$

where b_j is the jth residual force vector.

3. Damage identification

Eq. (4) can be expressed as

$$\begin{bmatrix} \Delta k_1^{\mathrm{T}} \\ \Delta k_2^{\mathrm{T}} \\ \vdots \\ \Delta k_n^{\mathrm{T}} \end{bmatrix} \phi_{dj} = \begin{Bmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jn} \end{Bmatrix}, \tag{5}$$

where Δk_i ($i = 1 \sim n$) is the stiffness change vector of the ith dof. Δk_i is nonzero only if the ith dof is damaged. By inspecting Eq. (5) it can be seen that the residual force vector b_j will have nonzero elements only in the damaged dofs. So we can find damaged elements according to the relation between the element number and the dof number. For the vibration data with measurement noises, the damaged dofs can be determined by the larger absolute elements in b_j.

For the convenience of actual engineering practices, the node residual force vector nb_j for the jth mode shape is defined as

$$nb_j = \begin{Bmatrix} nb_{j1} \\ nb_{j2} \\ \vdots \\ nb_{jq} \end{Bmatrix}, \tag{6}$$

where q is the number of nodes in the structural finite element model. Each element in nb_j is defined as the sum of the corresponding absolute values of elements in b_j associated with the dofs of this node. For example, assuming the 1st node is corresponding to the 1st and 2nd dofs, then the 1st node residual force is defined as

$$nb_{j1} = |b_{j1}| + |b_{j2}|. \tag{7}$$

Those nodes having larger values in nb_j are the most suspected damaged nodes and the damaged elements can be determined according to the relation between the element number and the node number.

3.1. The algebraic solution of the residual force equation

While the suspected damaged elements are determined by using the above described localization algorithm, a simple approach to obtain damage extents termed the algebraic solution is by directly solving the residual force equation. The perturbed global stiffness matrix ΔK can be expressed as

$$\Delta K = \sum_{l=1}^{N} \alpha_l K_l, \tag{8}$$

where α_l ($\alpha_l \in [0, 1]$) and K_l are the lth elemental damage parameter and stiffness matrix, respectively, N is the number of elements. The value of α_l is 0 if the lth element is undamaged and α_l is 1 if the lth element is completely damaged. From Eq. (8) we can obtain

$$\Delta k_i = \sum_{l=1}^{N} \alpha_l k_i^l, \tag{9}$$

where k_i^l is the ith stiffness vector in the lth element stiffness matrix. Both single damage and multiple damages are discussed in the following derivation.

For the case of single damage occurring in the structure, without loss of generality, assume that the qth element including the ith dof is damaged and the damage extent is α_q. Eq. (9) reduces to

$$\Delta k_i = \alpha_q k_i^q. \tag{10}$$

From Eq. (5) it is obvious that

$$\Delta k_i^{\mathrm{T}} \phi_{dj} = b_{ji}. \tag{11}$$

Substituting Eq. (10) into Eq. (11) the damage extent can be obtained as

$$\alpha_q = \frac{b_{ji}}{(k_i^q)^{\mathrm{T}} \phi_{dj}}. \tag{12}$$

For the case of multiple damages occurring in the structure, assume that the number of the damaged elements is r and the damage extents are $\alpha_1, \alpha_2, \ldots, \alpha_r$, respectively, then Eq. (9) becomes

$$\Delta k_i = \sum_{l=1}^{r} \alpha_l k_i^l. \tag{13}$$

Correspondingly, r equations associated with the damaged dof are selected from Eq. (5) to calculate the damage extent, i.e.,

$$\begin{bmatrix} \Delta k_1^{\mathrm{T}} \\ \Delta k_2^{\mathrm{T}} \\ \vdots \\ \Delta k_r^{\mathrm{T}} \end{bmatrix} \phi_{dj} = \begin{Bmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jr} \end{Bmatrix}. \tag{14}$$

Substituting Eq. (13) into Eq. (14) yields

$$\begin{bmatrix} \sum_{l=1}^{r} \alpha_l k_1^{l\,\mathrm{T}} \phi_{dj} \\ \sum_{l=1}^{r} \alpha_l k_2^{l\,\mathrm{T}} \phi_{dj} \\ \vdots \\ \sum_{l=1}^{r} \alpha_l k_r^{l\,\mathrm{T}} \phi_{dj} \end{bmatrix} = \begin{Bmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jr} \end{Bmatrix}. \tag{15}$$

Eq. (15) can be rewritten as

$$\begin{bmatrix} k_1^{1\,\mathrm{T}} \phi_{dj} & k_1^{2\,\mathrm{T}} \phi_{dj} & \cdots & k_1^{r\,\mathrm{T}} \phi_{dj} \\ k_2^{1\,\mathrm{T}} \phi_{dj} & k_2^{2\,\mathrm{T}} \phi_{dj} & \cdots & k_2^{r\,\mathrm{T}} \phi_{dj} \\ \vdots & \vdots & \vdots & \vdots \\ k_r^{1\,\mathrm{T}} \phi_{dj} & k_r^{2\,\mathrm{T}} \phi_{dj} & \cdots & k_r^{r\,\mathrm{T}} \phi_{dj} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{Bmatrix} = \begin{Bmatrix} b_{j1} \\ b_{j2} \\ \vdots \\ b_{jr} \end{Bmatrix}. \tag{16}$$

The damage extents can be easily obtained by solving the linear Eq. (16) using the Gauss elimination method.

3.2. *The natural frequency sensitivity method*

The algebraic solution of the residual force equation needs less computation efforts than that of the minimum norm solution and the minimum rank solution. However, as described in the introduction, the algebraic solution also depends on the accuracy of the residual force vector; in other words, mode shape noises have quite large effects on its damage detection result. Those damage quantification methods using mode shape information are all inevitably affected by the large measurement noises of mode shapes. So a better choice to obtain the damage extents is only using the natural frequency information because the lower natural frequencies can be measured very precisely. In this paper, the natural frequency sensitivity method [6,7] is also employed to identify damages after damages are located using the node residual force vector. Assuming that the number of measured natural frequencies is m, the damage quantification formulation from measured modal frequencies with the first-order approximation is as follows:

$$\begin{Bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_m \end{Bmatrix} = S_f \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{Bmatrix}, \tag{17}$$

where S_f denotes the first-order sensitivity matrix of natural frequencies and $\Delta\lambda_j$ ($j = 1\sim m$) is the change of the natural frequency before and after damage. From Eq. (17), the damage extents can be obtained as

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{Bmatrix} = S_f^{+} \begin{Bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_m \end{Bmatrix}. \tag{18}$$

When the damage is not small, a second-order approximation on the damage [8] can be performed or an iteration scheme [9] can be used to estimate damage extents more precisely, which are not further discussed in this paper.

4. Mode shape expansion

Lim and Kashangaki [10] used the best achievable technique to locate damages. This paper makes use of it to expand the incomplete measured mode shape. From Eq. (3), one obtains

$$(K_u - \lambda_{dj} M)^{-1} \Delta K \phi_{dj} = \phi_{dj}. \tag{19}$$

Substituting Eq. (8) into Eq. (19) yields

$$L_{jl} \gamma_{jl} = \phi_{dj}, \tag{20}$$

where

$$L_{jl} = (K_u - \lambda_{dj} M)^{-1} K_l, \quad l = 1 - N, \tag{21}$$

$$\gamma_{jl} = \alpha_l \phi_{dj}, \quad l = 1 - N. \tag{22}$$

Eq. (20) shows that the mode shape of the damaged structure is the linear combination of the nonzero columns of the matrix L_{jl} associated with the damaged elements. So Eq. (20) can be used to expand mode shapes after the probably damaged elements are identified preliminary. The elements used to mode shape expansion can be down selected by the best achievable technique. Eq. (20) can be partitioned into measured dofs, represented by m, and unmeasured dofs, represented by s, to obtain

$$\begin{bmatrix} L_{jl}^m \\ L_{jl}^s \end{bmatrix} \gamma_{jl} = \begin{Bmatrix} \phi_{dj}^m \\ \phi_{dj}^s \end{Bmatrix}. \tag{23}$$

The best achievable eigenvector for the measured dofs ϕ_{dj}^{ma} can be obtained as

$$\phi_{dj}^{ma} = \overline{L_{jl}^m} (\overline{L_{jl}^m})^+ \phi_{dj}^m, \tag{24}$$

where $\overline{L_{jl}^m}$ is the matrix L_{jl}^m where the zero columns have been removed to enhance computational efficiency, and the superscript '+' indicates the pseudo inverse of a matrix. The distance between the vector ϕ_{dj}^m and ϕ_{dj}^{ma} can be computed using the Frobenious norm

$$d_{jl} = \left\| \phi_{dj}^m - \phi_{dj}^{ma} \right\|_F, \tag{25}$$

where $\|\cdot\|_F$ represents the Frobenious norm. If the number of measured modes is m, an $N \times m$ matrix of d's can be constructed as

$$D = \begin{bmatrix} d_{11} & d_{21} & \cdots & d_{m1} \\ d_{12} & d_{22} & \cdots & d_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ d_{1N} & d_{2N} & \cdots & d_{mN} \end{bmatrix}. \tag{26}$$

Searching for values that are considerably smaller than others in the matrix D and the columns of their corresponding matrices, $\overline{L_{jl}}$ are used as the basis vectors to expand the mode shape. Without loss of generality, assuming that the matrices associated with the smaller elements in D are $\overline{L_{j1}}, \overline{L_{j2}}, \ldots, \overline{L_{je}}$, the matrix used to expand the jth mode shape can be obtained as

$$E_j = \left[\overline{L_{j1}} \quad \overline{L_{j2}} \quad \cdots \quad \overline{L_{je}} \right]. \tag{27}$$

The mode shape expansion formulation is as follows:

$$\phi_{dj}^s = E_j^s (E_j^m)^+ \phi_{dj}^m. \tag{28}$$

5. Numerical example

A plane truss structure (shown in Fig. 1) is taken as an example to verify the proposed methods. The basic parameters of the structure are as follows: $E = 200\,\text{GPa}$, $\rho = 7.8 \times 10^3\,\text{kg/m}^3$, $L = 1\,\text{m}$ and $A = 0.004\,\text{m}^2$. The relation between the element number and the node number is listed in Table 1. Assume that only the 1st, 2nd, 3rd, 4th and 5th nodes are measured locations and the other nodes are unmeasured sites. Natural frequencies and mode shapes are contaminated with 0.05% and 5% random noises, respectively. Three damage cases are studied in this work. The first damage case assumes a single damage occurs in the 10th element with a stiffness loss of 10%. The second damage case assumes element 16 is damaged with a stiffness loss of 10%. The third damage case assumes that elements 9 and 10 are damaged at the same time with stiffness losses both of 10%. The ratios of frequency changes before and after damage for these three damage cases are shown in Table 2.

For the first damage case, mode 4 is used to calculate the node residual force vector because it undergoes a significant shift in frequency according to Table 2. The node residual force vector of mode 4 is shown in Fig. 2. Nodes 2 and 3 are the most suspected damaged nodes from Fig. 2 since their residual force values are much larger than others. According to Table 1, element 10 is the corresponding element associated with nodes 2 and 3. By using Eq. (12), the algebraic solution of the damage extent can be obtained as $\alpha_{10} = 0.1035$ (3.5%). The value in bracket denotes the comparative error between the calculated value and the assumed value. The minimum rank solution by the MREU technique is shown in Fig. 3 and the damage extent of element 10 is $\alpha_{10} = 0.0743$ (25.7%). Using the natural frequency sensitivity method, damage extent of element 10 can be obtained as $\alpha_{10} = 0.1025$ (2.5%). According to the above results of the three damage quantification methods, conclusion can be easily drawn that the natural frequency sensitivity method has the best precision and the MREU technique has the worst.

For the second damage case, mode 1 is used to obtain the node residual force vector since it undergoes a significant shift in frequency from Table 2. The node residual force vector is shown in Fig. 4. Nodes 5, 7 and 8 are most probably the damaged nodes for their node residual force values are considerably larger than others. According to Table 1, elements 16 and 19 are the suspected damaged elements. The algebraic solution can be calculated using Eq. (17) to be $\alpha_{16} = 0.0903$ (9.7%) and $\alpha_{19} = 0.0494$. The result shows that the algebraic solution is fallible because element 19 is falsely identified. Results obtained by MREU using mode 1 and using modes 1 and 2 are shown in Figs. 5 and 6. According to Figs. 5 and 6, the better result is obtained when only mode 1 is used because the number of modes equals the number of nonzero perturbations in this case. Damage extent of element 16 in Fig. 5 is $\alpha_{16} = 0.0823$ (17.7%). Using the natural frequency sensitivity method, one obtains $\alpha_{16} = 0.1076$ (7.6%) and $\alpha_{19} = 0.0032$. Element 16 is identified from the damage quantification results to be the genuine damaged element and element 19 is excluded. From the above results, one can see that only the natural frequency sensitivity method is the most feasible and has the best accuracy for damage identification.

In the third damage case, mode 4 is used to calculate the node residual force vector and the result is shown in Fig. 7. Nodes 2, 3 and 9 are damaged according to Fig. 7 and their corresponding elements are 9, 10 and 11. The algebraic solutions from Eq. (17) are $\alpha_9 = 0.0965$ (3.5%), $\alpha_{10} = 0.1054$ (5.4%) and $\alpha_{11} = 0.0409$. Again, element 11 is falsely identified. Results by MREU using modes 4 and 5 and using modes 4, 5 and 6 are shown in Figs. 8 and 9. It is obvious that the MREU technique is valid only when the number of mode shapes used to calculate equals the rank of the perturbed matrix. This is a potential drawback to any minimum rank optimal matrix update technique, because in practice the expected order of the damage will typically be unknown.

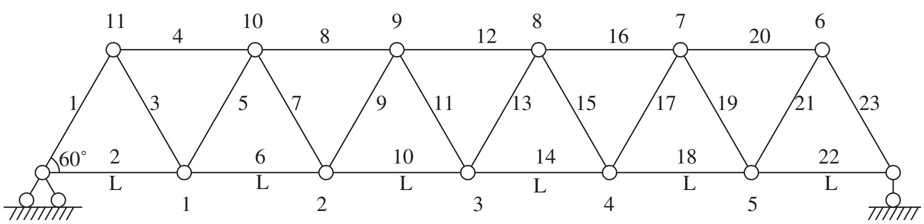

Fig. 1. A plane truss structure.

Table 1
Nodes that are related to each element in the truss

Element number	Corresponding nodes
1	11
2	1
3	1, 11
4	10, 11
5	1, 10
6	1, 2
7	2, 10
8	9, 10
9	2, 9
10	2, 3
11	3, 9
12	8, 9
13	3, 8
14	3, 4
15	4, 8
16	7, 8
17	4, 7
18	4, 5
19	5, 7
20	6, 7
21	5, 6
22	5
23	6

Table 2
The ratios of frequency changes before and after damage

Mode	Damage in element 10 (%)	Damage in element 16 (%)	Damages in element 9 and 10 (%)
1	0.17	1.67	0.34
2	0.32	0.42	0.69
3	0.01	0.60	0.02
4	0.73	0.06	1.72
5	0.52	0.25	0.54
6	0.55	0.76	0.58

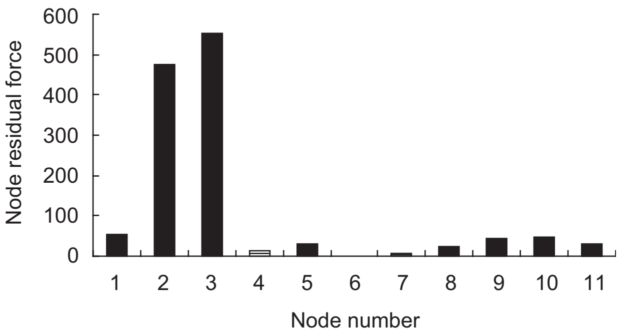

Fig. 2. The node residual force vector for mode 4 when element 10 is damaged.

Damage extents of elements 9 and 10 from Fig. 8 are $\alpha_9 = 0.079$ (21%) and $\alpha_{10} = 0.0735$ (26.5%), which have large errors. Results by using the natural frequency sensitivity method are $\alpha_9 = 0.1154$ (15.4%), $\alpha_{10} = 0.0952$ (4.8%) and $\alpha_{11} = 0.0091$. Elements 9 and 10 are identified from the damage quantification results to be the

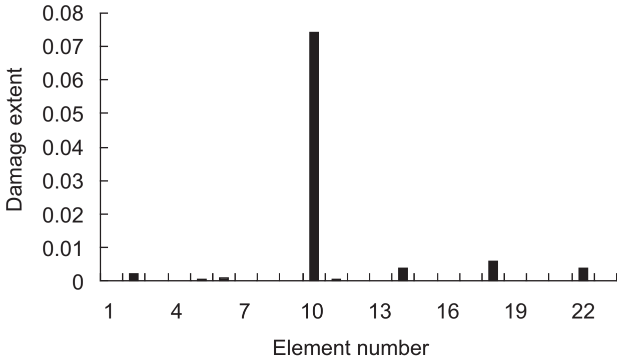

Fig. 3. Result of MREU using mode 4 when element 10 is damaged.

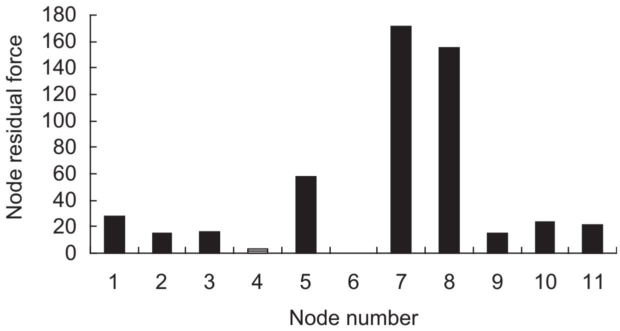

Fig. 4. The node residual force vector for mode 1 when element 16 is damaged.

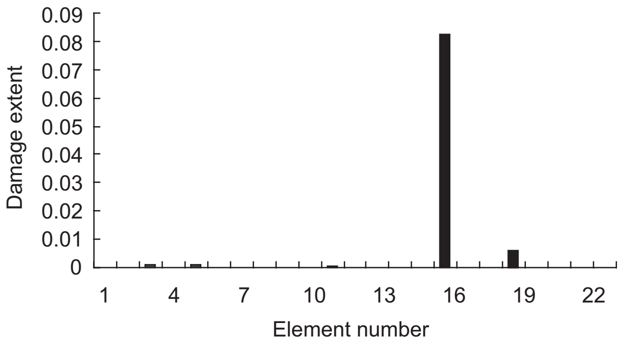

Fig. 5. Result of MREU using mode 1 when element 16 is damaged.

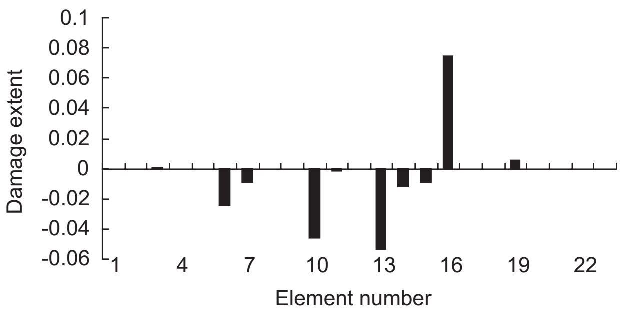

Fig. 6. Result of MREU using mode 1 and 2 when element 16 is damaged.

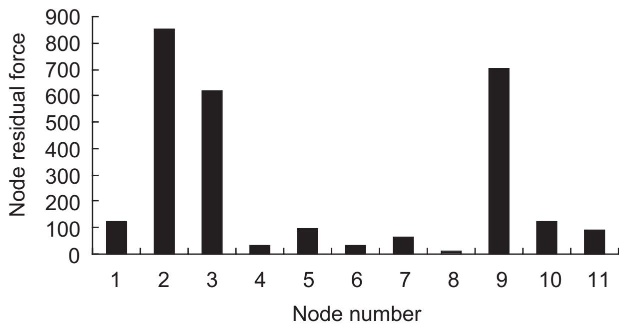

Fig. 7. The node residual force vector for mode 4 when element 9 and 10 are damaged.

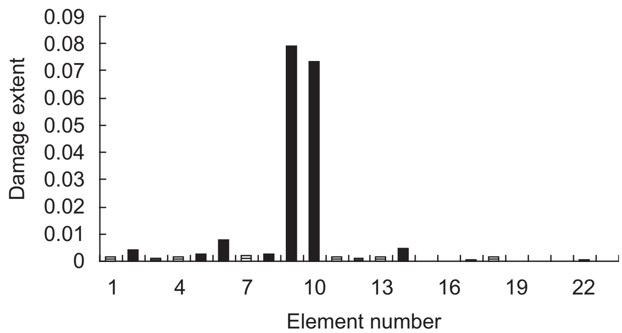

Fig. 8. Result of MREU using mode 4 and 5 when element 9 and 10 are damaged.

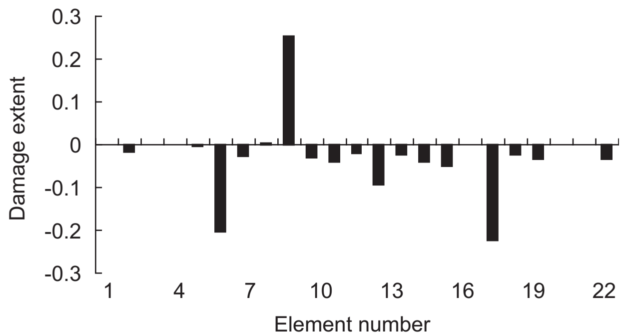

Fig. 9. Result of MREU using mode 4, 5 and 6 when element 9 and 10 are damaged.

genuine damaged elements and element 11 is excluded. On condition that the suspected damaged elements are determined preliminarily by the node residual force vector, the above results show that the natural frequency sensitivity method is the most feasible technique in engineering practice.

6. Conclusion

Structural damage identification methods based on the residual force vector are studied in this paper. The node residual force vector is defined according to the residual force vector to localize damages preliminarily. In order to solve the incomplete measurement problem, a mode shape expansion algorithm based on the best achievable technique is presented. After the suspected damaged elements are determined using the node residual force vector, three different damage quantification methods are used to identify damages more precisely. A plane truss structure is analyzed as a numerical example to demonstrate these damage detection methods with measurement noises.

Results of numerical example show that the node residual force vector can preliminarily locate the probable damages in measured locations or unmeasured locations, but the absolutely accurate damage localization is impossible for the measured mode shapes have large measurement noises.

The algebraic solution of the residual force vector is the simplest damage quantification technique among these damage detection methods. This advantage is especially obvious when the structural damages are separate. But this method often identifies false damages when the measurements have noises.

The MREU technique can identify structural damages only when the number of mode shapes used in the calculation equals the rank of the perturbed matrix. Since in practice the expected rank of the perturbed matrix is unknown, the MREU technique is limited to be used in actual engineering practice. Even if this precondition is satisfied, the damage extents obtained by MREU often have large errors with measurement noises.

On condition that the suspected damaged elements are determined preliminary by the node residual force vector, the natural frequency sensitivity method is the most practical technique because it can not only obtain damage extents of the genuine damaged elements accurately, but also exclude the false damaged elements. This method is robust because the lower natural frequencies of structures can be measured very precisely. The drawback of this method is a second-order approximation or an iteration scheme must be performed with increasing damages, which adds to the computation efforts. This coupled damage detection method would have broad foreground in the actual engineering practices.

Acknowledgments

This work is supported in part by the TRAPOYT, NSFC, GDSF and the Foundation of Zhongshan University Advanced Research Center.

References

[1] J.-C. Chen, J.A. Garba, On-orbit damage assessment for large space structures, *AIAA Journal* 26 (9) (1988) 1119–1126.

[2] D.C. Zimmerman, M. Kaouk, Structural damage detection using a minimum rank update theory, *Journal of Vibration and Acoustics* 116 (2) (1994) 222–231.

[3] S.W. Doebling, Minimum-rank optimal update of elemental stiffness parameters for structural damage identification, *AIAA Journal* 34 (12) (1996) 2615–2621.

[4] D.-Y. Chiang, W.-Y. Lai, Structural damage detection using the simulated evolution method, *AIAA Journal* 37 (10) (1999) 1331–1333.

[5] C. Mares, C. Surace, An application of genetic algorithms to identify damage in elastic structures, *Journal of Sound and Vibration* 195 (2) (1996) 195–215.

[6] R.M. Lin, H. Du, J.H. Ong, Sensitivity based method for structural dynamic model improvement, *Computer & Structures* 47 (3) (1993) 349–369.

[7] A. Messina, A.I. Jones, J.E. Williams, Damage detection and localization using natural frequency changes, in: *Proceedings of Conference on Identification in Engineering Systems*, Swansea, UK (1996) 67–76.

[8] A. Messina, J. E Williams, T. Contursi, Structural damage detection by a sensitivity and statistical-based method, *Journal of Sound and Vibration* 216 (5) (1996) 791–808.

[9] Z.Y. Shi, S.S. Law, L.M. Zhang, Structural damage detection from modal strain energy change, *Journal of Engineering Mechanics, ASCE* 126 (12) (2000) 1216–1223.

[10] T.W. Lim, T.A.L. Kashangaki, Structural damage detection of space truss structures using best achievable eigenvectors, *AIAA Journal* 32 (5) (1994) 1049–1057.