Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 305 (2007) 481–491
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Robust synchronization of chaotic horizontal platform systems with phase difference

Xiaofeng Wu[a,b,*], Jianping Cai[c], Muhong Wang[b]

[a]Center for Control and Optimization, South China University of Technology, Guangzhou 510640, China
[b]College of Electronic Engineering, Naval University of Engineering, Wuhan 430033, China
[c]Department of Applied Mechanics and Engineering, Zhongshan (Sun Yat-sen) University, Guangzhou 510275, China

Received 27 August 2006; received in revised form 8 April 2007; accepted 15 April 2007
Available online 13 June 2007

Abstract

This paper studies a robust synchronization of non-autonomous chaotic systems with parameter mismatch. In the synchronization scheme, a linear state error feedback control is used to couple the master and slave horizontal platform systems excited by harmonic external forces, between which there exists a phase mismatch. A new definition of global synchronization with error bound is introduced. Using Lyapunov's stability theory, the sufficient synchronization criteria for the scheme are proven and the corresponding synchronization error bound is estimated. The synchronization criteria are further optimized by optimally designing a quadratic Lyapunov function to more precisely estimate the synchronization error bound. The illustrative simulations verify the effectiveness of these criteria. The estimated synchronization error bound is compared with numerical one in the examples.
© 2007 Elsevier Ltd. All rights reserved.

1. Introduction

In the past decade, the research on synchronization has intensively focused on the autonomous chaotic systems, e.g. Chua's circuit, Lorenz oscillator, and Chen system, etc. [1–6]. Lately, more and more non-autonomous chaotic systems have been found in engineering and physics [7–12]. This has motivated the researchers to make efforts in studying the sufficient synchronization criteria for the various non-autonomous synchronization schemes [13–17].

However, these non-autonomous synchronization schemes assume that two non-autonomous chaotic systems to be synchronized are identical except for their initial states. In practice, when two identical systems are placed in a synchronization scheme, parameter mismatch between the systems often occurs because of the inevitable perturbation in operations, which may destroy the synchronization [2]. From the theoretical point of view, research on synchronization of two chaotic systems with parameter mismatch is more challenging.

For a class of autonomous chaotic system, Lur'e system, robust synchronization for parameter mismatch has been investigated in the synchronization schemes with feedback control [3,4] or replacing variables control

*Corresponding author. Center for Control and Optimization, South China University of Technology, Guangzhou 510640, China.
E-mail addresses: wuxiaof@21cn.com, mathwxf@sina.com (X. Wu).

0022-460X/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2007.04.034

[5]. Nevertheless, to our best knowledge, research on chaos synchronization of two non-autonomous systems with parameter mismatch has received little attention up to now.

In 2003, Ge et al. [10] studied a chaos synchronization scheme composed of two non-autonomous horizontal platform systems unidirectionally coupled by linear state error feedback control. They numerically verified that the scheme can achieve chaos synchronization whether there is a phase mismatch between two coupled systems or not, provided that the coupling strength is large enough. The so-called phase mismatch here implies that there exists a phase difference between the external harmonic excitations of two coupled systems. It should be pointed out that the phase mismatch also appears in other non-autonomous systems and may affect chaos synchronization [18]. Ge et al. [10] detected the coupling strengths resulting in chaos synchronization according to the criterion that the largest Lyapunov exponent of the slave system is negative. However, this kind of synchronization criterion has been confirmed to be only necessary but not sufficient for chaos synchronization [1].

Very recently, the sufficient synchronization criteria for two identical (without phase mismatch) non-autonomous horizontal platform systems unidirectionally coupled by linear state error feedback control have been obtained [17]. In this paper, we intend to prove the sufficient synchronization criteria for the synchronization scheme with phase mismatch. A new definition of global synchronization with error bound is first introduced because the complete (zero error) synchronization of two non-identical systems cannot be achieved by linear state error feedback control [2], which is different from Ref. [17]. Based on the definition, the sufficient synchronization criteria are investigated with help of Lyapunov's stability theory and the corresponding synchronization error bound is theoretically estimated. The synchronization criteria are further optimized by optimally designing a quadratic Lyapunov function to improve the precision of the estimated synchronization error bound. Finally some examples are simulated to verify the obtained criteria and illustrate the difference between the estimated and numerical synchronization error bounds.

The rest of the paper is organized as follows. Section 2 presents a master–slave synchronization scheme and the corresponding time-varied error system. In Section 3, the sufficient synchronization criteria for the scheme are proven. The obtained synchronization criteria are optimized in Section 4. Some examples are simulated and analyzed in Section 5. The concluding remarks are described in the final section.

2. Synchronization scheme and error system

The horizontal platform system with harmonic external excitation ($h\cos\omega t$) is described by the following equations [10]:

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= -ax_2 - b\sin x_1 + l\cos x_1\sin x_1 + h\cos\omega t,\end{aligned} \tag{1}$$

where a, b, h, and ω are positive constants and l is a constant.

Let $x=(x_1,x_2)^{\mathrm{T}}\in R^2$. The vector form of the system (1) is

$$\dot{x} = Ax + f(x) + m(t), \tag{2}$$

with

$$A=\begin{pmatrix}0 & 1\\ 0 & -a\end{pmatrix},\quad f(x)=\begin{pmatrix}0\\ -b\sin x_1 + l\sin x_1\cos x_1\end{pmatrix},\quad m(t)=\begin{pmatrix}0\\ h\cos\omega t\end{pmatrix}. \tag{3}$$

Now we construct a synchronization scheme for the master (x) and slave (z) systems coupled by a linear state error feedback control $u(t)=K(x-z)$ as follows:

$$\begin{aligned}M &: \dot{x} = Ax + f(x) + m(t),\\ S &: \dot{z} = Az + f(z) + m'(t) + u(t),\\ C &: u(t) = K(x-z),\end{aligned} \tag{4}$$

where $z = (z_1, z_2)^T \in R^2$, $K = (k_{ij})_{2\times 2}$ is the constant feedback gain matrix, and

$$m'(t) = \begin{pmatrix} 0 \\ h\cos(\omega t + \varphi) \end{pmatrix} \tag{5}$$

is the external excitation to slave system.

Hence, there exists a phase difference φ between the external excitation of the master system and that of the slave systems. Assume φ is a constant and $0 \leqslant \varphi \leqslant 2\pi$ [10].

Define the error variable $e = x - z$. From the scheme (4) we can obtain a dynamical error system:

$$\dot{e} = (A - K)e + f(x) - f(z) + m(t) - m'(t) = (A - K + Q(t))e + m(t) - m'(t), \tag{6}$$

where

$$Q(t) = \begin{pmatrix} 0 & 0 \\ q(t) & 0 \end{pmatrix}, \quad q(t) = \frac{-b(\sin x_1 - \sin z_1) + l(\sin x_1 \cos x_1 - \sin z_1 \cos z_1)}{x_1 - z_1}, \tag{7}$$

and

$$m(t) - m'(t) = \Delta m = h\begin{pmatrix} 0 \\ \cos\omega t - \cos(\omega t + \varphi) \end{pmatrix}. \tag{8}$$

Let $||\cdot||$ denote the Euclidean norm of the vector. For the synchronization scheme (4), our ideal object is to choose the feedback matrix K such that $||e|| \to 0$ as $t \to \infty$. However, the error variable e cannot tend asymptotically to zero for the non-identical master–slave systems. Therefore, a concept of synchronization with error bound must be introduced here.

Definition. The synchronization scheme (4) achieves the synchronization with error bound δ if for any finite initial conditions $(x(0), z(0))$, there exists a real constant $\delta > 0$ and a $T \geqslant 0$ such that $||x(t) - z(t)|| = ||e|| \leqslant \delta$ for all $t > T$.

This definition suggests a synchronization for which the trajectory of error system will globally uniformly converge into a small circle $C_\delta = \{e : ||e|| = \delta\}$.

3. Sufficient synchronization criteria

Two lemmas are first given, where Lemma 1 is obvious and Lemma 2 is quoted from Ref. [18].

Lemma 1. $||m(t) - m'(t)|| \leqslant h\varphi$ *for* $\varphi \in [0, 2\pi]$.

Lemma 2. *For* $q(t)$ *defined by Eq.* (7), *certainly*,

$$|q(t)| \leqslant b + |l|. \tag{9}$$

Let $I_2 \in R^{2\times 2}$ *be the unit matrix. The following theorem gives the sufficient criterion for synchronization with error bound in the form of linear matrix inequality.*

Theorem 1. *If there exists the positive definite symmetric matrix* $P \in R^{2\times 2}$, *feedback gain matrix* $K \in R^{2\times 2}$ *and a constant* $\alpha > 0$ *such that for any time* $t > 0$,

$$[A - K + Q(t)]^T P + P[A - K + Q(t)] + 2h\alpha I_2 < 0, \tag{10}$$

then the synchronization scheme (4) *achieves synchronization with error bound* $\varphi\sigma(P, \alpha)$, *where*

$$\sigma(P, \alpha) = \frac{\beta(P)}{\alpha} \quad \text{with} \quad \beta(P) = \lambda_{\max}(P)\left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{1/2}, \tag{11}$$

$\lambda_{\max}(P)$ *and* $\lambda_{\min}(P)$ *are the maximal and minimal eigenvalues of the matrix* P, *respectively.*

Proof. Take the quadratic Lyapunov function

$$V(e) = e^{\mathrm{T}}Pe, \quad 0 < P = P^{\mathrm{T}} \in R^{2\times 2},$$

which is positive definite, derescent and radially unbounded.

The derivative of $V(e)$ for the time along the trajectory of the error system (6) equals:

$$\begin{aligned}\dot{V}(e) &= \dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e},\\ &= [(A-K+Q)e + \Delta m]^{\mathrm{T}}Pe + e^{\mathrm{T}}P[(A-K+Q)e+\Delta m],\\ &= e^{\mathrm{T}}\left[(A-K+Q)^{\mathrm{T}}P + P(A-K+Q)\right]e + 2\Delta m^{\mathrm{T}}Pe,\\ &= e^{\mathrm{T}}\left[(A-K+Q)^{\mathrm{T}}P + P(A-K+Q) + 2h\alpha I_2\right]e - 2h\alpha e^{\mathrm{T}}e + 2\Delta m^{\mathrm{T}}Pe,\\ &< -2h\alpha e^{\mathrm{T}}e + 2\Delta m^{\mathrm{T}}Pe.\end{aligned}$$

Since $\Delta m^{\mathrm{T}}Pe \in R$, it follows from Lemma 1 that

$$2\Delta m^{\mathrm{T}}Pe \leqslant \left\|2\Delta m^{\mathrm{T}}Pe\right\|\| \leqslant 2\|\Delta m^{\mathrm{T}}\|\|P\|\|e\| \leqslant 2hr\|e\|,$$

where $r = \varphi\lambda_{\max}(P)$. Hence, $\dot{V}(e) < 2h||e||[r - \alpha||e||]$.

Obviously, $\dot{V}(e) < 0$ on condition that $||e|| > r/\alpha$, i.e. $\dot{V}(e) < 0$ for any e outside of ball B_1 (see Fig. 1). Thus, if a_1 is selected such that the ellipsoid

$$E(a_1) = \{e : e^{\mathrm{T}}\mathrm{Pe} \leqslant a_1\} \supset B_1 = \left\{e : e^{\mathrm{T}}e \leqslant \frac{r^2}{\alpha^2}\right\},$$

and is the smallest, then for any e outside of the ellipsoid $E(a_1)$ it is confirmed that $\dot{V}(e) < 0$ and the trajectory of the error system will uniformly enter the ellipsoid $E(a_1)$ for any finite $e(0)$.

If a_2 is determined such that the ball:

$$B_2(a_2) = \left\{e : e^{\mathrm{T}}e \leqslant a_2\right\} \supset E(a_1),$$

and is the smallest, then the radius $\sqrt{a_2}$ of ball $B_2(a_2)$ is synchronization error bound which we are able to find out.

Again the matrix P is positive definite, so there exists an orthogonal transformation $F \in R^{2\times 2}$ such that for a new vector $y = (y_1, y_2)^{\mathrm{T}} \in R^2$ and $e = Fy$,

$$e^{\mathrm{T}}Pe = \lambda_1 y_1^2 + \lambda_2 y_2^2 = \frac{y_1^2}{\left(1/\sqrt{\lambda_1}\right)^2} + \frac{y_2^2}{\left(1/\sqrt{\lambda_2}\right)^2},$$

where $\lambda_i > 0$ ($i = 1, 2$) are the eigenvalues of the matrix P.

It has been known that:

$$\lambda_{\max}(P) = \max\{\lambda_i, i = 1, 2\}, \quad \lambda_{\min}(P) = \min\{\lambda_i, i = 1, 2\}.$$

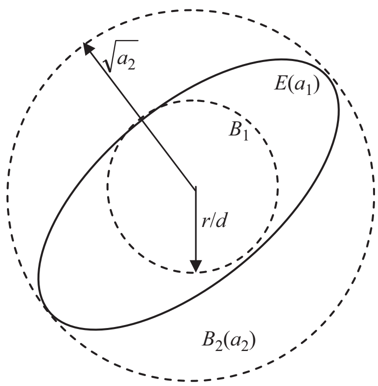

Fig. 1. Illustration of ball B_1, $B_2(a_2)$ and ellipsoid $E(a_1)$.

Hence, the maximal ($R_{\max}$) and minimal ($R_{\min}$) radius of the ellipsoid $E(a_1)$ equals:

$$R_{\max} = \frac{\sqrt{a_1}}{\sqrt{\lambda_{\min}(P)}} = \sqrt{a_2}, \quad R_{\min} = \frac{\sqrt{a_1}}{\sqrt{\lambda_{\max}(P)}} = \frac{r}{\alpha},$$

respectively.

Thus,

$$\sqrt{a_2} = \frac{\gamma}{\alpha}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} = \varphi\sigma(P, \alpha). \qquad \square$$

The following theorem gives an algebraic sufficient criterion for the synchronization with error bound and is derived from Theorem 1.

Theorem 2. *If the symmetric positive definite matrix* $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix}$, *feedback gain matrix* $K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$ *and the constant* $\alpha > 0$ *are selected such that*

$$\begin{aligned} &\Omega_1 = -k_{11}p_{11} - k_{21}p_{12} + h\alpha + |p_{12}|(b + |l|) < 0, \\ &\Omega_2 = p_{12}(1 - k_{12}) - p_{22}(a + k_{22}) + h\alpha < 0, \\ &4\Omega_1\Omega_2 > \left[\left|p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) - p_{22}k_{21}\right| + p_{22}(b + |l|)\right]^2. \end{aligned} \tag{12}$$

then the synchronization scheme (4) *achieves synchronization with error bound* $\varphi\sigma(P, \alpha)$, *where* σ *is defined by* (11).

Proof. From Eqs. (3) and (7) we have

$$\begin{aligned} &(A - K + Q)^{\mathrm{T}}P + P(A - K + Q) + 2h\alpha I_2 \\ &= \begin{pmatrix} -2k_{11}p_{11} + 2p_{12}(q - k_{21}) + 2h\alpha & p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21}) \\ p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21}) & 2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22}) + 2h\alpha \end{pmatrix}. \end{aligned}$$

The above symmetric matrix is negative definite if and only if

$$\begin{aligned} &-2k_{11}p_{11} + 2p_{12}(q - k_{21}) + 2h\alpha < 0, \\ &2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22}) + 2h\alpha < 0, \\ &\left[-2k_{11}p_{11} + 2p_{12}(q - k_{21}) + 2h\alpha\right]\left[2p_{12}(1 - k_{12}) - 2p_{22}(a + k_{22}) + 2h\alpha\right] \\ &- \left[p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21})\right]^2 > 0. \end{aligned} \tag{13}$$

Since $p_{22} > 0$, it follows from Lemma 2 that

$$\begin{aligned} &-2k_{11}p_{11} + 2p_{12}(q - k_{21}) + 2h\alpha \leqslant 2\Omega_1, \\ &\left|p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) + p_{22}(q - k_{21})\right| \leqslant \left|p_{11}(1 - k_{12}) - p_{12}(k_{11} + k_{22} + a) - p_{22}k_{21}\right| + p_{22}(b + |l|). \end{aligned}$$

Hence, for any $t > 0$, the inequalities (13) hold provided that the conditions (12) are satisfied. $\square$

Remark 1. The estimated synchronization error bound $\varphi\sigma(P, \alpha)$ given by Theorem 1 is linearly proportional to phase difference φ. This is because the matrix P and constant α determined by Eq. (10) are irrelevant to the phase difference φ, thus $\sigma(P, \alpha)$ and φ are independence of each other.

4. The optimized sufficient synchronization criteria

We must say that the synchronization error bound $\varphi\sigma$ is only an estimation of the real synchronization error bound. In order to improve the quality of the estimation, it is always expected that the estimated synchronization error bound is reduced to approach the real one. Since the synchronization error bound $\varphi\sigma$ depends upon the choice of the matrix P and constant α, an optimization issue is suggested as follows: select the matrices $P = P^T > 0$, K and constant α such that the synchronization conditions (12) are satisfied and the function $\sigma(P, \alpha) = \beta(P)/\alpha$ is as small as possible.

For $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} > 0$, it is easily known that the eigenvalues of the matrix P equal:

$$\lambda_1(P) = \frac{p_{11} + p_{22} + \sqrt{(p_{11} - p_{22})^2 + 4p_{12}^2}}{2}, \quad \lambda_2(P) = \frac{p_{11} + p_{22} - \sqrt{(p_{11} - p_{22})^2 + 4p_{12}^2}}{2},$$

with $\lambda_1 > \lambda_2$. Thus, it follows from Eq. (11) that $p_{12} = 0$ is necessary to the minimal $\beta(P)$.

Let $y = p_{22}/p_{11}$ with $p_{22} > 0$ and $p_{11} > 0$. Then, $\beta(P)$ with $p_{12} = 0$ can be represented as

$$\beta(P) = \begin{cases} p_{11} y^{3/2} & \text{as} \quad y > 1, \\ p_{11} & \text{as} \quad y = 1, \\ p_{11} y^{-1/2} & \text{as} \quad 0 < y < 1. \end{cases}$$

Obviously the above $\beta(P)$ is continuous for y and takes the minimal value at $y = 1$.

Hence, we have

Lemma 3. *A necessary condition for the minimal $\beta(P)$ is that $P = pI_2$ with $p > 0$.*

According to Lemma 3, the synchronization criterion in Theorem 2 can be optimized based on the consideration of reducing the estimated synchronization error bound, which is described in the following theorem where $\sigma = (p/\alpha) > 0$.

Theorem 3. *If the feedback gain matrix $K = (k_{ij})_{2\times 2}$ and the constant $\sigma > 0$ are selected such that*

$$\begin{aligned} \Omega_3 &= -k_{11} + \frac{h}{\sigma} < 0, \\ \Omega_4 &= -(a + k_{22}) + \frac{h}{\sigma} < 0, \\ 4\Omega_3\Omega_4 &> [|1 - k_{12} - k_{21}| + (b + |l|)]^2, \end{aligned} \tag{14}$$

then the synchronization scheme (4) *achieves the synchronization with error bound $\varphi\sigma$.*

The following results are related to some simple feedback gain matrix and derived from Theorem 3.

Corollary 1. *If the constant $\sigma > 0$ is selected, then the synchronization scheme* (4) *achieves the synchronization with error bound $\varphi\sigma$ provided that any of the following conditions is satisfied*:

(i) $K = \mathrm{diag}\{k_1, k_2\}$,

$$k_1 > \frac{h}{\sigma}, \quad k_2 > \frac{h}{\sigma} - a, \quad \left(k_1 - \frac{h}{\sigma}\right)\left(k_2 + a - \frac{h}{\sigma}\right) > \frac{1}{4}(b + |l|)^2, \tag{15}$$

(ii) $K = kI_2$,

$$k > \frac{h}{\sigma} + \frac{1}{2}\left(\sqrt{a^2 + (b + |l|)^2} - a\right), \tag{16}$$

(iii) $K = \mathrm{diag}\{k, 0\}$,

$$k > \frac{h}{\sigma} + \frac{1}{4}(b + |l|)^2 \frac{\sigma}{a\sigma - h} \quad \textit{with} \quad \sigma > \frac{h}{a}. \tag{17}$$

From Eqs. (15)–(17) we know that in the synchronization scheme, the larger feedback gains must be chosen to reduce the synchronization error bound.

5. Simulations and analyses

For simulation we take the parameters of the master and slave horizontal platform systems $a = 4/3$, $b = 3.776$, $l = 4.6 \times 10^{-6}$, $h = 34/3$, $\omega = 1.8$ and $\varphi = 0.1$. As the initial states $(x_1(0), x_2(0)) = (-3.4, 2.1)$ and $(z_1(0), z_2(0)) = (0.78, -2.9)$, the trajectories of both master and uncontrolled slave systems are chaotic [10]. They separate randomly and remarkably in the course of time, as shown in Fig. 2.

Appling the synchronization criteria (16) and (17), we can obtain the following synchronization conditions,

(i)
$$K = kI_2,\ k > \frac{34}{3\sigma} + 1.336 \quad \text{with} \quad \sigma > 0, \tag{18}$$

(ii)
$$K = \mathrm{diag}(k, 0),\ k > \frac{34}{3\sigma} + \frac{10.696\sigma}{4\sigma - 34} \quad \text{with} \quad \sigma > 8.5. \tag{19}$$

Take $K = kI_2$ with $k = 7(\sigma = 2)$. The simulation verifies that the trajectories of the master and slave systems tend asymptotically to the synchronization with the small non-zero error, as shown in Fig. 3.

The similar results are shown in Fig. 4 by taking $K = \mathrm{diag}(k, 0)$ with $k = 20$ $(\sigma = 9.9)$.

In order to illustrate the difference between the estimated synchronization error bound and the real one, we take the feedback gain matrices:

(i)
$$K = kI_2,\ k = \frac{34}{3\sigma} + 1.337, \quad \text{with} \quad \sigma > 0, \tag{20}$$

(ii)
$$K = \mathrm{diag}(k, 0),\ k = \frac{34}{3\sigma} + \frac{10.696\sigma}{4\sigma - 34} + 0.001, \quad \text{with} \quad \sigma > 8.5. \tag{21}$$

Let $||e||_{Tm} = \max_{t \geqslant T} \sqrt{(x_1 - z_1)^2 + (x_2 - z_2)^2}$ be the real synchronization error bound, where T represents a time threshold after which the synchronization error will be converged and stabilized in a small area (in our simulations, take $T = 8000$ time unit).

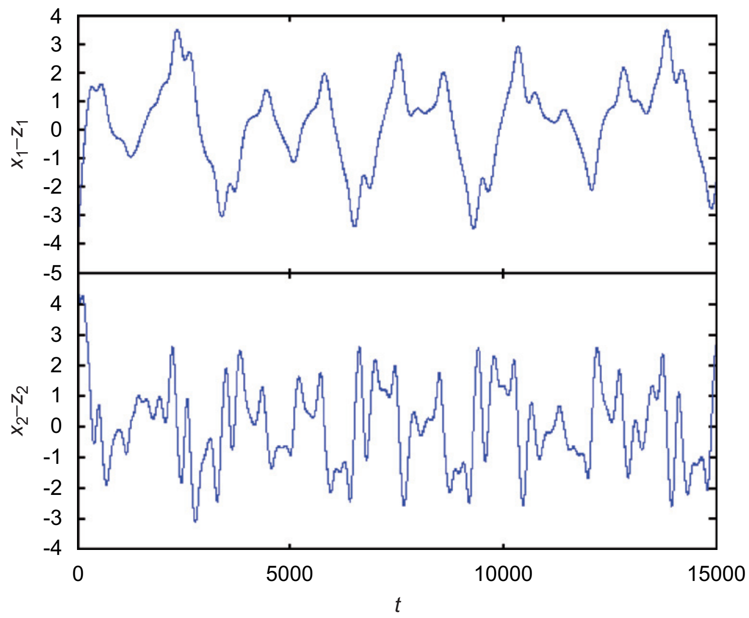

Fig. 2. The trajectory of the error system for the isolated master–slave systems.

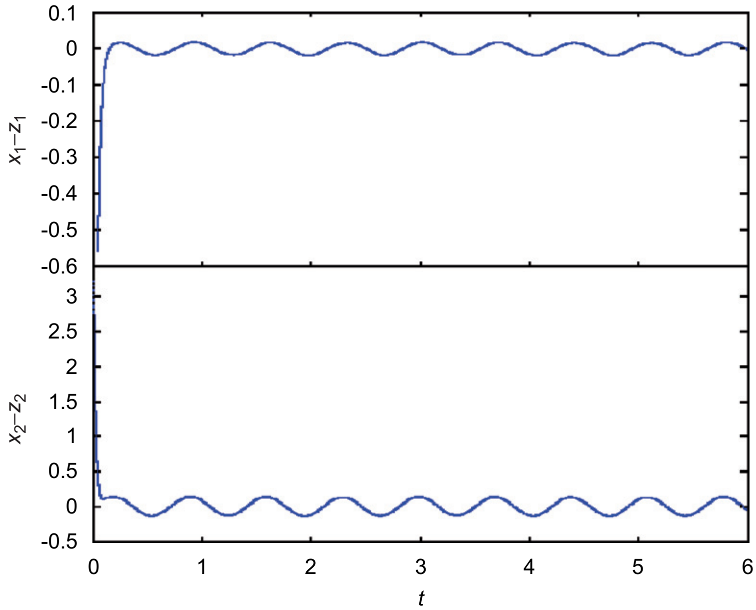

Fig. 3. The trajectory of the error system for the synchronization scheme with $K = 7I_2$.

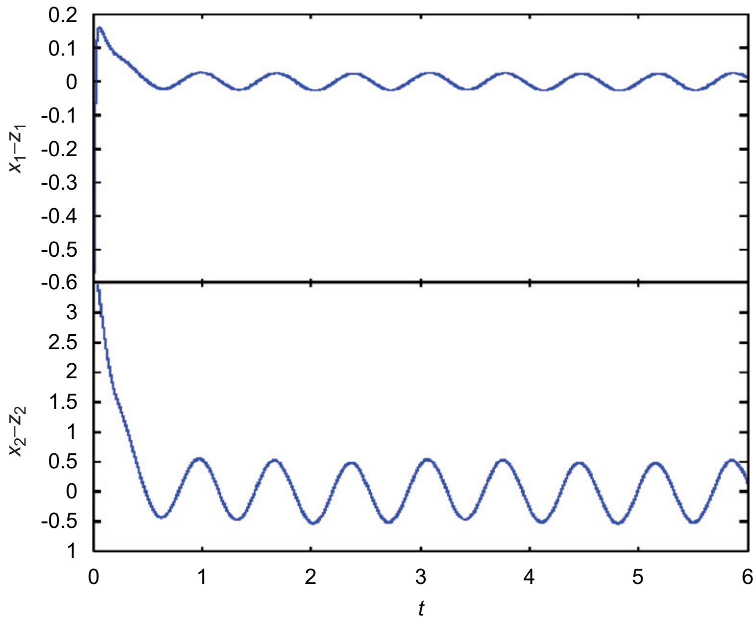

Fig. 4. The trajectory of the error system for the synchronization scheme with $K = \text{diag}\{20, 0\}$.

For a given σ, the feedback gains can be determined by Eqs. (20) and (21), so the real synchronization error bound $||e||_{Tm}$ can numerically be solved on condition that the obtained feedback gains and compared with the estimated one.

According to the above method, we first consider the synchronization condition (20) and choose $\sigma \in [0.1, 3.5]$. The numerical and estimated synchronization error bound for the various σ are solved and shown in Fig. 5. From the simulation results it can be seen that the selection of the smaller σ (corresponds to

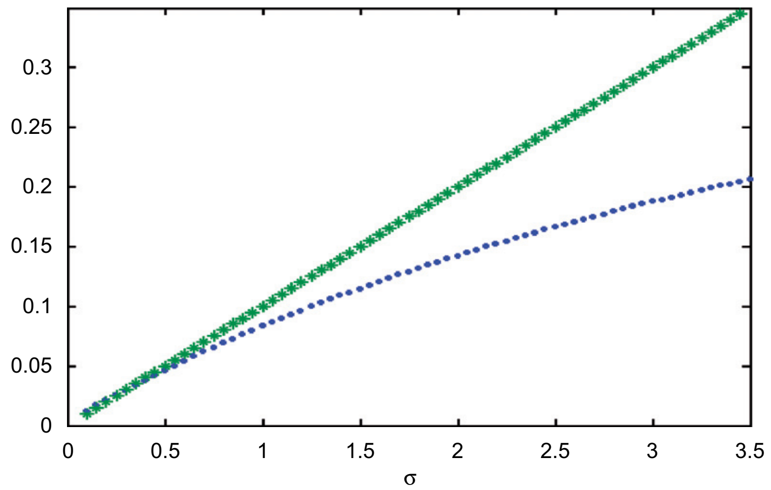

Fig. 5. The estimated and numerical synchronization error bounds for the synchronization condition $K = kI_2$, $k = 34/3\sigma + 1.337$ with $\sigma \in [0.1, 3.5]$. The curves plotted by "*" and "·" represent the estimated and numerical synchronization error bounds, respectively.

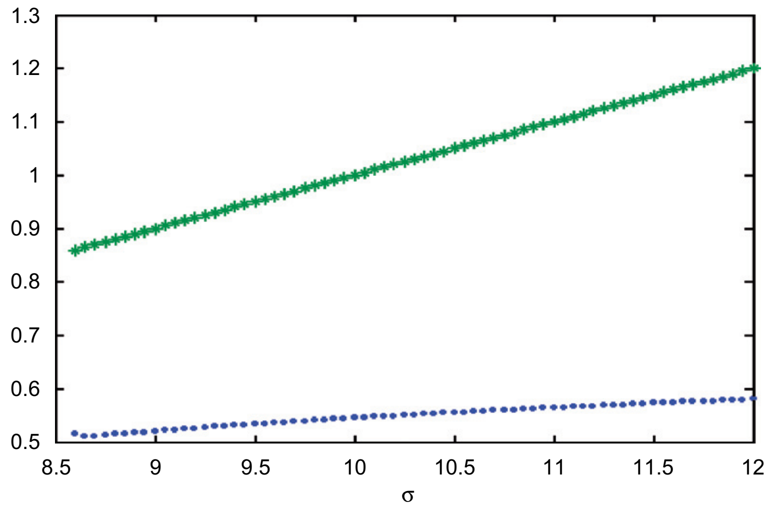

Fig. 6. The estimated and numerical synchronization error bounds for the synchronization condition $K = \mathrm{diag}\{k, 0\}$, $k = 34/3\sigma + 10.696\sigma/(4\sigma - 34) + 0.001$ with $\sigma \in [8.6, 12]$. The curves plotted by "*" and "·" represent the estimated and numerical synchronization error bounds, respectively.

the larger feedback gain k) will lower the numerical synchronization error bounds and improve the precision of the estimated synchronization error bounds up to a tiny difference from the numerical one.

Using the synchronization condition (21), the numerical and estimated synchronization error bounds for the various σ are shown in Fig. 6, where $\sigma \in [8.6, 12]$. Obviously the difference between the numerical and estimated synchronization error bounds is slowly decreased following the reduction of σ.

Now we analyze the influence of phase difference φ on the difference between the numerical and estimated synchronization error bound. Take $\varphi \in [0.05, 0.6]$, $\sigma = 0.5$ for the synchronization condition (20), and $\sigma = 8.8$ for the condition (21). The simulation results are shown in Figs. 7 and 8.

It should be indicated that all of our simulations have shown that the master–slave systems with phase difference cannot achieve the complete (zero error) synchronization, ever though the feedback gains are chosen to be large enough. This result is different from that of Ref. [10].

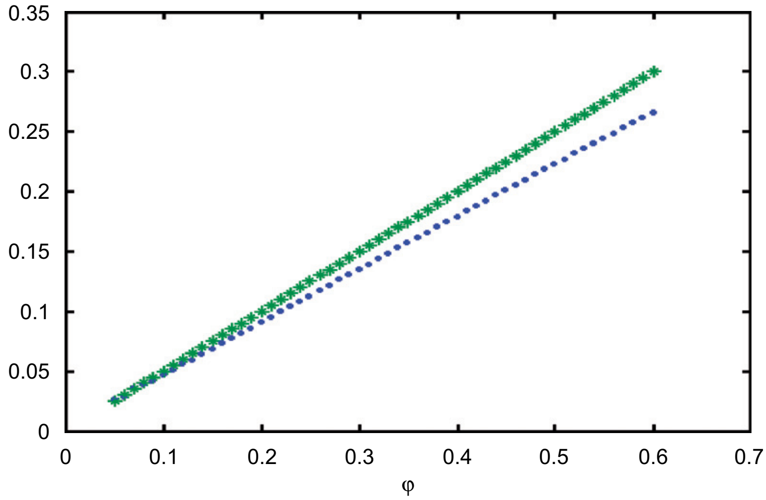

Fig. 7. The estimated and numerical synchronization error bounds for the synchronization condition $K = 24I_2$. The curves plotted by "*" and "$\cdot$" represent the estimated and numerical synchronization error bounds, respectively.

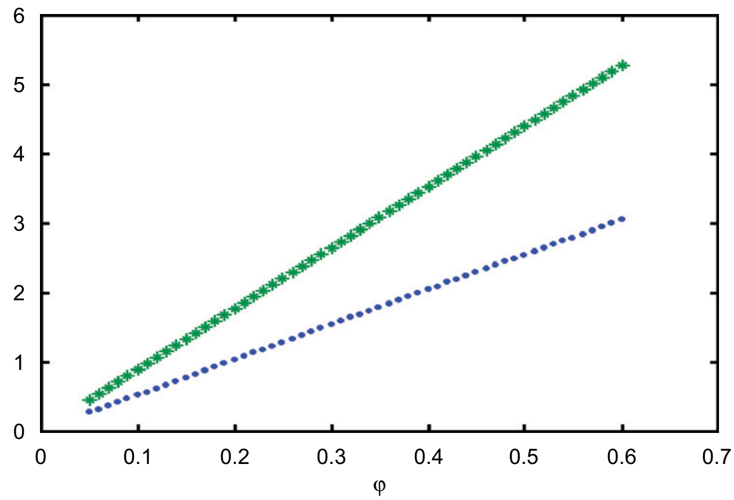

Fig. 8. The estimated and numerical synchronization error bounds for the synchronization condition $K = \text{diag}\{79.3, 0\}$. The curves plotted by "*" and "$\cdot$" represent the estimated and numerical synchronization error bounds, respectively.

6. Conclusions

The sufficient synchronization criteria for the non-autonomous horizontal platform systems with phase difference were obtained in the linear feedback scheme. The work was based on a new definition of chaos synchronization with error bound, which is global and relevant to phase difference. The estimation of the synchronization error bound for the synchronization criteria is linearly related to phase difference. Aiming at more precisely estimating synchronization error bound, the synchronization criteria were further optimized. These criteria are in the algebraic form, so can conveniently be applied to design and analysis of the linear state error feedback controller. The examples showed that it is possible to synchronize the master–slave horizontal platform systems with phase difference up to a small synchronization error, which means the robustness of the synchronization for the phase mismatch. The real and estimated synchronization error bounds are numerically compared in the examples.

Acknowledgments

Research is supported by National Nature Science Foundation of China under Grant no. 60674049 and China Postdoctoral Science Foundation under Grant no. 2005038177.

References

[1] S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares, C.S. Zhou, The synchronization of chaotic systems, *Physics Reports* 366 (2002) 1–101.
[2] P.F. Curran, L.O. Chua, Absolute stability theory and the synchronization problem, *International Journal of Bifurcation and Chaos* 7 (1997) 1375–1382.
[3] J.A. K .Suykens, P.F. Curran, L.O. Chua, Robust synthesis for master–slave synchronization of Lur'e system, *IEEE Transactions on Circuits Systems– I* 46 (1999) 841–850.
[4] X.F. Wu, J.P. Cai, Y. Zhao. Revision, improvement of a theorem for robust synchronization of non-identical Lur'e systems, *IEEE Transactions on Circuits Systems—II* 52 (2005) 429–432.
[5] X.F. Wu, M.H. Wang, Robust synchronization of chaotic Lur'e systems via replacing variables control, *International Journal of Bifurcation and Chaos* 16 (2006) 3421–3433.
[6] G.P. Jiang, G. Chen, W.K. Tang, A new criterion for chaos synchronization using linear state feedback control, *International Journal of Bifurcation and Chaos* 13 (2003) 2343–2351.
[7] S.R. Bishop, M.J. clifford, Zone of chaotic behavior in the parametrically excited pendulum, *Journal of Sound and Vibration* 189 (1996) 142–147.
[8] H.K. Chen, Chaotic and chaos synchronization of symmetric gyro with linear-plus-cubic damping, *Journal of Sound and Vibration* 255 (2002) 719–740.
[9] L.J. Chen, J.B. Li, Chaotic behavior and subharmonic bifurcations for a rotating pendulum equation, *International Journal of Bifurcation and Chaos* 14 (2004) 3477–3488.
[10] Z.M. Ge, T.C. Yu, Y.S. Chen, Chaos synchronization of a horizontal platform system, *Journal of Sound and Vibration* 268 (2003) 731–749.
[11] F. M: Kakmeni, S. Bowong, C. Tchawous, E. Kaptouom, Chaos control and synchronization of a Φ^6-Van der Pol oscillator, *Physics Letters A* 332 (2004) 305–323.
[12] C. Schäfer, M.G. Rosenblum, H. Abel, J. Kurths, Synchronization in the human cardiorespiratory system, *Physical Review E* 60 (1999) 857–870.
[13] E.M. Elabbasy, H.N. Agiza, M.M. EI-Dessoky, Global synchronization criterion and adaptive synchronization of new chaotic system, *Chaos, Solitons and Fractals* 23 (2005) 1299–1309.
[14] X.F. Wu, J.P. Cai, M.H. Wang, Global chaos synchronization of the parametrically excited Duffing oscillators by linear state error feedback control, *Chaos, Sotitons and Fractal*s (2006), doi:10.1016/j.chaos.2006.06.014.
[15] J.H. Park, Stability criterion for synchronization criterion for linearly coupled unified chaotic systems, *Chaos, Solitons and Fractals* 23 (2005) 1319–1325.
[16] J.T. Sun, Y.P. Zhang, Some simple global synchronization criterion for coupled time-varied chaotic systems, *Chaos, Solitons and Fractals* 19 (2004) 93–98.
[17] X.F. Wu, J.P. Cai, M.H. Wang, Master–slave chaos synchronization criteria for the horizontal platform systems via linear state error feedback control, *Journal of Sound and Vibration* 295 (2006) 378–387.
[18] H.W. Yin, J.H. Dai, H.J. Zhang, Phase affect of two coupled periodically driven Duffing oscillators, *Physical Review E* 58 (1998) 5683.