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Short Communication

# Instantaneous structural intensity by the harmonic wavelet transform

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**Abstract**

The wavelet transform is an analysis method to measure the transient signals instead of the conventional Fourier analysis, which is difficult to obtain high-definition time–frequency maps for rapidly changing signals. In this paper, we present a new method for the calculation of instantaneous structural intensity on the beam by using the wavelet transform and the details of the computational algorithm to obtain structural intensity by the harmonic wavelet transform is described. By applying this method for the data measured by hammering the beam structure, we confirmed that instantaneous intensity obtained by using the harmonic wavelet transform is a useful tool for analyzing the propagation of flexural waves on the beam structure.



## 1. Introduction

It is difficult to obtain time–frequency maps for rapidly changing transient signals. Thus, the wavelet transform is applied to the analysis of transient waves propagating in a dispersive medium. The calculation methods to measure the structural intensity were proposed by Pavic [1] and Verheij [2], respectively, which are based on the Fourier transform technique. But their calculation methods cannot be applied for measuring transient structural vibration because they are used for obtaining time-averaged intensity. The wavelet transform is the recent mathematical technique, which allows us to unfold a non-stationary signal into both space and scale and it is very promising for analyzing non-stationary signals. Authors proposed the method for measuring instantaneous intensity of impulsive sound by using the harmonic wavelet transform proposed by Prof. D.E. Newland in their papers [3,4]. In this paper, we also attempt to apply this wavelet transform method to obtain instantaneous intensity of structural vibration.

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## 2. Definition of the structural intensity on the beam by the wavelet transform

### 2.1. Finite difference approximations of the structural intensity

For a beam, the intensity is transported by shear forces and bending moments shown as follows [5]:

$$\begin{aligned} i(x,t) &= i_{sf}(x,t) + i_{bm}(x,t) \\ &= -B\left[\frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial t \partial x}\right], \end{aligned} \tag{1}$$

where $i_{sf}$ is the term for intensity due to shear forces, $i_{bm}$ is the term for intensity due to bending moment, $B$ is the bending stiffness of the beam given by $B = E'h^3/12(1-\sigma^2)$, and $w$ is the normal displacement of the beam.

By using finite difference approximations given by

$$w \approx (w_2 + w_3)/2, \tag{2.1}$$

$$\frac{\partial w}{\partial x} \approx \frac{w_3 - w_2}{d}, \tag{2.2}$$

$$\frac{\partial^2 w}{\partial x^2} \approx \frac{w_4 - w_3 - w_2 + w_1}{2d^2}, \tag{2.3}$$

$$\frac{\partial^3 w}{\partial x^3} \approx \frac{w_4 - 3w_3 + 3w_2 - w_1}{d^3}, \tag{2.4}$$

where $w_1$, $w_2$, $w_3$ and $w_4$ are normal displacements at points equally spaced on the beam shown in Fig. 1.

When we let $v = \dot{w}$ ($\dot{w}$: derivative of $w$), the structural intensity in Eq. (1) can be given by

$$\begin{aligned} i(t) &\approx -\frac{B}{d^3}\left[\frac{v_2 + v_3}{2}\int_{-\infty}^{t}\{v_4 - 3v_3 + 3v_2 - v_1\}\,d\tau - \frac{v_3 - v_2}{2}\int_{-\infty}^{t}\{v_4 - v_3 - v_2 + v_1\}\,d\tau\right] \\ &= -\frac{B}{d^3}\left[v_3(t)\int_{-\infty}^{t}\{v_3(\tau) - 2v_2(\tau) + v_1(\tau)\}\,d\tau - v_2(t)\int_{-\infty}^{t}\{v_4(\tau) - 2v_3(\tau) + v_2(\tau)\}\,d\tau\right], \end{aligned} \tag{3}$$

where $v_l (l = 1–4)$ is the surface velocity measured on the beam and $d$ is the separation of them. From this equation, the structural intensity can be measured by using an equally spaces linear array of accelerometers.

### 2.2. Error in structural intensity calculation

The time-averaged intensity of the flexural wave becomes $\langle i(x,t) \rangle = B\omega_0 k^3 A^2$ [6] for the sinusoidal signal given by $w(x,t) = A\cos(\omega_0 t - kx)$, where $A$ is an amplitude of normal displacement, $\omega_0$ is an angular frequency and $k$ is a wavenumber. By using the finite difference approximation given by Eq. (3), the time average of the

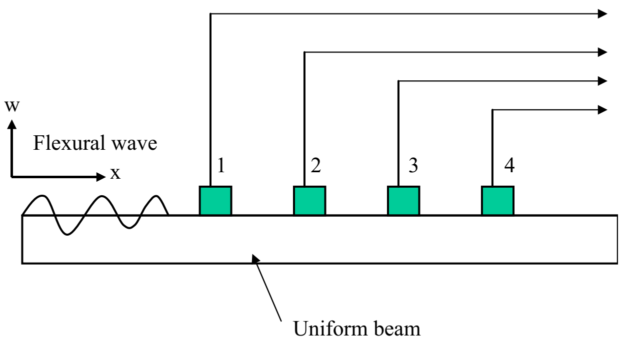


Fig. 1. Schematic diagram of the transducer array for the measurement of structural intensity.

structural intensity becomes

$$\langle i(t)\rangle = \frac{2A^2B\omega_0[\cos(kd)-1]\sin(kd)}{d^3}. \tag{4}$$

From which, the error in dB of the structural intensity can be estimated as

$$\text{Error} = 10\log_{10}\left(\frac{2[\cos(kd)-1]\sin(kd)}{k^3d^3}\right). \tag{5}$$

The error of the intensity estimated from Eq. (5) is shown in Fig. 2. From which, it is seen that the error of the intensity stays less than 1.5 dB when satisfying $kd<1$.

### 2.3. Structural intensity by the wavelet transform

Using finite difference approximation, the structural intensity can be obtained by the Fourier transform technique. Instead of the Fourier transform technique, we use the wavelet transform to obtain instantaneous structural intensity given by

$$W_i(s,\tau) = s\int_{-\infty}^{+\infty} i(t)\psi^*[s(t-\tau)]\,dt, \tag{6}$$

where $s$ and $\tau$ determine the scaling and translation of the mother wavelet and $\psi^*$ represents the complex conjugate of the mother wavelet function $\psi$.

The wavelet transform can be equivalently written by using the Fourier transform as [7]

$$W_i(s,\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} I(\omega)\Psi^*\left(\frac{\omega}{s}\right)\exp(j\omega\tau)\,d\omega, \tag{7}$$

where $I(\omega)$ and $\psi(\omega)$ are Fourier transforms of $i(t)$ and $\psi(t)$, and $\omega$ is an angular frequency of the spectrum.

When we let $x(t) = dv/dt$, the Fourier transform of $x(t)$ can be given by $X(\omega) = j\omega V(\omega)$, then Fourier transform of $i(t)$ becomes

$$\begin{aligned} I(\omega) &\approx -\frac{B}{d^3}\left[\frac{X_3(\omega)}{j\omega} * \frac{X_3(\omega)-2X_2(\omega)+X_1(\omega)}{\omega^2} - \frac{X_2(\omega)}{j\omega} * \frac{X_4(\omega)-2X_3(\omega)+X_2(\omega)}{\omega^2}\right] \\ &= j\frac{B}{d^3}\left[\frac{X_3(\omega)}{\omega} * \frac{X_3(\omega)-2X_2(\omega)+X_1(\omega)}{\omega^2} - \frac{X_2(\omega)}{\omega} * \frac{X_4(\omega)-2X_3(\omega)+X_2(\omega)}{\omega^2}\right], \end{aligned} \tag{8}$$

where $X_l(\omega)(l = 1\sim4)$ is the Fourier transform of acceleration measured on each point on the beam and the symbol * denotes the convolution of two functions.

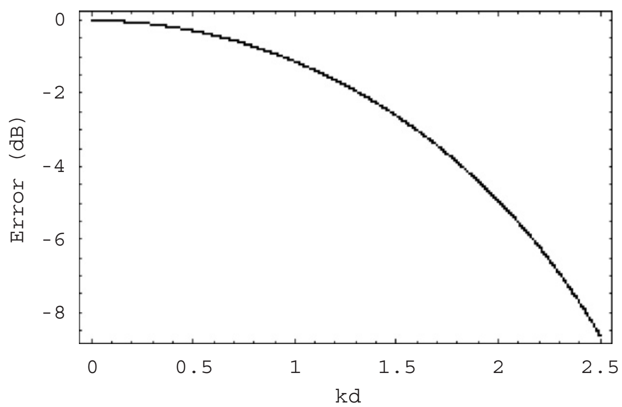


Fig. 2. Error of the structural intensity as a function of *kd*.

By inserting Eq. (8) into Eq. (7), the wavelet transform of the structural intensity can be given by

$$
\begin{aligned}
W_i(s,\tau) = & -\frac{\mathrm{j}B}{2\pi d^3}\int_{-\infty}^{+\infty}\left(\frac{X_2(\omega)}{\omega} * \frac{X_4(\omega)-2X_3(\omega)+X_2(\omega)}{\omega^2}\right)\Psi^*\left(\frac{\omega}{s}\right)\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega \\
& + \frac{\mathrm{j}B}{2\pi d^3}\int_{-\infty}^{+\infty}\left(\frac{X_3(\omega)}{\omega} * \frac{X_3(\omega)-2X_2(\omega)+X_1(\omega)}{\omega^2}\right)\Psi^*\left(\frac{\omega}{s}\right)\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega. \qquad (9)
\end{aligned}
$$

If we suppose $i(s,\tau) = \mathrm{Re}[W(s,\tau)]$, where Re is the real part of the complex number, the instantaneous intensity of flexural waves on the beam structure can be calculated.

## 3. Numerical calculation procedures by using the harmonic wavelet transform

We select the mother wavelet function, which is named harmonic wavelet by Newland [8], shown as follows:

$$\psi(t) = (\mathrm{e}^{\mathrm{j}4\pi t} - \mathrm{e}^{\mathrm{j}2\pi t})/\mathrm{j}2\pi t. \qquad (10)$$

As the Fourier transform of $\psi(t)$ can be given by

$$
\begin{aligned}
\Psi(\omega) &= 1 \quad (2\pi \leqslant \omega < 4\pi) \\
&= 0 \quad (\text{otherwise}). \qquad (11)
\end{aligned}
$$

When we let $s = 2^m$, where $m$ is an integer named a level according to Newland [8], which corresponds to the octave band number ($m > 0$), $\Psi(\omega/s)$ is identically zero outside $2\pi 2^m \leqslant \omega < 2\pi 2^{m+1}$. Then the wavelet transform of the instantaneous intensity $i(s,\tau)$ at $s = 2^m$ can be given by

$$
\begin{aligned}
i(2^m,\tau) = & \frac{B}{2\pi d^3}\mathrm{Im}\int_{2\pi 2^m}^{2\pi 2^{m+1}}\left(\frac{X_2(\omega)}{\omega} * \frac{X_4(\omega)-2X_3(\omega)+X_2(\omega)}{\omega^2}\right)\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega \\
& - \frac{B}{2\pi d^3}\mathrm{Im}\int_{2\pi 2^m}^{2\pi 2^{m+1}}\left(\frac{X_3(\omega)}{\omega} * \frac{X_3(\omega)-2X_2(\omega)+X_1(\omega)}{\omega^2}\right)\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega, \qquad (12)
\end{aligned}
$$

where Im is an imaginary part of the complex number.

According to the Newland's algorithm [8], this integral calculation can be performed by using the FFT algorithm shown as in the following procedures:

(1) Compute the discrete Fourier transforms of $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ to obtain $X_l(n)(l = 1\sim 4)$, $(n = 0\sim N-1))$, where $N$ is a number of sampling points satisfying $N \geqslant 4$, which equals a power of 2.
(2) Compute the convolution of $\hat{x}_l(n) = X_{l+2}(n)/n\Delta\omega$ and $\hat{y}_l(n) = (X_{4-l}(n) - 2X_{3-l}(n) + X_{2-l}(n)/n^2\Delta\omega^2)$ ($l = 0$ or 1), where $\Delta\omega = 2\pi\Delta f$ ($\Delta f = f_s/N$, $f_s$:sampling frequency) via a cyclic convolution given by

$$z_l(n) = \sum_{i+j=n(\mathrm{mod}\ N)}^{N-1} \hat{x}_l(i)\hat{y}_l(j), \qquad (13)$$

by setting that $n\Delta\omega = 1$ at $n = 0$.
(3) Compute $z(n) = z_0(n) - z_1(n)$.
(4) Conduct the inverse discrete Fourier transform (IDFT) of a $N/2$-length data $\{z(0), \mathrm{z}(1), z(2), \ldots, z(N/2)\}$ according to the following algorithm:

- First we let $w_0 = z(0)$ and $w_1 = z(1)$.
- Conduct the 2-points IDFT of $\{z(2), z(3)\}$ to obtain $\{w_2, w_3\}$.
- Compute 4-points IDFT of $\{z(4), z(5), z(7)\}$ to obtain $\{w_4, w_5, w_6, w_7\}$.
- ⋮
- Compute $N/4$ points IDFT of $\{z(N/4), \ldots, z(N/2-1)\}$ to obtain $\{w_{N/4}, \ldots, w_{N/2-1}\}$.

- Finally, we let $w_{N/2} = z(N/2)$.
- Replace, respectively, high $N/2-1$ components with Hermitian conjugates as

$$w_j = w^*_{N-j}(j = N/2 + 1 \sim N - 1). \tag{14}$$

Imaginary parts of the sequence of $\{w_k\}$ multiplied by $B/2\pi d^3$ yields the instantaneous intensity of a signal by the harmonic wavelet transform given by Eq. (12).

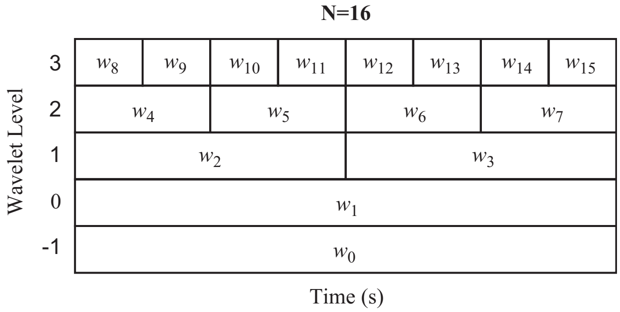


Fig. 3. Grid base for plotting the harmonic wavelet transform ($N = 16$).

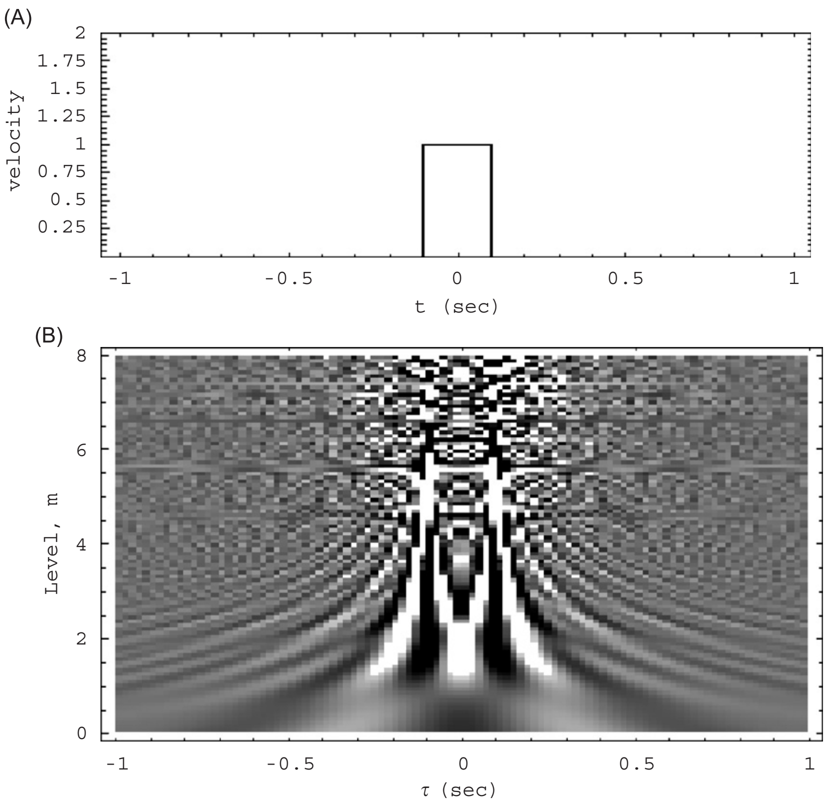


Fig. 4. Sample signal (A) and the time–frequency density map of the instantaneous structural intensity obtained by the calculation (B): (A) waveform of the sample signal and (B) density map of the structural intensity.

Finally plot the obtained result $w_{2^m+i}$ $(i = 0\sim2^m-1)$ by using the grid base divided into $2^m$ segments, because there are $2^m$wavelets in the unit interval for the octave band, for plotting wavelet amplitude as shown in Fig. 3, which is the case for $N = 16$, where number on the vertical axis corresponds the octave band given by $2^m\Delta f\sim2^{m+1}\Delta f$ (Hz).

By applying this calculation method for the sample signal given by

$$\dot{w}(x,t) = A[\text{sgn}(t-kx+\varepsilon) - \text{sgn}(t-kx-\varepsilon)], \tag{15}$$

where $\dot{w}(x,t)$ is the surface velocity on the beam at the position $(x,t)$, $A$ is an amplitude of the signal, sgn is a function defined as $\text{sgn}(t) = 1(t>0)$, $\text{sgn}(t) = -1(t<0)$, $k$ is a wavenumber given by $k = \omega/c_B$, ($c_B$: flexural wave speed) the structural intensity result can be obtained by using Mathematica 4.2 as an energy density map shown in Fig 4 for the cases of $A = 0.5$, the thickness of the beam, $h = 5$ mm, the Young's modulus, $E' = 20\times10^{10}\,\text{N/m}^2$, the Poisson's ratio, $\sigma = 0.3$, and mass density, $\rho = 7.8\times10^3\,\text{kg/m}^3$, when we select the length of space between the accelerometers to be $d = 40$ mm for satisfying $kd\ll1$.

The wavenumber of the flexural wave can be given by [9]

$$k = \sqrt[4]{\frac{4\pi^2\rho h}{B}f^2}. \tag{16}$$

In this figure, (A) shows the waveform of the sample signal and (B) shows the time–frequency density map. In Fig. 5, The instantaneous intensities at $m = 3$ for cases $\varepsilon = 0.01$ and $\varepsilon = 0.1$ are shown as (A) and (B), respectively, where the upper figure shows the waveform and the lower figure shows the instantaneous intensity. From which, it is seen that the intensity of pulse signal can be localized in time by using the wavelet transform analysis.

## 4. Instantaneous structural intensity obtained from the data of flexural waves propagating on the beam

### 4.1. Measurement system of flexural waves propagating on the uniform beam

As shown in the upper figure of Fig. 6, a uniform beam made of stainless steel was used for the experiment, where the width of the beam is 20 mm, its thickness is 5 mm, the Young's modulus, $E' = 20\times10^{10}\,\text{N/m}^2$, the Poisson ratio, $\sigma = 0.3$ and the mass density is $\rho = 7.8\times10^3$ (kg/m$^3$).

Four miniature accelerometers (TEAC, Type 706) were mounted firmly on the beam with the separation of them to be 40 mm and a hammer blow was applied to the free end of the beam by the impulse hammer as shown in the lower figure in Fig. 6.

The separation of accelerometers were selected to satisfy $kd<1$ for the frequency band less than $10^{10} = 1024$ Hz. The beam was clamped at one end by the vise. The total length from the clamped end to the free end of the beam was 0.92 m.

The accelerometer No.1 is situated at the distance of 0.6 m from the free end of the beam. We conducted the experiment for the uniform beam first and then for the beam with damping layer on both sides with a thickness of 5 mm. A hammer blow was applied to the beam at the tip of the free end in a direction perpendicular to the beam surface. Signals from accelerometers were amplified and converted to digital signal with the sampling frequency of 16 kHz. The sampling size of the data for conducting the wavelet transform by the computer was selected to be $N = 8192$.

### 4.2. Calculation results by the harmonic wavelet transform

By using the harmonic wavelet transform algorithm, instantaneous intensity of the beam was calculated for both cases, which are for the uniform beam and for the beam with damping layers on both sides. Fig. 7 shows the time–frequency map calculated by the harmonic wavelet transform, where the upper figure is for the time history of acceleration on each measured point of the beam and the lower figure is for the calculation result by using the harmonic wavelet transform. In the lower figure, the horizontal axis shows the time in seconds and the vertical axis shows a band number (wavelet level) of the wavelet transform. From the density map,

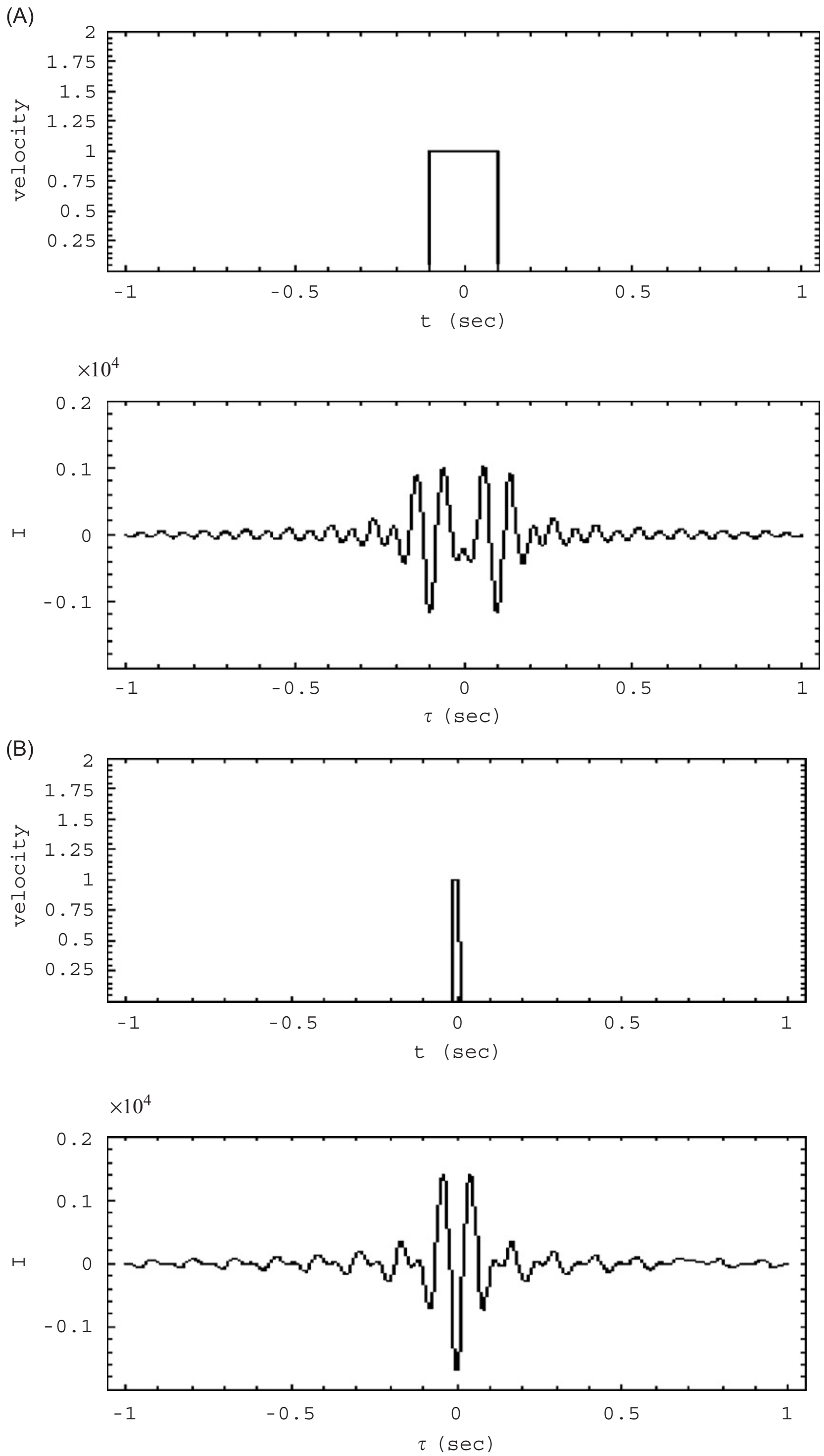


Fig. 5. Calculation results of the instantaneous structural intensity for two different signals, (A) $m = 3$, $\varepsilon = 0.1$, (B) $m = 3$, $\varepsilon = 0.01$.

multiple reflections of the flexural wave from the boundaries of a finite length system can be observed with different phase velocities, following the initial impact as intensity components experience alternate their values. Fig. 8 also shows the time–frequency map obtained for the beam with damping layers. From which,

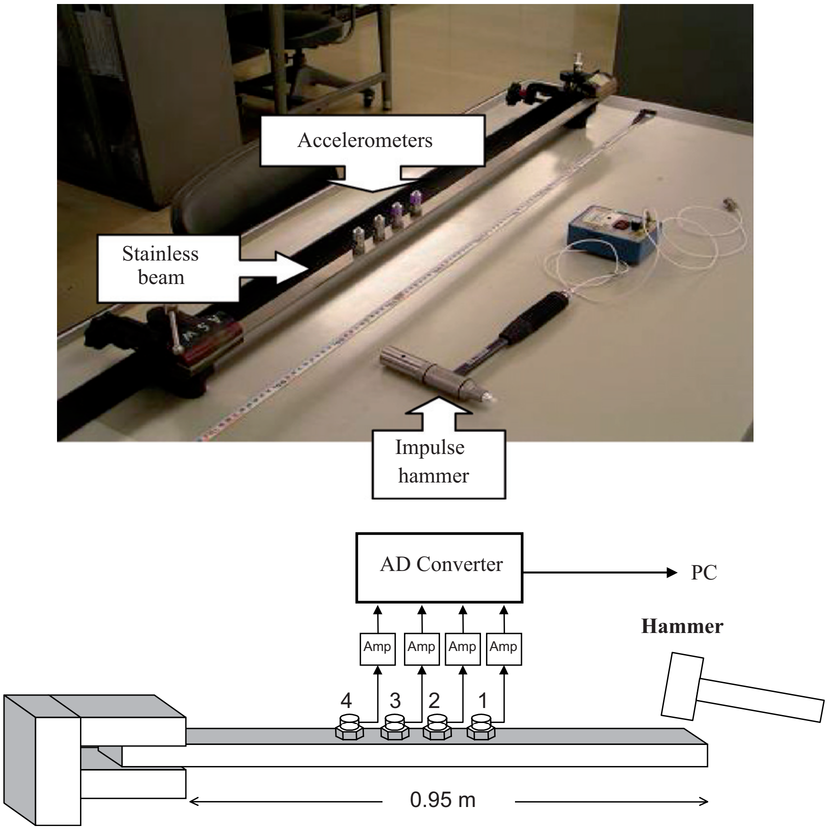


Fig. 6. Photo of the stainless beam used for the experiment (upper figure) and the schematic diagram of the experimental setup (lower figure).

it is clearly seen that the high frequency progressive wave components are leading the wave group and decays faster that the lower frequency waves, while the low-frequency components are traveling behind them. Thus, it is considered that each individual progressive wave component continuing the initial impact propagates with different speed as predicted by the theory of flexural waves [9].

By introducing the center frequency of the octave band at the level. $m$ given by $f = \sqrt{2} \cdot 2^m$ into the following formula for the arrival time of the flexural wave $T_0$:

$$T_0 = L \cdot \sqrt[4]{\frac{\rho h}{4\pi^2 B f^2}}, \tag{17}$$

where $L$ is the separation of the excited point and the measured point, the arrival time satisfies the relation given by $T_0 \propto 1/\sqrt{2^m}$.

Fig. 9 shows the instantaneous intensity at the wavelet level 4 and the wavelet level 7, which corresponds to the octave bands, the frequency range of them are 16∼32 and 128∼256 Hz, respectively, for the uniform beam and the uniform beam with damping layers. From these results, it can be seen that the progressive impulse flexural wave decays rapidly for higher frequency and higher damping material as predicted by the theory given by $-1/\dot{w} \cdot \mathrm{d}\dot{w}/\mathrm{d}t = \eta\omega/2$ ($\eta$: loss factor), which is the time-rate-decay of flexural vibration [9]. As the initial instance of the transient signal can be clearly detected, the arrival time of a flexural wave for each frequency band can be obtained as shown in the Table 1. From which, it can be shown that the flexural wave at $m = 7$ arrives 2.8 times faster than the flexural wave at $m = 4$ for the uniform beam, that is almost equal to the ratio $\sqrt{2^7}/\sqrt{2^4} = 2.82$, predicted from Eq. (17). Similarly, the ratio

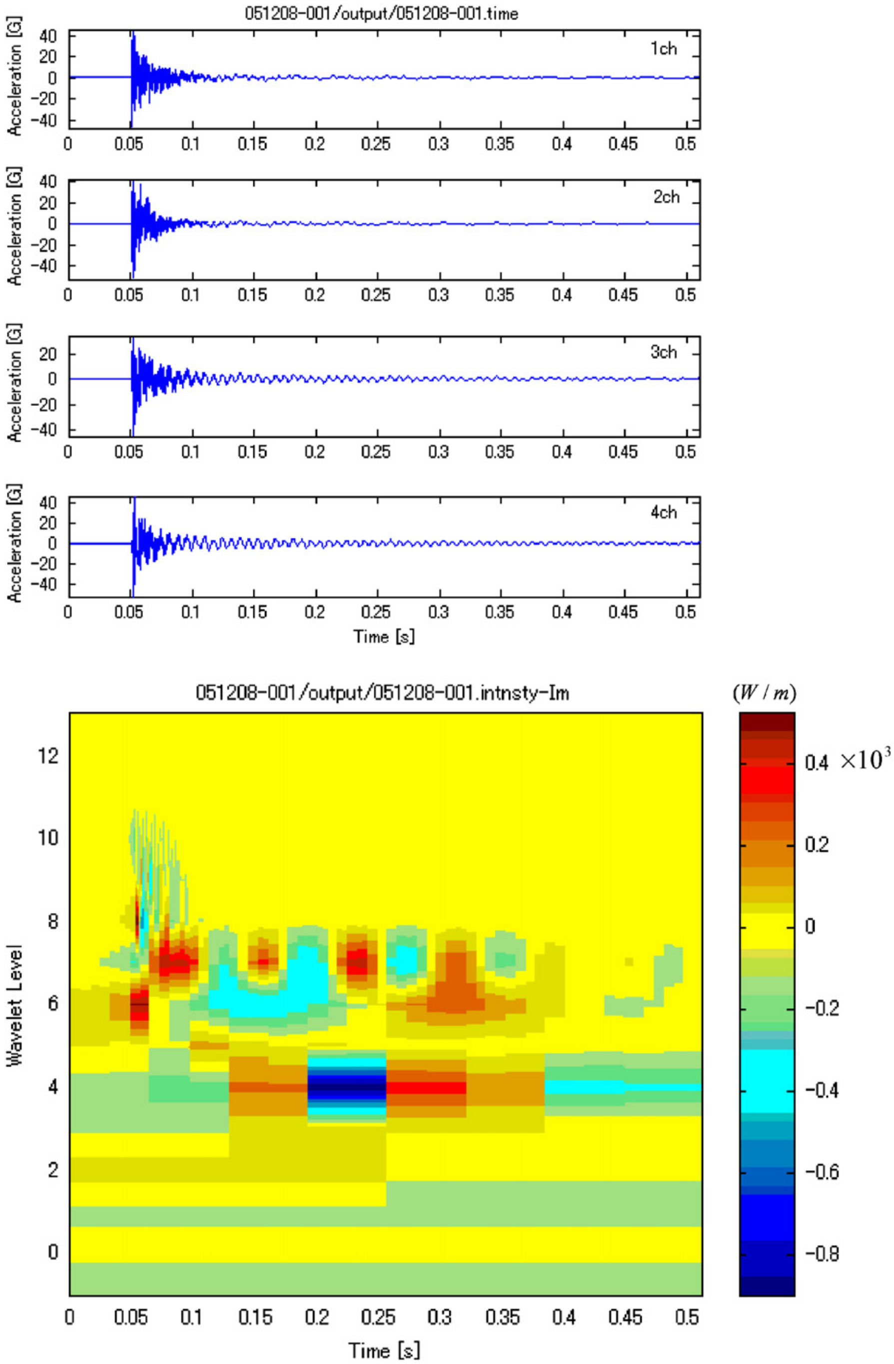


Fig. 7. Time history of the structural vibration at each measuring point and the time–frequency map of the instantaneous structural intensity obtained by using the harmonic wavelet transform (uniform beam).

of arrival times for the damped beam becomes 2.6, that is smaller than the undamped case, which coincides to the theory that the speed of the flexural wave increases with increasing the value of damping factor [10].

Hence it is seen that flexural waves propagating in the beam structure travel faster at higher frequencies than lower frequencies as predicted by the theory.

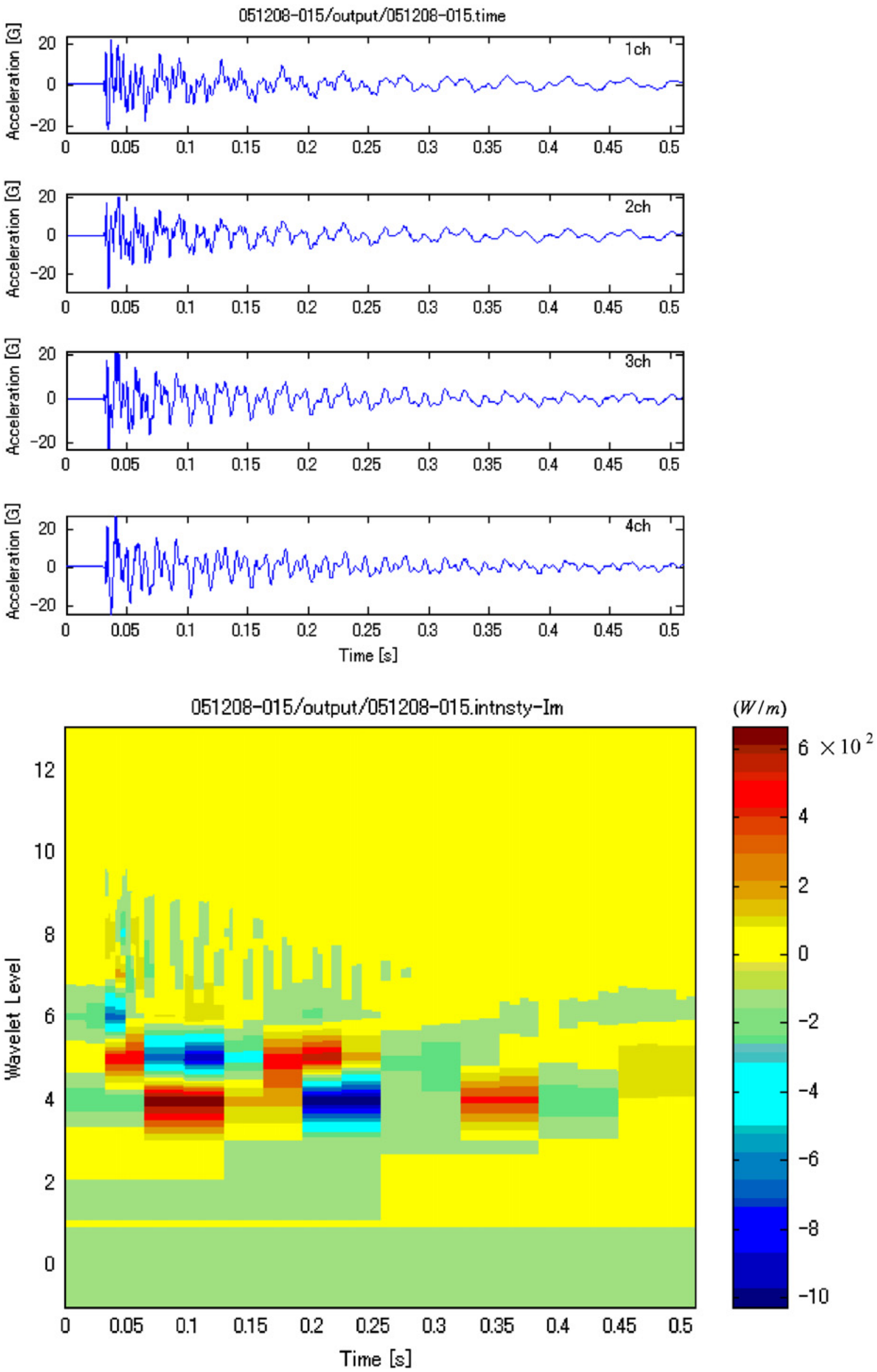


Fig. 8. Time history of the structural vibration at each measuring point and the time–frequency map of the instantaneous structural intensity obtained by using the harmonic wavelet transform (uniform beam with damping layers on both sides).

## 5. Conclusion

The measurement of the structural intensity for the beam by using the harmonic wavelet transform shows that flexural waves propagating in the beam structure travel faster at higher frequencies than lower frequencies

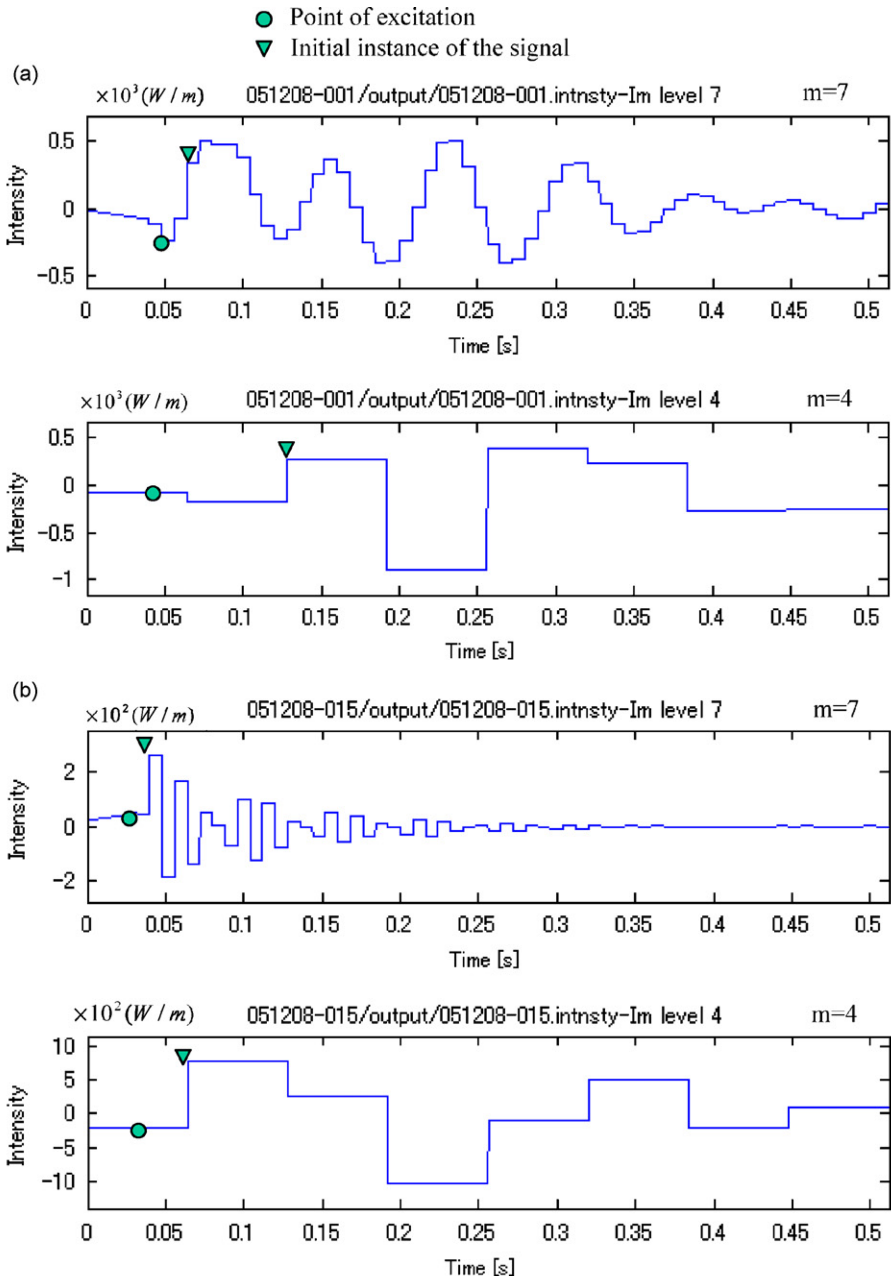


Fig. 9. Instantaneous intensity at the levels 4 and 7, for the uniform beam (a) and the uniform beam with damping layers (b).

Table 1
Arrival time of the flexural wave

| Level | (a) Arrival time | (b) Arrival time |
|---|---|---|
| $m = 4$ | 0.088 (s) | 0.031(s) |
| $m = 7$ | 0.031 (s) | 0.012(s) |
| $T_{m=4}/T_{m=7}$ | 2.8 | 2.6 |

as predicted by the theory. From these results, it is considered that the wavelet intensity analysis gives us an effective tool for the analysis of transient waves on the beam structures.

## Acknowledgment

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