Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 306 (2007) 389–399
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Flexural wave propagation and localized vibration in narrow Mindlin's plate☆

Hu Chao, Chen Tao*, Han Gang, Huang Wenhu

Department of Aerospace Engineering and Mechanics, Harbin Institute of Technology, Harbin 150001, China

Received 24 February 2005; received in revised form 11 May 2005; accepted 10 July 2006

Abstract

In this paper, based on Mindlin's theory of thick plates, using Hamiltonian formulism, the elastic wave propagation and localized vibration in narrow plates with free boundary conditions are investigated. The existence of localized vibration mode and propagation mode is analyzed. The dispersion relation of propagation modes in strip plates is deduced from eigenfunction expansion method. And it is compared with the dispersion relation that is gained using Mindlin's thick plate theory. Based on the two kinds of theories, dispersion relation curves differ much under short wave. Cutoff frequencies are higher under Hamilton formulism. But dispersion curves are almost same under long wave.

1. Introduction

The classical theory of thin plates has been deduced by Lagrange–German in 19th century. It has limitation when the classical theory is applied to analyze dynamic elasticity under free boundary conditions. Sometimes, it cannot satisfy boundary conditions at angular points. In the middle of 20th century, Reissner put forward the static equation of plates and in that the effect of transverse shear deformations are accounted [1,2]. Subsequently, Mindlin undertook a systematic investigation of dynamics of a plate. The Mindlin theory is a better approximation of the underlying mechanics in a thin plate, but introduces analytical complications as compared with the simpler formulation. Midlin's theory contains two rotations as field variables in addition to the transverse displacement and includes rotary inertia and shear effects which are ignored in the Kirchhoff theory [3,4]. It makes up insufficiency of the classical plate theory to some degree. It can be used in a large range of the frequency. So the analytic result is close to practical result in engineering [6,7].

With the development of modern science and technology, composite structure is usually applied to engineering. Transverse shearing modules of composite plate are usually small, so thick plate theory ought to be used in dynamical analysis. Meanwhile, strip plate is applied to aircraft structure and large-scale building structure. Strip plate is usually simplified to beam in structural analysis and vibration control. In classical Kirchhoff and Euler–Bernoulli theory [8,9], flexural wave numbers of rectangle beam or plate are

☆The project is supported by the National Natural Science Foundation of China (19972018).
*Corresponding author. Tel.: +86 451 89264194.
E-mail addresses: huchao@hope.hit.edu.cn (H. Chao), chen_tao1024@hit.edu.cn (C. Tao).

0022-460X/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2006.07.011

$k = (\rho h \omega^2 / D)^{1/4}$. Plate theory adopts plane stress hypothesis and here the beam theory is based on uniaxial stress hypothesis. Uniaxial stress hypothesis is reasonable for small sectional components, like, beam and rod. Uniaxial stress hypothesis results in error, even structure vibration instability, so further research is needed.

In the past, semi-analytic method was applied to solve bending vibration of plates. This method has many limitations, such as the difficulty in analyzing complicated boundary value problems and cannot lose any mode of motion. Using Hamiltonian formulation the problem, which may not be solvable by the classical method, can be solved [10–12].

In this paper, based on Mindlin's theory of thick plates, using Hamiltonian formulation, the elastic wave propagation and localized vibration in narrow plates with free boundary conditions are investigated. The existence of localized vibration mode and propagation mode are analyzed. The dispersion relation of elastic waveguide in the plates is given at free boundary value. As examples, numerical results of dispersion relations in the plates are graphically presented.

2. State vectors of elastic plates

By using Mindlin's theory of plates, in rectangular coordinate system the displacement components u_x, u_y, u_z are given by [5,12]

$$u_x = -z\varphi_x(x, y, t), \quad u_y = -z\varphi_y(x, y, t), \quad u_z = w(x, y, t), \tag{1}$$

where w is transverse displacement φ_x and φ_y refer to rotation about x- and y-axis of normal line in xz and yz plane, respectively. h is the height of the plate. $w(x, y, t)$, φ_x and φ_y are shown in Fig. 1. So bending moment and shear in plates can be expressed as follows:

$$M_x = \int_{-h/2}^{h/2} z\sigma_x \, \mathrm{d}z = -D\left(\frac{\partial \varphi_x}{\partial x} + v\frac{\partial \varphi_y}{\partial y}\right), \tag{2a}$$

$$M_y = \int_{-h/2}^{h/2} z\sigma_y \, \mathrm{d}z = -D\left(\frac{\partial \varphi_y}{\partial y} + v\frac{\partial \varphi_x}{\partial x}\right), \tag{2b}$$

$$M_{xy} = M_{yx} = \int_{-h/2}^{h/2} z\sigma_{xy} \, \mathrm{d}z = -\frac{(1-v)}{2} D\left(\frac{\partial \varphi_y}{\partial x} + \frac{\partial \varphi_x}{\partial y}\right), \tag{2c}$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz} \, \mathrm{d}z = C\left(\frac{\partial w}{\partial x} - \varphi_x\right), \tag{2d}$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz} \, \mathrm{d}z = C\left(\frac{\partial w}{\partial y} - \varphi_y\right), \tag{2e}$$

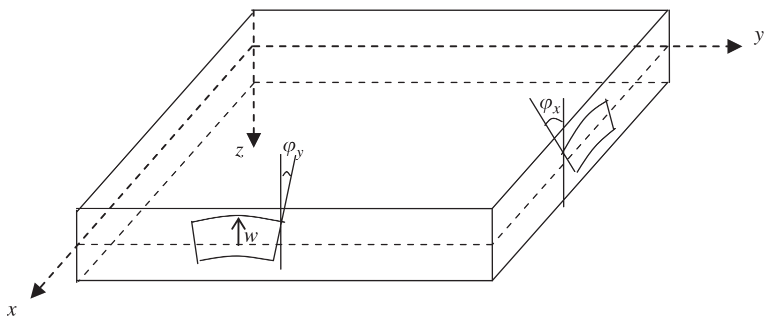

Fig. 1. Mindlin's plate.

where D is bending rigidity of plates, $D = Eh^3/12(1-v^2)$, $C = \kappa Gh$, $G = Eh/2(1+v)$, where κ is shear reduced coefficient, $\kappa = \pi^2/12$.

For using Hamilton formulism, the coordinate x is analog to time variable. Thus in state space, generalized displacement variables $q = (w, \varphi_x, \varphi_y)^T$, generalized velocity $\dot{q} = \partial q/\partial x = (\dot{w}, \dot{\varphi}_x, \dot{\varphi}_y)^T$. Using expression of strain energy and kinetic energy of plates, Lagrangian density function is given by

$$L(q,\dot{q}) = \frac{1}{2}D\left[\dot{\varphi}_x^2 + \left(\frac{\partial \varphi_y}{\partial y}\right)^2 + 2v\dot{\varphi}_x\frac{\partial \varphi_y}{\partial y}\right] + \frac{Gh^3}{24}\left(\frac{\partial \varphi_x}{\partial y} + \dot{\varphi}_y\right)^2 + \frac{1}{2}C\left[(\dot{w} - \varphi_x)^2 + \left(\frac{\partial w}{\partial y} - \varphi_y\right)^2\right] + \frac{1}{2}\rho h\omega^2 w^2 + \frac{1}{2}\rho J\omega^2\varphi_x^2 + \frac{1}{2}\rho J\omega^2\varphi_y^2, \tag{3}$$

where J is the rotary inertia of plates $J = h^3/12$. ρ is density, ω is frequency.

In phase space, generalized displacement and generalized momentum refer to $q = (w, \varphi_x, \varphi_y)^T$, $p = (p_w, p_{\varphi_x}, p_{\varphi_y})^T$, respectively. The components of generalized momentum are given by

$$p_w = \frac{\partial L(q,\dot{q})}{\partial \dot{w}} = Q_x, \tag{4a}$$

$$p_{\varphi_x} = \frac{\partial L(q,\dot{q})}{\partial \dot{\varphi}_x} = -M_x, \tag{4b}$$

$$p_{\varphi_y} = \frac{\partial L(q,\dot{q})}{\partial \dot{\varphi}_y} = -M_{xy}. \tag{4c}$$

The generalized velocity is denoted by generalized displacement and momentum. From Eq. (2) and the Hamiltonian formulism, the following expression can be obtained:

$$\dot{w} = \frac{1}{C}Q_x + \varphi_x, \tag{5a}$$

$$\dot{\varphi}_x = -\frac{1}{D}M_x - v\frac{\partial \varphi_y}{\partial y}, \tag{5b}$$

$$\dot{\varphi}_y = -\frac{2}{(1-v)D}M_{xy} - \frac{\partial \varphi_x}{\partial y}. \tag{5c}$$

The Hamiltonian function of flexural vibration of plates is given by [9]

$$H(q,p) = p^T\dot{q} - L(q,\dot{q}) = \frac{1}{2}\frac{1}{C}Q_x^2 + Q_x\varphi_x + \frac{1}{2}\frac{1}{D}M_x^2 + vM_x\frac{\partial \varphi_y}{\partial y} + \frac{1}{(1-v)D}M_{xy}^2 + M_{xy}\frac{\partial \varphi_x}{\partial y} - \frac{1}{2}D(1-v^2)\left(\frac{\partial \varphi_y}{\partial y}\right)^2 - \frac{1}{2}C\left(\frac{\partial w}{\partial y} - \varphi_y\right)^2 - \frac{1}{2}\rho h\omega^2 w^2 - \frac{1}{2}\rho J\omega^2\varphi_x^2 - \frac{1}{2}\rho J\omega^2\varphi_y^2. \tag{6}$$

Let the state variable be given by $v = (q,p)^T = (w, \varphi_x, \varphi_y, Q_x, -M_x, -M_{xy})^T$

$$\dot{p}_w = -\frac{\partial H}{\partial w} = -C\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial \varphi_y}{\partial y}\right) + \rho h\omega^2 w, \tag{7a}$$

$$\dot{p}_{\varphi_x} = -\frac{\partial H}{\partial \varphi_x} = -Q_x + \frac{\partial M_{xy}}{\partial y} + \rho J\omega^2\varphi_x, \tag{7b}$$

$$\dot{p}_{\varphi_y} = -\frac{\partial H}{\partial \varphi_y} = -C\frac{\partial w}{\partial y} - D(1-v^2)\left(\frac{\partial^2 \varphi_y}{\partial y^2}\right) + C\varphi_y + \rho J\omega^2\varphi_y + v\frac{\partial M_x}{\partial y}. \tag{7c}$$

In phase space, from Eqs. (5) and (7). Single-frequency flexural wave of plates can be written:

$$\dot{v} = Hv = \mu v, \tag{8}$$

where

$$H = \begin{bmatrix} 0 & 1 & 0 & 1/C & 0 & 0 \\ 0 & 0 & -v\partial/\partial y & 0 & 1/D & 0 \\ 0 & -\partial/\partial y & 0 & 0 & 0 & 2/(1-v)D \\ -C\partial^2/\partial y^2 - \rho h\omega^2 & 0 & C\partial/\partial y & 0 & 0 & 0 \\ 0 & -\rho J\omega^2 & 0 & -1 & 0 & -\partial/\partial y \\ -C\partial/\partial y & 0 & -(1-v^2)D\partial^2/\partial y^2 + C - \rho J\omega^2 & 0 & -v\partial/\partial y & 0 \end{bmatrix}.$$

The relation of cross eigenvalue λ and longitudinal eigenvalue μ are given by

$$(\mu^2 + \lambda^2 + k_1^2)(\mu^2 + \lambda^2 + k_2^2)(\mu^2 + \lambda^2 + k_3^2) = 0. \tag{9a}$$

So

$$\lambda_n^2 = -\mu^2 - k_n^2 \quad (n = 1, 2, 3), \tag{9b}$$

where

$$k_{1,2}^4 - \frac{D\rho h\omega^2 + C\rho J\omega^2}{CD}k_{1,2}^2 + \frac{\rho h\omega^2(\rho J\omega^2 - C)}{CD} = 0, \tag{10a}$$

$$k_3^2 = \frac{2(\rho J\omega^2 - C)}{D(1-v)}. \tag{10b}$$

2.1. Definition of zero eigensolution

Zero eigenvalue is important in elastic mechanics. For elastodynamics of rectangle domain, it will have the solution of zero eigenvalue under free boundary conditions. Then the following equation is satisfied:

$$H\psi^{(0)} = 0, \tag{11}$$

$$\dot{v} = Hv, \ v = [w, \varphi_x, \varphi_y, Q_x, -M_x, -M_{xy}]^{\mathrm{T}}.$$

When the two sides of plates are free, expressions for boundary conditions are given by

$$D(1-v^2)\frac{\partial \varphi_y}{\partial y} - vM_x = D(1-v^2)\frac{\partial q_3}{\partial y} + vp_2 = 0, \tag{12a}$$

$$\frac{\partial w}{\partial y} - \varphi_y = \frac{\partial q_1}{\partial y} - q_3 = 0, \tag{12b}$$

$$-M_{xy} = p_3 = 0. \tag{12c}$$

After analyzing, linear independent basic eigensolutions is given by

$$\psi_0^{(1)} = (w, 0, \varphi_y, 0, -M_x, 0)^{\mathrm{T}}, \tag{13a}$$

$$\psi_0^{(2)} = (0, \varphi_x, 0, Q_x, 0, -M_{xy})^{\mathrm{T}}, \tag{13b}$$

where

$$w = w_0^{(1)} = \left[\frac{D(1-v)k_3^2 - 2Dk_1^2}{2Ck_1}\right] \cos(k_1 y) + \left[\frac{D(1-v)k_3^2 - 2Dk_2^2}{2Ck_2}\right] \delta \cos(k_2 y),$$

$$\delta = \delta(k_1, k_2) = -\frac{k_1 \cos(k_1 a)}{k_2 \cos(k_2 a)}, \quad \varphi_x = \varphi_{x0}^{(2)} = \cos(k_3 y), \quad \varphi_y = \varphi_{y0}^{(1)} = \sin(k_1 y) + \delta \sin(k_2 y),$$

$$Q_x = Q_{x0}^{(2)} = -C \cos(k_3 y), \ M_x = M_{x0}^{(1)} = -vDk_1 \cos(k_1 y) - vDk_2 \delta \cos(k_2 y),$$

$$M_{xy} = M_{xy0}^{(2)} = \frac{(1-v)D}{2} k_3 \sin(k_3 y).$$

For satisfying the boundary conditions, we must have $k_3 a = n\pi$. $\psi_0^{(2)}$ satisfies Eqs. (12a) and (12b). For Eq. (12c) to be satisfied vibration frequency of plates is given by

$$\omega^2 = \frac{1}{\rho J}\left[(1-v)D\frac{n^2\pi^2}{2a^2} + C\right] \quad (n = 0, 1, 2, \ldots). \tag{14}$$

According to boundary conditions of plates, when vibration has the relation $k_{1,2}$, frequency obtained

$$\omega^2 = \frac{D}{\rho J} \frac{k_1^2 k_2 \tan(k_1 a) - k_1 k_2^2 \tan(k_1 a)}{k_2 \tan(k_1 a) - k_1 \tan(k_2 a)}. \tag{15}$$

Then, Jordan zero-order eigenvectors are given by

$$v_0^{(1)} = \psi_0^{(1)}, \tag{16a}$$

$$v_0^{(2)} = \psi_0^{(2)}. \tag{16b}$$

The $v_0^{(1)}$ and $v_0^{(2)}$ denote a kind of vibration mode. It is not propagation in the x direction, i.e., standing wave is homogeneous along x-axis and oscillatory along y-axis. Transverse displacements of $\psi_0^{(1)}$ is w, the corresponding rotational angle is φ_y and bending moment is M_x. The $\psi_0^{(2)}$ has the rotation angle φ_y, shear Q_x and bending moment M_{xy}.

First-order zero eigenvector $\psi_1^{(1)}$ satisfies

$$H\psi_1^{(1)} = \psi_0^{(1)}, \tag{17}$$

$$\psi_1^{(1)} = (0, \varphi_x, 0, Q_x, 0, -M_{xy})^{\mathrm{T}}, \tag{18}$$

where

$$\varphi_x = \varphi_{x1}^{(1)} = -\frac{1}{k_1}\cos(k_1 y) - \delta\frac{1}{k_2}\cos(k_2 y), \quad M_x = M_{xy1}^{(1)} = -D(1-v)\sin(k_1 y) - \delta D(1-v)\sin(k_2 y),$$

$$Q_x = Q_{x1}^{(1)} = \frac{D(1-v)k_3^2 - 2Dk_1^2 - 2C}{2k_1}\cos(k_1 y) + \delta\frac{D(1-v)k_3^2 - 2Dk_2^2 - 2C}{2k_2}\cos(k_2 y).$$

A solution of the original equation is

$$v_1^{(1)} = \psi_1^{(1)} + x\psi_0^{(1)}, \tag{19a}$$

$$v_1^{(1)} = [xw_0^{(1)}, \varphi_{x1}^{(1)}, x\varphi_{y0}^{(1)}, Q_{x1}^{(1)}, -xM_{xy0}^{(1)}, -M_{xy1}^{(1)}]^{\mathrm{T}}. \tag{19b}$$

The $v_1^{(1)}$ is a kind of non-propagation vibration mode. It denotes rigidity rotation in xoz plane. Through analysis, the sub-eigensolution chain can be obtained until $\psi_2^{(1)}$ break off.

For seeking the first-order zero eigenvector $\psi_1^{(2)}$ we have

$$H\psi_1^{(2)} = \psi_0^{(2)}, \tag{20}$$

$$\psi_1^{(2)} = (w, 0, \varphi_y, 0, -M_x, 0)^{\mathrm{T}}, \tag{21}$$

where

$$w = w_1^{(2)} = \frac{1}{(k_s^2 - k_3^2)}\left\{1 + \frac{2C - (1+v)D(k_3^2 - k_s^2)}{\left[2Dv(k_3^2 - k_s^2) - 2C\right]}\right\}\cos(k_3 y),$$

$$\varphi_y = \varphi_{y1}^{(2)} = \frac{2C - (1+v)D(k_3^2 - k_s^2)}{\left[2Dv(k_3^2 - k_s^2) - 2C\right]k_3}\sin(k_3 y),$$

$$M_x = M_{x1}^{(2)} = -D\left[1 + v\frac{2C - (1+v)D(k_3^2 - k_s^2)}{\left[2Dv(k_3^2 - k_s^2) - 2C\right]}\right]\cos(k_3 y).$$

A solution of the original equation is

$$v_1^{(2)} = \psi_1^{(2)} + x\psi_0^{(2)}, \tag{22a}$$

$$v_1^{(2)} = [w_1^{(2)}, x\varphi_{x0}^{(2)}, \varphi_{y1}^{(2)}, xQ_{x0}^{(2)}, -M_{xy1}^{(2)}, -xM_{xy0}^{(2)}]^{\mathrm{T}}. \tag{22b}$$

The $v_1^{(2)}$ is also a kind of non-propagation vibration mode. It denotes rigidity rotation in *xoz* plane. Through analysis, the sub-eigensolution chain can be obtained until $\psi_2^{(2)}$ break off.

For zero eigenvalue, the dynamical mode corresponds to the modal frequency of vibration in plates. For example, the first modal frequency is referred to as inherent frequency in engineering. Then integrals of all mechanical variables are equal to zero along y-axis.

2.2. Definition of nonzero eigensolution

Considering symmetrical condition, the eigensolution of flexural vibration of symmetrical plates $\psi_n = (q_n, p_n)^{\mathrm{T}}$ is written as

$$w = A_{11}\cosh(\lambda_1 y) + A_{12}\cosh(\lambda_2 y) + A_{13}\cosh(\lambda_3 y), \tag{23a}$$

$$\varphi_x = A_{21}\cosh(\lambda_1 y) + A_{22}\cosh(\lambda_2 y) + A_{23}\cosh(\lambda_3 y), \tag{23b}$$

$$\varphi_y = A_{31}\sinh(\lambda_1 y) + A_{32}\sinh(\lambda_2 y) + A_{33}\sinh(\lambda_3 y), \tag{23c}$$

$$Q_x = A_{41}\cosh(\lambda_1 y) + A_{42}\cosh(\lambda_2 y) + A_{43}\cosh(\lambda_3 y), \tag{23d}$$

$$-M_x = -A_{51}\cosh(\lambda_1 y) - A_{52}\cosh(\lambda_2 y) - A_{53}\cosh(\lambda_3 y), \tag{23e}$$

$$-M_{xy} = -A_{61}\sinh(\lambda_1 y) - A_{62}\sinh(\lambda_2 y) - A_{63}\sinh(\lambda_3 y), \tag{23f}$$

where $\lambda_n^2 = -\mu^2 - k_n^2$; $A_{mn}(m = 1, 2, \ldots 6,\ n = 1, 2, 3)$ are mode coefficients and they are not independent. Based on analysis, we see only three coefficients are independent.

Substituting Eq. (23) into Eq. (8), the relational expressions of mode coefficients can be obtained:

$$\frac{A_{2n}}{A_{1n}} = \frac{\mu\left[2C(Dk_n^2 - \rho J\omega^2) - D(1+v)\rho h\omega^2\right]}{D(1-v)(Dk_n^2 - \rho J\omega^2)(k_n^2 - k_3^2)}, \quad \frac{A_{3n}}{A_{1n}} = \frac{\lambda_n\left[2C(Dk_n^2 - \rho J\omega^2) - D(1+v)\rho h\omega^2\right]}{D(1-v)(Dk_n^2 - \rho J\omega^2)(k_n^2 - k_3^2)},$$

$$\frac{A_{4n}}{A_{1n}} = \frac{\mu\rho\omega^2\left[Dh(1-v)(Dk_n^2 - \rho J\omega^2) + 2C(D(h - Jk_n^2) + \rho J^2\omega^2\right]}{D(1-v)(Dk_n^2 - \rho J\omega^2)(k_n^2 - k_3^2)},$$

$$\frac{A_{6n}}{A_{1n}} = \frac{\mu\lambda_n D(1-v)\left[D(1+v)\rho h\omega^2 - 2C(Dk_n^2 - \rho J\omega^2)\right]}{D(1-v)(Dk_n^2 - \rho J\omega^2)(k_n^2 - k_3^2)}, \quad (n = 1, 2),$$

$$A_{13} = 0, \quad \frac{A_{33}}{A_{23}} = \frac{-\mu}{\lambda_3}, \quad \frac{A_{43}}{A_{22}} = -C, \quad \frac{A_{53}}{A_{23}} = -\mu D(1-v), \quad \frac{A_{63}}{A_{23}} = \frac{D(1-v)(2\mu^2 + k_3^2)}{2\lambda_3}.$$

To simplify the expression, the following relations will be used:

$$\frac{\left[2C(Dk^2-\rho J\omega^2)-D(1+v)\rho h\omega^2\right]}{(Dk^2-\rho J\omega^2)}=\frac{\left[2C^2-D(1+v)(Ck^2-\rho h\omega^2)\right]}{C}.$$

The equation of eigenvalue under free boundary conditions is given by

$$\begin{aligned}&\lambda_1 k_1^2[2k_1^2-k_s^2(1-v)]^{-1}\{Ck_2^2k_s^2+D(k_2^2-k_s^2)[k_1^2k_2^2+(1-v)\mu^2]\}\{D\mu^2(1-v)[k_s^2(1-v)-2k_1^2]\\&+2k_s^2[C-D(k_1^2-k_s^2)]v^2-2Dk_1^2k_2^2(1-v^2)+Dk_2^2k_s^2(1-v-4v^2+2v^3)\}c\tanh(\lambda_2 a)\\&-\lambda_2k_2^2[2k_2^2-k_s^2(1-v)]^{-1}\{Ck_1^2k_s^2+D(k_1^2-k_s^2)[k_1^2k_2^2+(1-v)\mu^2]\}\{D\mu^2(1-v)[k_s^2(1-v)-2k_2^2]\\&+2k_s^2[C-D(k_2^2-k_s^2)]v^2-2Dk_1^2k_2^2(1-v^2)+Dk_1^2k_s^2(1-v-4v^2+2v^3)\}c\tanh(\lambda_1 a)+D^2(1-v)^2\\&\times\lambda_1\lambda_2\lambda_3(k_1^2-k_2^2)k_s^4\mu^2c\tanh(\lambda_3 a)=0,\end{aligned}\tag{24a}$$

where $k_s^2=\rho h\omega^2/C$. Considering existence of waveguide along x-axis, from Eq. (24a), and defining μ by $\mu=\mathrm{i}k$, a dispersion relation by using Hamilton formulism are obtained. Here k is wave number of elastic waves in plates.

Using Mindlin's plate theory the dispersion relations of flexural waves in strip plates can be presented as

$$\begin{aligned}&\left(k^2-\frac{k_1^2}{1-v}\right)^2(k_s^2-k_1^2)k_2^2\lambda_1^{-1}c\tanh(\lambda_1 a)-(k^2-\frac{k_2^2}{1-v})^2(k_s^2-k_2^2)k_1^2\lambda_2^{-1}c\tanh(\lambda_2 a)\\&+k^2k_s^2(k_1^2-k_2^2)\lambda_3c\tanh(\lambda_3 a)=0.\end{aligned}\tag{24b}$$

In Figs. 2–7, c is phase velocity. Figs. 2, 4 and 6 represent dispersion relations of flexural waves in strip plates by using Hamilton formulism. Figs. 3, 5 and 7 show dispersion relations of flexural waves of strip plates under Mindlin theory. Comparing the dispersion relations of flexural waves under two kinds of theories, it is easy to get some results: dispersion relations have great differences under short wave, namely, anterior some low frequency modes have differences in some degree. Cutoff frequencies are higher under Hamilton formulation. Dispersion relations are almost same under long wave, namely high-frequency mode has difference [13].

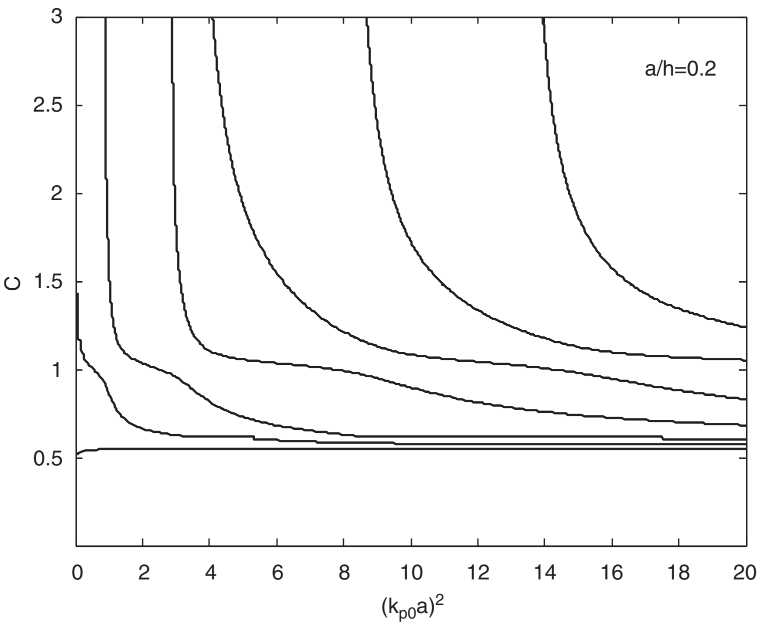

Fig. 2. Dispersion relations of flexural wave in strip plates.

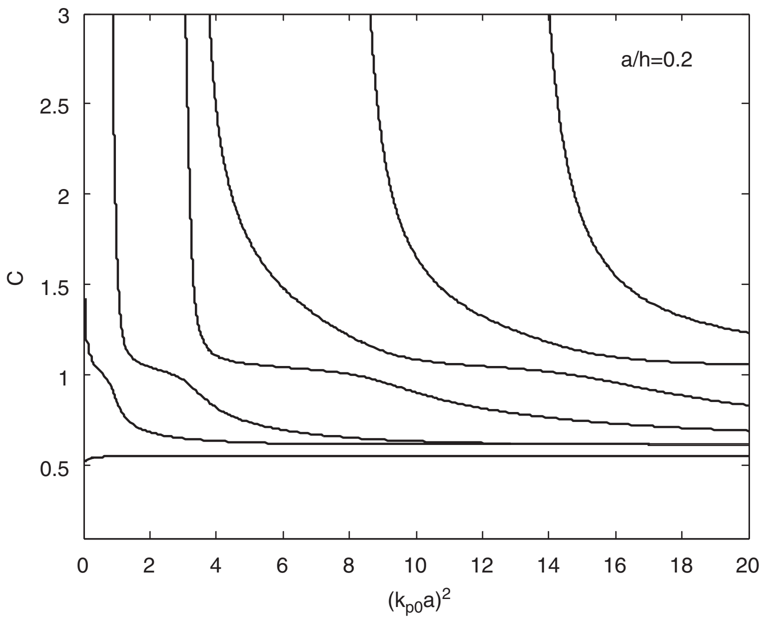

Fig. 3. Dispersion relations of flexural wave in strip plates.

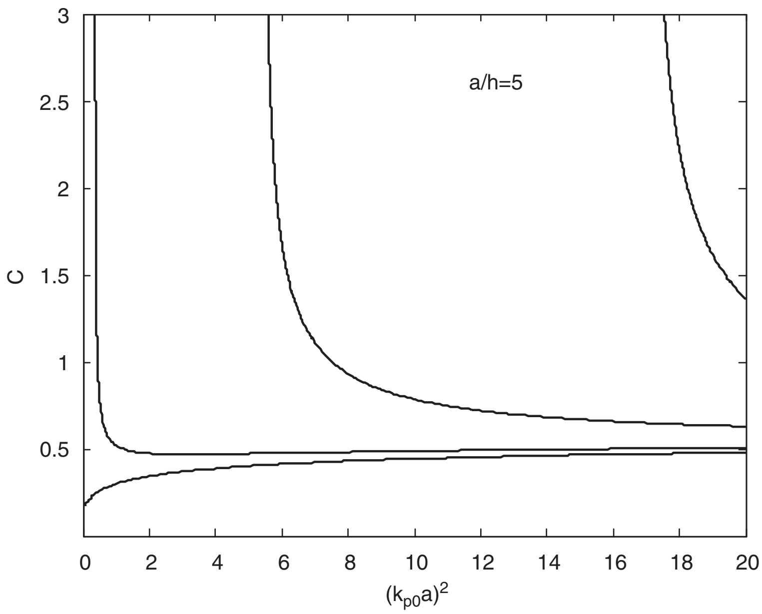

Fig. 4. Dispersion relations of flexural wave in strip plates.

3. Analyses and discussion

It can be seen that when dynamics of elastic plates are considered, the translation of rigid body and static rotation do not exist, but non-propagation modes exist. For example, the oscillation which is standing waves at y-axis direction is homogeneous on x-axis.

Based on the Hamiltonian formulism, beside the extended mode at the x direction can be determined; the localized vibration also can be sought. But using Mindlin's theory of plates the extended mode can be

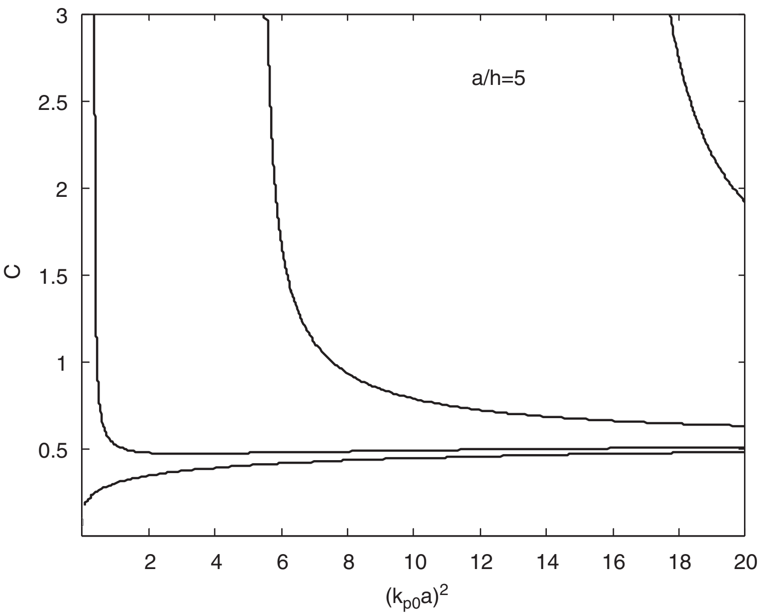

Fig. 5. Dispersion relations of flexural wave in strip plates.

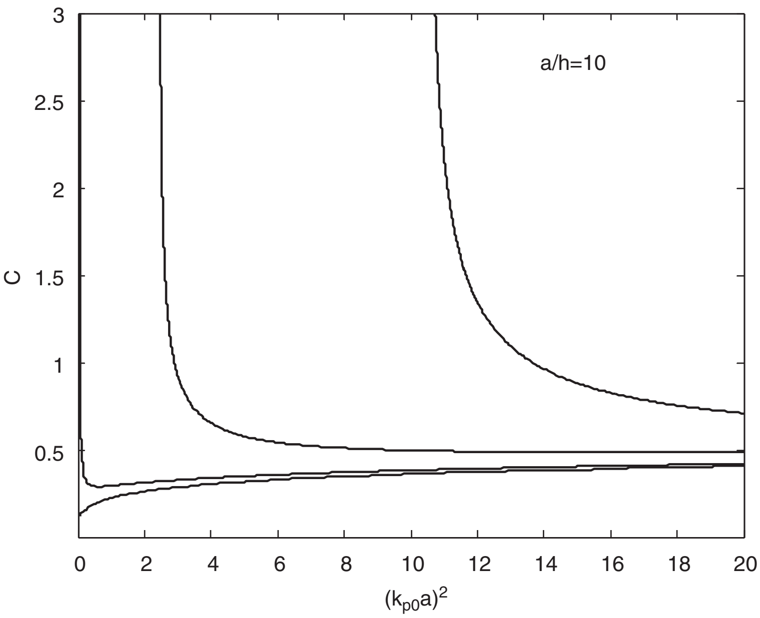

Fig. 6. Dispersion relations of flexural wave in strip plates.

determined only, the some of localized vibrations may be lost. Although Mindlin's theory of plates has limitations, it can be used to analyze the dynamics of plates at high frequencies.

The dynamical mode of zero eigenvalues corresponds with free frequency of vibrations in general mechanics. For example, the frequency of the first mode in engineering is treated as free frequency of structure in some cases. But integral of any mechanical quantity is zero along the y direction.

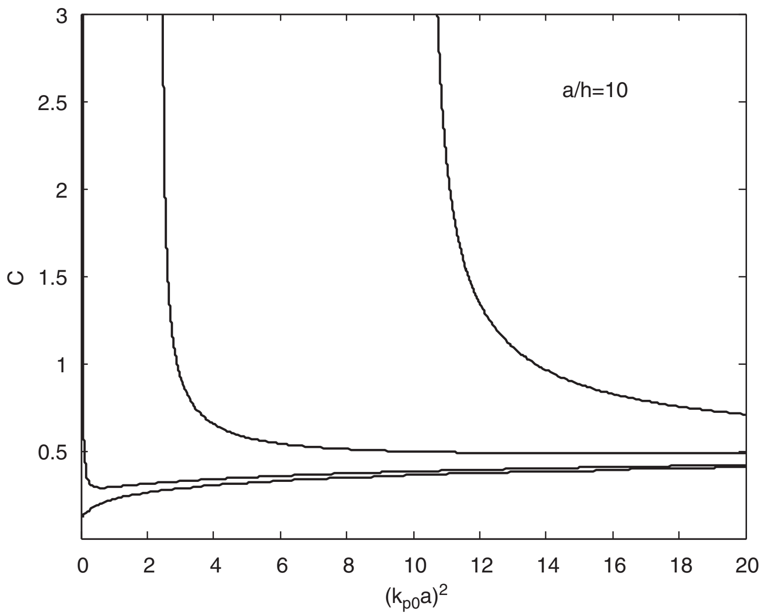

Fig. 7. Dispersion relations of flexural wave in strip plates.

For non-zero eigenvalues, when imaginary part is greater than zero and real part is less than zero, it denotes propagation decaying modes in x-axis positive direction, or the unstable state of resonance. When imaginary part is less than zero and real part is greater than zero, it indicates decaying propagation modes in x-axis negative directions, or else the accumulable state of energy.

When the real part is equal to zero, it represents the propagation mode in x-axis, positive or negative direction. When the real part is less than zero, it expresses localized vibrations.

When the real part is greater than zero, it denotes destabilization of the resonance. Then the vibration frequency of structures must satisfy dispersion relations, namely, circular frequency of vibrations, wave number of incident waves, and geometric parameters of structures must fit the dispersion equation.

The analytical method and numerical results in this paper are expected to apply to dynamics and vibration control of aircraft structures.

References

[1] E. Reissner, The effect of transverse shear deformation on the bending of elastic plates, *Journal of Applied Mechanics* 12 (2) (1945) 69–77.
[2] E. Reissner, On the theory of transverse bending of elastic plates, *International Journal of Solids and Structure* 12 (8) (1976) 545–554.
[3] D.L. Russel, L.W. White, The bowed narrow plate modelé, *Elec. Journal of Differential Equations* (27) (2000) 1–19.
[4] D.E. Newland, Wavelet analysis of vibration, *Transactions of the ASME Journal of Vibration and Acoustics* 116 (1994) 409–417.
[5] R.D. Mindlin, Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates, *Transactions of the ASME Journal of Applied Mechanics* 18 (2) (1951) 31–38.
[6] Y.H. Pao, Diffractions of flexural waves by a cavity in an elastic plate, *Journal of AIAA* 2 (1) (1964) 2000–2010.
[7] D.K. Liu, C. Hu, Scattering of flexural wave and dynamic stress concentrations in Mindlin's thick plates, *Acta Mechanica Sinica* 12 (2) (1996) 69–185.
[8] A.N. Andrew, Flexural waves on narrow plates, *Journal of the Acoustical Society of America* 113 (5) (2003) 2647–2658.
[9] P.C. Xu, S.K. Datta, Guided waves in a bonded plate: a parametric study, *Journal of Applied Physics* 67 (11) (1990) 6779–6786.
[10] S.P. Timoshenko, J.N. Goodier, *Theory of Elasticity*, McGraw-Hill, New York, 1951.

[11] W.X. Zhong, *A New Systematic Methodology for Theory of Elasticity*, Publishing House of Dalian University of Technology, Lioaning, 1995 (in Chinese).
[12] W.X. Zhong, W.A. Yao, New solution system for plate bending and its application, *Acta Mechanica Sinica* 31 (2) (1999) 173–184 (in Chinese).
[13] Z.X. Hang, *Introduction of the Cutoff Frequency of Waveguide*, Publishing House of China Measurement, Beijing, 1991 (in Chinese).