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Journal of Sound and Vibration 306 (2007) 897–907
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Dynamic response of a piezoelectric rod with thermal relaxation

Tianhu He*, Li Cao, Shirong Li

School of Science, Lanzhou University of Technology, Lanzhou 730050, PR China

Received 27 June 2006; received in revised form 18 May 2007; accepted 30 June 2007

Abstract

Based on the Lord and Shulman generalized thermo-elastic theory with one relaxation time, the dynamic thermal and elastic responses of a piezoelectric rod fixed at both ends and subjected to a moving heat source are investigated. The generalized piezoelectric–thermoelastic coupled governing equations for piezoelectric rod are formulated. By means of Laplace transformation and numerical Laplace inversion the governing equations are solved. Numerical calculation for stress, displacement and temperature within the rod is carried out and displayed graphically. The effect of moving heat source speed on temperature, stress and temperature is studied. It is found from the distributions that the temperature, thermally induced displacement and stress of the rod are found to decrease at large source speed.

1. Introduction

To eliminate the paradox inherent in the classical uncoupled and coupled thermoelastic theories that heat propagates with an infinite speed due to the diffusion type heat conduction equation, the generalized thermoelastic theories were introduced by Lord and Shulman [1] (L–S) and Green and Lindsay [2] (G–L) in 1960s. The L–S theory postulated a wave-type heat conduction law to replace the classical Fourier's law. This law is the same as that suggested by Cattaneo [3] and Vernotte [4]. It contains the heat flux vector as well as its time derivative and contains also a new constant that acts as a relaxation time. The G–L theory modified both the energy equation and the Duhamel–Neumann relation, and allows two relaxation times. This two generalized theories can both ensure finite propagation speed of thermoelastic waves.

Numerous works had been devoted to problems involving a moving heat source due to its extensive engineering applications, such as continuous annealing after cold working, pulsed-laser cutting and welding, and high speed machining and grinding, etc. Al-Huniti et al. [5] studied the dynamic responses of a copper rod due to a moving heat source under the wave type heat conduction model. In Ref. [5], by means of the Laplace transform the temperature was obtained directly from the heat conduction equation.

Piezoelectric ceramics and composites have been extensively used in many engineering applications such as sensors, actuators, intelligent structures, etc. Mindlin [6] first proposed a thermo-piezoelectricity theory and also established the governing equations of a thermo-piezoelectric plate [7]. Nowacki [8,9] has investigated the

*Corresponding author. Tel.: +86 0931 2973780; fax: +86 0931 2976040.
E-mail address: heth@lut.cn (T. He).

0022-460X/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2007.06.018

Nomenclatures

c_{ijkl}	elastic constants
C_E	specific heat at constant deformation
D_i	components of electric displacement
E_i	components of electric field vector
h_{ijk}	piezoelectric constants
p_i	pyroelectric constants
Q	strength of the applied heat source per unit mass where $i, j, k, l = 1,2,3$
t	time
T	absolute temperature
T_0	initial environment temperature, chosen as $\|(T-T_0)/T \ll 1\|$
u_i	components of displacement vector
ε_{ij}	components of strain tensor
θ	temperature increment $\theta = T-T_0$
κ_{ij}	coefficients of thermal conductivity
λ_{ij}	thermal modulus
ρ	mass density
σ_{ij}	components of stress tensor
τ	thermal relaxation time
τ_{ik}	dielectric constants
φ	electric potential function

physical laws for the thermo-piezoelectric materials. Chandrasekharaiah [10] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances on the basis of the first and the second thermodynamics laws. Compared with the investigation of propagation of thermoelastic waves in elastic media, similar investigation in piezoelectric media is much fewer. Majhi [11] introduced a potential function to deal with the transient thermal response of a semi-infinite piezoelectric rod subjected to a local heat source based on L–S theory. Although the temperature distribution was given in Ref. [11], the result is unreasonable. He et al. [12] used Laplace transform and state-space method to solve the dynamic response of a semi-infinite piezoelectric rod subjected to a thermal shock at one end based on the G–L theory, and one-dimensional (1D) analytical solution was obtained and displayed graphically.

In this paper, based on the L–S theory, the dynamic response of a finite piezoelectric rod fixed at both ends and subjected to a moving heat source is investigated. The piezoelectric–thermoelastic coupled equations are formulated and the equations are solved by means of Laplace transform. Then the numerical Laplace inversion is carried out to obtain the temperature, displacement and stress distributions in the piezoelectric rod.

2. Basic equations

In the absence of body force and free charge, the generalized thermo-piezoelectric governing differential equations are:

(a) Strain–displacement relations

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

(b) Constitutive equation

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - h_{kij}D_k - \lambda_{ij}T, \quad E_i = h_{ikl}\varepsilon_{kl} + \tau_{ik}D_k - p_i\theta. \tag{2}$$

(c) Motion equation

$$\sigma_{ij,j} = \rho\ddot{u}_i. \tag{3}$$

(d) Gauss equation and electric field relation

$$D_{i,i} = 0, \quad E_i = -\varphi_{,i}. \tag{4}$$

(e) Heat conduction equation

$$\kappa_{ij}\theta_{,ij} = \left(1 + \tau\frac{\partial}{\partial t}\right)(\rho C_E\dot{\theta} + T_0\lambda_{kl}\dot{\varepsilon}_{kl} - T_0 p_k\dot{\varphi}_{,k} - Q). \tag{5}$$

In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the derivative with respect to time.

We shall consider a thin piezoelectric rod. Let the piezoelectric rod polarization direction be parallel with the axial direction.

For 1D problem we assume displacement components of the form

$$u_x = u(x,t), \quad u_y = u_z = 0. \tag{6}$$

From Gauss's law, since there is no free charge inside the piezoelectric rod, for 1D case, we have

$$\frac{\partial D}{\partial x} = 0 \tag{7}$$

which leads to $D = D(t)$. For simplification, we would like to keep $D = \text{const}$ along the piezoelectric rod.

For 1D case, the first equation in Eqs. (2), (3) and (5) reduce to

$$\sigma = c_{11}\frac{\partial u}{\partial x} - \lambda_{11}T - hD, \tag{8}$$

$$c_{11}\frac{\partial^2 u}{\partial x^2} - \lambda_{11}\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\kappa_{11}\frac{\partial^2 T}{\partial x^2} = \left(1 + \tau\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial T}{\partial t} + \lambda_{11}T_0\frac{\partial^2 u}{\partial x\partial t} - Q\right). \tag{10}$$

For simplifications we introduce the following non-dimensional variables

$$x^* = c_0\eta_0 x, \quad u^* = c_0\eta_0 u, \quad t^* = c_0^2\eta_0 t, \quad \tau^* = c_0^2\eta_0\tau, \quad \theta^* = \frac{\theta}{T_0},$$
$$\sigma^* = \frac{\sigma}{c_{11}}, \quad D^* = \frac{hD}{c_{11}}, \quad Q^* = \frac{Q}{\kappa_{11}T_0c_0^2\eta_0^2}, \quad c_0 = \sqrt{\frac{c_{11}}{\rho}}, \quad \eta_0 = \frac{\rho C_E}{\kappa_{11}}. \tag{11}$$

In terms of these non-dimensional variables in Eq. (11), Eqs. (8)–(10) take the following forms (dropping the asterisks for convenience)

$$\sigma = \frac{\partial u}{\partial x} - b\theta - D, \tag{12}$$

$$\frac{\partial^2 u}{\partial x^2} - b\frac{\partial\theta}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2\theta}{\partial x^2} = \left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial\theta}{\partial t} + g\frac{\partial^2 u}{\partial x\partial t} - Q\right), \tag{14}$$

where $b = \lambda_{11}T_0/c_{11}$ and $g = \lambda_{11}/\rho C_E$. The piezoelectric rod is assumed to be initially at rest and has a reference temperature T_0 and zero temperature velocity such that the initial conditions are determined as

$$u(x,0) = \dot{u}(x,0) = 0, \quad \theta(x,0) = \dot{\theta}(x,0) = 0. \tag{15}$$

Assuming the piezoelectric rod is fixed at both ends with a non-dimensional length l and both ends are heat insulated, the boundary conditions can be given as

$$u(0,t) = u(l,t) = 0, \quad \frac{\partial\theta(0,t)}{\partial x} = \frac{\partial\theta(l,t)}{\partial x} = 0. \tag{16}$$

The piezoelectric rod is subjected to a moving plane heat source of constant strength releasing its energy continuously while moving along the positive direction of the x-axis with a constant velocity υ. This moving heat source is assumed to be the following non-dimensional form:

$$Q = Q_0\delta(x - \upsilon t), \tag{17}$$

where Q_0 is constant and δ is the delta function. Applying the Laplace transform defined by

$$L[f(t)] = \bar{f}(p) = \int_0^\infty \mathrm{e}^{-pt} f(t)\,\mathrm{d}t, \quad \mathrm{Re}(p)>0 \tag{18}$$

to Eqs. (12)–(14) with Eq. (15), we obtain

$$\bar{\sigma} = \frac{\mathrm{d}\bar{u}}{\mathrm{d}x} - b\bar{\theta} - \frac{D}{p}, \tag{19}$$

$$\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}x^2} - b\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}x} = p^2\bar{u}, \tag{20}$$

$$\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2} = (1+\tau p)\left(p\bar{\theta} + gp\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} - \gamma\,\mathrm{e}^{-(p/\upsilon)x}\right), \tag{21}$$

where $\gamma = Q_0/\upsilon$.

The boundary conditions in Eq. (16) can be transformed to

$$\bar{u}(0,p) = \bar{u}(l,p) = 0, \quad \frac{\mathrm{d}\bar{\theta}(0,p)}{\mathrm{d}x} = \frac{\mathrm{d}\bar{\theta}(l,p)}{\mathrm{d}x} = 0. \tag{22}$$

3. Solutions in the Laplace domain

Eliminating $\bar{\theta}$ between Eqs. (20) and (21), we obtain the following equation satisfied by $\bar{u}$

$$\frac{\mathrm{d}^4\bar{u}}{\mathrm{d}x^4} - m_1\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}x^2} + m_2\bar{u} = m_3\,\mathrm{e}^{-(p/\upsilon)x}, \tag{23}$$

where

$$m_1 = (1+gb)(1+\tau p)p + p^2, \quad m_2 = p^3(1+\tau p), \quad m_3 = \frac{b\gamma p(1+\tau p)}{\upsilon}.$$

The general solution of Eq. (23) is

$$\bar{u} = C_1\,\mathrm{e}^{-k_1x} + C_2\,\mathrm{e}^{k_1x} + C_3\,\mathrm{e}^{-k_2x} + C_4\,\mathrm{e}^{k_2x} + C_5\,\mathrm{e}^{-(p/\upsilon)x}, \tag{24}$$

where C_i ($i = 1,2,3,4$) are parameters depending on p to be determined from the boundary conditions and $C_5 = m_3/[(p/\upsilon)^4 - m_1(p/\upsilon)^2 + m_2]$. k_1 and k_2 are the roots of the characteristic equation

$$k^4 - m_1k^2 + m_2 = 0. \tag{25}$$

k_1 and k_2 are given by

$$k_1 = \sqrt{\frac{m_1 + \sqrt{m_1^2 - 4m_2}}{2}}, \quad k_2 = \sqrt{\frac{m_1 - \sqrt{m_1^2 - 4m_2}}{2}}. \tag{26}$$

Similarly, eliminating $\bar{u}$ between Eqs. (20) and (21), we obtain the following equation satisfied by $\bar{\theta}$:

$$\frac{\mathrm{d}^4\bar{\theta}}{\mathrm{d}x^4} - m_1\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2} + m_2\bar{\theta} = m_4\,\mathrm{e}^{-(p/\upsilon)x}, \tag{27}$$

where $m_4 = \gamma p^2(1+\tau p)(1 - 1/\upsilon^2)$. The general solution of Eq. (27) is

$$\bar{\theta} = C_{11}\,\mathrm{e}^{-k_1x} + C_{22}\,\mathrm{e}^{k_1x} + C_{33}\,\mathrm{e}^{-k_2x} + C_{44}\,\mathrm{e}^{k_2x} + C_{55}\,\mathrm{e}^{-(p/\upsilon)x}, \tag{28}$$

where C_{ii} ($i = 1,2,3,4,5$) are parameters depending on p. Substituting $\bar{u}$ from Eq. (24) and $\bar{\theta}$ from Eq. (28) into Eq. (20), we can find the following relationships:

$$C_{11} = -\frac{k_1^2 - p^2}{bk_1}C_1, \quad C_{22} = \frac{k_1^2 - p^2}{bk_1}C_2, \quad C_{33} = -\frac{k_2^2 - p^2}{bk_2}C_3, \quad C_{44} = \frac{k_2^2 - p^2}{bk_2}C_4, \quad C_{55} = \frac{p(\upsilon^2 - 1)}{b\upsilon}C_5. \tag{29}$$

In order to determine the parameters C_i ($i = 1,2,3,4$) and C_{ii} ($i = 1,2,3,4$), we need to consider the boundary conditions in Eq. (16) and we get

$$C_1 + C_2 + C_3 + C_4 = -C_5, \tag{30}$$

$$C_1\,\mathrm{e}^{-k_1 l} + C_2\,\mathrm{e}^{k_1 l} + C_3\,\mathrm{e}^{-k_2 l} + C_4\,\mathrm{e}^{k_2 l} = -C_5\,\mathrm{e}^{-(p/\upsilon)l}, \tag{31}$$

$$-C_{11}k_1 + C_{22}k_1 - C_{33}k_2 + C_{44}k_2 = (p/\upsilon)C_{55}, \tag{32}$$

$$-C_{11}k_1\,\mathrm{e}^{-k_1 l} + C_{22}k_1\,\mathrm{e}^{k_1 l} - C_{33}k_2\,\mathrm{e}^{-k_2 l} + C_{44}k_2\,\mathrm{e}^{k_2 l} = (p/\upsilon)C_{55}\,\mathrm{e}^{-(p/\upsilon)l}. \tag{33}$$

Solving Eqs. (30)–(33) with Eq. (29), we obtain C_i ($i = 1,2,3,4$) as the following:

$$\begin{aligned}
C_1 &= \frac{(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})},\\
C_2 &= -\frac{(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})},\\
C_3 &= -\frac{(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})},\\
C_4 &= \frac{(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})}.
\end{aligned} \tag{34}$$

Substituting C_i ($i = 1,2,3,4$) into Eq. (24), we obtain

$$\begin{aligned}
\bar{u} = {} & \frac{(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})}\,\mathrm{e}^{-k_1 x} - \frac{(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})}\,\mathrm{e}^{k_1 x}\\
& - \frac{(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})}\,\mathrm{e}^{-k_2 x} + \frac{(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})}\,\mathrm{e}^{k_2 x} + C_5\,\mathrm{e}^{-(p/\upsilon)x}.
\end{aligned} \tag{35}$$

From the relationships between C_i and C_{ii} in Eq. (29), we get

$$\begin{aligned}
C_{11} &= -\frac{(k_1^2 - p^2)(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{bk_1(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})},\\
C_{22} &= -\frac{(k_1^2 - p^2)(k_2^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_1 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{bk_1(k_1^2 - k_2^2)(\mathrm{e}^{k_1 l} - \mathrm{e}^{-k_1 l})},\\
C_{33} &= \frac{(k_2^2 - p^2)(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{bk_2(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})},\\
C_{44} &= \frac{(k_2^2 - p^2)(k_1^2 - p^2/\upsilon^2)(\mathrm{e}^{-k_2 l} - \mathrm{e}^{-(p/\upsilon)l})C_5}{bk_2(k_1^2 - k_2^2)(\mathrm{e}^{k_2 l} - \mathrm{e}^{-k_2 l})}.
\end{aligned} \tag{36}$$

Substituting C_{ii} ($i = 1,2,3,4$) into Eq. (28), we obtain

$$\begin{aligned}\bar{\theta} = & -\frac{(k_1^2 - p^2)(k_2^2 - p^2/\upsilon^2)(e^{k_1 l} - e^{-(p/\upsilon)l})C_5}{bk_1(k_1^2 - k_2^2)(e^{k_1 l} - e^{-k_1 l})} e^{-k_1 x} - \frac{(k_1^2 - p^2)(k_2^2 - p^2/\upsilon^2)(e^{-k_1 l} - e^{-(p/\upsilon)l})C_5}{bk_1(k_1^2 - k_2^2)(e^{k_1 l} - e^{-k_1 l})} e^{k_1 x} \\ & + \frac{(k_2^2 - p^2)(k_1^2 - p^2/\upsilon^2)(e^{k_2 l} - e^{-(p/\upsilon)l})C_5}{bk_2(k_1^2 - k_2^2)(e^{k_2 l} - e^{-k_2 l})} e^{-k_2 x} + \frac{(k_2^2 - p^2)(k_1^2 - p^2/\upsilon^2)(e^{-k_2 l} - e^{-(p/\upsilon)l})C_5}{bk_2(k_1^2 - k_2^2)(e^{k_2 l} - e^{-k_2 l})} e^{k_2 x} \\ & + \frac{p(\upsilon^2 - 1)}{b\upsilon} C_5 e^{-(p/\upsilon)x}. \end{aligned} \tag{37}$$

Substituting $\bar{u}$ in Eq. (35) and $\bar{\theta}$ in Eq. (37) into Eq. (19), we obtain

$$\begin{aligned}\bar{\sigma} = & -\frac{p^2(k_2^2 - p^2/\upsilon^2)(e^{k_1 l} - e^{-(p/\upsilon)l})C_5}{k_1(k_1^2 - k_2^2)(e^{k_1 l} - e^{-k_1 l})} e^{-k_1 x} - \frac{p^2(k_2^2 - p^2/\upsilon^2)(e^{-k_1 l} - e^{-(p/\upsilon)l})C_5}{k_1(k_1^2 - k_2^2)(e^{k_1 l} - e^{-k_1 l})} e^{k_1 x} \\ & + \frac{p^2(k_1^2 - p^2/\upsilon^2)(e^{k_2 l} - e^{-(p/\upsilon)l})C_5}{k_2(k_1^2 - k_2^2)(e^{k_2 l} - e^{-k_2 l})} e^{-k_2 x} + \frac{p^2(k_1^2 - p^2/\upsilon^2)(e^{-k_2 l} - e^{-(p/\upsilon)l})C_5}{k_2(k_1^2 - k_2^2)(e^{k_2 l} - e^{-k_2 l})} e^{k_2 x} \\ & - p\upsilon C_5 e^{-(p/\upsilon)x} - \frac{D}{p}. \end{aligned} \tag{38}$$

4. Numerical inversion of the transforms

In order to determine the temperature, displacement and stress distributions in the piezoelectric rod, $\bar{\theta}$, $\bar{u}$ and $\bar{\sigma}$ must be inverted from Laplace domain back into the time domain. However, these solutions are too complicated to be inverted directly and hence, no analytic solutions are possible. Therefore, the Riemann-sum approximation method is used to obtain numerical results. In this method, any function $\bar{f}(x,p)$ in Laplace domain can be inverted to the time domain as [13]

$$f(x,t) = \frac{e^{\beta t}}{t}\left[\frac{1}{2}\bar{f}(x,\beta) + \mathrm{Re}\sum_{n=1}^{N}\bar{f}\left(x,\beta + \frac{in\pi}{t}\right)(-1)^n\right], \tag{39}$$

where Re is the real part and i is the imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of β satisfies the relation $\beta t \approx 4.7$ [13].

5. Numerical results and discussion

In terms of the Riemann-sum approximation defined in Eq. (39), numerical Laplace inversion is performed to obtain the non-dimensional temperature, displacement and stress in the piezoelectric rod. In the calculation, the material constants of the piezoelectric rod necessary to be known are given as [14]

$$c_{11} = 74.1 \times 10^9\,\mathrm{N\,m^{-2}}, \quad \rho = 7600\,\mathrm{kg\,m^{-3}}, \quad C_E = 420\,\mathrm{J\,kg^{-1}\,K^{-1}}, \quad T_0 = 293\,\mathrm{K},$$
$$\lambda_{11} = 0.621 \times 10^6\,\mathrm{N\,K^{-1}\,m^{-2}}.$$

The other constants are taken as

$$Q_0 = 10, \quad \tau = 0.05, \quad l = 10, \quad D = 1 \times 10^{-6}.$$

Numerical calculation is carried out for two different cases. The first case is investigating how the non-dimensional temperature, displacement and stress vary with different time when the moving heat source velocity keeps constant. The second case is investigating how the non-dimensional temperature, displacement and stress vary with different moving heat source velocity when the time keeps constant. In the first case, we consider three different time instants $t = 1$, 2 and 3, while the constant heat source velocity is $\upsilon = 1$. In the second case, we consider three different heat source velocities $\upsilon = 1$, 2 and 3, while the constant time instant is $t = 1$. The numerical results are obtained and presented graphically in Figs. 1–6.

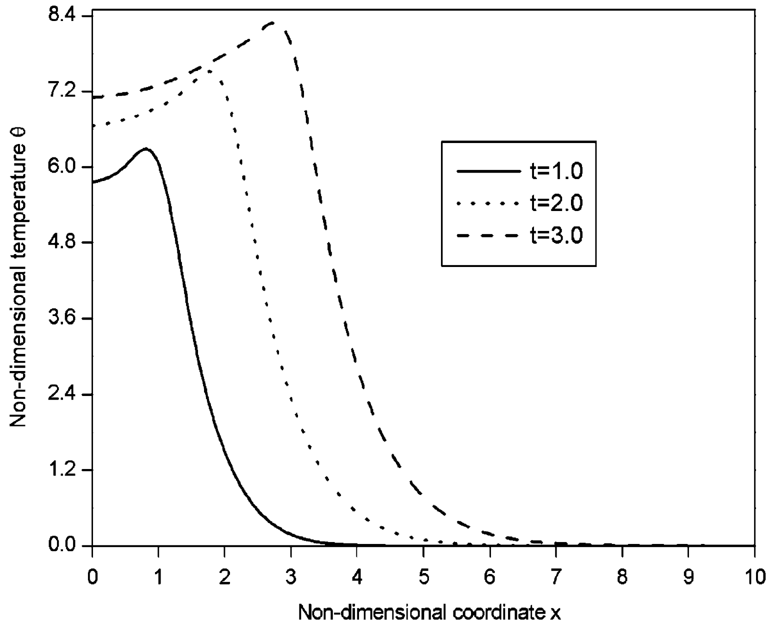

Fig. 1. Non-dimensional temperature distributions at $\upsilon = 1$.

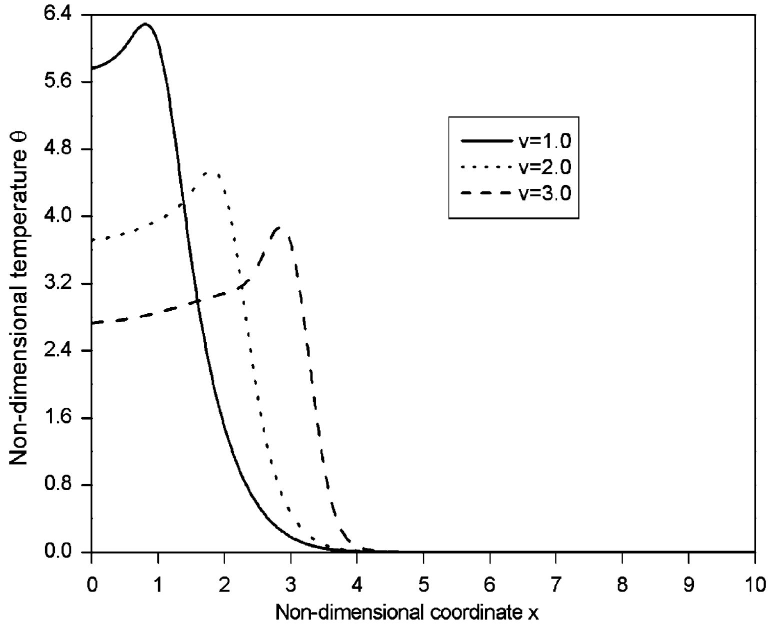

Fig. 2. Non-dimensional temperature distributions at $t = 1$.

Fig. 1 shows the non-dimensional temperature variation in the piezoelectric rod for the first case. In Fig. 1, the non-dimensional temperature distributions at $t = 1$, 2 and 3 are represented by the solid line, dot line and dash line, respectively. From Fig. 1 we observe that the temperature increases as the time increases. The peak value of non-dimensional temperature in solid line, dot line and dash line occurs at $x = 1$, 2 and 3, respectively. This can be understood clearly from Eq. (18) that because the applied heat source moves with

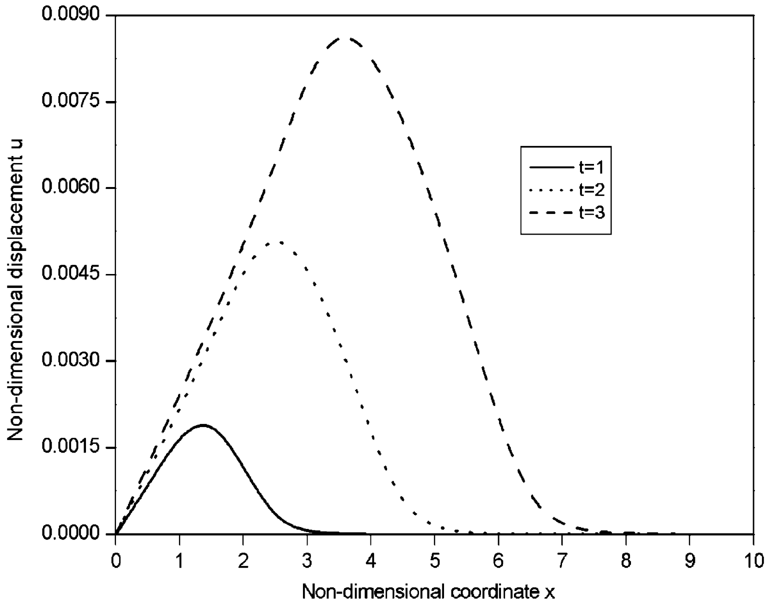

Fig. 3. Non-dimensional displacement distributions at $\upsilon = 1$.

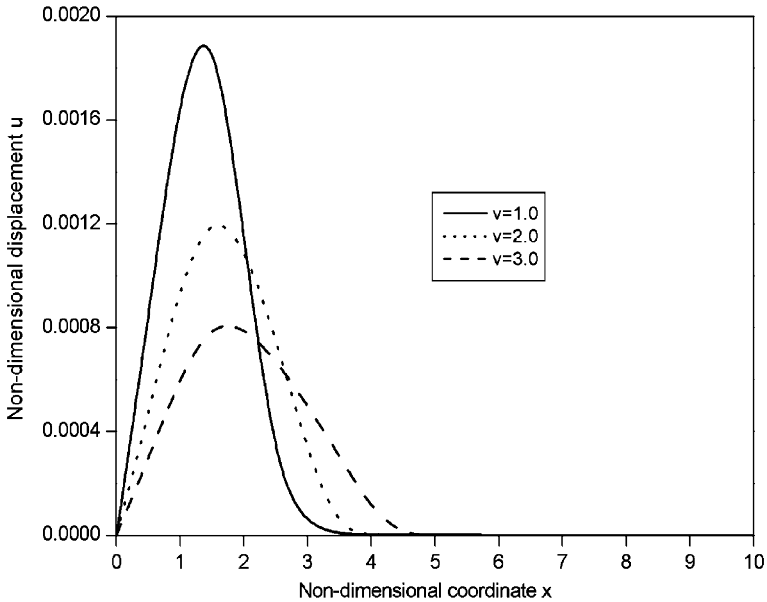

Fig. 4. Non-dimensional displacement distributions at $t = 1$.

a constant velocity, υ, thus, once the time instant t is given, the distance that heat source moves across is $x = \upsilon t$. At location $x = \upsilon t$ heat source releases its maximum energy, which leads to a higher local temperature.

Fig. 2 shows the non-dimensional temperature distribution in the piezoelectric rod for the second case. It can be found from Fig. 2 that temperature decreases as the moving heat source velocity increases. For the same time duration, the heat source releases the same amount of energy. However, the intensity of the released energy per unit rod length decreases as the source speed increases. As a result, each location in the thermal disturbed region receives less amount energy as the source speed increases. This in turn leads to a reduction in the local temperature distribution within the rod.

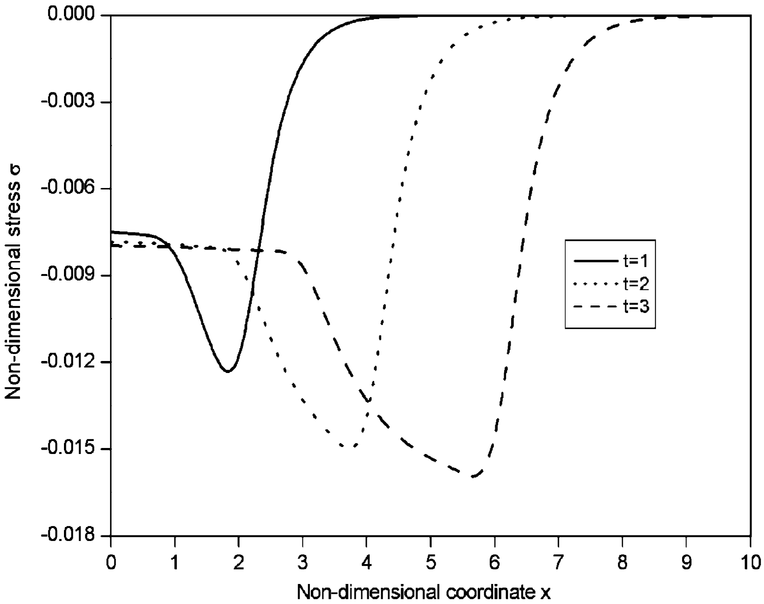

Fig. 5. Non-dimensional stress distributions at $\upsilon = 1$.

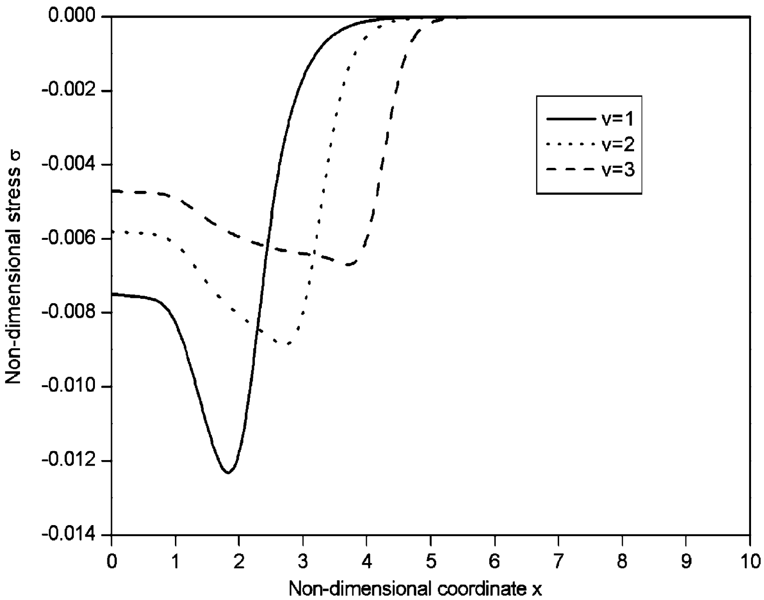

Fig. 6. Non-dimensional stress distributions at $t = 1$.

Figs. 3 and 4 show the non-dimensional displacement distributions for the first case and second case, respectively. From Fig. 3 it can be found that displacement increases as the time increases. This indicates as the time increases heat disturbed region evolves deeper in the piezoelectric rod. Due to the applied heat source the rod undergoes thermal expansion deformation. As time increases the thermal expansion deformation accumulates. Therefore, the displacement increases. From Fig. 4 it can be observed that displacement value decreases as the moving heat source velocity increases. This results from the reduction of the heat energy intensity per unit rod length at large velocity. It can be also found from Figs. 3 and 4 that the displacement value is kept zero at $x = 0$ and 10, which coincides with the boundary conditions that the rod is fixed at both ends.

Figs. 5 and 6 show the non-dimensional stress distributions for the two cases, respectively. From Fig. 5 we can find that the stress in the rod is compressive. The absolute value of the stress increases as the time increases. This is because the rod is fixed at both ends, which leads to the thermal expansion deformation being restrained to develop along the rod elongation, therefore, compressive thermal stress occurs in the piezoelectric rod. It can be seen from Fig. 6 that the absolute of the stress decreases as the velocity increases. The similar reason for this can be found from the above descriptions.

From Figs. 1–6 we can notice that once the time instant is given, the non-zero values of non-dimensional temperature, displacement and stress are only in a finite region and outside this region the results are all zero. This accounts for heat propagation speed being finite in the rod, also this is entirely different from the classical heat conduction theories, which predict an infinite speed. Because of the finite heat propagation speed, the heat disturbed region is bounded when the time instant is given, which results in the thermally induced displacement and stress are also in a bounded region.

6. Conclusions

The dynamic thermo-elastic responses of a piezoelectric rod subjected to a moving heat source is investigated based on the L–S generalized thermo-elastic theory. The piezoelectric–thermoelastic coupled governing equations are formulated. By means of Laplace transform and Riemann-sum approximation, the equations are solved and the numerical results of non-dimensional temperature, displacement and stress are obtained and displayed graphically. From the graphs, we can arrive at the following conclusions.

1. In all figures under the generalized theory of L–S theory, it is clear that all distributions considered have a non-zero value only in a bounded region of space. Outside this region the values vanish identically and this means that the region has not felt thermal disturbance yet. From the distributions of temperature, it can be found the wave type heat propagation in the piezoelectric–thermoelastic medium. The heat wavefront moves forward with a finite speed with the passage of time. This is not the case for the coupled theory, where infinite speed of propagation is inherent and hence all the considered functions have non-zero (although may be very small) value for any point in the medium. This indicates that the generalized heat conduction mechanism is completely different from the classical Fourier's in essence.
2. At $x = \upsilon t$ non-dimensional temperature attains its peak value once the time instant is given.
3. The values of non-dimensional temperature, displacement and stress decrease as the moving heat source increases.

Acknowledgement

The authors would like to thank the support of the National Natural Science Foundation of China (10602021, 10472039) and China Postdoctoral Science Foundation (20060400209).

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