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Journal of Sound and Vibration 306 (2007) 940–945
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Efficient enforced motion analysis of full-scale vehicle structures with global and local structural damping

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Received 21 September 2006; received in revised form 6 May 2007; accepted 12 May 2007
Available online 30 July 2007

Abstract

A new approach for the enforced motion analysis of large-scale vehicle structures with the large mass technique is presented. A vehicle finite element model includes global and local structural damping to reduce the vibration level. In order to avoid possible numerical inaccuracy of the large mass approach, the modal frequency problem is partitioned into the low-frequency mode part and the flexible mode frequency mode part, in which the FRRA algorithm is employed for the solution of the flexible mode part. A numerical example shows an outstanding performance compared to traditional industry methods.

1. Introduction

Vehicle structural systems are subjected to various load and boundary conditions. Boundary conditions are classified as single-point constraints, multi-point constraints, and enforced motion. In particular, the enforced motion is used when base motion is specified, in which the displacement, velocity, or acceleration at nodes is specified for dynamic response analysis [1]. In vehicle dynamic response analysis, the enforced motion analysis has been one of the essential tools for improving ride quality because there is a need to enforce the motion of tires to simulate the vehicle travelling over a prescribed surface.

For the enforced motion analysis, large mass, large stiffness, or Lagrange multiplier techniques have been widely used [1,2] to solve the frequency response problem. If the added stiffness or mass is sufficiently stiff or massive, the reaction force from the actual structure will not significantly affect the input motions. The large mass method has more of an advantage than the other method since it is easy to estimate a good value for the large mass, which is approximately 10^6-10^7 times the mass of the entire structure. However, the conventional approach, which factorizes the coefficient matrix, may lead to inaccurate results [1,2] because the finite element (FE) matrices introduce ill-conditioning due to significantly larger masses compared to the mass of the structure. Even worse, as the size of the FE model increases, the factorization cost of the coefficient matrix at each excitation frequency increases significantly.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.05.028

This paper presents an efficient numerical method for the dynamic response analysis of large-scale vehicle structures subjected to enforced motion, in which the large mass approach is used to describe the enforced motion. A vehicle structure considers local and global structural dampings in order to reduce the vibration level. In addition, this paper utilizes the fast frequency response analysis (FFRA) algorithm for solving the enforced motion problem in order to improve the performance significantly compared to the conventional approach in commercial FE softwares.

2. Enforced motion analysis using the FFRA algorithm

Fig. 1 illustrates the large mass technique concept in the enforced motion analysis. The FE model of a vehicle structure is connected to a point with a large mass instead of restraining at a ground point. The large mass represents the base for which the motion is to be specified. If a very large mass M_0 is attached to a degree of freedom and the load $P(t)$ is applied to the same degree of freedom, a desired or enforced acceleration $\ddot{u}(t)$ can be produced since $P(t) = M_0\ddot{u}(t)$.

In the FE method that includes both a vehicle structure and large masses, the direct frequency response analysis (FRA) with local and global structural damping [3,4] can be represented as

$$\left[-\omega^2 M + (1 + \mathrm{i}\gamma)K + \mathrm{i}K_s\right]X(\omega) = P(\omega), \quad (1)$$

where ω is the excitation frequency, $P(\omega) \in \mathbb{C}^{n\times nf}$ is the excitations force, and $X(\omega) \in \mathbb{C}^{n\times nf}$ is the frequency response matrix, where nf is the number of load cases. K, and $K_s \in \mathbb{R}^{n\times n}$ are the FE stiffness, and local structural damping matrix, respectively. n Represents the number of FE degrees of freedom. The M includes both the FE structural mass and the large masses M_0. The scalar γ is the global structural damping coefficient. Structural damping encompasses energy dissipation related to the internal structure of a vibrating body. It is especially intended to account for energy loss due to the hysteresis of elastic materials experiencing cyclic stress [4,5]. In this paper, we consider only frequency-independent damping material.

Today, since the size of industry FE models tends to increase for more accurate analysis, solving these very large FE systems of equations at many frequencies ω is still expensive. Instead, modal frequency response analysis has been used [6]. With the generalized eigenvalue problem $K\Phi = M\Phi\Lambda$, in which $\Phi \in \mathbb{R}^{n\times m}$ is the eigenvector matrix, $\Lambda \in \mathbb{R}^{m\times m}$ is the diagonal eigenvalue matrix and m is the number of modes obtained up to cutoff frequency ($m \ll n$), the modal frequency response problem is represented as

$$\left[-\omega^2 I + (1 + \mathrm{i}\gamma)\Lambda + \mathrm{i}\bar{K}_s\right]Z(\omega) = F(\omega), \quad (2)$$

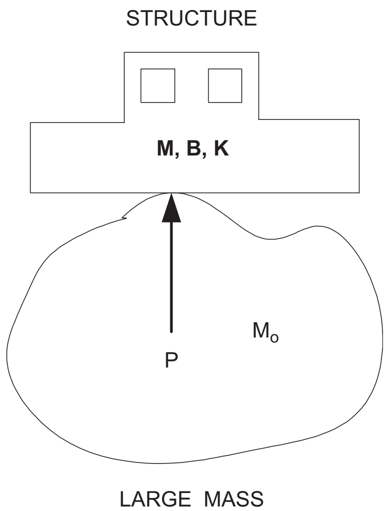

Fig. 1. Large mass approach ($M_0 \gg M$) for the enforced motion analysis.

where $\Phi^{\mathrm{T}} M \Phi = I$, $\Phi^{\mathrm{T}} K \Phi = \Lambda \in \mathbb{R}^{m \times m}$, and $X(\omega) = \Phi Z(\omega)$, $F(\omega) = \Phi^{\mathrm{T}} P(\omega) \in \mathbb{C}^{m \times nf}$. When the local structural damping matrix K_s does not exist, solving the modal frequency response problem is very inexpensive because the coefficient matrix becomes diagonal. However, when the local structural damping matrix is non-zero, the modal local structural damping matrix $\bar{K}_s = \Phi^{\mathrm{T}} K_s \Phi \in \mathbb{R}^{m \times m}$ becomes a fully populated matrix, which resulting in expensive computational cost.

The accuracy of the enforced motion analysis with the large mass approach increases as the mass M_0 increases compared to the mass of the structure [1,2]. However, due to significantly large mass M_0 compared to the mass of the structure, large-scale factors may produce numerical problems in the factorization of the coefficient matrix of Eq. (2). Therefore, in order to avoid the numerical inaccuracy or difficulty of factorization related to large numbers, the frequency response problem is partitioned into the *low-frequency mode* part l, corresponding to modes with very low natural frequencies due to large masses, and the *flexible mode* part f, corresponding to the rest of the global modes, in which $l \ll f$.

Then, the modal frequency response problem (2) can be represented in the form

$$A(\omega) Z = \left[\begin{array}{c|c} A_{ll} & A_{lf} \\ \hline A_{fl} & A_{ff} \end{array}\right] \left[\begin{array}{c} Z_l \\ \hline Z_f \end{array}\right] = \left[\begin{array}{c} F_l \\ \hline F_f \end{array}\right], \tag{3}$$

where $A(\omega)$ represents $(-\omega^2 I + (1 + \mathrm{i}\gamma)\Lambda + \mathrm{i}\bar{K}_s)$. The subscripts ll, ff, lf and fl represent the low-frequency mode part, the remaining or flexible mode part, and the interaction part between those two parts, respectively. Z_l and Z_f are the low-frequency mode and the flexible mode parts of the structure solution, respectively. F_l is the force matrix in the low-frequency mode part and F_f is the force matrix in the flexible mode part. The submatrices A_{ll}, A_{lf}, A_{fl} and A_{ff} can be defined as follows:

$$\begin{aligned} A_{ll} &= -\omega^2 I_{ll} + (1 + \mathrm{i}\gamma)\Lambda_{ll} + \mathrm{i}\bar{K}_{s,ll}, \\ A_{lf} &= \mathrm{i}\bar{K}_{s,lf}, \quad A_{fl} = A_{lf}^{\mathrm{T}} \\ A_{ff} &= -\omega^2 I_{ff} + (1 + \mathrm{i}\gamma)\Lambda_{ff} + \mathrm{i}\bar{K}_{s,ff}. \end{aligned} \tag{4}$$

Eq. (3) is solved by representing the upper part and the lower part separately. The lower part of Eq. (3) is expressed as

$$Z_f = A_{ff}^{-1}(F_f - A_{fl} Z_l). \tag{5}$$

Substituting Eq. (5) into the upper part of Eq. (3) yields

$$(A_{ll} - A_{lf} A_{ff}^{-1} A_{fl}) Z_l = F_l - A_{lf} A_{ff}^{-1} F_f. \tag{6}$$

Note that both Eqs. (5) and (6) require the inverse of the submatrix A_{ff}, which is the biggest sub-matrix. Therefore, it is essential to obtain $A_{ff}^{-1} F_f$ and $A_{ff}^{-1} A_{fl}$ efficiently.

Recently, Kim [4] developed the FFRA algorithm, which is an efficient modal frequency response problem reformulation to obtain high performance while producing exactly the same solution as the direct method that factorizes the coefficient matrix. In this point, the FFRA algorithm can be utilized to obtain $A_{ff}^{-1} F_f$ and $A_{ff}^{-1} A_{fl}$ efficiently from the following linear system of equations:

$$A_{ff}[V_1, V_2] = \left[F_f, A_{fl}\right]. \tag{7}$$

First, Eq. (7) can be rewritten as

$$[-\omega^2 I_{ff} + C][V_1, V_2] = \left[F_f, A_{fl}\right] \tag{8}$$

by introducing the complex symmetric and frequency-independent matrix C as [3,4]

$$C = (1 + \mathrm{i}\gamma)\Lambda_{ff} + \mathrm{i}\bar{K}_{s,ff}. \tag{9}$$

Then, the complex symmetric matrix eigenvalue problem for C is solved as

$$C\Phi_C = \Phi_C \Lambda_C \quad \text{such that} \quad \Phi_C \Phi_C^{\mathrm{T}} = \Phi_C^{\mathrm{T}} \Phi_C = I, \tag{10}$$

where Λ_C is the complex eigenvalue matrix and Φ_C is the corresponding complex eigenvector matrix. Substituting $[V_1, V_2] = \Phi_C[W_1, W_2]$ into Eq. (8) and premultiplying by Φ_C^{T} gives

$$\Phi_C^{\mathrm{T}}[-\omega^2 I_{ff} + C]\Phi_C[W_1, W_2] = \Phi_C^{\mathrm{T}}\left[F_f, A_{fl}\right]. \tag{11}$$

Finally, Eq. (11) results in

$$\left[-\omega^2 I_{ff} + \Lambda_C\right][W_1, W_2] = \Phi_C^{\mathrm{T}}\left[F_f, A_{fl}\right]. \tag{12}$$

where $\Lambda_C = \Phi_C^{\mathrm{T}} C \Phi_C$ from Eq. (10). $[W_1, W_2]$ is obtained inexpensively because of the diagonal coefficient matrix in Eq. (12).

Then $[V_1, V_2]$ is obtained from the back-transformation as $[V_1, V_2] = \Phi_C[W_1, W_2]$. From the solution $[V_1, V_2] = \left[A_{ff}^{-1}F_f, A_{ff}^{-1}A_{fl}\right]$, Eq. (6) can be rewritten as follows.

$$(A_{ll} - A_{lf}V_2)Z_l = F_l - A_{lf}V_1. \tag{13}$$

Since the dimension l of the coefficient matrix is very small compared to the flexible mode part f, it is inexpensive to solve the linear system (13) with the direct method.

After the low-frequency mode part solution Z_l is obtained, the flexible mode part Z_f is obtained from Eq. (5) in the form

$$Z_f = V_1 - V_2 Z_l. \tag{14}$$

The final modal solution for the entire structure becomes

$$Z(\omega) = \begin{bmatrix} Z_l \\ Z_f \end{bmatrix}. \tag{15}$$

Note that, in the conventional approach, the modal frequency response problem in Eq. (2) needs to be solved with $O(m^3)$ operations to factorize the coefficient matrix. Even worse, one needs to solve Eq. (2) for many excitation frequencies, hundreds of excitation frequencies. However, in the new approach of this paper, the complex symmetric matrix eigenvalue problem in Eq. (10) needs to be solved only one time with $O(f^3)$ operations for the flexible mode part, because C is frequency independent. Then, at each excitation frequency ω, Eq. (12), which has a diagonal coefficient matrix, Eq. (13), which has a very small dimension l, and Eq. (14), which requires only one matrix–matrix multiplication, need to be solved. This significantly reduces computational costs compared to the method that requires $O(m^3)$ operations at each frequency.

3. Accuracy and performance

As a numerical example, an industry automobile FE model is selected to evaluate the performance and accuracy of the new approach developed in this paper. An HP rx5670 with 900 MHz Itanium II processor is used to evaluate the performance of algorithm.

A full size vehicle FE model subjected to enforced motion has a total of 36 large masses that are attached to the four wheels. This FE model has 2,091,329 FE degrees of freedom to describe the structural system. The frequency range of interest is from 50 to 250 Hz with a 0.5 Hz increment, so that the global cutoff frequency is set to 400 Hz. The number of global modes m obtained from the generalized eigenvalue problem $K\Phi = M\Phi\Lambda$ is 3091. In the FE model, the global structural damping γ and local structural damping K_s are included. Table 1 summarizes the analysis information for the FE model.

Table 2 represents the timing profile of the main steps in the algorithm for the enforced motion analysis of the FE model. Since this model has several large masses, the coefficient matrix of the modal frequency response problem is partitioned into the low-frequency mode part and the flexible mode part. The size of the low-frequency mode part is 36 and the size of the flexible mode part is 3055, in order to avoid the numerical difficulty of factorization due to large masses. Modes below 1 Hz are considered to be the low-frequency mode part.

In step (1), in order to obtain V_1 and V_2 of Eq. (7), the complex symmetric matrix eigenvalue problem for the flexible mode part C is solved, which takes 39% of the analysis time. To solve the complex symmetric

Table 1
FE model analysis information

Degrees of freedom	2,091,329 dof
Damping	Global structural damping γ
	Structural damping K_s
Large masses	36
Excitation frequency range	50–250 Hz with 0.5 Hz increment
Global cutoff frequency	400 Hz
Number of load cases	3
Number of global modes	3,091

Table 2
Timing profile [hh:mm:ss] of the enforced motion analysis for the FE model

Step	Operation	FFRA	Direct method
	complex symmetric matrix eigenvalue problem		
(1)	$C\Phi_C = \Phi_C \Lambda$	2:51	–
	for each excitation frequency ω		
	for $i = 1, n$freq		
(2.1)	$A_{ff}[V_1, V_2] = [F_f, A_{fl}]$	3:08	1:43:02
(2.1.1)	$P = \Phi_C^{\mathrm{T}}[F_f, A_{fl}]$	(0:21)	
(2.1.2)	$\Phi_C^{\mathrm{T}} A_{ff} \Phi_C [W_1, W_2] = P$	(2:26)	
(2.1.3)	$[V_1, V_2] = \Phi_C [W_1, W_2]$	(0:21)	
(2.2)	$(A_{ll} - A_{lf} V_2) Z_l = F_l - A_{lf} V_1$	0:06	0:06
(2.3)	$Z_f = V_1 - V_2 Z_l$	0:11	0:11
	end		
Total		7:16	1:43:19

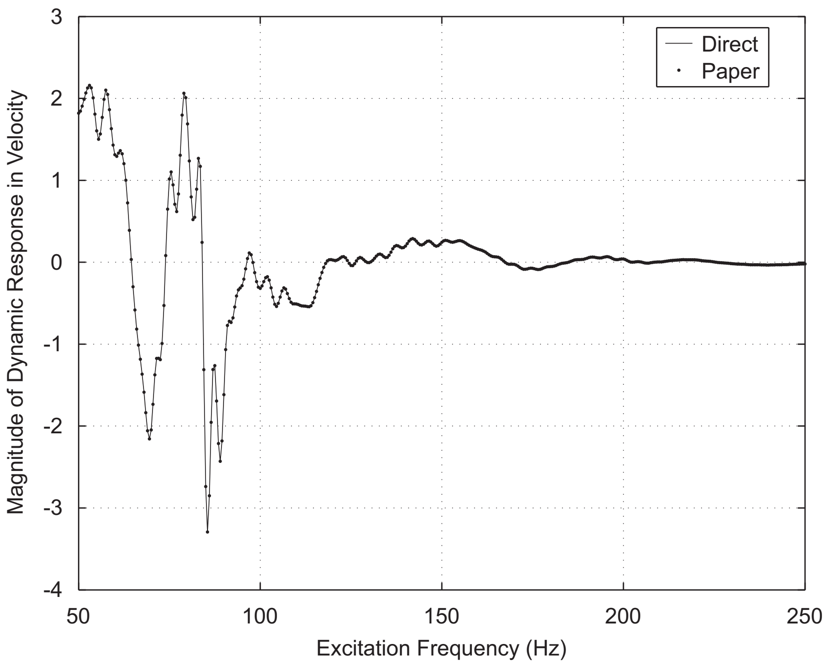

Fig. 2. The magnitude of velocity of the structural system subjected to the enforced motion.

matrix eigenvalue problem efficiently, complex symmetric Householder method [4] is implemented with Level 3 BLAS (matrix–matrix multiplication) operations [7].

Once V_1 and V_2 are obtained in step (2.1) of the FFRA algorithm, the linear system for the low-frequency mode part is solved in step (2.2). Since the number of low-frequency modes, 36, is much smaller than the number of flexible modes of the structure, it is very inexpensive to solve this linear system. Then, the flexible mode part solution Z_f is computed in step (2.3), which requires only matrix multiplication.

To evaluate the performance of the new approach in this paper, step (2.1) is solved with the direct method, ZSYSV in LAPACK [9] that factorize the coefficient matrix at each frequency, instead of using the FFRA algorithm. Table 2 shows the elapsed time spent for step (2.1) with the direct method for 401 excitation frequencies, from which 50 to 250 Hz with a 0.5 Hz increment. The new approach reduces the time spent in the frequency response analysis by more than 93% compared to the traditional industry method.

The magnitude of the dynamic response in velocity subjected to the enforced motion is illustrated in Fig. 2. For this prescribed enforced motion condition, this structural system has high peaks between 50 and 100 Hz range, so designers need to modify the design to reduce the peak in this range. In this figure, the results from the direct method and the new approach of this paper are the same. This numerical model does not suffer from numerical problems in the direct method problems.

4. Conclusion

An efficient numerical method for the dynamics response analysis of large-scale vehicle structures subjected to the enforced motion is developed, in which local and global structural damping are considered. The large mass approach is employed to describe the enforced motion, but has possible numerical inaccuracy due to large-scale factors in the factorization of the coefficient matrix. In order to avoid the numerical inaccuracy of factorization related to large numbers, to improve performance, the frequency response problem is partitioned into the low-frequency mode part and the flexible mode part. The FFRA algorithm is employed for the solution of the flexible mode part. With the FFRA algorithm, the new approach provides an outstanding performance as well as a high degree of accuracy compared to the traditional industry method, which requires coefficient matrix factorization at each frequency.

Acknowledgment

This work was supported by the faculty research fund of Konkuk University in Seoul, Korea, in 2007.

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