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Applying a fuzzy-set-based method for robust estimation of coupling loss factors

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Abstract

Finite element models have been used by many authors to provide accurate estimations of coupling loss factors. Although much progress has been achieved in this area, little attention has been paid to the influence of uncertain parameters in the finite element model used to estimate these factors. It is well known that, in the mid-frequency range, uncertainty is a major issue. In this context, a spectral element method combined with a special implementation of a fuzzy-set-based method, which is called the transformation method, is proposed as an alternative to compute coupling loss factors. The proposed technique is applied to a frame-type junction, which can consist of two beams connected at an arbitrary angle. In this context, two problems are investigated. In the first one, the influence of the confidence intervals of the coupling loss factors on the estimated energy envelopes assuming a unit power input is considered. In the other problem the influence of the envelope of the input power obtained considering the confidence intervals of the coupling loss factors is also taken into account. The estimates of the intervals are obtained by using the spectral element method combined with a fuzzy-set-based method. Results using a Monte Carlo analysis for the estimation of the coupling loss factors under the influence of uncertain parameters are shown for comparison and verification of the fuzzy method.

1. Introduction

Statistical energy analysis (SEA) has been established as a powerful technique to address dynamic problems in the high-frequency range. Finite element analysis (FEA) has also been considered as a standard methodology to assess problems in the low-frequency range. Broadly speaking, both techniques work well in their respective frequency ranges. On the other hand, it is also well known that in order to extend the applicability of SEA and FEA methods, i.e., SEA for low and mid-frequency ranges and FEA for mid and high frequencies, some theoretical and numerical restrictions are found.

Basically, for SEA applications, if the main assumptions are filled out, which is usually the case only at high frequencies, the method works according to the theory [1]. For FEA, two main restrictions are well known: the first concerns the number of elements needed to describe the characteristic wavelength of the propagating

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vibrational waves, which in the mid-high-frequency range becomes much smaller than the dimensions of the structure. Therefore, a good model requires an excessively large number of finite elements. The second limitation is due to the influence of the parameter uncertainty upon the dynamic responses.

Moreover, and in the same context, FEA has also been successfully applied for the estimation of the SEA coupling loss factors (CLFs) between two subsystems [2–5]. The vibrational energy transmission between strongly coupled subsystems based on energy flow methods (EFM) has also been investigated using results of FEA, for example the energy influence coefficients (EIC) and power influence coefficients (PIC) derived from modal analysis [6,7]. Recently, the so called virtual/experimental SEA based on FEA was also proposed to estimate the coupling loss factors [8]. In this method, the idea adopted to estimate the coupling loss factors is based on the experimental concept of the power injection method (PIM). In order to assess those factors, the energy responses of a number of subsystems are found considering spatially incoherent excitation (*rain-on-the-roof*) applied to a given subsystem. By doing so, the input powers and energies can be estimated by postprocessing the FEA data. In Ref. [9], the estimation of the coupling loss factors is computed using the spectral element method (SEM), where continuous elements replace an infinite number of finite elements, leading to an exact solution under the assumptions of the underlying theory.

Although some progress has been achieved, little attention has been paid, in this context, to the influence of the uncertain parameters in the finite element or the spectral element model used to estimate the coupling loss factors. This paper addresses the assessment of the coupling loss factors taking into account the influence of the uncertainty of parameters. Other related work in the literature can be found in Refs. [10,11], where statistical and interval analysis of energy flow between uncertain vibrating systems are investigated.

For this sake, in this paper the spectral element method is combined with a special implementation of the extension principle of fuzzy sets [12] in order to form what we have called the spectral element method combined with fuzzy-set-based method, or spectral element method/fuzzy, for simplicity. The proposed methodology is applicable in early design stages, when statistical data is not yet available. Thus, the main objective in this paper is to provide information about the confidence limits of the SEA coupling loss factors estimated by the spectral element method combined with a fuzzy-set-based method and their effect in the energy levels predicted using a SEA model.

In Section 2, a brief review of the spectral element method applied to beams is given. In Section 3, the extension principle is reviewed. In Section 4, the spectral element method/fuzzy method is summarized. Finally, a test problem with an application of the spectral element method/fuzzy method for the estimation of SEA coupling loss factors for frame-type coupling is presented.

2. The spectral element method applied to beams

In this section, the spectral element method is applied to beams based on the elementary Bernoulli–Euler theory, which implies that the effects of the shear deformation and the rotational inertia are neglected. Spectral elements for higher-order beam theories can be found in the literature [13]. The basic idea of the spectral element method is to combine the advantages of analytical spectral analysis with the efficiency and organization of the finite element method (FEM). The main advantage of the spectral element method in comparison to FEM is the fact that the spectral element dynamic stiffness matrix is computed in the frequency domain, which allows the stiffness and the inertia of the distributed-parameter system to be described exactly. Thus, it is not necessary to refine the mesh as the wavelength becomes smaller.

The spectral element method is completely formulated for spatial frame-type structures, but for two and three-dimensional elements such as plates, shells and solids, no general formulation exists. Some spectral element formulations exist for plates and shells, but with severe limitations concerning the geometry and the boundary conditions of these elements. For example, simple thin flat plate spectral elements exist for structures that are simply supported along two parallel sides (Levy plates), but general plate elements depend on Fourier expansions which make the formulation more complex, the resulting dynamic stiffness matrices much bigger, and results are not analytically exact any more. In general, spectral elements can only be assembled along one dimension, and analytical solutions must be found in the plane orthogonal to this direction. The main drawback of spectral element method is that it is only applicable to simple geometries and, sometimes, specific boundary conditions.

For the Euler–Bernoulli beam element, the following equation of motion can be derived,

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 v}{\partial x^2}\right]+\rho A\frac{\partial^2 v}{\partial t^2}=q(x,t)\equiv q_v-\frac{\partial q_\phi}{\partial x}, \tag{1}$$

where E is the Young's modulus, I is the inertia moment and A is the area of the cross-section, ρ is the mass density, v is the transverse displacement,and $q(x,t)$ is the external force, which can be separated in the distributed transverse load $q_v(x,t)$ and distributed torque q_ϕ.

Now, assuming that the beam treated here has constant properties along its length, the following homogeneous differential equation is defined:

$$\frac{\mathrm{d}^4\hat{v}}{\mathrm{d}x^4}-\beta^4\hat{v}=0. \tag{2}$$

Considering Eq. (2), the particular solutions are found based on solutions of the two equations described by

$$\frac{\mathrm{d}^2\hat{v}}{\mathrm{d}x^2}+\beta^2\hat{v}=0 \quad\text{and}\quad \frac{\mathrm{d}^2\hat{v}}{\mathrm{d}x^2}-\beta^2\hat{v}=0, \tag{3}$$

with the following wavenumbers:

$$k_1=\pm\beta,\quad k_2=\pm\mathrm{i}\beta \quad\text{and}\quad \beta^2\equiv\sqrt{\frac{\omega^2\rho A-\mathrm{i}\omega\eta A}{EI}}, \tag{4}$$

where η is the material loss factor. Following that, the complete solution can be written using the spectral representation as

$$v(x,t)=\Sigma[\mathbf{A}\mathrm{e}^{-\mathrm{i}\beta x}+\mathbf{B}\mathrm{e}^{-\beta x}+\mathbf{C}\mathrm{e}^{\mathrm{i}\beta x}+\mathbf{D}\mathrm{e}^{\beta x}]\mathrm{e}^{\mathrm{i}\omega t}, \tag{5}$$

where **A**, **B**, **C** and **D** are the complex wave amplitudes at each frequency.

In order to develop the dynamic stiffness for a 2-noded element of length L, with displacements $\hat{v}_i$ and node rotation $\hat{\phi}_i$, the following expressions are defined:

$$\hat{v}(0)\equiv\hat{v}_1,\quad \hat{\phi}(0)\equiv\hat{\phi}_1,\quad \hat{v}(L)\equiv\hat{v}_2 \quad\text{and}\quad \hat{\phi}(L)\equiv\hat{\phi}_2. \tag{6}$$

By using Eqs. (6) and (5), the coefficients **A**, **B**, **C** and **D** can be found. By using Eq. (5), the displacements $\hat{v}_i$ and $\hat{\phi}_i$ at any arbitrary point along the beam can be calculated by

$$\hat{v}(x)=\hat{g}_1(x)\hat{v}_1+\hat{g}_2(x)\hat{\phi}_1+\hat{g}_3(x)\hat{v}_2+\hat{g}_4(x)\hat{\phi}_2, \tag{7}$$

with $\hat{\phi}(x)=\partial\hat{v}(x)/\partial x$. The functions $\hat{g}_i$ are the frequency-dependent shape functions, which can be found in Ref. [13]. The nodal loads are then written in terms of the displacement degrees of freedom (dofs) as

$$\hat{V}(x)=-EI\frac{\partial^2\hat{\phi}}{\partial x^2},\quad \hat{M}(x)=EI\frac{\partial\hat{\phi}}{\partial x}. \tag{8}$$

Applying the boundary conditions to the beam, the dynamic stiffness relation can be written as

$$\begin{Bmatrix}\hat{V}_1\\ \hat{M}_1\\ \hat{V}_2\\ \hat{M}_2\end{Bmatrix}=\frac{EI}{L^3}[\hat{k}^B]\begin{Bmatrix}\hat{v}_1\\ \hat{\phi}_1\\ \hat{v}_2\\ \hat{\phi}_2\end{Bmatrix}, \tag{9}$$

which can be re-written in a compact form as

$$\{\hat{F}\}=\frac{EI}{L^3}[\hat{k}^B]\{\hat{u}\}, \tag{10}$$

where $(EI/L^3)[\hat{k}^B]$ is defined as the dynamic stiffness matrix of the *two-noded* Bernoulli–Euler beam element. $\hat{k}^B$ is a [4 × 4] symmetric matrix which, in general, is complex. For the *throw-off* elements, waves propagate in only one direction and, hence, they can be obtained by setting $\mathbf{C}=\mathbf{D}=0$ in Eq. (5). The *throw-off* elements are used to represent infinite beams, which conduct energy out of the system.

When it can be formulated, a spectral element is equivalent to an infinite number of finite elements, as shown by Doyle [13]. Thus, the problem of mesh refinement for higher frequencies is no longer an issue. This is the reason for using spectral element method to model the frame structure and obtain the coupling loss factors in this work. For more complex structures, such as 3D frames, please refer to Refs. [14,24].

3. Fuzzy sets using the extension principle

Fuzzy set theory has been proposed in the literature as a methodology that can be very helpful to analyze systems with respect to uncertain model parameters. In the context of engineering applications, a fuzzy model adopted for a dynamic problem can be used to determine the range of results or simply the intervals of confidence, considering non-uniform materials, manufacturing tolerances and other uncertainties.

The fuzzy concept has its foundation in the fuzzy logic theory introduced by Zadeh [12]. One important aspect in this theory is that incomplete information can be described in a non-probabilistic form. Such an idea has become very popular, especially when applied to engineering problems. This is in contrast with the probabilistic theory, where more information is needed in the design process in the form of probability density functions (PDF). On the other hand, considering an initial phase of a design process, when little information is available, the concept of possibility theory adopted in the fuzzy-set methods is, in general, more appropriate.

In terms of framework, a fuzzy set is also considered as an extension of the classical set theory. In this context, Zadeh's extension principle [12] provides the fundamental basis for fuzzy-set methods, as it states that real valued functions can be extended to functions of fuzzy numbers. Fuzzy-set-based methods, often referred to as possibility theory, emerged from the work of Zadeh [12]. Fuzzy set theory is especially well suited for dealing with forms of uncertainty that are inherently non-statistical in nature. Instead of producing single intervals as outputs, possibility theory based on fuzzy sets permits gradations of possibility.

In the following, we recall the extension principle (see Ref. [15]). From a mathematical point of view, we can define $\tilde{A}_1, \ldots, \tilde{A}_d$ as the d fuzzy sets with the membership functions $\mu_1, \ldots, \mu_d$ defined on the universes $X_1, \ldots, X_d$, respectively, and $f : X_1 \times \cdots \times X_d \to Y$ as the objective function that maps the universes $X_1 \times \cdots \times X_d$ over the universe Y, i.e., $y = f(x_1, \ldots, x_d)$ and $y \in Y$. Thus, the fuzzy image $\tilde{B}$ can be obtained from

$$\tilde{B} = \{(y, \mu_{\tilde{B}}(y)) | y = f(x_1, \ldots, x_d), (x_1, \ldots, x_d) \in X_1 \times \cdots \times X_d\},$$

with

$$\pi(x_1, \ldots, x_d) = \min(\mu_1(x_1), \ldots, \mu_d(x_d)),$$
$$\mu_{\tilde{B}}(y) = \begin{cases} \sup\limits_{y=f(x_1,\ldots,x_d)} (\pi(x_1, \ldots, x_d)) & \text{if } \exists y = f(x_1, \ldots, x_d), \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Considering that the fuzzy sets $\tilde{A}_1, \ldots, \tilde{A}_d$ are convex fuzzy sets with compact support and the objective function defined as $f : X_1 \times \cdots \times X_d \to Y$ is continuous, then the following *alternative formulation* is equivalent to Eqs. (11) [16].

$$\tilde{B} = \{(y, \mu_{\tilde{B}}(y)) | y \in Y\}, \quad \text{with}$$
$$\mu_{\tilde{B}}(y) = \begin{cases} \sup\{\alpha | y \in B_\alpha\} & \text{if} y \in B_0, \\ 0 & \text{otherwise,} \end{cases}$$
$$B_\alpha = \left[\min_{\mathbf{x} \in \Omega_\alpha} f(\mathbf{x}), \max_{\mathbf{x} \in \Omega_\alpha} f(\mathbf{x})\right], \quad 0 \leqslant \alpha \leqslant 1. \tag{12}$$

In Eq. (12), $\Omega_\alpha = (A_1)_\alpha \times \cdots \times (A_d)_\alpha$, $0 \leqslant \alpha \leqslant 1$, denote the interval boxes formed by the α-cuts $(A_1)_\alpha, \ldots, (A_d)_\alpha$.

The extension principle can be more easily treated by a numerical algorithm based upon the alternative formulation than the original formulation by Zadeh, Eq. (11). On the other hand, modern algorithms for fuzzy sets are based on point-based methods, such as the transformation method proposed by Hanss [17,18].

In the context of this present work, Eq. (12) is applied based on the idea that the d uncertain input parameters of the dynamic system problem $\tilde{p}_1, \ldots, \tilde{p}_d$ are discretized into α-cuts to form the boxes Ω_α.

Furthermore, to compute the upper and lower bounds $[a, b]$ of the α-cuts, we have to solve a global optimization problem that finds the *minimum* and *maximum* taking into account the influence of the uncertain input parameters.

3.1. Implementation of the extension principle

In what follows, a brief description of the general transformation method is presented. However, instead of applying the proposed method as presented by Hanss [17,18], an improvement suggested by Klimke [19] is introduced, which is called the transformation method without recurring permutations or simply the *gtrmr*.

The general transformation method proposed by Hanss [17,18] can be considered as the first implementation of the extension principle that discretizes the convex fuzzy sets into α-cuts, and then discretizes the α-cuts into sets of points. In the work by Hanss [17] this process is described based on subdividing the axis for the degree of membership μ into a number of m segments, equally spaced by $\Delta_\mu = 1/m$. Such an alternative is based on the practice of sampling analog signals, which consists in describing the fuzzy number in a discrete form. Moreover, the $m+1$ levels of membership μ_j for a given α-cut at the level α-level $\in [0, 1]$ are then given by

$$\mu_j = \frac{j}{m}, \quad j = 0, 1, \ldots, m. \tag{13}$$

One important point to add is that, in the fuzzy application, the intervals of confidence are simply called α-cuts with the α-level $\alpha = j/m \in [0, 1]$. According to Hanss [17], this discretization is also called in the literature as α-cut representation or α-sublevel technique. In this case, the fuzzy number to be implemented either using an approximation by discrete numbers or decomposed into a number of intervals, for instance $[a^j, b^j]$, $a^j \leqslant b^j$, $j = 0, 1, \ldots, m$ given by α-cuts and the α-levels μ_j. To illustrate this idea, Fig. 1 shows an implementation of a fuzzy number $\tilde{p}$ using decomposition into intervals by the discrete fuzzy numbers representation [17].

In the same context, Hanss [17] also proposed the reduced transformation method, which can be considered as an improved version of the classical fuzzy weighted averages (FWA) algorithm by Dong and Wong [20] that ensures convexity of the fuzzy results. Only the lower and upper bounds of the interval at membership level μ_j for the ith uncertain model parameter are considered. In the case of the general transformation method, the mid-point between the lower and upper bounds is also considered. Fig. 2 shows the idea of removing recurring points in detail.

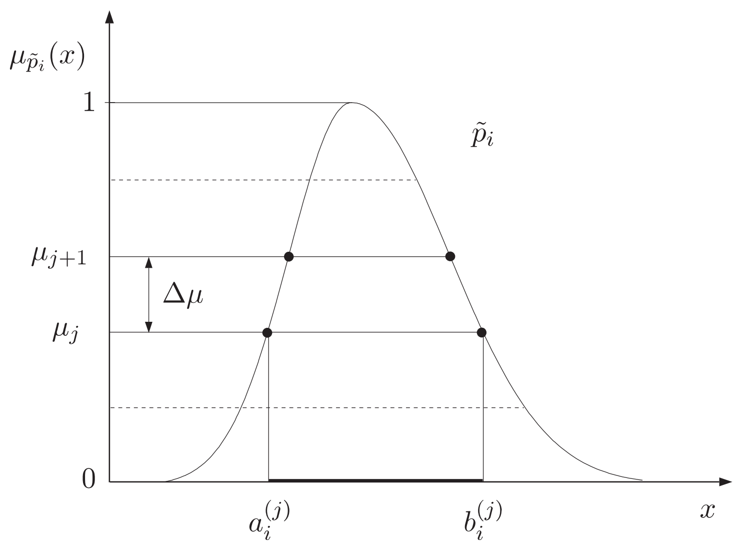

Fig. 1. Implementation of a fuzzy number $\tilde{p}$ decomposed into intervals.

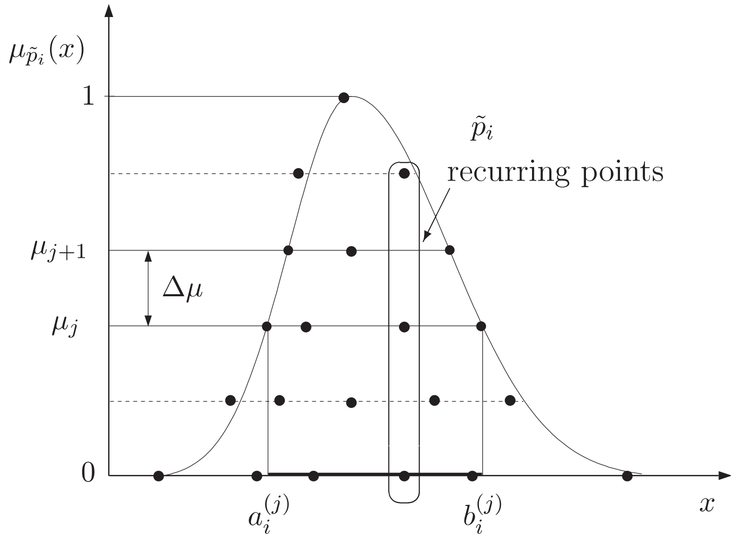

Fig. 2. General transformation method and its recurring points [19].

Nevertheless, due to the discretization scheme, only convex fuzzy sets $\tilde{A}$ with bounded support and a single value $\overline{m}$ with $\mu_{\tilde{A}}(\overline{m}) = 1$ may be used as inputs. In terms of accuracy, the results can be made arbitrarily accurate by letting the number of α-cuts m tend to infinity, but at the cost of a higher computational cost.

One important point to mention, in terms of practical applications, is that the number of function evaluations N is responsible for the cost of the general transformation method. Here, instead of applying the original method proposed by Hanss [17], the general transformation method that eliminates recurring permutations proposed by Klimke [19] is adopted.

In order to check the number of function evaluations N, the following expressions are used for the *gtrm*:

$$N = \sum_{k=1}^{m} k^d \tag{14}$$

and for the *gtrmr*,

$$N = (m-1)^d + m^d, \tag{15}$$

with m denoting the number of α-cuts and d the number of uncertain parameters.

For instance, as shown in the work of Klimke [19], for $d = 2$ and 100 α-cuts, the number of function evaluations of the original form of the general transformation method (*gtrm*) is 17.1 times bigger than N for the proposed method avoiding additional functions evaluations for recurring combinations (*gtrmr*). For the same example, however, with 10 α-cuts, a factor of 2.1 is found. Therefore, one general conclusion is that, in the case where the number of α-cuts is large compared with the number of uncertain parameters, a better performance is found with the *gtrmr*. However, in the case of a less regular distribution of the inner points, it is suggested to increase the number of α-cuts instead of keeping it constant as in the original method.

In addition, it is important to point out that the recent extended transformation method proposed by Hanss [21] cannot reduce the computational effort to less than $N = m^d$ function evaluations (even if monotonicity is detected). For non-monotonic behavior, the computational effort is significantly higher, i.e., $N = \sum_{k=1}^{m} m^d$. In the case of the reduced transformation method, this number for the dimension d is defined by $N = m2^d$. In this paper, the general transformation method considering recurring permutations according to Klimke [19] is chosen to be combined with the spectral element method.

3.2. Computing the SEA coupling loss factors using the spectral element method combined with a fuzzy-set-based method

In this section, a *step-by-step* procedure is presented for the implementation of the spectral element method combined with the general transformation method without recurring permutations *gtrmr* in the context of the fuzzy-set theory. For additional information about the performance of the *gtrmr*, see Ref. [15], where the present method is also compared to the reduced method, the sparse grids approach and to the *crude* Monte Carlo analysis. In Ref. [15] it was demonstrated that even when considering many samples using Monte Carlo simulation, for the two tests proposed, fuzzy-set-based methods (gtrmr) and sparse grids performed better than Monte Carlo.

At first, a brief review of the main equations proposed by Stimpson and Lalor [22] for performing a sensitivity analysis of the coupling loss factors is given. According to the PIM the inverse of the energy matrix should be solved to obtain the coupling loss factors. The sensitivity of the inverse of the energy matrix with respect to the different subsystems energies is used. The sensitivity of element E_{ij} of the normalized energy matrix with respect to element E_{mn} can be given by [22]

$$\frac{\partial [E^n]_{ij}^{-1}}{\partial E^n_{mn}} = -\sum_{p=1}^{N}\sum_{q=1}^{N}[E^n]_{ip}^{-1}\frac{\partial [E^n]_{pq}}{\partial E^n_{mn}}[E^n]_{qj}^{-1}, \tag{16}$$

where the contributions of the p and q lines and columns of the energy matrix are incorporated. In this expression, the term E_{ij} is the energy of subsystem i when power is input into subsystem j and the normalized term E^n_{ij} is given by

$$E^n_{ij} = \frac{\omega E_{ij}}{P^i_{in}}. \tag{17}$$

For the coupling loss factors, a similar expression could be written as

$$\frac{\partial \eta_{ji}}{\partial E^n_{mn}} = \sum_p \sum_q \eta_{jq}\delta_{mp}\delta_{nq}\eta_{pi}, \quad i\neq j, \tag{18}$$

where δ_{ij} is Kronecker delta. For a two-subsystem SEA model this expression can still be simplified and written as

$$\frac{\partial \eta_{ji}}{\partial E^n_{mn}} \cong \eta_{mi}\eta_{jn}, \quad i\neq j. \tag{19}$$

Using the fact that the coupling loss factor matrix is predominated by the main diagonal, the last equation is predominated by the terms $n = j$ and $m = i$, and thus the variations in η_{ji} is given by

$$\Delta\eta_{ji} \cong E^n_{ij}\eta_{ii}\eta_{jj}, \quad i\neq j, \tag{20}$$

where $\Delta\eta_{ji}$ is defined as the variation in the factor η_{ji}. By making use of Eq. (20), the coupling loss factors can be computed by [22]

$$\eta^{\mathrm{SL}}_{ji} \cong \frac{E^n_{ij}}{E^n_{ii}E^n_{jj}}, \tag{21}$$

where η^{SL}_{ji} gives the coupling loss factor estimated by Stimpson and Lalor.

In the approximation of Eq. (21), the terms $E_{ji}, i\neq j$ are not taken into account when calculating η_{ji}, i.e. E_{21} is not needed for the calculation of η_{21}. When Eq. (21) is used, it is not necessary to solve the linear system of equations of the PIM method and, thus, the condition number of the PIM energy matrix is not relevant, which could, otherwise, reach high values and cause numerical instability. Using the spectral element method, nodal forces are applied to two beams of the L-beam structure used in this paper at the two different directions and the frequency response functions (FRFs) are calculated. Then, these frequency response functions are used to calculate the total energies (kinetic + potential) of the different subsystems of the L-beam model, see Ref. [24].

These subsystems are characterized by the longitudinal and transverse wave propagation in each beam. Then, these energies are inserted in Eq. (21) for the calculation of the different coupling loss factors.

In what follows, Eqs. (21) and (17) are used for the computation of the coupling loss factors using the proposed spectral element method/fuzzy method. As described in Ref. [15], this can be done by a three-step procedure:

Step 1: *Discretization process*. First, the frequency range $[f_0, f_1]$ is divided into s logarithmically spaced steps or 1/3-octave bands, which is more convenient for SEA application, with the central frequency defined by f_i, $i = 1, \ldots, s$. The d uncertain input parameters $\tilde{p}_1, \ldots, \tilde{p}_d$ are discretized into N discrete parameter vectors $\mathbf{p}_j$, $j = 1, \ldots, N$, $\mathbf{p}_j \in \Omega_0$ where for the general case, $\Omega_\alpha = (A_1)_\alpha \times \cdots \times (A_d)_\alpha$, $0 \leqslant \alpha \leqslant 1$ denote the interval boxes formed by the α-cuts $(A_1)_\alpha, \ldots, (A_d)_\alpha$. In this work, the transformation method removing the recurring permutations is adopted. The number of discrete parameter vectors N is determined by the number of α-cuts m chosen for the fuzzy number discretization.

Step 2: *Model evaluation*. The $\mathrm{CLF}(f_i, \mathbf{p}_j)$ is computed for all $s \cdot N$ permutations. An efficient implementation may vectorize the calls to the spectral element method model to treat multiple discrete frequencies, or alternatively, several sets of parameter permutations at once.

Step 3: *Coupling loss factor confidence limits*. In this case, the resulting $\mathrm{CLF}(f_i, \mathbf{p}_j)$ are used to compute an approximate envelope. For each α-cut $\in [0, 1]$, where α must match the cuts selected for the discretization with $\mathrm{CLF}_\alpha(f_i) = [\mathrm{CLF}_{\alpha,\min}(f_i), \mathrm{CLF}_{\alpha,\max}(f_i)]$, with

$$\mathrm{CLF}_{\alpha,\min}(f_i) = \min_{\mathbf{p}_j \in \Omega_\alpha} \mathrm{CLF}(f_i, \mathbf{p}_j) \tag{22}$$

and

$$\mathrm{CLF}_{\alpha,\max}(f_i) = \max_{\mathbf{p}_j \in \Omega_\alpha} \mathrm{CLF}(f_i, \mathbf{p}_j). \tag{23}$$

At the end of the process, the fuzzy-valued coupling loss factor response at any given frequency f_i can be composed from the α-level sets of $\mathrm{CLF}(f_i)_\alpha$. Furthermore, the coupling loss factor envelopes for a given interval of confidence α are easily obtained by plotting the two curves of the minimum and the maximum coupling loss factor values $\mathrm{CLF}_{\alpha,\min}(f_i)$ and $\mathrm{CLF}_{\alpha,\max}(f_i)$, respectively, over the frequencies f_i, $i \in [f_0, f_1]$. Therefore, considering that the coupling loss factors are the main SEA parameters, the idea to assess the envelopes is to provide more robust coupling loss factors to build an SEA model. In the next section, the proposed method is applied to a frame-type structure consisting of two beams connected at an arbitrary angle.

4. Application to frame-type structures

4.1. Problem definition

In order to show the applicability of the proposed spectral element method/fuzzy method for the estimation of SEA coupling loss factors of frame-type junctions, the example shown in Fig. 3 is presented.

Table 1 shows the physical properties for beams 1 and 2 with mean value and standard deviation for each uncertain input parameter. Different parameterization schemes may be used for the frame junction depending on the physics of the problem at hand. For the proposed example, the Young's modulus $\tilde{E}$, mass density $\tilde{\rho}$ and

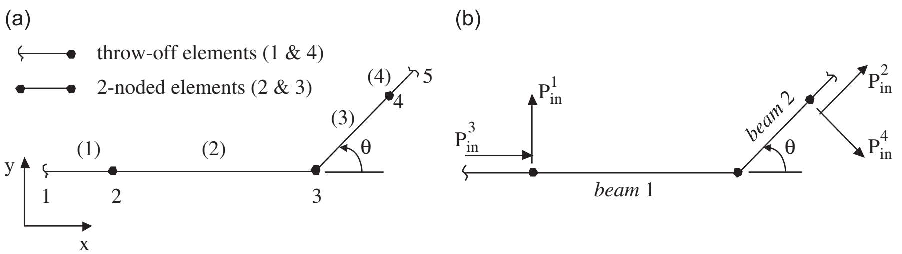

Fig. 3. Spectral element method (SEM) model for two beams connected at an arbitrary angle.

Table 1
Physical properties: beams with non-deterministic input parameters

Parameter	Mean value $\bar{m}$	Standard deviation	Dimension
$E_{1,2}$	2.62×10^9	10%	N/m^2
$\rho_{1,2}$	1280	10%	kg/m^3
$A_{1,2}$	1×10^{-4}	0	m^2
$I_{z1,z2}$	8.33×10^{-10}	1%	m^4
$L_{1,2}$	100	0	m

moment of inertia in z direction $\tilde{I}_z$ are treated as uncertain parameters. In this simulation, $\tilde{E}_1 = \tilde{E}_2 = \tilde{E}$, $\tilde{\rho}_1 = \tilde{\rho}_2 = \tilde{\rho}$ and $\tilde{I}_{z1} = \tilde{I}_{z2} = \tilde{I}_z$. For the length of the rods $L_{1,2}$ and the cross-section areas $A_{1,2}$, the mean values are assumed. In this study a simple, and not very realistic, uncertainty model was used only for the sake of illustrating the proposed procedure.

The L-beam model shown in Fig. 3(b) consists of two semi-infinite beams, respectively, beam 1 and beam 2 connected at an arbitrary angle θ. In this example an angle of 60° is assumed. However, it is important to stress that it is straightforward to address arbitrary angles and an arbitrary number of beams converging to the junction with the SE model described here.

The SE model is composed of two 2-noded and two throw-off spectral elements as presented in Fig. 3(a). Using SEA methodology, four subsystems are defined: two for longitudinal waves and another two for the flexural waves in the x–y plane. To assess the total energy in each subsystem using the spectral element method, power is input into each subsystem and then the energies are found for each input, as shown in Fig. 3(b). The coupling loss factors for these subsystems are defined as follows:

- η_{B1B2}: coupling loss factor between flexural waves incident at *beam* 1 and flexural waves transmitted to *beam* 2.
- η_{B1L2}: coupling loss factor between flexural waves incident at *beam* 1 and longitudinal waves transmitted to *beam* 2.
- η_{L1B2}: coupling loss factor between longitudinal waves incident at *beam* 1 and flexural waves transmitted to *beam* 2.
- η_{L1L2}: coupling loss factor between longitudinal waves incident at *beam* 1 and longitudinal waves transmitted to *beam* 2.

As discussed in Ref. [24], in terms of deterministic response, a good agreement with the coupling loss factors was obtained using the spectral element method with the simplified expressions of Stimpson and Lalor [22] and using the analytical expressions of Cremer and Heckel [23]. In the next section, the SEA coupling loss factors will be estimated using the proposed spectral element method/fuzzy method with the simplified expression defined in Ref. [22], so that a more robust estimation of the SEA coupling loss factors can be expected, instead of just obtaining deterministic values.

4.2. Estimation of coupling loss factors using the proposed spectral element method/fuzzy method

In this section, the coupling loss factors estimated via the proposed spectral element method/fuzzy method are presented. Coupling loss factor confidence limits are provided in order to be used in SEA models. A frequency range is chosen from 1 Hz to 5 kHz using 1/3-octave bands.

In addition, for each frequency band analyzed, the mean square energy values were computed by averaging over 10 frequency lines. Figs. 4 and 5 show the main results found using the S&L approximation with spectral element method/fuzzy method.

The energies for bending and longitudinal waves in beams 1 and 2 were computed from the spectral element solution using the methodology explained by Ahmida and Arruda [24]. With an internal loss factor of $\eta = 0.001$ in each beam, the energy in the throw-off elements could be neglected due to the length of the

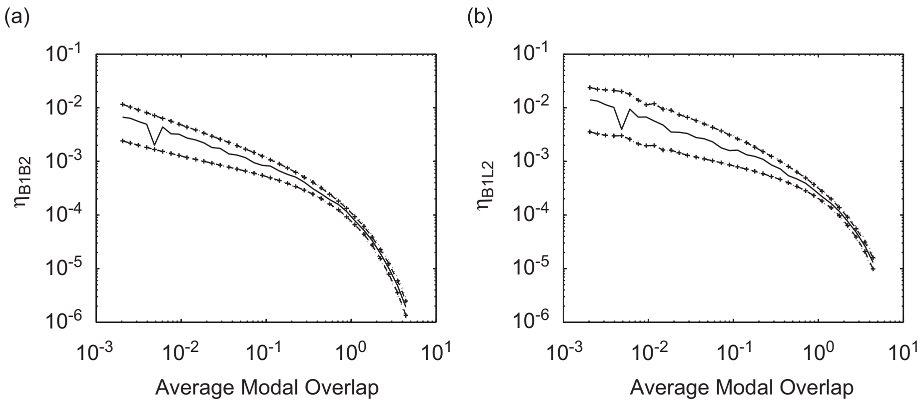

Fig. 4. Estimation of coupling loss factors using S&L approximation and their envelopes: (a) η_{B1B2} and (b) η_{B1L2}. spectral element method-nominal (solid), spectral element method/fuzzy (dotted-line) and spectral element method/Monte Carlo (dashed +).

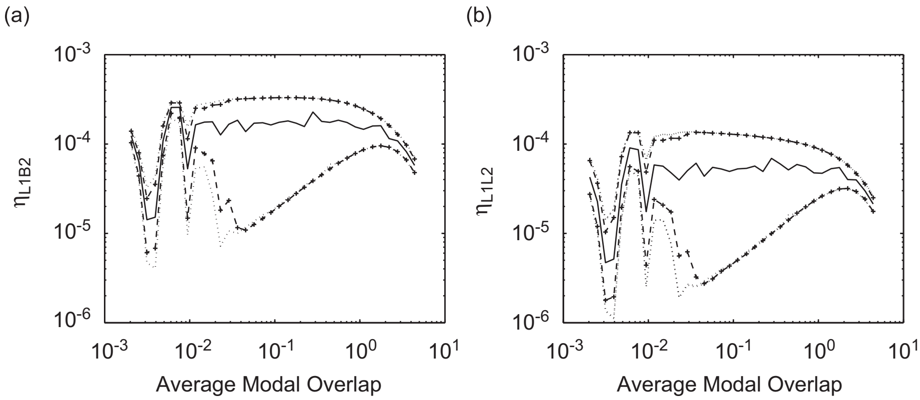

Fig. 5. Estimation of coupling loss factors using S&L approximation and their envelopes: (a) η_{L1B2} and (b) η_{L1L2}. spectral element method-nominal (solid), spectral element method/fuzzy (dotted-line) and spectral element method/Monte Carlo (dashed +).

two-noded elements, which is equal to 100 m. The number of α-cuts used in the Fuzzy method is $m = 7$ and the membership functions are quasi-Gaussian shaped, clipped at plus and minus 3 standard deviations. The results include the *mean*, *maximum* and *minimum* values for the coupling loss factors found for beam 1 and beam 2 plotted versus the average modal overlap (AMO). In order to compute the modal overlap factor (MOF), the following expression is adopted [1]:

$$\mathrm{MOF} = \omega\eta n, \tag{24}$$

where η is the loss factor and n is the total modal density of the L-beam system, which is composed of the modal densities for the longitudinal and transversal waves. To assess the AMO, the central frequency band is considered, and, for each frequency band analyzed, the mean values were computed over 10 frequency lines within the band.

Also, a Monte Carlo analysis (MC) was conducted to check the results found using the proposed spectral element method/fuzzy method. For the Monte Carlo analysis, considering the same uncertain parameters defined in Table1, Gaussian normal distributions clipped at 3σ-bounds were adopted. They were generated by the pseudo-random number generator of MATLAB. In addition, to determine the required sample size, the following expression, described by Maglaras [25], was used:

$$N = \frac{1 - P}{P - \mathrm{COV}_P^2}, \tag{25}$$

with P defined as the anticipated probability of failure and COV_P the desired coefficient of variation of probability of failure. The coefficient of variation is defined as the standard deviation divided by the mean. For instance, taking a probability of failure of 0.1 and a coefficient of variation of 0.1, we find, substituting these values, a minimum required N of 900. In our case, just for simplicity, we assume a sample size of $N = 1000$.

In general, Figs. 4 and 5 show that the spectral element method/fuzzy and the spectral element method/Monte Carlo yielded quite similar results, specially for higher AMO. However, in Figs. 5(a) and (b), for the longitudinal waves, some difference at low frequencies can be observed between the spectral element method/fuzzy and spectral element method/Monte Carlo results. In this context, it is important to add that, on the basis of the transformation method, the overestimation effect is avoided. Additional work and comparison of spectral element method/fuzzy with the classical spectral element method/Monte Carlo can be found in the work of Nunes et al. [15], where accuracy, performance and scalability have been investigated.

In the next section, where the main purpose is to find the energy envelopes using the robust coupling loss factors obtained in Figs. 4 and 5, we have used only the coupling loss factors found with the spectral element method/fuzzy method. In order to assess the energy levels for the four sub-systems described above, conventional SEA was adopted.

4.3. Estimation of the subsystem energies using the coupling loss factors obtained with the SEM/fuzzy method

Basically, in this section, the energy levels for two cases are estimated. In the first case, the coupling loss factors obtained with the spectral element method/fuzzy method are combined with a unit power input to the subsystem associated with transverse waves on beam 1 to assess the energy level envelopes, i.e., *maximum*, *mean* and *minimum* values. In this regard, it is important to stress that, for the other 3 subsystems, the input powers are not considered. In the second case, the power input envelopes due to the coupling loss factor variation are taken into account when computing the energy envelope. The second case is treated in order to show that the uncertainty in the input power due to coupling loss factor variation plays an important role in SEA predictions. In a recent paper presented by Davis [26], the uncertainty in the predictions is presented as three separate problems. The first problem concerns the uncertainty in the input power, which is shown to be the major issue. The second problem regards the uncertainty in the transfer functions, and, finally, the third deals with uncertainties in the definition of the SEA model and its subsystems. Here, the two examples proposed are mainly focused on two of those three concerns, namely the uncertainty in the input power and the uncertainty in the transfer functions, due to the influence of the uncertain physical parameters.

Figs. 6 and 7 show the results for the transversal and longitudinal vibration energies for beams 1 and 2 for the first case, i.e., the energies were computed for a unit power input to the subsystem associated with transverse waves on beam 1. In this first example, it is important to add that the effects of uncertain parameters increases when the central frequency increases, which can be clearly observed in Figs. 6(b) and 7(b).

In the second example, the energy limits are obtained using the same coupling loss factors, but including the envelopes obtained with the input powers, i.e., input powers *max*, *min* and *mean*. To assess those values of input power, for each point mobility, the respective force and velocity are assessed using the following expression:

$$P_{in} = \tfrac{1}{2}\,\mathrm{Re}(Fv^*), \tag{26}$$

where F is the point force, in this example simulated with 1 N, Re denotes the real part of a complex quantity and v^* is the complex conjugate of the velocity at the same point where the force acts. If the displacement u is assessed, velocity can be computed as $v = \mathrm{i}\omega u$, which leads to $P_{\mathrm{in}}/\omega = -F\,\mathrm{Im}(u)/2$, where Im is the imaginary part of the displacement at the node where the force is applied, which is always negative, so that the input power is always positive.

Figs. 8 and 9 show the energy levels for the beams 1 and 2 using the coupling loss factors and power input estimated with spectral element method/fuzzy. It should be noted that the envelope energies found considering the power input to each subsystem have a greater influence in the final result than the direct effect of the coupling loss factor variation.

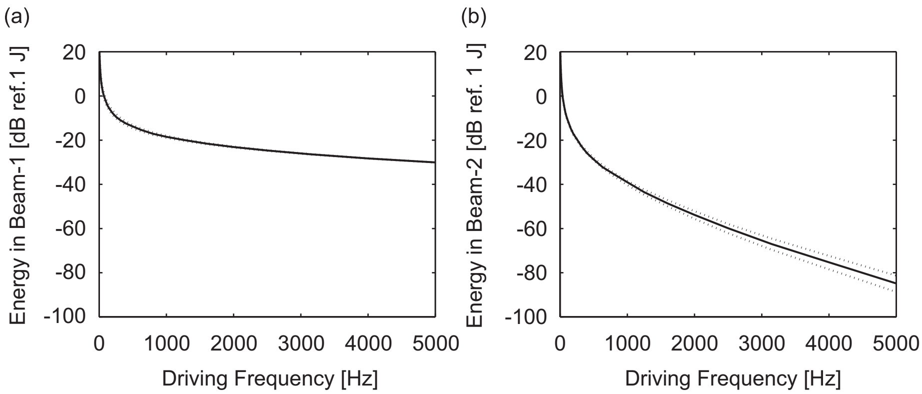

Fig. 6. Energy levels for the beams 1 and 2 using coupling loss factors estimated with spectral element method/fuzzy: (a) transversal energy in beam 1 and (b) transversal energy in beam 2. *Max* and *Min* energies (dotted-line) and nominal energy (solid).

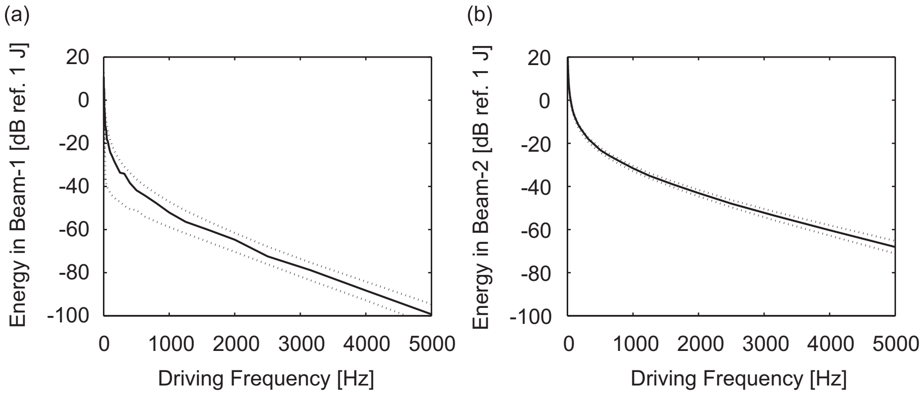

Fig. 7. Energy levels for the beams 1 and 2 using coupling loss factors estimated with spectral element method/fuzzy: (a) longitudinal energy in beam 1 and (b) longitudinal energy in beam 2. *Max* and *Min* energies (dotted-line) and nominal energy (solid).

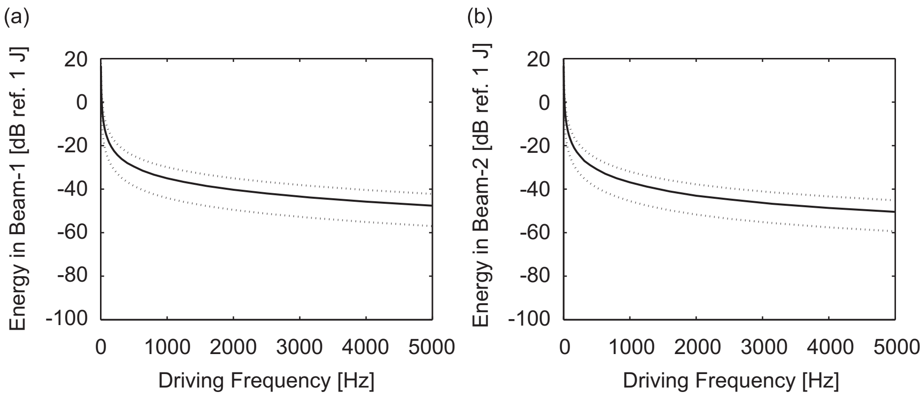

Fig. 8. Energy levels for the beams 1 and 2 using coupling loss factors and power input estimated with spectral element method/fuzzy: (a) transversal energy in beam 1 and (b) transversal energy in beam 2. *Max* and *Min* energies (dotted-line) and nominal energy (solid).

To summarize, both cases show that, using the spectral element method/fuzzy method, it is possible to take into account the influence of the uncertain input parameters into a SEA model using a fuzzy description of the parameter uncertainty. In addition, one important fact to add from the second case is that it is recommended that such envelopes consider the input power envelopes. This is clarified based on results presented in

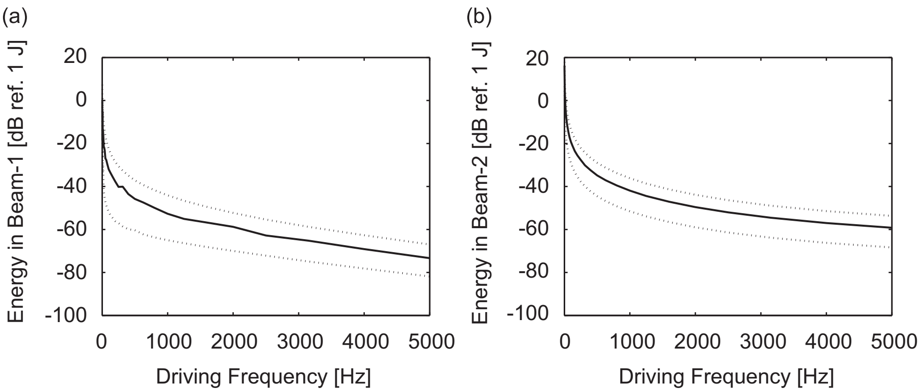

Fig. 9. Energy levels for the beams 1 and 2 using coupling loss factors and power input estimated with spectral element method/fuzzy: (a) longitudinal energy in beam 1 and (b) longitudinal energy in beam 2. *Max* and *Min* energies (dotted-line) and nominal energy (solid).

Figs. 6(a) and 8(b). Thus, both coupling loss factor and input power envelopes can be computed via the proposed spectral element method/fuzzy method, and the envelopes of the energy levels of subsystems computed by SEA can be estimated.

5. Concluding remarks

The spectral element method (SEM) combined with a special implementation of fuzzy-set-based method is proposed as an alternative approach to compute the SEA coupling loss factors. The spectral element method is used to overcome some problems of excessive model size due to mesh refinement at higher frequencies, usually observed in FEA. The fuzzy set approach based on the extension principle is used to treat the influence of model parameter uncertainty. The idea is to estimate the coupling loss factors and their confidence limits based upon possible parameter variations in the early stages of product development.

A simple numerical example consisting of two semi-infinite beams connected at an arbitrary angle was used to assess the coupling loss factor variation in a frame-type junction due to physical parameter uncertainty. The obtained results showed that, taking into account the influence of the uncertain input parameters according to Table 1, the results presented significant variations in comparison with the nominal values. These variations affect the subsystem energies predicted by the SEA model.

In this simple example, the variations assumed for the input uncertain parameters are taken as *possible* to occur. When a new project is starting, not much information is available and uncertainty is predicted based upon the experience in former projects. On the one hand, as the development cycle evolves and more experimental data becomes available, a more accurate parameter variation prediction can be made. The pressure to develop new products in shorter time means, in most cases, that there is no time to set up many experiments to provide probability density functions (PDF) of model parameters. However, in the case when input data with the associated probability density functions are available, more accurate models can be obtained by combining both theories, i.e., *possibility* and *probability*. Therefore, for an initial estimation of the influence of uncertain input parameters, the spectral element method combined with fuzzy-set-based method proposed here can be helpful to address uncertainty in frame-type structures.

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