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Analytical modelling and extraction of the modal behaviour of a cantilever beam in fluid interaction

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Abstract

When carrying out vibration health monitoring (VHM) of a structure it is usually assumed that the structure is in the absence of fluid interaction and/or that any environmental effects which can cause changes in the vibration response of the structure either remain constant or are negligible. In general, the natural frequencies of a structure are the first candidates to be considered for damage features. But the natural frequencies would also change as a result of the interaction of the structure with a fluid/gas environment. For the purpose of VHM, one needs the pure structural natural frequencies corresponding to conditions when the structure does not interact with the environment. Therefore, in certain cases when the above assumptions cannot be made it becomes necessary to extract values of natural frequencies of the structure if it were in the absence of fluid interaction from those values measured. This paper considers the case of a cantilever beam in contact with a fluid cavity giving rise to strong structural/fluid vibration interaction and develops a method by which the natural frequencies of the beam in the absence of fluid interaction can be obtained from those of the beam in interaction.

1. Introduction

Vibration health monitoring (VHM) of structures is based upon monitoring aspects of the vibration signature of the structure and relating any changes in these to the introduction, or progression, of damage. The normal manner in which VHM is applied is to lightly excite the structure and record any changes, which occur to the modal parameters, which are then related to changes (damage) which are (have) occurred in the structure. Damage in a structure, which can be either concentrated (in the form of a crack) or distributed (corrosion, erosion), normally leads to alterations of the stiffness and/or mass, which in turn results in changes in the vibration response of the structure. A number of authors suggest the use of the natural frequencies as a first resort for damage features, because they are easy and straightforward to obtain from experiment [1,2]. Beams are simple structures, which lend themselves well to VHM in which changes in natural frequency are

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used as the basis for damage detection [2–4]. An important condition for VHM is that any environmental effects, such as temperature stressing and/or structural/fluid interaction, remain constant, or better still are not present since these effects alone can cause changes to the vibration signature and therefore obscure any changes due to defects in the structure.

This paper considers a general idealised case of a simple cantilever beam in free lateral vibration and interacting with a rectangular fluid cavity. Previous studies of cantilevers in interaction with a fluid have been related either to the case where the fluid medium has been infinite [5], i.e., no wall reflecting the fluid pressure waves, and the celebrated work of Sarkar and Paidoussis [6] in which cantilever pipes have been conveying fluid. In the present study, linear vibrations are assumed because in general a number of structural elements, and beams in particular, demonstrate linear behaviour especially for small amplitude vibrations, as is normally the case for the purpose of VHM. Nonlinearities and nonlinear behaviour can become important for cases of large amplitude vibrations, e.g. close to some of the natural frequencies of the structures during modal testing for the purpose of predicting how the structure will behave under the action of large and multipoint exciting forces [7]. This paper details the development of a method for extracting the natural frequencies of the beam in the absence of fluid interaction (which can be then used as the basis of VHM) from those values obtained from the beam/fluid cavity coupled system.

The paper commences by presenting a comprehensive analysis of the coupled free vibration of a cantilever beam in interaction with a fluid-filled fluid cavity. The analysis is based upon a similar analysis of a circular plate in interaction with a fluid cavity [8], which produced natural frequencies of the coupled system and was in good agreement with values obtained using finite element analysis and experiments. The analysis is based upon the basic equations describing the free vibration of the beam and of the fluid, which are then combined using the Galerkin approach. The analysis proceeds to establish the relative energy between the beam and fluid whilst in interaction and finally describes a method whereby the natural frequencies of the beam in the absence of fluid interaction can be extracted from knowledge of the natural frequencies of the coupled system together with known parameters of the fluid cavity.

2. Free vibration analysis of a beam in interaction with a fluid cavity

Fig. 1 shows an idealised model of a coupled structural/fluid vibration interacting system in the form of a cantilever beam, of length a, interacting with a enclosed rectangular fluid (air) cavity of length a and depth l. The vibration of the beam is a function of the spatial coordinate, x, and time, t, only whilst the pressure in the fluid due to the ensuing vibration of the beam is a function of the spatial coordinates, x and y, and time, t, only. The beam is located at $y = l$. Both the beam and fluid cavity will have the same width (along the o–z

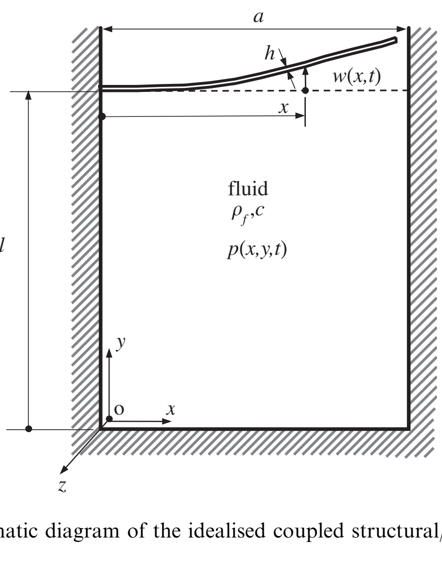

Fig. 1. Schematic diagram of the idealised coupled structural/fluid system.

axis shown) and for small linear vibration of the system, any vortex effects at the free end of the beam ($x = a$) are assumed negligible. In this idealised system, any fluid pressure acting on the open top of the beam will be treated as negligible on the basis that the fluid is air (or a gas) and these pressure waves will travel off to infinity.

Hence, the equation of motion, describing the free small lateral vibration, $w = w(x, t)$, of a cantilever beam of constant rectangular cross section in interaction with the fluid in a rectangular cavity shown in Fig. 1, is [9]

$$\frac{\partial^4 \bar{w}}{\partial \bar{x}^4} = \frac{-\rho_d h a^4}{EI'}\frac{\partial^2 \bar{w}}{\partial t^2} + \left.\frac{pa^3}{EI'}\right|_{\bar{y}=1}, \tag{1}$$

where $\bar{w} = w/a$, $\bar{x} = x/a$, and $\bar{y} = y/l$ are dimensionless displacement and coordinates, E is Young's modulus and I' is the second moment of area of the beam per unit width about the neutral axis of bending ($= h^3/12$) and ρ_d is the beam material mass density; h is the beam thickness, a is the length of the beam, equal to the horizontal length of the fluid cavity; l is the depth of the fluid cavity and p is the oscillatory pressure of the fluid loading the beam.

Now writing

$$\bar{w} = \sum_{s=1}^{\infty} \chi_s \psi_s \, \mathrm{e}^{\mathrm{i}\omega t}, \tag{2}$$

where $\psi_s = \psi_s(\bar{x})$ is the sth natural mode shape of the beam in the absence of fluid interaction and χ_s is a constant associated with that mode, generally referred to as the *mode shape coefficient* for the sth mode. It is therefore assumed that (in accordance with the Galerkin approximation) that the natural mode shapes of the structure are unaffected by the presence of the fluid pressure acting on the structure. For a particular value of s, the natural frequency of free undamped vibration ω_s, is then [9]

$$\omega_s = \xi_s^2 \sqrt{\frac{EI'}{\rho_d h a^4}}, \tag{3}$$

where ξ_s is the non-dimensional natural frequency of the beam for the sth mode. For a cantilever beam whose boundary conditions are

$$\bar{w}|_{\bar{x}=0} = \left.\frac{\partial \bar{w}}{\partial \bar{x}}\right|_{\bar{x}=0} = \left.\frac{\partial^2 \bar{w}}{\mathrm{d}\bar{x}^2}\right|_{\bar{x}=1} = \left.\frac{\partial^3 \bar{w}}{\mathrm{d}\bar{x}^3}\right|_{\bar{x}=1} = 0$$

for all time, the values of ξ_s are the roots of the equation

$$\cos\,\xi_s \cosh \xi_s = -1$$

and the first ten roots ($s = 1,2,3,\ldots,10$) are listed in Table 1.

The sth natural mode shape of the cantilever is [9]

$$\psi_s = \sin\,\xi_s\bar{x} - \sinh \xi_s\bar{x} - \mu_s(\cos\,\xi_s\bar{x} - \cosh \xi_s\bar{x}), \tag{4}$$

where $\mu_s = (\sin\,\xi_s + \sinh \xi_s)/(\cos\,\xi_s + \cosh \xi_s)$.

Table 1
Non-dimensional natural frequencies, ξ_s, of a cantilever beam in the absence of fluid interaction

s	ξ_s	s	ξ_s
1	1.8751	6	17.2789
2	4.6940	7	20.4203
3	7.8548	8	23.5620
4	10.9955	9	26.7036
5	14.1372	10	29.8453

For a particular mode of vibration for the beam in the absence of fluid interaction:

$$\frac{\partial^4 \psi_s}{\partial \bar{x}^4} = \frac{\omega_s^2 \rho_d h a^4}{EI'} \psi_s. \tag{5}$$

Therefore, combination of Eqs. (1), (2) and (5) gives

$$\sum_{s=1}^{\infty} [(\omega_s^2 - \omega^2)\chi_s \psi_s] \, \mathrm{e}^{\mathrm{i}\omega t} = \left. \frac{p}{\rho_d h a} \right|_{\bar{y}=1}. \tag{6}$$

We shall now establish the form of the fluid pressure, p, acting on the beam. Consider the fluid cavity shown in Fig. 1, whose velocity potential, $\phi = \phi(x, y, t)$ is described by the standard two spatial dimension wave equation

$$\frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} + \left(\frac{a}{l}\right)^2 \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} = \left(\frac{a}{c}\right)^2 \frac{\partial^2 \bar{\phi}}{\partial t^2}, \tag{7}$$

where $\bar{\phi} = \phi/ac$, $\bar{y} = y/l$ and c is speed of sound.

The general solution to Eq. (7), assuming the function of time to being harmonic, is

$$\bar{\phi} = \sum_{q=1}^{\infty} A_q \bar{\phi}_{xq} \bar{\phi}_{yq} \, \mathrm{e}^{\mathrm{i}\omega t}, \tag{8}$$

where $\bar{\phi}_{xq}$ is a function of $\bar{x}$ only and $\bar{\phi}_{yq}$ is a function of $\bar{y}$ only and A_q is a constant for a particular value of q. The velocity potential function yields the velocity components of the fluid in the x and y directions as v_{xq} and v_{yq}, respectively, viz.,

$$\begin{aligned} v_{xq} &= \frac{\partial \phi}{\partial x} = c \frac{\partial \bar{\phi}}{\partial \bar{x}}, \\ v_{yq} &= \frac{\partial \phi}{\partial y} = \frac{ac}{l} \frac{\partial \bar{\phi}}{\partial \bar{y}}. \end{aligned} \tag{9}$$

Therefore, substituting Eq. (8) into Eq. (7) and imposing the boundary conditions

$$v_x|_{\bar{x}=0} = v_x|_{\bar{x}=1} = 0 \quad \text{for all } \bar{y},$$

$$v_y|_{\bar{y}=0} = 0 \quad \text{for all } \bar{x}$$

gives $\bar{\phi}_{xq} = \cos(q-1)\pi\bar{x}$ and $\bar{\phi}_{yq} = \cos(\alpha_q \bar{y})$ where $\alpha_q = (l/a)\sqrt{\lambda^2 - \gamma_q^2}$, $\lambda = \omega a/c$ and $\gamma_q = (q-1)\pi$. Substituting the above expressions for $\bar{\phi}_{xq}$ and $\bar{\phi}_{yq}$ in Eq. (8) gives

$$\bar{\phi} = \sum_{q=1}^{\infty} A_q [\cos(q-1)\pi\bar{x}][\cos(\alpha_q \bar{y})] \, \mathrm{e}^{\mathrm{i}\omega t}. \tag{10}$$

At $\bar{y} = 1$, v_y and the lateral velocity of the beam are equal, i.e.,

$$\begin{aligned} \left. \frac{\partial \phi}{\partial y} \right|_{\bar{y}=1} &= \frac{\partial w}{\partial t}, \\ \left. \frac{c}{l} \frac{\partial \bar{\phi}}{\partial \bar{y}} \right|_{\bar{y}=1} &= \frac{\partial \bar{w}}{\partial t} \quad \text{for } 0 \leqslant \bar{x} \leqslant 1. \end{aligned} \tag{11}$$

Combining Eqs. (2), (10) and (11) and using the orthogonal properties of eigenfunctions gives

$$A_q = -i \left(\frac{l}{a}\right) \lambda \frac{\sum_{s=1}^{\infty} \chi_s k_{qs}}{\alpha_q I_q \sin \alpha_q}, \tag{12}$$

where

$$k_{qs} = \int_0^1 \psi_s \bar{\phi}_{xq} \, \mathrm{d}\bar{x} \tag{13}$$

the value of which can be obtained through standard numerical integration, and,

$$I_q = \int_0^1 \bar{\phi}_{xq}^2 \, \mathrm{d}\bar{x}.$$

Now the fluid pressure, p, at the surface of the beam is given by

$$p|_{\bar{y}=1} = -ac\rho_f \frac{\partial \bar{\phi}}{\partial t}\bigg|_{\bar{y}=1},$$

where ρ_f is the fluid density. Therefore, combining Eqs. (10) and (12) give

$$p|_{\bar{y}=1} = -\omega^2 al\rho_f \sum_{s=1}^{\infty}\sum_{q=1}^{\infty} \frac{\chi_s k_{qs}\bar{\phi}_{xq}}{(\alpha_q \tan \alpha_q)I_q} \mathrm{e}^{\mathrm{i}\omega t}. \tag{14}$$

Substituting Eq. (14) into Eq. (6) gives

$$\sum_{s=1}^{\infty}(\omega_s^2 - \omega^2)\chi_s\psi_s = -\omega^2 \frac{\rho_f l}{\rho_d h} \sum_{s=1}^{\infty}\sum_{q=1}^{\infty} \frac{\chi_s k_{qs}\bar{\phi}_{xq}}{(\alpha_q \tan \alpha_q)I_q}.$$

Multiplying both sides by $\bar{\phi}_{xq}$ and integrating between $0 \leqslant \bar{x} \leqslant 1$ gives

$$\sum_{s=1}^{\infty} \chi_s k_{qs}\left\{\omega_s^2 - \omega^2\left[1 - \frac{\rho}{(\alpha_q \tan \alpha_q)}\right]\right\} = 0, \quad q = 1, 2, 3\ldots, \tag{15}$$

where $\rho = \rho_f l/\rho_d h$ is the ratio of the mass of the fluid to the mass of the beam.

Since, from Eq. (3), $\omega_s^2 = \xi_s^4(EI'/\rho_d ha^4)$ then we introduce η instead of ω by the relation

$$\omega^2 = \eta^4 \frac{EI'}{\rho_d ha^4}. \tag{16}$$

Hence, Eq. (15) can be re-expressed as

$$\sum_{s=1}^{\infty} \chi_s k_{qs}\left\{\xi_s^4 - \eta^4\left[1 - \frac{\rho}{(\alpha_q \tan \alpha_q)}\right]\right\} = 0, \quad q = 1, 2, 3, \ldots,$$

which can be represented in matrix form as

$$\begin{bmatrix} a_{11}(\xi_1,\eta) & a_{12}(\xi_2,\eta)\cdots & a_{1n}(\xi_n,\eta) \\ a_{21}(\xi_1,\eta) & a_{22}(\xi_2,\eta)\cdots & a_{2n}(\xi_n,\eta) \\ \vdots & \vdots & \vdots \\ \cdots & a_{qs}(\xi_2,\eta) & \cdots \\ \vdots & \vdots & \vdots \\ a_{n1}(\xi_1,\eta) & a_{n2}(\xi_2,\eta)\cdots & a_{nn}(\xi_n,\eta) \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_s \\ \vdots \\ \chi_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, \tag{17}$$

i.e.

$$\mathbf{A}(\xi_s, \eta)\boldsymbol{\chi} = 0, \tag{18}$$

where

$$a_{qs}(\xi_s, \eta) = k_{qs}\left\{\xi_s^4 - \eta^4\left[1 - \frac{\rho}{(\alpha_q \tan \alpha_q)}\right]\right\}. \tag{19}$$

Hence, values of η can be obtained (iterated upon) which renders the determinant of the matrix.

$\mathbf{A}(\xi_s,\eta)$ equal to zero. Consequently, for each of these values of η the corresponding normalised values of mode shape coefficients $\chi_1,\chi_2,\ldots\chi_n$, are obtained. The determinant of this matrix equation is obtained by performing the LU decomposition [10,11], whereupon the value of the determinant is the product of the diagonal terms. Subsequently, these root values of η which render the determinant zero are substituted back into Eq. (18) to obtain the corresponding vector of the mode shape coefficients, $\boldsymbol{\chi}$, (normalised to χ_1 in the first instance and then to the largest value,) which describe which structural modes are present and dominant.

In Eq. (19) above, it is noted that significant deviation between η and ξ_s will occur around values of α_q equal to 0, π, 2π, etc. while minimum deviation will be around values of α_q equal to 0.5π, 1.5π, etc.

We shall now develop and postulate the parameters, which will give rise to conditions of strong structural/fluid vibration interaction. Fig. 2 shows a plot of a natural frequency, ω_s, of the beam in the absence of fluid interaction and a natural frequency, ω_m, of the fluid cavity if the beam was rigid. Both of these natural frequencies are plotted to a base of the controlling parameters, l; the depth of the cavity. Now, for any value of s, the natural frequency of the beam in the absence of fluid interaction is given by Eq. (3) and this value is independent of the depth l. A natural frequency of the solid bounded fluid cavity is obtained by now imposing the condition that $(c/l)(\partial\bar{\phi}/\partial\bar{y})|_{\bar{y}=1} = 0$ for all $\bar{x}$, upon Eq. (10).

This results in

$$\alpha_q = \frac{l}{a}\sqrt{\frac{\omega_m^2 a^2}{c^2} - \gamma_q^2} = m\pi, \quad m = 1, 2, \text{ etc.}$$

Note, in the special case(s) when $m = 0$, this would imply only horizontal fluid modes with the fluid having zero axial component of velocity, this having no interaction with the lateral vibration of the beam. Therefore, for $m \geqslant 1$,

$$\omega_m = \frac{c}{a}\sqrt{\left(\frac{m\pi a}{l}\right)^2 + \gamma_q^2}. \tag{20}$$

Reference to Eq. (20) and Fig. 2 demonstrates that as the value of l increases all values of ω_m decrease and will, for appropriate values of $l = l_c$ correspond to values ω_s of the beam in the absence of fluid interaction. In such circumstances we will then have strong structural/fluid vibration interaction characterised by a region of ‘‘veering’’ whence at $l = l_c$ the strongly interacting system will exhibit two natural frequencies close to each other and in the region where $\omega_m = \omega_s$. Around such a condition experiments on the system would show there to be two natural frequencies of the system which are very close in value; one would represent a strongly coupled structural/fluid (st/fl) mode and the other a strongly coupled fluid/structural (fl/st) mode. Therefore for $l = l_c$ we can equate Eqs. (3) and (20) to give

$$\xi_s^2 \frac{h}{a^2}\sqrt{\frac{E}{12\rho_d}} = \frac{c}{a}\sqrt{\left(\frac{m\pi a}{l_c}\right)^2 + \gamma_q^2}.$$

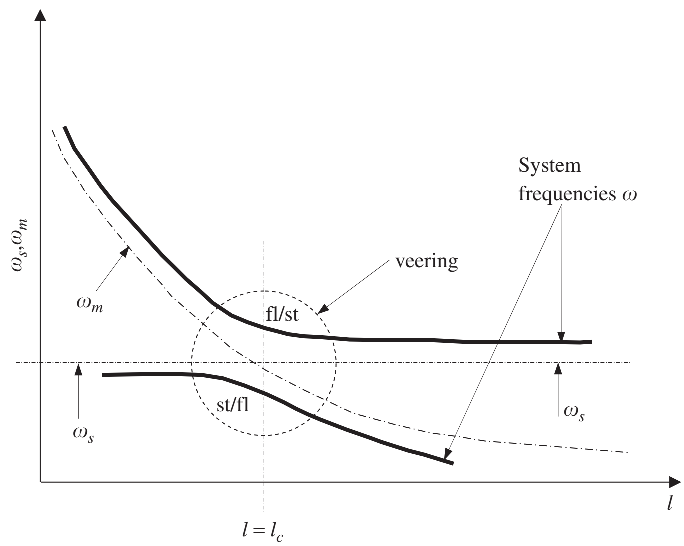

Fig. 2. Condition for strong structural/fluid vibration interaction.

For the special case where the system is designed such that a particular natural frequency of the beam in the absence of fluid interaction, ω_s, is equal to a natural frequency of the totally bounded fluid cavity, ω_m, which exhibits no horizontal component of fluid velocity, i.e. $\gamma_q = 0$, then,

$$\bar{l}_c = \frac{l_c}{a} = \frac{m\pi c}{\xi_s^2}\left(\frac{a}{h}\right)\sqrt{\frac{12\rho_d}{E}}. \tag{21}$$

Also, rearranging Eq. (20) the non-dimensional frequency parameter, β_m, relating to a rigidly bounded fluid cylindrical cavity is obtained and equal to

$$\beta_m = \frac{\omega_m l}{\pi c} = \sqrt{m^2 + \frac{\bar{l}^2}{\pi^2}\gamma_q^2}, \tag{22}$$

where $\bar{l} = l/a$. The parameter β_m can then be used as an indication of the characteristic of the fluid vibration. For this special case where $\gamma_q = 0$, from Eq. (22), $\beta_m = m$.

Prior to examining the characteristics of the coupled vibration, the convergence of the analysis presented will be investigated.

For this exercise, and all subsequent results, we will adopt the following properties and parameters; $c = 343\,\text{m/s}$ (air), $\rho_d = 7800\,\text{kg/m}^3$, $E = 210\,\text{GN/m}^2$ and $(a/h) = 100$.

2.1. Convergence

Referring to Eq. (18), the accuracy and range of values of η obtained will be influenced by the equal number of rows and columns selected for iteration of the determinant of matrix $\mathbf{A}(\xi_s,\eta)$ equal to zero, i.e., the value of n. Accordingly, Table 2 shows the results obtained for the first two values of η when n is set at values ranging from 2 to 10. In this example, the dimension, $\bar{l}_c$ is selected in accordance with Eq. (21), such that strong coupling exists between the first natural frequency of the beam in the absence of fluid interaction ($s = 1$) and the first natural frequency of the completely bounded cavity ($m = 1$, $q = 1$). In this case the value of $\bar{l}_c$ is calculated to being 20.461. The first two root values of η obtained from Eq. (18) are found to being around the first natural frequency of the cantilever in the absence of fluid interaction, ξ_1. These two values of η can therefore be compared with those for the cantilever in the absence of fluid interaction; $\xi_1 = 1.8751$. It is immediately noted that the first value of η is lower than ξ_1 while the second is higher. This is in accordance with the observation shown in Fig. 2 around $\bar{l} = \bar{l}_c$.

From Table 2, it is seen that convergence, with respect to n, is extremely fast; requiring only $n = 4$ for convergence to two decimal places for these first two roots.

2.2. Energy analysis

In this study, since in all cases we are dealing with some degree of structural/fluid vibration interaction, it would be erroneous to describe any mode of vibration as either purely a structural mode or a fluid mode. Rather reference will be made to the modes as either structural/fluid (st/fl), to denote modes which are predominantly structural with fluid interference, and likewise fluid/structural (fl/st) to denote modes which are predominantly fluid but with structural interference. In an attempt to quantify the degree of coupling and describe whether modes are mainly structural or fluid, attention will be drawn to the distribution of vibration kinetic energy between the structural and fluid components of the system.

Table 2
Convergence

$N = 2$	$n = 4$	$n = 6$	$n = 8$	$n = 10$
1.7876	1.7845	1.7840	1.7837	1.7836
1.9612	1.9646	1.9652	1.9654	1.9655

For the beam the maximum kinetic energy of vibration per unit width, T_p, is calculated from

$$T_p = \sum_{s=1}^{n} T_{ps},$$

where from Eq. (2),

$$T_{ps} = \frac{1}{2}\rho_d h a^3 \omega^2 \int_0^1 [\chi_s \psi_s]^2 \,\mathrm{d}\bar{x}.$$

For the fluid the maximum kinetic energy per unit width, T_f, is calculated from

$$T_f = \sum_{q=1}^{n} T_{fq}^{y} + \sum_{q=1}^{n} T_{fq}^{x},$$

where

$$T_{fq}^{y} = \frac{1}{2}\rho_f a l \int_0^1 \int_0^1 (v_{yq})^2 \,\mathrm{d}\bar{y}\,\mathrm{d}\bar{x}$$

and

$$T_{fq}^{x} = \frac{1}{2}\rho_f a l \int_0^1 \int_0^1 (v_{xq})^2 \,\mathrm{d}\bar{y}\,\mathrm{d}\bar{x}$$

and both v_{yq} and v_{xq} are now obtained from Eqs. (9). Therefore, a set of relative kinetic energy vectors for the fluid and beam can be written as

$\mathbf{KE}_{fq}^{y} = \{\mathrm{KE}_{f1}^{y}, \mathrm{KE}_{f2}^{y}, \ldots \mathrm{KE}_{fq}^{y}, \ldots \mathrm{KE}_{fn}^{y}\}$ = vector of relative vertical kinetic energy components (q) of the fluid,
$\mathbf{KE}_{fq}^{x} = \{\mathrm{KE}_{f1}^{x}, \mathrm{KE}_{f2}^{x}, \ldots \mathrm{KE}_{fq}^{x}, \ldots \mathrm{KE}_{fn}^{x}\}$ = vector of relative horizontal kinetic energy components (q) of the fluid, and
$\mathbf{KE}_{ps} = \{\mathrm{KE}_{p1}, \mathrm{KE}_{p2}, \ldots \mathrm{KE}_{ps}, \ldots \mathrm{KE}_{pn}\}$ = vector of relative kinetic energy of lateral vibration components (s) of the beam,
where $\mathrm{KE}_{fq}^{y} = T_{fq}^{y}/[T_f + T_p] \times 100\%$, $\mathrm{KE}_{fq}^{x} = T_{fq}^{x}/[T_f + T_p] \times 100\%$ and $\mathrm{KE}_{ps} = T_{ps}/[T_f + T_p] \times 100\%$.

Using the above relative percentage energies, the characteristics of a cantilever beam in strong interaction with a fluid cavity as described is investigated. Consider the case where $\bar{l}_c$ is 20.461 which, as before, results in a condition of strong coupling between the first mode of the beam in the absence of fluid interaction ($s = 1$), and the first ($m = 1$) axial mode of the fluid cavity if the beam is assumed rigid. In all cases the ratio of $a/h = 100$ and only roots of the system matrix Eq. (18), η, which are close to those corresponding to the first two frequency roots of the beam in the absence of fluid interaction; ξ_1 and ξ_2, equal to 1.8751 and 4.6940, respectively, will be considered.

Table 3 lists details of all coupled modes of vibration up to the 8th mode. From this table it is evident that the modal energy of the subsystems renders an excellent means of describing the degree of coupling and dominance of the structure or fluid. Also, calculation of the associated value of β_m from Eq. (22), using the computed value of frequency, ω, of the coupled system indicates the number of vertical waves associated with the horizontal component of the vibration of the fluid. It is also interesting to note that for modes with a strong or moderate structural component, the vector of mode shape coefficients $\boldsymbol{\chi}$ is very well defined. For example for the first two coupled modes (1 and 2) with frequencies around that corresponding to the first natural frequency of the beam in the absence of fluid interaction, $\boldsymbol{\chi} = \{1,0,0,\ldots\}^{\mathrm{T}}$ and for mode 8 which is once again structurally dominant at a frequency close to that of the second natural frequency of the beam in the absence of fluid interaction, $\boldsymbol{\chi} = \{0,1,0,0,0,\ldots\}^{\mathrm{T}}$. On the other hand, for modes of a strong fluid nature, such as modes 3–7 in Table 2, it is observed that the vector $\boldsymbol{\chi}$ indicates significant contributions from more than one structural mode, e.g., mode 6 where $\boldsymbol{\chi} = \{0.98,1,0.023,\ldots\}^{\mathrm{T}}$. This observation of the form of the vector $\boldsymbol{\chi}$ for modes with a strong or moderate structural energy component will be seen to be important when

Table 3
Modes of free vibration of structural/fluid interacting system with associated energy vectors

η, β_m	$\boldsymbol{\chi}$, $\mathbf{KE}_{ps}$, $\mathbf{KE}^y_{fq}$, $\mathbf{KE}^x_{fq}$	Mode description
Coupled mode 1 1.7840, 0.9051	$\boldsymbol{\chi}=\{1,0,0,\dots\}^T$ $\mathbf{KE}_{ps}=\{44.84,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{55.16,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\dots\}$	Strongly coupled fl/st mode at $s=1$, $q=1$, $m\sim 1$
Coupled mode 2 1.9652, 1.0984	$\boldsymbol{\chi}=\{1,0,0,\dots\}^T$ $\mathbf{KE}_{ps}=\{51,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{49,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Coupled st/fl mode at $s=1$, $q=1$, $m\sim 1$
Coupled mode 3 2.6600, 2.0123	$\boldsymbol{\chi}=\{1,0.0522,\sim 0,\dots\}^T$ $\mathbf{KE}_{ps}=\{1.64,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{98.36,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Weakly coupled fl/st mode. Almost total fluid vertical energy at $q=1$, $m\sim 2$
Coupled mode 4 3.2513, 3.0066	$\boldsymbol{\chi}=\{1,0.16,\sim 0,\sim 0,\dots\}^T$ $\mathbf{KE}_{ps}=\{0.53,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{99.47,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Weakly coupled fl/st mode. Almost total fluid vertical energy at $q=1$, $m\sim 3$
Coupled mode 5 3.7521, 4.004	$\boldsymbol{\chi}=\{1,0.39,0.014,\sim 0,\dots\}^T$ $\mathbf{KE}_{ps}=\{0.27,0.04,0,\dots\}$ $\mathbf{KE}^y_{fq}=\{99.69,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{0,0,0,\dots\}$	Weakly coupled fl/st mode Almost total fluid vertical energy at $q=1$, $m\sim 4$
Coupled mode 6 4.1936, 5.0017	$\boldsymbol{\chi}=\{0.98,1,0.023,\dots\}^T$ $\mathbf{KE}_{ps}=\{0.16,0.17,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{99.67,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Weakly coupled fl/st mode. Almost total fluid vertical energy at $q=1$, $m\sim 5$
Coupled mode 7 4.5898, 5.9915	$\boldsymbol{\chi}=\{0.155,1,0,\dots\}^T$ $\mathbf{KE}_{ps}=\{0.1,4.4,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{95.5,\sim 0,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Weakly coupled fl/st mode. Almost total fluid vertical energy at $q=1$, $m\sim 6$ but with 4.4% interaction with the beam at $s=2$
Coupled mode 8 4.6946, 6.268	$\boldsymbol{\chi}=\{0,1,0,\dots\}^T$ $\mathbf{KE}_{ps}=\{94.82,\sim 0,\dots\}$ $\mathbf{KE}^y_{fq}=\{5.18,\sim 0,\dots\}$ $\mathbf{KE}^x_{fq}=\{\sim 0,\sim 0,\sim 0,\dots\}$	Weakly coupled st/fl mode. Almost total beam energy at $s=2$ with slight interaction with fluid vertical energy at $q=1$, $m\sim 6$

Table 4
Non-dimensional natural frequencies, mode shape coefficients and relative structural vibration energy at coupled modes around frequencies close to those of the structure in the absence of fluid interaction

η (ξ_s)	$\boldsymbol{\chi}$, KE_{ps}	Mode description
1.7840, ($\xi_1 = 1.8751$)	$\boldsymbol{\chi} = \{1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p1} = 44.84$	Strongly coupled fl/st around the 1st natural mode of the structure
$^{+}$1.9652, ($\xi_1 = 1.8751$)	$\boldsymbol{\chi} = \{1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p1} = 51$	Strongly coupled st/fl around the 1st natural mode of the structure
$^{+}$4.6945, ($\xi_2 = 4.6940$)	$\boldsymbol{\chi} = \{0,1,0,\ldots\}^{\mathrm{T}}$ $KE_{p2} = 94.815$	Weakly coupled st/fl mode. Almost total beam energy at the 2nd structural mode
$^{+}$7.8489, ($\xi_3 = 7.8548$)	$\boldsymbol{\chi} = \{0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p3} = 98.403$	Weakly coupled st/fl mode. Almost total beam energy at the 3rd structural mode
$^{+}$10.9581, ($\xi_4 = 10.9955$)	$\boldsymbol{\chi} = \{0,0,0,1,0,0\ \ldots\}^{\mathrm{T}}$ $KE_{p4} = 97.566$	Weakly coupled st/fl mode. Almost total beam energy at the 4th structural mode
$^{+}$14.1298, ($\xi_5 = 14.1372$)	$\boldsymbol{\chi} = \{0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p5} = 55.223$	Strongly coupled st/fl around the 5th natural mode of the structure
14.1358, ($\xi_5 = 14.1372$)	$\boldsymbol{\chi} = \{0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p5} = 34.15$	Strongly coupled fl/st around the 5th natural mode of the structure
$^{+}$17.2751, ($\xi_6 = 17.2789$)	$\boldsymbol{\chi} = \{0,0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p6} = 76.45$	Moderately coupled st/fl around the 6th natural mode of the structure
17.2893, ($\xi_6 = 17.2789$)	$\boldsymbol{\chi} = \{0,0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p6} = 11.26$	Moderately coupled fl/st around the 6th natural mode of the structure
$^{+}$20.4135, ($\xi_7 = 20.4203$)	$\boldsymbol{\chi} = \{0,0,0,0,0,0,1,0,0\ldots\}^{\mathrm{T}}$ $KE_{p7} = 74.98$	Moderately coupled st/fl around the 7th natural mode of the structure
20.4223, ($\xi_7 = 20.4203$)	$\boldsymbol{\chi} = \{0,0,0,0,0,0,1,0,0\ldots\}^{\mathrm{T}}$ $KE_{p7} = 15.57$	Moderately coupled fl/st around the 7th natural mode of the structure
23.5455, ($\xi_8 = 23.5620$)	$\boldsymbol{\chi} = \{0,0,0,0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p8} = 29.92$	Moderately coupled fl/st around the 8th natural mode of the structure
$^{+}$23.5546, ($\xi_8 = 23.5620$)	$\boldsymbol{\chi} = \{0,0,0,0,0,0,0,1,0,0,\ldots\}^{\mathrm{T}}$ $KE_{p8} = 73.56$	Moderately coupled st/fl around the 8th natural mode of the structure

considering the inverse problem, i.e., the problem of being able to extract the natural frequencies of what the beam would be in the absence of fluid interaction, ξ_s, when one has obtained values of the natural frequencies of the coupled system, η. To reinforce this observation, Table 4 lists details of the modes of vibration, which have strong or moderate structural energy components and therefore have natural frequencies close to those corresponding to natural frequencies of the beam in the absence of fluid interaction. In all of these cases it is observed that the vectors of mode shape coefficients contain one relevant unity and all other components are zero (to 3 decimal places).

3. The inverse extraction problem

In the foregoing analysis, known values of the structural natural frequencies together with the parameters of the fluid cavity were used as the input to computed values of natural frequencies of the structural/fluid coupled system and corresponding vector of structural mode shape coefficients from which the relative levels of vibration kinetic energy were obtained for the structure and the fluid. As was seen from Tables 2 and 3, a significant difference occurs between the natural frequencies of the coupled system, where there is significant relative vibration energy associated with the structure and the natural frequencies of the structure in the absence of fluid interaction. Fig. 3 shows a plot of the two values of KE_{p1}, around the first natural frequency of the beam in the absence of fluid interaction, against η and superimposed is the value of $\xi_1 = 1.8751$.

In such modes, it was observed that the vector of mode shape coefficients, $\boldsymbol{\chi}$, was well defined with a single unity at the structural mode in question and zeros for all other components. This, as will be seen, is very important with respect to solving the *inverse problem*.

In structural modal analysis and vibration-based damage detection it is the values of the structural natural frequencies alone, ξ_s, which are of importance. Accordingly, in the case where the structure is in interaction with a fluid cavity (as is the case here), accepting the coupled natural frequencies obtained from experiments upon the coupled system to being sufficiently accurate estimates to the structural natural frequencies, could have grave consequences and give erroneous information. Accordingly, the inverse problem in this case is defined as extracting the structural natural frequencies, ξ_s, from the values of the coupled natural frequencies, η, obtained from experiments and known parameters of the fluid cavity in interaction.

To achieve this, Eq. (18) can be written in the form

$$\mathbf{A}(\xi_s, \eta)\boldsymbol{\chi} = [\mathbf{K\Omega} - \mathbf{\Theta K}]\boldsymbol{\chi} = 0, \tag{23}$$

where

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12}\cdots & k_{1n} \\ k_{21} & k_{22}\cdots & k_{2n} \\ \vdots & \vdots & \vdots \\ \cdots & k_{qs} & \cdots \\ \vdots & \vdots & \vdots \\ k_{n1} & k_{n2}\cdots & k_{nn} \end{bmatrix}, \tag{24}$$

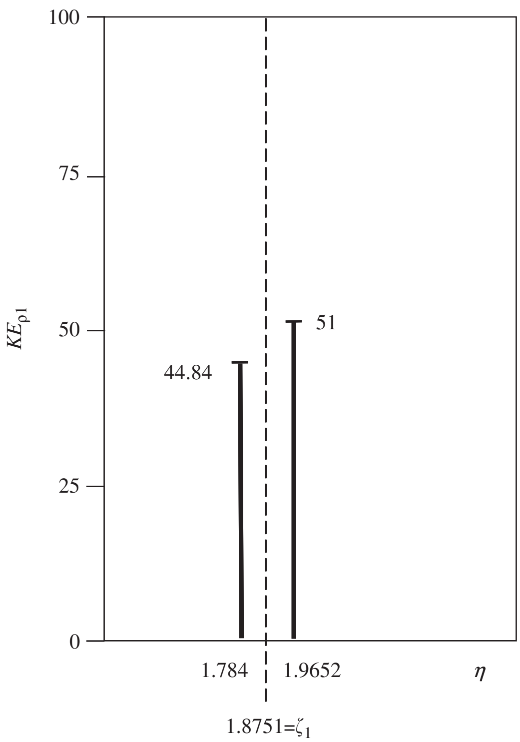

Fig. 3. Plot of KE_{p1} against η showing the position of ξ_1.

where $k_{qs} = \int_0^1 \psi_s \bar{\phi}_{xq}\, d\bar{x}$ as before, Eq. (13).

$$\mathbf{\Theta} = \eta^4 \begin{bmatrix} \theta_1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \theta_2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \theta_{q=s} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \theta_n \end{bmatrix} \quad \text{where } \theta_i = \left[1 - \frac{\rho}{(\alpha_i \tan \alpha_i)}\right]$$

and

$$\mathbf{\Omega} = \begin{bmatrix} \xi_1^4 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \xi_2^4 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \xi_s^4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \xi_n^4 \end{bmatrix}$$

is a diagonal matrix containing values of the non-dimensional natural frequencies of the structure alone. Now introducing the matrix **B** as

$$\mathbf{B} = \mathbf{K}^{-1}\mathbf{\Theta}\mathbf{K}.$$

Eqs. (18) and (23) can be written in the form

$$[\mathbf{B} - \mathbf{\Omega}]\boldsymbol{\chi} = 0. \tag{25}$$

Note that the matrix **K** contains the values of ξ_s from Eqs. (4) and (13). However, it will be assumed that the influence of small changes of ξ_s (due to the effect of damage to the structure) on the normal eigenfunctions, Eq. (4), and hence k_{qs} from Eq. (13) are negligible and the same form of the **K** matrix, based upon Eqs. (4) and (13), is used irrespective of any changes to ξ_s. This assumption will be examined further at a later stage in this paper. Therefore, the matrix **B** above is assumed to contain only values of the coupled natural frequencies of the system, η, and pre-known dimensional and physical parameters of the beam and fluid cavity. On the other hand, the matrix **Ω** contains only the unknown values of the non-dimensional natural frequencies of the structure in the absence of fluid interaction, which we aim to determine. Accordingly, Eq. (18) can now be expressed in the following form:

$$\begin{bmatrix} (b_{11} - \xi_1^4) & b_{12} & \cdots & b_{1s} & \cdots & b_{1n} \\ b_{21} & (b_{22} - \xi_2^4) & \cdots & b_{2s} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{s1} & b_{s2} & \cdots & (b_{ss} - \xi_s^4) & \cdots & b_{sn} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} & \cdots & (b_{nn} - \xi_n^4) \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_s \\ \vdots \\ \chi_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{Bmatrix}. \tag{26}$$

Now recalling that at coupled natural frequencies which are characterised by significant vibration energy of the structure around natural frequencies which are close to those of the structure in the absence of fluid interaction, the vector of mode shape coefficients, χ, is shown to be well defined with a single unity at the structural mode in question and zeros for all other components to within 3 decimal places. Upon that basis, and from matrix Eq. (26) above:

$$\xi_s = \sqrt[4]{b_{ss}}. \tag{27}$$

To test the feasibility and accuracy of the inverse methodology presented above, Eqs. (17)–(19) are used to compute values of natural frequencies, η, of a coupled beam/fluid interacting system with relevant input values of natural frequencies in the absence of fluid interaction, ξ_s, of the cantilever beam (from Table 1), the standard eigenfunctions relating to the beam in the absence of fluid interaction and the completely rigidly enclosed fluid cavity, and other pre-known parameters of the fluid cavity contained in the matrix **A**. The computed values of η are then used to generate the respective values of $\%KE_{ps}$. For the modes, which have the largest value of KE_{ps} for each value of s, the respective value of η is then used to determine the respective **B** matrix, Eqs. (25) and (26), along with the standard eigenfunctions relating to the beam in the absence of fluid interaction and the completely rigidly enclosed fluid cavity, and other pre-known parameters of the fluid cavity as before. Subsequently for the respective value of s, the natural frequency in the absence of fluid interaction, ξ_s, is then calculated from Eq. (27) and compared to that value used in Eqs. (17)–(19). Two examples will be considered. The first example will apply the methodology to the case where the beam is undamaged and therefore the values of ξ_s are those listed in Table 1. The second example will consider the case where damage to the beam is simulated by assuming that the values of ξ_s are all reduced by 10%.

3.1. Beam with no defect

The above inverse methodology will first be tested by considering the coupled modes labeled $^+$ in Table 4. In these cases, it can be seen from Table 4 that these modes represent those in which there is the highest proportion of structural vibration energy for each of the first 8 normal modes of the structure. Therefore, for these modes, Table 5 tests the above inverse methodology by computing the first eight natural frequencies of the beam in the absence of fluid interaction from Eq. (26) and comparing these values to those presented in Table 1 for the beam having no defects and used in Eqs. (17)–(19) for the purpose of computing the respective value of η.

Table 5 demonstrates the accuracy of the above inverse methodology to obtaining the structural natural frequencies from values of natural frequencies of the coupled system together with pre-determined physical parameters of the structure and the fluid cavity.

3.2. Beam with defect

This case will test the inverse methodology by considering the case where the beam has some defect(s) such that the natural frequencies of it in the absence of fluid interaction natural frequencies, ξ_s, are all reduced by 10%. In this case, once again, it will be assumed that such defect(s) do not incur significant change to the contents of the matrix **K** of Eq. (24). Therefore, the same form of the eigenfunction, Eq. (4), is used with the original values of ξ_s to generate the values of k_{ps} contained in Eq. (19). However, values of ξ_s reduced by 10% are used in the section $\{\xi_s^4 - \eta^4[1 - (\rho/(\alpha_q \tan \alpha_q))]\}$ of Eq. (19) and the inverse process described in Section 3.1

Table 5
Comparison between exact values of ξ_s and those values obtained from Eq. (25) when the exact values of coupled natural frequency, η, are introduced in Eq. (25)

s	KE_{ps}	η	$\xi_s = \sqrt[4]{b_{ss}}$	ξ_s (Table 1)
1	51	1.9652	1.8750	1.8751
2	94.815	4.6945	4.6941	4.6940
3	98.403	7.8489	7.8548	7.8548
4	97.566	10.9581	10.9955	10.9955
5	55.233	14.1298	14.1372	14.1372
6	76.45	17.2751	17.2789	17.2789
7	74.98	20.4135	20.4204	20.4203
8	73.56	23.5546	23.5620	23.5620

is repeated. In addition, in order to arrange that there will exist strong coupling between the first modes of the beam and fluid cavity as with the examples so far, for this case the non-dimensionalised depth of the fluid cavity is, from Eq. (21), $\bar{l}_c = 25.26$. Table 6 lists the results for this test.

From Table 6, it is seen that the method performs extremely well for the case where the beam has defect(s) giving rise to reductions in natural frequency from the undamaged beam values, and the matrices **K** and **B** described by Eqs. (24) and (25), respectively, are generated using the eigenfunctions of Eq. (4) for the beam in the undamaged state.

3.3. Sensitivity to errors in values of η—robustness of the methodology

In practice it is not possible to measure values of natural frequency to exacting values. Therefore the question arises as to what level of error will result in the values of ξ_s if the error in η is varied between say ±0.5%. Accordingly Table 7 lists the values of ξ_s obtained when η is varied between ±0.5% from the values listed and used in Table 5. In Table 7, the symbol $\xi_s{}^+$ is used to denote the value of ξ_s obtained when η is increased by 0.5% and the symbol $\xi_s{}^-$ to denote the value of ξ_s obtained when η is reduced by 0.5% from the value presented in Table 5.

Tables 7 demonstrates the sensitivity of the extracted values of the natural frequencies of the beam in the absence of fluid interaction to errors in the coupled natural frequencies being used in the construction of the **B** matrix described in Eq. (25). Note that, in general, the deviation of the estimated values of ξ_s (from Eq. (27)) from these exact values from Table 1 is for the most part within the same percentage deviation of the values of coupled natural frequencies used to construct the **B** matrix, namely, ±0.5%. The exception however is for the cases of the first and fifth natural frequencies of the beam; s equal to 1 and 5. Upon reflection it is noted from Table 5, that the first and fifth modes of the beam are the ones in strongest interaction with the fluid, demonstrated by having values of KE_{ps} equal to 51% and 55.233%, respectively.

Table 6
Comparison for a beam with defect reducing all natural frequencies by 10%

S	KE_{ps}	η	$\xi_s = \sqrt[4]{b_{ss}}$	$0.9\xi_s$ (Table 1)
1	51.03	1.7773	1.6874	1.6876
2	93.796	4.2259	4.2247	4.2247
3	98.367	7.0650	7.0693	7.0693
4	95.545	9.8932	9.8960	9.8960
5	50.08	12.7181	12.7235	12.7236
6	76.498	15.5493	15.5509	15.5509
7	48.427	18.3734	18.3783	18.3783
8	43.960	21.2035	21.2058	21.2056

Table 7
Sensitivity of ξ_s to a ±0.5% change in η

S	ξ_s (Table 1)	$\xi_s{}^+$	$\xi_s{}^-$
1	1.8751	1.8951 (1%)	1.8511 (−1.2%)
2	4.6940	4.7057 (0.25%)	4.6685 (−0.5%)
3	7.8548	7.8942 (0.5%)	7.8144 (−0.5%)
4	10.9955	11.0375 (0.4%)	10.9505 (−0.4%)
5	14.1372	14.2553 (0.8%)	14.0637 (−0.5%)
6	17.2789	17.3645 (0.5%)	17.1773 (−0.6%)
7	20.4203	20.5193 (0.5%)	20.3188 (−0.5%)
8	23.5620	23.6617 (0.4%)	23.4413 (−0.5%)

Figures in brackets are the % differences.

4. Conclusions

A method has been developed from which estimates of natural frequencies of a cantilever beam in the absence of fluid interaction can be computed from values of natural frequency when the beam is part of a structural/fluid vibrating interacting system of which details of the fluid sub-system are known. These estimates extracted were found to be in extremely close agreement with the exact theoretical values and also found to be stable when the values of the coupled natural frequencies used were in slight error. By being able to extract these natural frequencies of the structure in the absence of fluid interaction, the practice of VHM can be used for beam like structures when it is found that the structure is part of a coupled structural/fluid interacting system whose natural frequencies are removed for those of the beam alone. This is an important step towards the development of practically applied VHM.

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