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Short Communication

Uncertainty quantification using interval modeling with performance sensitivity

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Abstract

In this paper an interval modeling approach for uncertainty quantification of a structure with significant parameter variation is presented. Model uncertainty can be categorized as dominant uncertainty due to structural variation, such as joint uncertainty and temperature change, and minor uncertainty associated with other factors. In this paper, a singular value decomposition (SVD) technique is used to decompose parameter variations into principal components that are weighted based on the sensitivity of the performance metric to parameter variations. From this process, parameter bounds in the form of an interval model are generated and each interval corresponds to one identified bounded uncertainty parameter with its associated principal direction. The proposed approach can be used to differentiate between dominant and minor uncertainties. A beam structure with an attached subsystem proposed by Sandia National Laboratories is used to demonstrate this approach.

1. Introduction

Model validation and uncertainty quantification of structural dynamic problems are of great interest to both government and industry [1–3]. Structural uncertainty and the variability associated with physical parameters and environmental changes raise issues concerning reliability, safety, and performance. Thus, uncertainty quantification and model validation of structural dynamic problems play a key role in addressing these issues.

Recently, a model validation workshop [4] was organized by Sandia National Laboratories to address the problem of certification of structures under various forms of uncertainty. In this paper, an interval modeling technique for uncertainty quantification of the structural dynamics problem proposed by Sandia [5] is described. Following their formulation, an integrated system, consisting of a beam structure and an attached subsystem, shown in Fig. 1, is the test structure used for study. In this model the physical elements of the attached three degrees of freedom subsystem, shown in Fig. 2, are the only ones exhibiting significant parameter variations, while all other parameters are known.

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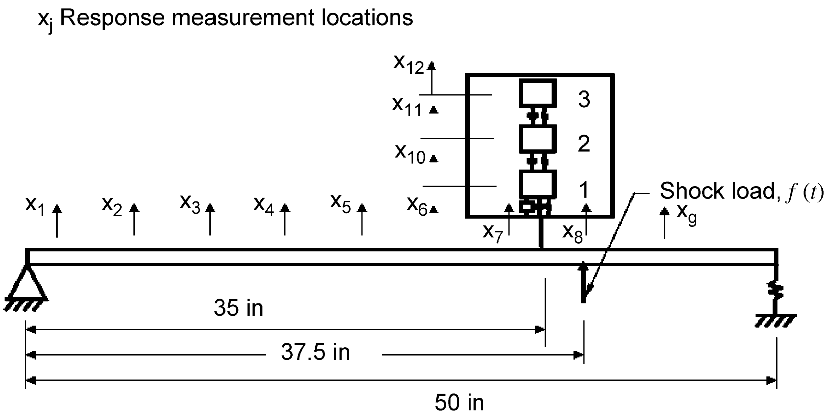

Fig. 1. A beam structure with an attached subsystem.

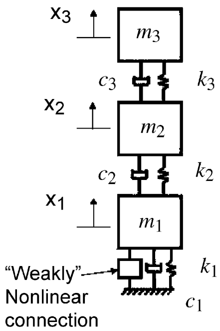

Fig. 2. Three degrees-of-freedom subsystem.

In the process of certifying structures for use in harsh dynamic environments it is often required that not only the main structure be capable of withstanding the loads but also all the attached substructures. To ensure survivability of all the substructures, Sandia in Ref. [5] has chosen a performance metric in terms of the maximum acceleration magnitude of mass 3, top of the substructure, under a shock force at position x_8, with the shock input profile prescribed. In this paper, a singular value decomposition (SVD) technique is applied to extract the principal components of parameter variations. For this study, the uncertain parameters are the identified modal parameters (frequency, damping, and mode shape), 15 parameters in total. To incorporate the performance metric in the SVD solution, the sensitivity of the performance metric to each modal parameter is computed, and used as a weighting factor in the SVD process. From this process, an interval model is generated with its associated principal direction [6,7]. It is this identified interval model that is used to model all parameter variations. The approach is presented next.

2. Model uncertainty quantification

In this section, an uncertainty quantification approach is presented using the example proposed by Sandia [5]. The data that are used for model uncertainty quantification are based on the identified modal parameters from 60 virtual experiments [5], generated from 20 identical systems selected from a virtual pool and three levels of random excitation applied at mass 2. These identified modal parameters were provided by Sandia [5] for uncertainty quantification. The modal parameter vector of the subsystem is defined as

$$p = \left[\omega_1, \omega_2, \omega_3, \xi_1, \xi_2, \xi_3, \phi_{11}, \phi_{21}, \phi_{31}, \phi_{12}, \ldots, \phi_{33}\right]^{\mathrm{T}}, \tag{1}$$

where ω_i is the ith natural frequency, ξ_i the ith damping ratio, and ϕ_{ji} the jth component of the ith mode shape. To quantify the parameter uncertainty, the deviation of the parameter vector from the nominal vector for the

jth test is defined as

$$\Delta p_j = p_j - p_0, \quad j = 1, \ldots, n, \quad p_0 = \frac{1}{n}\sum_{j=1}^{n} p_j, \tag{2}$$

where p_j is the jth identified parameter vector, and p_0 the nominal parameter vector which is computed as the average from $n = 60$ experiments. Using the changes from the nominal values an uncertainty matrix is defined as

$$\Delta P = [\Delta p_1, \Delta p_2, \ldots, \Delta p_n]. \tag{3}$$

To incorporate the effects of the performance metric, the sensitivity of the performance metric is used as a weighting factor. For example, in the Sandia problem, the performance metric is the maximum acceleration magnitude of mass 3 due to a shock input at position x_8. The sensitivity of the performance index to the jth component of the ith chosen subsystem p^i is defined as

$$s_{ij} = \frac{1}{a(p^i)}\left|\frac{\partial a(p^i)}{\partial p^{ij}}\right|\sigma_j, \quad i = 1, \ldots, n_s, \tag{4}$$

where $a(p^i)$ is the maximum acceleration magnitude with subsystem parameter vector p^i, p^{ij} is the jth component of parameter vector p^i, σ_j is the standard deviation of the jth vector component, and n_s is the number of parameter vectors. This sensitivity represents a percentage change weighted by σ_j to account for the size of the parameter variation. The average sensitivity corresponding to the jth parameter vector component is defined as

$$s_j = \frac{1}{n_s}\sum_{i=1}^{n_s} |s_{ij}|. \tag{5}$$

To weight the degree of variation in each parameter, an initial weighting matrix is computed as

$$\Delta P^1 = W_1^{-1}\Delta P, \tag{6}$$

where W_1 is a diagonal matrix with its jth diagonal element as the standard deviation σ_j. The weighting matrix including sensitivity is computed as

$$\Delta P^W = W_2 \Delta P^1, \tag{7}$$

where W_2 is a diagonal matrix with its jth diagonal element s_j. In this paper, a SVD technique [6] is used to generate an optimal linear interval model that represents the parameter vector with uncertainty, as

$$P = \left\{ p | p = p_0 + \sum_{j=1}^{15} \alpha_j q_j, \quad \alpha_j \in \left[\alpha_j^-, \alpha_j^+\right] \right\}, \tag{8}$$

where α_j are the identified uncertainty parameters corresponding to the basis vectors q_j.

In summary, this SVD process involves the following computational steps:

(1) Use SVD to compute the basis matrix U^W for ΔP^W:

$$\Delta P^W = U^W S V^T, \quad S = \text{diag}[d_1, \ldots, d_{15}]. \tag{9}$$

(2) Compute the basis matrix U for ΔP:

$$U = W_1 W_2^{-1} U^W, \quad U = [q_1, \ldots, q_{15}]. \tag{10}$$

Since the singular values d_j are in descending order, this leads to a descending order of perturbation distribution in q_j.

(3) Compute the coordinate vector of Δp_i corresponding to the basis vectors q_j:

$$\beta_i = U^{-1} \Delta p_i. \tag{11}$$

(4) Represent each parameter vector as

$$p_i = p_0 + \sum_{l=1}^{15} \beta_i(l) q_l, \quad (12)$$

where $\beta_i(l)$ is the lth element of the coordinate vector β_i.

(5) Compute the parameter bounds as

$$\alpha_j^+ = \max\{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\}, \quad (13)$$

$$\alpha_j^- = \min\{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\}. \quad (14)$$

All the basis vectors, coordinates, and parameter bounds are normalized to the first interval length [7].

3. Discussion of results

Following the approach just discussed, results for the Sandia Challenge problem are discussed next. To begin the weighting factors from the performance sensitivity of maximum acceleration to modal parameters are computed for masses 1, 2, and 3 and then the sensitivity is averaged to get a single value. These sensitivities are used in Eq. (7) along with the 60 identified parameter vectors to compute the interval model. Fig. 3 shows the normalized interval lengths for each α_i; normalization is with respect to the first interval, i.e. $\alpha_1 = 1$. After examining Fig. 3, note that the second interval length drops to 20% of the first interval length, and the fifth interval length is around 3% of the first one. Consequently, the second and third intervals are significantly less important when used to describe parameter variation. The model uncertainty is dominated by the first uncertainty parameter α_1. Note that in this approach instead of studying the uncertainty in parameter space, one looks at the uncertainty in terms of the coordinates of the interval space α_i.

Fig. 4 shows three modal parameters of the 60 virtual experiments as functions of the first uncertainty parameter α_1. Variations in the natural frequencies are significant, around 100%, and increase linearly as the first uncertainty parameter α_1 increases. Also, natural frequency variations are the dominant uncertainty corresponding to variations in α_1. In contrast to frequency variations, damping and mode shape variations behave more like random variables, and they correspond to secondary uncertainties. Fig. 5 shows the natural frequencies of the second and third modes of the integrated system for the 60 calibration systems and the

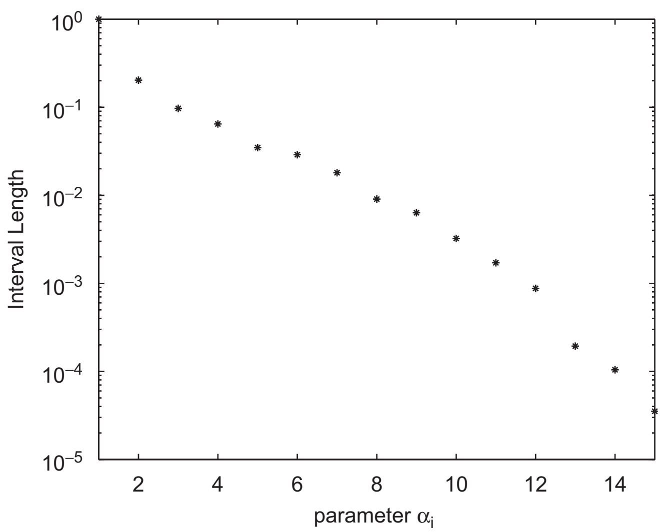

Fig. 3. Interval length of identified uncertainty parameters α_i.

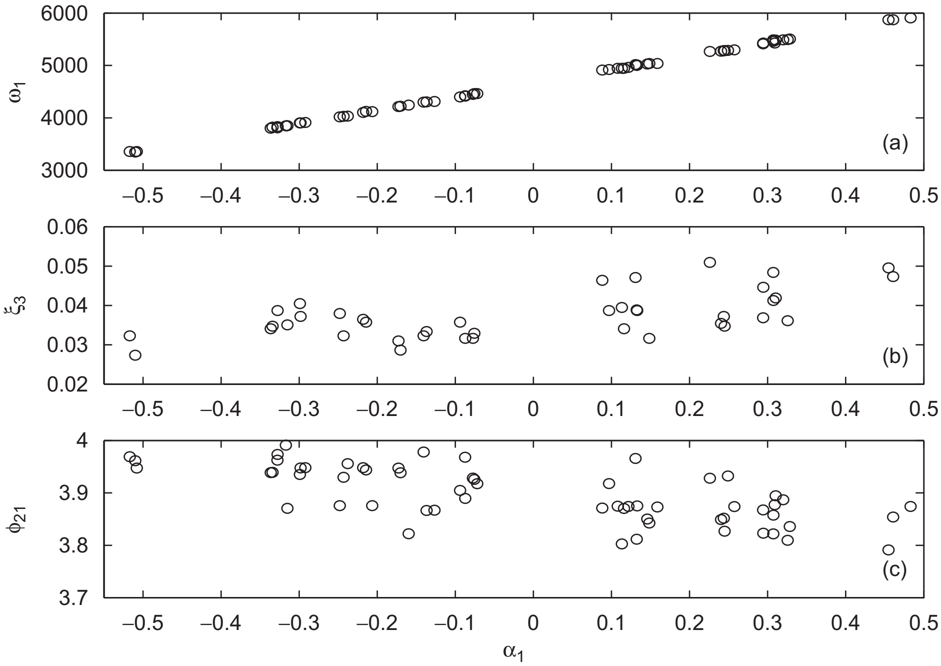

Fig. 4. Modal parameters of 60 subsystems: (a) natural frequency of 1st mode, (b) damping ratio of 3rd mode, and (c) 2nd mode shape coefficient of 1st mode.

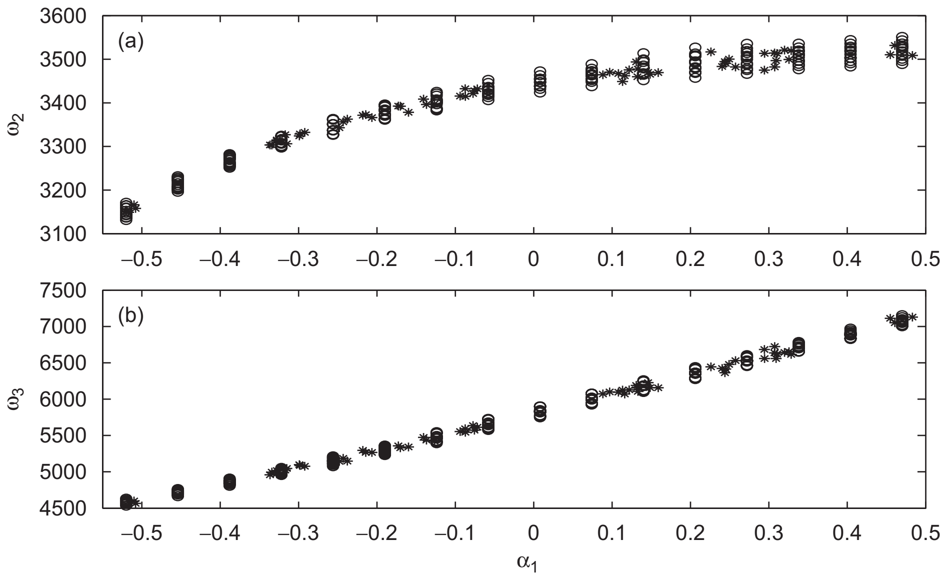

Fig. 5. Natural frequencies of integrated system: (a) second mode and (b) third mode. o is the reduced 4-intervals model and * is the raw data for 60 calibration systems.

reduced 4-intervals model shown in with circles. The identified interval model precisely represents and covers the original systems.

Figs. 6 and 7 show the identified interval lengths and the natural frequencies of the second and third modes of the integrated system when the weighting W_2 for performance sensitivity is not included in SVD process. The sixth interval length is still around 30% of the first interval length. However, the dominant uncertainty

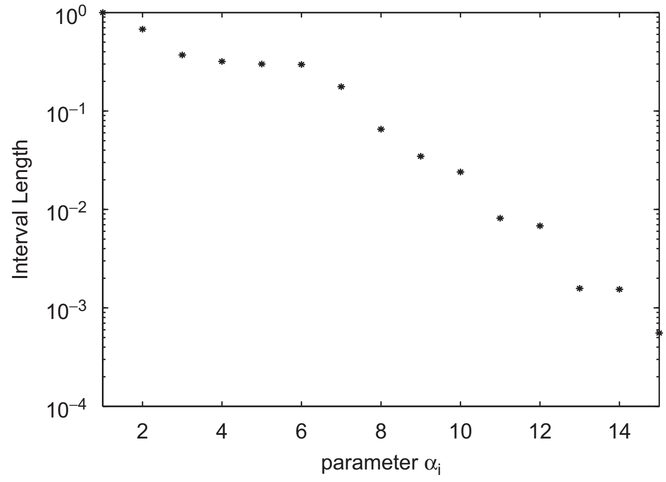

Fig. 6. Interval length of identified uncertainty parameters α_i.

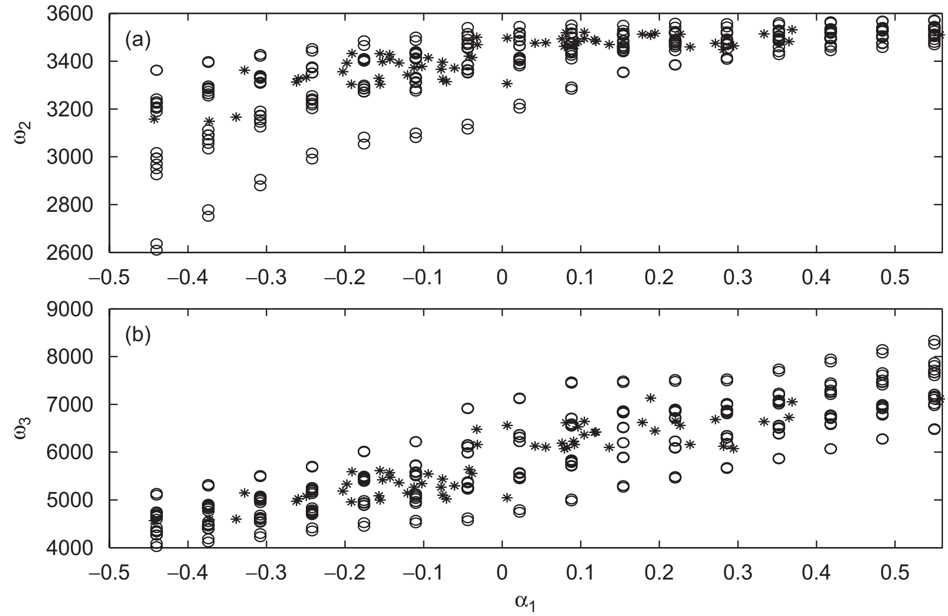

Fig. 7. Natural frequencies of integrated system: (a) second mode and (b) third mode. o is the reduced 5-intervals model and * is the raw data for 60 calibration systems.

corresponding to natural frequency variation cannot be distinguished from the secondary random uncertainty of damping ratios and mode shape coefficients. Compared with the preceding identified interval model, this identified interval model is too conservative to represent the original system.

Fig. 8 shows the maximum acceleration of the integrated system of the first interval model with sensitivity weighting when an impulse input is applied at position x_8. The identified interval model precisely represents the original systems.

4. Concluding remarks

A novel approach for uncertainty quantification of a system with a subsystem attached that exhibits significant parameter uncertainty was presented. The inclusion of the performance sensitivity weighting in

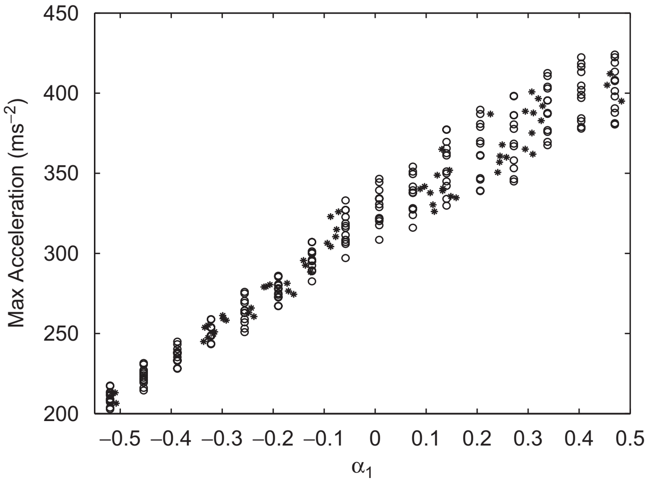

Fig. 8. Maximum acceleration of integrated system with impulse force. o is the reduced 5-intervals model and * is the raw data for 60 calibration systems.

singular value decomposition process can identify the dominant uncertainty from a secondary uncertainty. In addition, the identified interval model precisely represents the observed parameter variations with dominant systematic model uncertainty and secondary uncertainty. The results showed that the identified interval lengths correspond to the uncertainty associated with the direction of the identified basis vectors, and they can be used as an indicator to reduce the number of uncertain parameters given a performance metric.

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