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Thermal vibrational analysis for simply supported beam and clamped beam

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Abstract

The presented paper shows how the temperature field in beams has impact on vibrations of beams. Due to the relatively high possible temperature variations in beams we have developed the mathematical model where fundamental thermomechanical properties of state are functions of temperature such as modulus of elasticity, Poisson number, linear expansion coefficient, shear modulus and some other important thermodynamic properties of state. The detailed analysis shows, that also small changes of temperature cause significant changes of natural frequencies for beams. The comparison between our analytical model and experimental data shows good agreement. In the presented paper it is possible to find for the first time in scientific literature the application of the combination of statistical thermodynamics and the theory of vibration at the same time.

1. Introduction

Mechanics is a very important scientific discipline, developing at an ever increasing pace. Nonlinear theory of vibrations represents one of the most important areas [1] in modern science. From theoretical viewpoint, nonlinear mechanics constitutes an extremely complex mathematical problem. Particularly complex are the models of nonlinear vibrations in the area of chaos and bifurcations. In terms of the engineering approach, the theory of vibrations means a basic method in technical systems design. In the majority of cases, the impacts of temperature changes are ignored, yet many times they need to be taken into consideration. In such case the analyses of the above indicated problems become very complex.

Mechanical machines very often operate under diverse temperature conditions. In internal combustion engines, rocket systems, movement of the satellites, etc. the conditions are particularly temperature-sensitive. Thermodynamic effects are frequently ignored in research, which may yield totally incorrect results. Literature [2–6] shows that even the slightest temperature change leads to huge alteration of the clamped beam vibration properties. Contrary as in papers [1,2], the impact in the present paper is not neglected of a change in thermodynamic properties, which have to be taken into consideration at major temperature changes.

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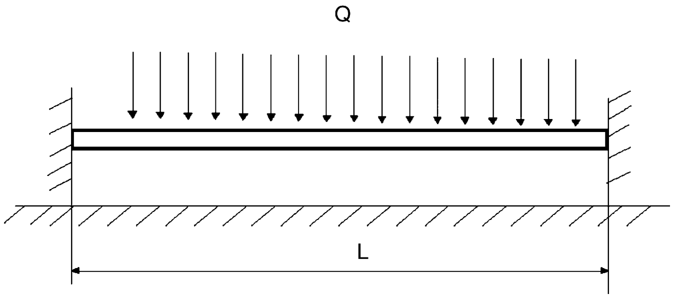

Fig. 1. Isotropic clamped beam.

In this article we have developed a dynamic thermo-vibrational model for the clamped (Fig. 1) and the simply supported isotropic beam (Fig. 2). Let us assume that the beam is homogenous, having the same temperature over its entire length. As a result of thermal expansion, an additional axial force F_T occurs:

$$F_T = \alpha\theta EA. \tag{1}$$

In Eq. (1) α is the linear thermal extension coefficient, θ is the temperature difference between the actual and initial or reference temperature. The equation by means of which we can resolve the problem using the axial force is as follows according to Timoshenko [4]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + F_T\frac{\partial^2 w(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0, \tag{2}$$

where E means Young modulus, I is the area moment of inertia, A the beam cross-section area, ρ the density of material, t the time and w the displacement. Using new variables, Eq. (2) can be written down in a slightly less complicated way as

$$w''''(x) + 2\gamma w''(x) - \beta^4 w(x) = 0. \tag{3}$$

In Eq. (3), the new symbols represent the following functional relations: $\beta^2 = \omega/c$, $c^2 = EI/\rho A$, $\gamma = F_T/2EI$.

Thus, a general solution to Eq. (3) is ($\lambda = \sqrt{\beta^4 + \gamma^2}$) [1–4]

$$\begin{aligned} w(x) = {} & C_1 \cos\left(\sqrt{\lambda + \gamma}x\right) + C_2 \cosh\left(\sqrt{\lambda - \gamma}x\right) \\ & + C_3 \sin\left(\sqrt{\lambda + \gamma}x\right) + C_4 \sinh\left(\sqrt{\lambda - \gamma}x\right). \end{aligned} \tag{4}$$

Using boundary conditions, the following solutions can be analytically computed ($\Gamma = L^2\gamma$, $\quad \Lambda = L^2\lambda$):

(a) Clamped beam:

$$\sqrt{\Lambda^2 - \Gamma^2}\{\cos(\Lambda + \Gamma)\cosh(\Lambda - \Gamma) - 1\} + \Gamma \sin(\Lambda + \Gamma)\sinh(\Lambda - \Gamma) = 0. \tag{5}$$

(b) Simply supported beam:

$$\sin(\Lambda + \Gamma) = 0. \tag{6}$$

The model presented here is fully analytical, but if we compare analytical results with the measured results it points to a large deviation from reality [1,2]. The biggest problem of this model is that in the mathematical model in question the clamped wall can fully withstand the beam for the beam to have a constant length all the time. The above assumption is not realistic. As a result, a new model was designed to reduce to at least to some extent the huge differences between the analytical results and the measured values.

2. The dynamic model for beams under thermal stresses

Figs. 3 and 4 illustrate a new rheological model for the clamped and the simply supported beam. To obtain better analytical results in comparison with experimental data, a spring is added with the spring constant K.

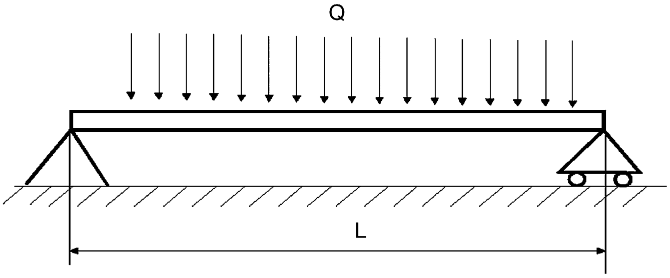

Fig. 2. Simply supported beam.

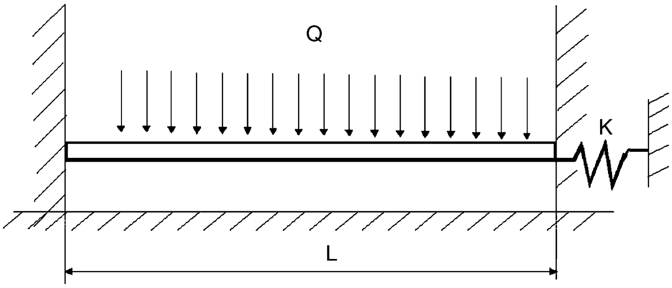

Fig. 3. New model of clamped beam.

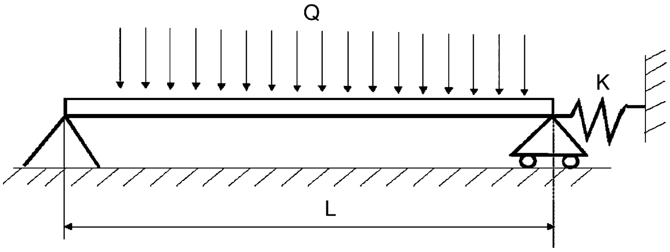

Fig. 4. New model of simply supported beam.

The model slightly differs from the model presented in paper [1], where the authors Marques and Inman integrated additional torsion springs into the rheological model.

In this case, force F_T can be computed in the following way [1]:

$$\varepsilon EA = \alpha\theta EA - K\delta, \quad K\delta = EA(\alpha\theta - \delta/L), \quad \delta = \frac{EAL\alpha\theta}{KL + EA}. \tag{7}$$

The reaction force computation can be as follows:

$$F_T = K\delta = \frac{EAL\alpha\theta}{L + EA/K}, \tag{8}$$

where the modulus of elasticity E and linear expansion coefficient α are temperature-dependent functions. An aluminium beam and copper beam with the dimensions indicated in Tables 1 and 2 were used for the computation.

3. Statistical thermomechanics

The need for mathematical modeling of thermomechanical properties of state arises in various fields [7–24]. Currently, a growing emphasis is placed on new materials, such as different alloys, polymers, plastic material,

Table 1
Fundamental constants for aluminum beam

	Beam
Length (m)	6.35×10^{-2}
Width (m)	2.04×10^{-2}
Thickness (m)	1.62×10^{-3}
Young modulus (N/m^2)	6.9×10^{10}
Volume expansion coefficient (1/K)	$24\times10^{-6}\,\mathrm{K}^{-1}$
Spring constant (N/m)	1.553×10^{5}
Density (kg/m^3)	2780

Table 2
Fundamental constants for copper beam

	Beam
Length (m)	6.35×10^{-2}
Width (m)	2.04×10^{-2}
Thickness (m)	1.62×10^{-3}
Spring constant (N/m)	1.553×10^{5}

ceramics [12,19]. Due to a large number of possible combinations of various components mathematical models are frequently used to predict thermodynamic properties. Mathematical modeling is often used also in some metallurgical processes, such as sintering, corrosion, welding, etc. Furthermore, analytical computation of thermodynamic properties of state in solids is of paramount importance also in a number of other fields such as planetary physics, for example. Another important area is the production of liquid–solid [20], solid–gaseous phase diagrams. At higher pressure and temperature bands, such measurements may be very costly, which is why mathematical models are often used instead.

Assume that each form of motion of energy is independent of the others; thus, the energy of the system of molecules can be written as a sum of individual contributions or decoupled forms of motion [9]:

(a) Vibration energy of molecules (E_{vib}) due to the relative motion of atoms inside the molecules.
(b) Potential energy (E_{pot}) of a system of molecules, which occurs due to the attractive or repulsive intermolecular forces in a system of molecules.
(c) Energy of electrons (E_{el}), which is concentrated in the electrons or in the electron shell of an atom or a molecule
(d) Nuclear energy (E_{nuc}), which is concentrated in the atom nuclei.

Now introduce the term of partition function Z [21], which applies to the system of particles at a certain volume V, temperature T and particle number N. Assuming that the energy spectrum is continuous and having the above assumptions we can then write the canonical partition function for the one-component system in an even easier manner [9]:

$$Z = \frac{1}{N!h^{Nf}} \int \cdots \int \exp\left(-\frac{E_{\text{vib}} + E_{el} + E_{\text{nuc}}}{K_{\text{B}}T}\right) \cdot \mathrm{d}\mathbf{p}_1 \cdot \mathrm{d}\mathbf{p}_2 \ldots \cdot \mathrm{d}\mathbf{p}_N \times \int \cdots \int \exp\left(-\frac{E_{\text{pot}}}{k_B T}\right) \mathrm{d}\mathbf{r}_1 \cdot \mathrm{d}\mathbf{r}_2 \ldots \mathrm{d}\mathbf{r}_N \tag{9}$$

The second term on the right-hand side in Eq. (9) is called the configurational integral, f is the number of degrees of freedom of an individual molecule, p is momentum, r is coordinate, E_{vib}, E_{el}, E_{nuc}, E_{pot} represents the vibrational energy, electron energy, nuclear energy of individual molecule and potential energy between two molecules.

Similarly, we can express also the partition function Z for a multi-component system of indistinguishable molecules:

$$Z = \frac{1}{\prod_i N! h^{N_i f_i}} \int .. \int \exp\left(-\frac{E_{\text{vib}} + E_{el} + E_{\text{nuc}}}{k_B T}\right) \mathrm{d}\mathbf{p}_1 \cdot \mathrm{d}\mathbf{p}_2 \ldots \mathrm{d}\mathbf{p}_N \\ \times \int .. \int \exp\left(-\frac{E_{\text{pot}}}{k_{\text{B}} T}\right) \cdot \mathrm{d}\mathbf{r}_1 \cdot \mathrm{d}\mathbf{r}_2 \ldots \mathrm{d}\mathbf{r}_N. \tag{10}$$

In Eq. (2) N_i is the number of molecules of the ith component, f_i is the number of degrees of freedom of the ith molecule. On the basis of all indicated generalizations we can write, using the canonical partition, the partition function Z of the one-component system as a product of partition functions:

$$Z = Z_0 Z_{\text{vib}} Z_{el} Z_{\text{nuc}} Z_{\text{conf}}. \tag{11}$$

For a system of many components the partition function Z [16,18] can be written as a product of partition functions for individual terms:

$$Z = \prod_i (Z_0 Z_{\text{vib}} Z_{el} Z_{\text{nuc}})_i Z_{\text{conf}} = \prod_i Z_i Z_{\text{conf}}. \tag{12}$$

3.1. Vibration properties of solids

Our thermodynamic system consists of N particles associated by attractive forces. Atoms in a crystal lattice are not motionless but they constantly thermally oscillate around their positions of equilibrium. At temperatures far below the melting point the motion of atoms is approximately harmonic [7,21,22]. This assembly of atoms has $3N-6$ vibration degrees of freedom. Ignore 6 vibration degrees of freedom and mark the number of vibration degrees of freedom with $3N$.

Through the knowledge of independent harmonic oscillators the distribution function Z [11] can be derived as follows:

$$Z = \left[\frac{\exp(-hv/2k_B T)}{1 - \exp(-hv/k_B T)}\right]^{3N}. \tag{13}$$

In Eq. (5) v is the oscillation frequency of the crystal. The term hv/k is the Einstein temperature θ_E.

In comparing the experimental data for simple crystals a relatively good matching with analytical calculations at higher temperatures is observed whereas at lower temperatures the discrepancies are higher. This is why Debye corrected the Einstein's model by taking account of the interactions between a number of quantized oscillators. The Debye approximation treats a solid as an isotropic elastic substance. Using the canonical distribution the partition function may be written as:

$$\ln Z = -\frac{9}{8} N \frac{\theta_D}{T} - 3N \ln\bigl(1 - \exp(-\theta_D/T)\bigr) + 3N \frac{T^3}{\theta_D^3} \int_0^{\theta_D/T} \frac{\xi^3}{\exp(\xi) - 1}\, \mathrm{d}\xi. \tag{14}$$

In Eq. (14) θ_D is the Debye temperature. By developing the third term in Eq. (14) into a series for a higher temperature range [11,23] we can write

$$\frac{\xi^3}{\exp(\xi) - 1} = \xi^2 - \frac{1}{2}\xi^3 + \frac{1}{12}\xi^4 - \frac{1}{720}\xi^6 + \cdots . \tag{15}$$

Using Eqs. (14) and (15) turns into the following expression:

$$\ln Z = -\frac{9}{8}N\frac{\theta_D}{T} - 3N\ln\left(1-\exp\left(-\frac{\theta_D}{T}\right)\right) + 3N\left(\frac{T}{\theta_D}\right)^3\left[\begin{array}{c}\frac{1}{3}\left(\frac{\theta_D}{T}\right)^3 - \frac{1}{8}\left(\frac{\theta_D}{T}\right)^4 + \frac{1}{60}\left(\frac{\theta_D}{T}\right)^5 \\ -\frac{1}{5040}\left(\frac{\theta_D}{T}\right)^7 + \frac{1}{272160}\left(\frac{\theta_D}{T}\right)^9 - ..\end{array}\right]. \tag{16}$$

The relation between the Einstein and Debye temperature may be written as $\theta_E = 0.7355\theta_D$.

The Debye characteristic temperature was determined by means of the Grüneisen independent constant γ

$$\theta_D = CV^{-\gamma}, \tag{17}$$

where C is the constant dependent on material. We developed a mathematical model for the calculation of thermodynamic properties of polyatomic crystals. The derivations of the Einstein and Debye equations, outlined in the previous paragraphs, apply specifically to monoatomic solids, i.e. those belonging to the cubic system. However, experiments have shown that the Debye equation represents the values of specific heat and other thermophysical properties for certain other monoatomic solids, such as zinc, which crystallizes in the hexagonal system. Suppose that the crystal contains N molecules, each composed of s atoms. Since there are Ns atoms, the crystal as a whole has $3Ns$ vibrational modes. A reasonable approximation is obtained by classifying the vibration into

- $3N$ lattice vibrations, which are the normal modes discussed in the Debye treatment (acoustical modes).
- independent vibrations of individual molecules in which bond angles and lengths may vary. there must be $3N(s-1)$ of these (optical modes). We expressed the optical modes using the Einstein model.

3.2. Crystal bonds

The analytical calculation of configuration integral in solids is a very difficult task. Most frequently numerical procedures are applied in practical computations by means of the Monte-Carlo method. Nevertheless, the presented method requires a lot of computer time with another serious drawback being also that it does not provide a functional dependence of thermodynamic properties on temperature and volume. Empirical equations are frequently used as well, though mostly without any theoretical basis built on a molecular view of the world. In the paper presented we used the perturbation Van der Waals (VDW) theory for solids [10,24] around the model of hard spheres to calculate the thermodynamic properties of state. In order to calculate the mixtures of atoms of hard spheres we obtain the configuration free energy for a certain binary crystal [9]:

$$A_{\text{conf}0} = Nk_BTx\left[\begin{array}{c}-3\ln\left(\frac{V^*-1}{V^*}\right) + 5.124\cdot\ln\ V^* \\ -20.78V^* + 9.52V^{*2} - 1.98V^{*3} \\ +C_0 + \psi_1\cdot\ln\ \psi_1 + \psi_2\cdot\ln\ \psi_2\end{array}\right], \tag{18}$$

$$C_0 = 15.022, \quad V^* = \frac{V}{V_0}, \quad V_0 = \frac{N\sigma^3}{\sqrt{2}}, \tag{19}$$

where σ means the Lennard-Jones parameter, V^* reduced volume and ψ_i a molar fracture of component 1. In case of a crystal formed of atoms of the same type the free energy can be written as:

$$A_{\text{conf}0} = Nk_BT\left[\begin{array}{c}-3\ln\frac{V^*-1}{V^*} + 5.124\cdot\ln\ V^* \\ -20.78V^* + 9.52V^{*2} \\ -1.98V^{*3} + C_0\end{array}\right]. \tag{20}$$

To calculate the perturbation contribution the VDW model was used. In most of the technical literature [13,24] the VDW model is treated only in relation to atomic structure, whereas we additionally presented the temperature-dependent coefficients:

$$A_{\mathrm{conf}1} = -\frac{a(\psi_1, \psi_2, T)}{V}. \tag{21}$$

The configuration integral is thus formed by the contribution of hard spheres and perturbation:

$$A_{\mathrm{conf}} = A_{\mathrm{conf}0} + A_{\mathrm{conf}1}. \tag{22}$$

In our case the coefficient a was determined as a temperature-dependent polynomial following a comparison between experimental data and analytical results:

$$a = a_0 + a_1 T + a_2 T^2. \tag{23}$$

Coefficients a_0, a_1, a_2 are obtained by numerical approximation and the comparison with thermodynamic data.

3.3. *Bulk and elastic modulus*

Using Eq. (16) we can calculate also some derived thermodynamic properties of state, which are very important in planning and monitoring the thermomechanical processes:

Isobaric heat capacity is very important for the calculation of some important parameters such as the thermal diffusivity of solids:

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p. \tag{24}$$

The bulk modulus relates the relative pressure change and the applied stress [15] as:

Isentropic bulk modulus:

$$K_s = -V\left(\frac{\partial p}{\partial V}\right)_s. \tag{25}$$

Isothermal bulk modulus:

$$K_s = -V\left(\frac{\partial p}{\partial V}\right)_T. \tag{26}$$

The modulus of elasticity E is also known as Young's modulus and is the measure of the elastic force of any substance, expressed by the ratio of a stress on given unit of the substance to the accompanying distortion. For highly symmetric solids the Young's modulus can be written in terms of bulk modulus and the Poisson ratio [15] as

$$E = 3K(1 - 2v). \tag{27}$$

Volumetric expansion coefficient β is expressed with the next express:

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,\psi}. \tag{28}$$

For many engineering applications the quantity of direct interest is not the coefficient of volume expansion β but rather the coefficient of linear expansion coefficient α:

$$\alpha = \frac{1}{V^{1/3}}\left(\frac{\partial V^{1/3}}{\partial T}\right)_p = \frac{1}{3}\beta, \tag{29}$$

where β is the volumetric expansion coefficient.

3.4. Murnaghan equation of the state for solids [25]

The internal energy of a solid u_T can be expressed as the sum of two terms:

$$u_T(v, T) = u_0(v) + u_D(T, v), \tag{30}$$

where u_0 represents the cohesive energy at zero temperature. On the basis of relations from classical thermodynamics ($-p = \partial u_t/\partial v|_S$) we can express the next expression:

$$p = p_0 + u_D \frac{\gamma}{V}, \tag{31}$$

where p_0 is the pressure at zero temperature and u_D is internal energy on the basis of Debye model [25]. The pressure at zero temperature p_0 is expressed with the next equation:

$$p_0 = K_0 \frac{3[(V/V_0)^{1/3} - 1]}{(V/V_0)^{2/3}} \mathrm{e}^{-\alpha}\left(1 - 0.15\alpha_t + 0.05\alpha_t^2\right), \tag{32}$$

where K_0 is the bulk modulus at zero temperature and α_t is the constant calculated from interatomic distance. How to determine all important constants is possibly to find in Ref. [25].

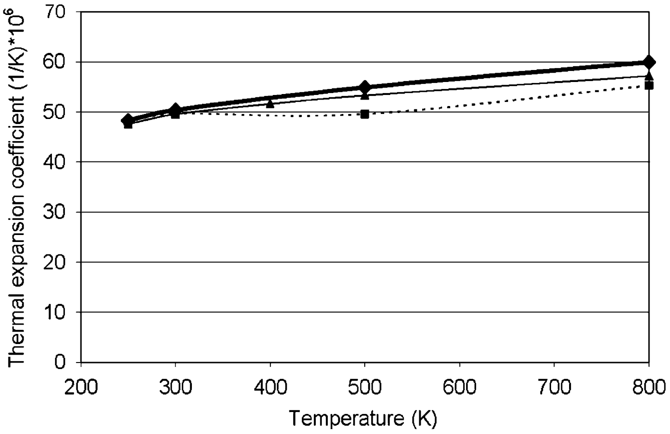

Fig. 5. Thermal expansion coefficient for copper: —◆—, EXP; ···■···, AO-HS; —▲—, AO-M.

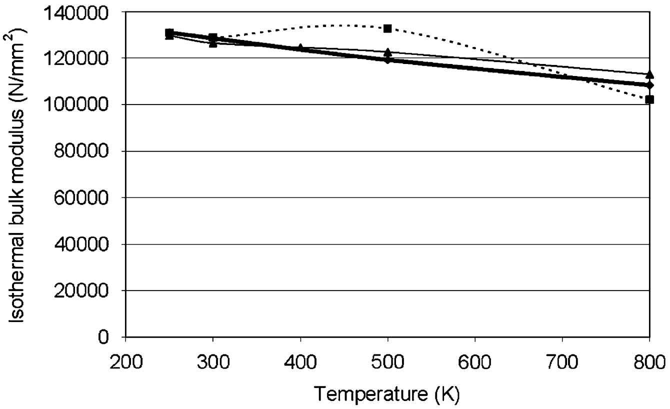

Fig. 6. Isothermal bulk modulus for copper: —◆—, EXP; ···■···, AO-HS; —▲—, AO-M.

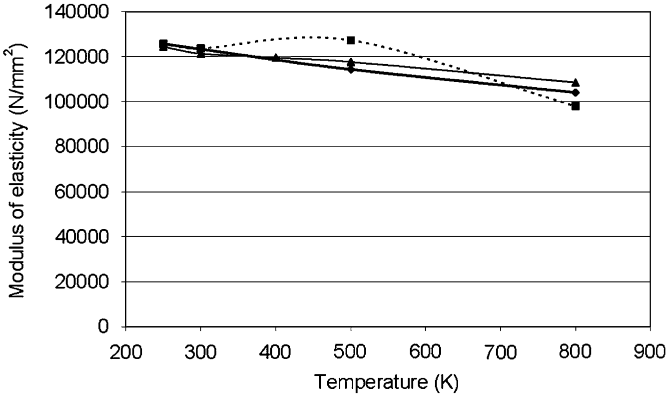

Fig. 7. Modulus of elasticity for copper: —◆—, EXP; ···■···, AO-HS; —▲—, AO-M.

Table 3
Results for simply supported aluminum beam

Temperature	Mode of vibration	Experimental results [2]	Analytical results Marques–Inman [2]	Analytical results Avsec–Oblak
23 °C	1	36.79444	31.6279	32.05
	2	95.7661	87.2021	88.66
	3	182.4597	170.9858	173.7
	4	294.8589	282.7049	287.26
29 °C	1	31.7540	29.2867	29.72
	2	88.2056	84.0921	85.421
	3	171.875	167.6089	170.294
	4	285.2823	279.1737	283.64
33 °C	1	29.2339	27.605	27.91
	2	84.1734	81.9485	83.185
	3	166.8347	165.3185	167.987
	4	280.2419	276.7942	279.14
36 °C	1	26.2097	26.266	26.56
	2	79.1331	80.3006	81.466
	3	161.7944	163.5791	166.09
	4	274.1935	274.9959	279.14
40 °C	1	22.6815	24.3582	24.56
	2	74.5968	78.0458	78.981
	3	156.25	161.2305	163.646
	4	268.6492	272.5976	276.4
42 °C	1	21.1694	23.3410	23.49
	2	71.0685	76.8919	77.906
	3	152.2177	160.043	162.981
	4	265.1210	271.3633	271.656
48 °C	1	24.6976	19.9576	19.91
	2	66.5323	73.3143	74.16
	3	144.1532	156.4261	158.63
	4	259.5766	267.681	271.656

4. Results and discussion

Thermal and caloric equations of state for solids are fundamental characteristics of matter defining thermodynamic properties over a wide range of temperatures. The principal problem of modern powerful theories is the necessity to take into account correctly the strong collective and interparticle interaction in dissordered media. Solids have been extensively studied with models on the basis of classical thermomechanics. The models are from the practical view more accurate and they allow simulation in wider temperature and pressure regions. Classical thermomechanics has no insight into the microstructure of the substance. But it allows the calculation of thermodynamic functions of state with asistance from measurement or empirical equations. Statistical thermodynamics, on the other hand, calculates the properties of state on the basis of molecular motions in space, and on the basis of the intermolecular interactions. This paper is one of the first attempts in determining how to calculate vibrational properties for beams on the basis of statistical thermodynamics, with the combination of analytical vibrational theory.

The presented mathematical model was used to calculate thermodynamic properties of state of pure aluminum and copper beam. Figs. 5–7 show the comparison between experimental results (Exp.), model on the basis of statistical thermodynamics and hard sphere theory (AO-HS) and model on the basis of statistical thermodynamics and Murnaghan model (AO-M). The presented results show relatively good agreement between analytical models and experimental data. The detailed analysis shows, that also small changes of temperature cause significant changes of natural frequencies for beams.

Table 1 contains the main data for the beam. An aluminium beam was chosen. The aluminium beam is very interesting, particularly due to relatively high expansion coefficients. Indicated in Table 3 are the results for the natural frequencies for the first four modes for the simply supported beam. Table 4 contains the results for the natural frequencies for the first four modes of vibration for the clamped beam. The analytical results are

Table 4
Results for isotropic aluminum clamped beam

Temperature	Mode of vibration	Experimental results [2]	Analytical results Marques–Inman [2]	Analytical results Avsec–Oblak
23 °C	1	36.79444	31.6279	32.05
	2	95.7661	87.2021	88.66
	3	182.4597	170.9858	173.7
	4	294.8589	282.7049	287.26
29 °C	1	31.7540	29.2867	29.72
	2	88.2056	84.0921	85.421
	3	171.875	167.6089	170.294
	4	285.2823	279.1737	283.64
33 °C	1	29.2339	27.605	27.91
	2	84.1734	81.9485	83.185
	3	166.8347	165.3185	167.987
	4	280.2419	276.7942	279.14
36 °C	1	26.2097	26.266	26.56
	2	79.1331	80.3006	81.466
	3	161.7944	163.5791	166.09
	4	274.1935	274.9959	279.14
40 °C	1	22.6815	24.3582	24.56
	2	74.5968	78.0458	78.981
	3	156.25	161.2305	163.646
	4	268.6492	272.5976	276.4
42 °C	1	21.1694	23.3410	23.49
	2	71.0685	76.8919	77.906
	3	152.2177	160.043	162.981
	4	265.1210	271.3633	271.656
48 °C	1	24.6976	19.9576	19.91
	2	66.5323	73.3143	74.16
	3	144.1532	156.4261	158.63
	4	259.5766	267.681	271.656

Table 5
Results for isotropic clamped copper beam

Temperature	Mode of vibration	Analytical results Avsec–Oblak
250 K	1	24.51
	2	67.56
	3	218.94
	4	327.06
300 K	1	17.898
	2	58.88
	3	207.16
	4	313.53
400 K	1	~0
	2	30.254
	3	95.93
	4	179.1

compared with the measured values [1] and with the analytical results of the authors Marques-Inman [1]. The comparison of the results indicates a satisfactory matching of the results for the new analytical model. Table 5 shows the analytical results for clamped copper beam under thermal effects on the basis of statistical thermomechanics.

5. Conclusion

The paper deals with the vibrations of the clamped and simply supported beams under the impact of temperature effects. The thermophysical properties of state, such as modulus of elasticity and linear expansion coefficient, are calculated on the basis of statistical thermomechanics. The analysis shows that a minor change in temperature results in a considerable alteration in natural frequencies of the beam. The comparison between experimental data and analytical results shows satisfactory agreement.

References

[1] L.D. Zavodney, The response of a single-degree-of-freedom system with quadratic and cubic non-linearities to a principal parametric resonance, Journal of Sound and Vibration 129(3) 417–442.

[2] R.F. Marques, D.J. Inman, An analytical model for a clamped isotropic beam under thermal effects, *Proceeedings of IMECE 2002, November* (2002) 17–22.

[3] R.F. Marques, D.J. Inman, D.A. Rade, Assesment of adaptive techniques for the control of structures subject to temperature variations, 12th ICSV Conference, Lisbon, Portugal, 2005.

[4] J. Avsec, M. Oblak, Vibrational analysis of temperature dependent materials for clamped isotropic beam under thermal effects, First ICOVIS Conference, Loughborough, England, 2006.

[5] S. Timoshenko, *Vibration Problems in Engineering*, Wiley, New York, 1974.

[6] B.A. Boley, J.H. Weiner, *Theory of Thermal Stresses*, Wiley, New York, 1966.

[7] R.J. Borg, G.J. Dienes, *The Physical Chemistry of Solids*, Academic Press Inc., Boston, 1992.

[8] C. Borgnakke, R.E. Sonntag, *Thermodynamic and Transport Properties* University of Michigan, Wiley, New York, 1997.

[9] J. Avsec, M. Marčič, Calculation of elastic modulus and other thermophysical properties for molecular crystals, *Journal of Thermophysics and Heat Transfer* 16 (3) (2002) 463–468.

[10] A. Donoun, C.F. Tejero, M. Baus, Van der Waals theory of solids, *Physical Review* 50 (4) (1994) 2913–2924.

[11] C. Garrod, *Statistical Mechanics and Thermodynamics*, Oxford University Press, New York, 1995.

[12] N.A. Gokcen, *Statistical Thermodynamics of Alloys*, Plenum Press, New York, 1986.

[13] T. Hino, J.M. Prausnitz, Solid–liquid equilibria for solutions of binary globular–protein mixtures, *A.I.Ch.E. Journal* 45 (3) (1999) 459–465.

[14] R.T. DeHoff, *Thermodynamics in Material Science*, McGraw-Hill, Singapore, 1993.

[15] L.D. Landau, E.M. Lifschitz, *Elastizitätstheorie*, Akademie-Verlag, Berlin, 1975.

[16] K. Lucas, *Statistical Thermodynamics*, Springer, New York, 1992.

[17] W.J. Moore, *Physical Chemistry*, Prentice-Hall, Englewood Cliffs, NJ, 1972.

[18] A. Münster, *Statistical Thermodynamics*, Vol. II, Springer, Berlin, 1974.
[19] A.D. Pelton, W.T. Thompson, *Phase Diagrams 1988. Progress in Solid State Chemistry*, Vol. 10, Part 3, Pergamon Press, Oxford.
[20] D.A. Porter, K.A. Easterling, *Phase Transformations in Metals and Alloys*, Chapman & Hall, London, 1993.
[21] N.A. Smirnova, *Methods of the Statistical Thermodynamics in the Physical Chemistry*, University of Moscow, Moscow, 1982.
[22] R.A. Swalin, *Thermodynamics of Solids*, Wiley, New York, 1972.
[23] M.W. Zemansky, R.H. Dittman, *Heat and Thermodynamics*, McGraw-Hill, New York, 1997.
[24] D.A. Young, Van der Waals theory of two-component melting, *Journal of Chemical Physics* 98 (2) (1993) 9819–9829.
[25] G.E. Hernandez, A Debye-Grüneisen correction to the Murnaghan equation of the state for solids, *Journal of Physical Chemistry of Solids* 59 (6-7) (1998) 867–870.