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Dynamic folding analysis for multi-folding structures under impact loading

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Abstract

The authors review the basic mechanisms for folding structures, such as a combination of pantographs which have unstable states under dynamic loading. The problem is considered in terms of the large displacement range within which a truss is allowed to be foldable and deployable, in the context of elastic stability and allows for dynamic control. There are several possible folding patterns that are identified at the unstable states and the authors put forward a new concept for this multi-folding of a pantograph structure using a simple model. The authors explore the critical dynamic and postbuckling effects through the concept of energy minimization. For comparisons with final large-deflection fold patterns, the authors use an original program for dynamic truss analysis. They demonstrate that the fold patterns change as a function of both the velocity of the dynamic loading and the dynamic geometry of the structure.

1. Introduction

For structural members and/or materials subject to impact loadings the structural design must try to successfully absorb and dissipate the impact energy. The designer should decide upon the best structural form without the possibility of fragmentation of the system and for minimal damage as well as aiming at minimal mass and maximum stiffness of the system. In the field of impact engineering, high-performance, energy absorbing members with localized foldings have been developed to absorb/disperse a large impact energy. If the folding pattern is controlled super-elastic behaviour, then the material and the members of the structure would, under impact loading, act as shock absorbers with the system being able to return to its initial form post impact. For material selection the use of porous and honeycomb materials, are good choices since they have *cellular micro-structures* which can easily deform under loading and thus act as impact buffers [1–5].

Recently, Holnicki et al. [6,7] designed an active shock-absorber system from which the authors put forward the concept of a pantograph truss to model *multi-folding of microstructures* (*MFM*). We have successfully carried out FEM simulations with geometric nonlinearity, allowing for contact between nodes which

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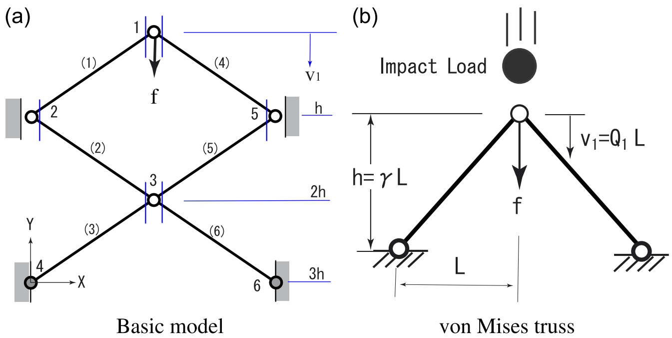

Fig. 1. Elastic folding systems subject to impact load f: (a) basic model; (b) von Mises truss.

correlated with control experiments of a basic pantograph shown in Fig. 1(a). To develop this new model, we needed to estimate the energy that initiates the multi-folding of the structure under impact and then identify the different possible folding patterns of the system. By controlling the stress of an individual member, we can actively control the folding characteristics and we simulate the effect of various impact scenarios. The repetitive use dissipaters in the honeycomb-like *cellular micro-structures* gives an additional value of energy dissipation. Careful pre-design for the optimal distribution of the maximum stress level in all controllable elements, triggers the desired sequence of local collapses. The high effectiveness of active impact energy absorption by yield stress adjustment demonstrates the potential for application of this system, e.g. in shock-absorbing systems.

In this paper, using the fundamental concept of MFM, the authors compare experimental, numerical and theoretical results of the folding of pantograph structures. The authors extend the discussion to include an understanding of the behavioural characteristics of the folding patterns using the theory of elastic stability and to identify and account for the folding patterns; and how to interpret the theoretical results through control experiments. The problems related to the elastic stability of a structure have been researched by many scientists in this field, notably Thompson and Stewart [8], who introduced nonlinear dynamics and chaos based upon the instability of a structural system. It is well known that a shallow arch or truss displays unstable bifurcation phenomena with regard to elastic nonlinearity [9–12].

This research puts forward a folding mechanism with nonlinear behaviour for an elastic folding truss subject to an impact load from the viewpoint of structural mechanics. This is done through the numerical analysis of dynamic instability. To observe large-scale dynamic displacement behaviour, the authors analyze the equilibrium equation with geometric nonlinearity for a basic folding model. The numerical methods for this folding are used in both the static and dynamic nonlinear analysis. In particular, the dynamic analysis uses a kinematic equation described by an ordinary differential equation. The authors have found that it is similar to the folding process for the multi-folding system and there are complex equilibrium curves during unstable state, i.e. post buckling 'snap-through behaviour'. In addition this paper puts forward the results of a multi-folding simulation.

2. Theory of elastic folding for multi-folding microstructure

In this section, we describe the basic mechanism for the MFM with geometric nonlinearlity.

2.1. Nonlinear problem of a simple folding truss

The authors consider an elastic bifurcation problem with a nonlinear equilibrium equation for a folding structure that allows for a large vertical displacement (such as *snap-through* behaviour) as shown in Fig. 1(b). In this paper, we do not allow for any horizontal displacement at any node in the model (see Fig. 1(a)). The building blocks of the basic model of Fig. 1(a) are the von Mises Truss shown in Fig. 1(b) which is formed as a 2-bar truss from bars of elastic stiffness EA, each element has a vertical projection of h, and horizontal projection, L with the ratio $\gamma = h/L$. For the pre impact condition there is no stress in the members and for the theoretical approach of the folding mechanism, we assume that the bar is not buckled and that it is perfectly elastic. In the case of the elastic buckling model we obtain the well-known *snap through* behaviour for the von Mises truss. The energy principle is applied to solve a memory force, taking into account geometrical nonlinearity, based on elastic stability [13]. In this paper, we assume that there is only a primary path without secondary bifurcation paths. The initial length of a bar is ℓ_0 and the length after the deformation for the ith element is defined as $\hat{\ell}_i$, where the following geometric rules apply:

$$\ell_0 = \sqrt{L^2 + h^2} = L\sqrt{1 + \gamma^2}, \tag{1}$$

$$\hat{\ell}_i = L\sqrt{1 + (\gamma - Q_1)^2}, \quad i = 1, 2, \tag{2}$$

where $Q_1 = v_1/L$ is denoted as the normalized vertical displacement for the generalized degree of freedom.

Using the definition of the Green's strain we obtain:

$$\varepsilon_i = \frac{1}{2}\left\{\left(\frac{\hat{\ell}_i}{\ell_0}\right)^2 - 1\right\}. \tag{3}$$

This strain is more useful than normal strain[1] for theoretically analyzing the folding mechanism. The total potential energy is then defined in the following:

$$\mathscr{V} = \sum_{i=1}^{2} \frac{EA\ell_0}{2}(\varepsilon_i)^2 - fQ_1L. \tag{4}$$

Rewriting Eq. (4), we obtain

$$\mathscr{V} = A_{11}Q_1^2 + A_{111}Q_1^3 + A_{1111}Q_1^4 - fLQ_1, \tag{5}$$

where

$$A_{11} = \beta L\gamma^2, \quad A_{111} = -\beta L\gamma, \quad A_{1111} = \frac{\beta L}{4}, \quad \beta = \frac{EA}{(1+\gamma^2)^{3/2}}.$$

Hence the nonlinear equilibrium equations based on the principle of the minimum potential energy are expressed in the following equation:

$$\mathscr{V}_1 \equiv \left(\frac{d\mathscr{V}}{dQ_1}\right) = 2A_{11}Q_1 + 3A_{111}Q_1^2 + 4A_{1111}Q_1^3 - fL = 0. \tag{6}$$

Using Eq. (6), the main equilibrium path is obtained in the following equation:

$$\begin{aligned} f(Q_1) &= \frac{2A_{11}Q_1 + 3A_{111}Q_1^2 + 4A_{1111}Q_1^3}{L} \\ &= \beta\, Q_1(Q_1 - \gamma)(Q_1 - 2\gamma). \end{aligned} \tag{7}$$

This equation is the primary path for a von Mises truss without a bifurcation path and is shown in Fig. 2.

For the limit point, it is useful to know the strain energy stored at the point of dynamic *snap-through* behaviour. Differentiating Eq. (7) and equating to zero we obtain

$$\mathscr{V}_{11} = \frac{df(Q_1)}{dQ_1} = \beta(3Q_1^2 - 6\gamma Q_1 + 2\gamma^2) = 0,$$

which gives

$$Q_1^{cr} = \left(\frac{3 - \sqrt{3}}{3}\right)\gamma. \tag{8}$$

[1]Numerical analysis in this paper has used normal strain ($\varepsilon_i = \hat{\ell}_i/\ell_0 - 1$).

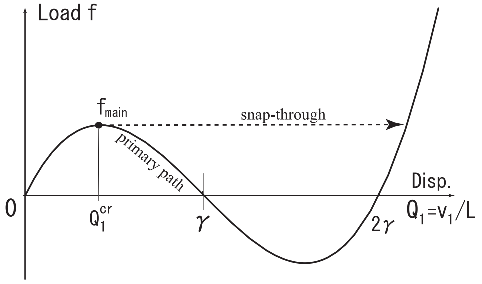

Fig. 2. Nonlinear equilibrium curve for von Mises truss.

Table 1
The coordinates of nodes

Node	X (m)	Y (m)
1	0.240	0.489
2	0.000	0.326
3	0.240	0.163
4	0.000	0.000
5	0.480	0.326
6	0.480	0.000

Then, by substituting Eq. (8) in Eq. (7), we obtain the maximum force for the primary path. The value of the maximum force is obtained as

$$f_{\max} = f(Q_1^{\mathrm{cr}}) = \frac{2\gamma^3}{3\sqrt{3}}\beta = \frac{2\gamma^3}{3\sqrt{3}}\frac{EA}{(1+\gamma^2)^{3/2}}. \tag{9}$$

This maximum force $f_{\max}$ and the critical displacement Q_1^{cr} give us important information on the physical behaviour with regard to the *snap-through* behaviour of the dynamic problem. If this system does not reach these critical values under dynamic loading then *snap-through* behaviour will not take place. The kinematic equation of the folding model is combined with the nonlinear relationships to give the following ordinary differential equation:

$$M\ddot{Q}_1 + C\dot{Q}_1 + \beta Q_1(Q_1 - \gamma)(Q_1 - 2\gamma) = 0. \tag{10}$$

This is a dynamic equation in which the stiffness has nonlinearity of order three for primary equilibrium path of statics analysis. From this equation, we obtain the dynamic instability of *snap-through* behaviour or *snap-back* behaviour and real physical motion.

Numerical investigations are undertaken for this model, allowing for geometrical nonlinearity, for both static and dynamic analysis in the paper. The numerical solutions to the dynamic analysis are a function of time, dynamic loading and folding.

3. Numerical analysis for folding truss with nodal contact

We now examine the folding of a pantographic truss allowing for contact between members and nodes in order to compare it with the experiment.

3.1. Static equilibrium analysis

The nodes for the FEM model are shown in Fig. 1(a) and the coordinates are given in Table 1. The extensional stiffness of all members in the model have $EA = 1$.

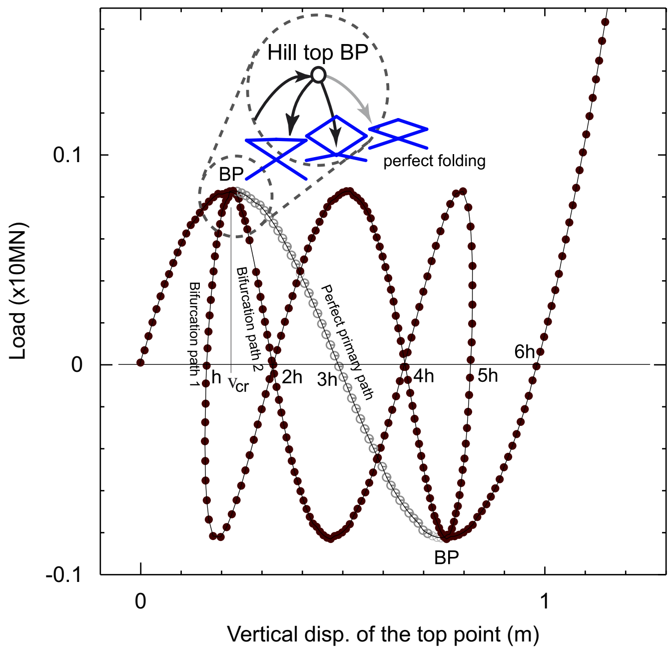

Fig. 3. Structural model and static equilibrium curves.

We consider the problem of contact under folding and introduce two virtual additional elements between the nodes 2 and 4 and nodes 5 and 6 in the basic model of Fig. 1(a). It is assumed that the damping factor of all elements is given by $C = 0.5$ and the mass $M = 1$. All units are normalized. If the value of the damping is small the system response appears as vibration motion analogous to the dynamics of a molecular model. In this idealized model, we have not considered the effects of gravity on the elements and the rolling hinges supported on the walls are assumed to be frictionless. This model therefore makes it possible to gain a physical understanding in which there are several equilibrium curves for the different folding patterns from the limit point (first peak of Fig. 3 below). This point is called the bifurcation point (BP) of *the Hill-top type* and the eigenvalue at this singular point becomes zero. There are several paths that can be followed from BP in addition to the primary path.

By using Eq. (8), we obtain the theoretical displacement v^{cr} for the critical state of the basic model when the load reaches the maximum point:

$$v_n^{cr} = \left(\frac{3-\sqrt{3}}{3}\right)(n \times h) = 0.2068(\text{m}) \quad \text{for the basic model } n = 3$$

and we can use Eq. (9) to obtain the maximum load on this system. The numerical result at the critical point is given by

$$v_{nu}^{cr} = 0.2128(\text{m}), \quad f_{\max} = 0.0821.$$

Eq. (8) is used to predict the approximate value for the critical state of the whole of this system even though the definition of the numerical and theoretical strains is different.

For the case of no contact the folding pattern is proportional to global deformation of the whole system. However, it has a high level of sensitivity and is very difficult to control. Other static paths of this system are shown as bifurcation paths, from BP (the first peak of the equilibrium curve in Fig. 3). Bifurcation path 1 in Fig. 3 is the line of lowest strength after BP and is a path where local buckling has occurred, i.e. local *snap-through* behaviour has occurred at the top two members. Bifurcation path 2 has a different type of local buckling which occurs in the bottom two members. Generally the folding in this system occurs in the elements that are directly subject to the impact loading.

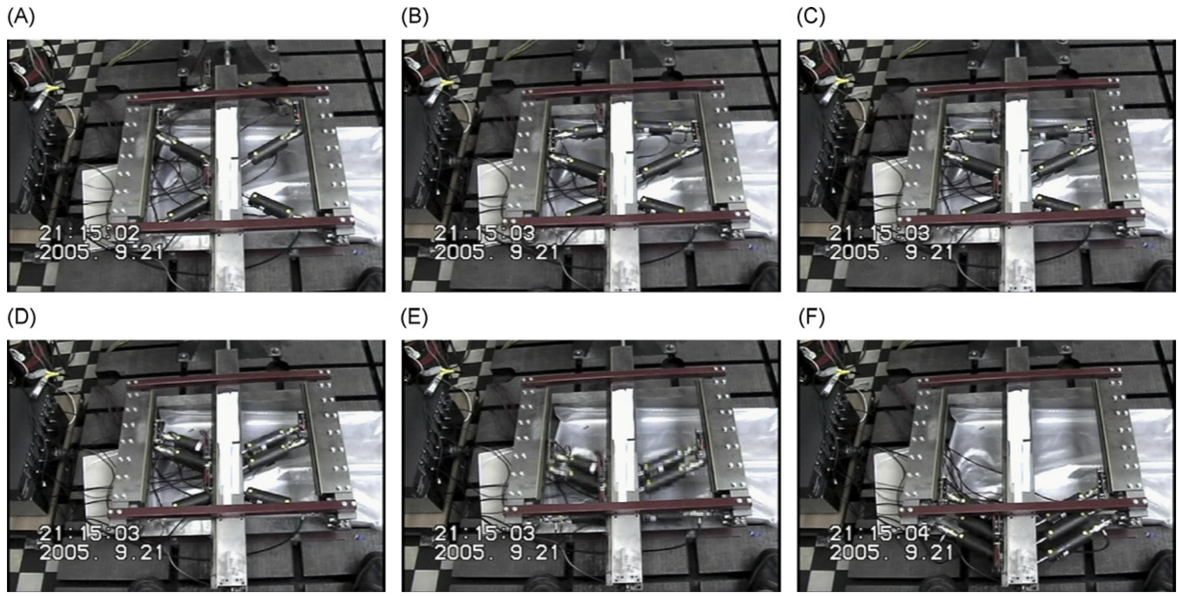

Fig. 4. Folding processes in the experiment of J. Holnicki and P. Pawlowski: (A) initial state; (B) top deformation; (C) the first *snap-through*; (D) contacting; (E) the second *snap-through*; (F) restable state.

3.2. Folding processes in the experiment

We know that there are three types of equilibrium paths for the elastic truss model in the previous subsection. Therefore, we have investigated the folding mechanisms making allowance for contact between nodes in the dynamic problem.

We have developed the active control concept for the multi-folding system under impact loads which has been carried out in a laboratory experiment described by Holnicki et al. [7]. The photographs of the experimental process show in Fig. 4. Picture (A) is the initial location of structural system prior to impact. For all joints displacement is possible in the vertical direction and the top joint is the point of application of the impact load. The six members have identical properties of absolvers (stiffness). In this paper, we confirm the process of the experimental folding process shown from Pictures (A) to (F) of Fig. 4.

3.3. Dynamic numerical analysis

Several equilibrium paths were identified using the static analysis of Section 3.1. We have obtained numerical results using dynamic analysis by incremental loading and displacement control methods. Firstly we describe the folding pattern for the dynamic analysis after postbuckling. The first fold for this pattern is the local snap-through behaviour of the top two members. There is more than one equilibrium curve as shown in Fig. 5(a). A process such as ‘a’ → ‘BP’ → ‘e’ → ‘h’ in the figure, is obtained by using the dynamic loading control. It is shown as two *snap-throughs* from ‘BP’ to ‘e’ and ‘e’ to ‘h’. As can be seen in the figure, point ‘h’ is corresponds to the final point and here all members are in contact. The path followed by using dynamic displacement control, shows a solid line from ‘a’ → ‘BP’ → ‘c’ → ‘CP’ → ‘e’ → ‘f’ → ‘g’ → ‘h’ in Fig. 5(a). Here ‘CP’ means Contact Point on folding members.

In particular, the trace of the path ‘d–e–f’ corresponds to the equilibrium curve of a von Mises truss model (see Fig. 2). There are several paths crossing at the CP point as shown at ‘d’ in Fig. 5(a) The paths have higher stiffnesses compared with the static paths. There is an increase in the stiffness of the system as members come into contact with each other during the folding process as shown by ‘d’ in Fig. 5(b). The numerical simulation of the complete folding process is shown in Fig. 5(b). This figure displays seven folding processes from ‘a’ to ‘g’. The change of displacement reduces between *snap throughs* as can be seen in Fig. 5(b) e.g. from ‘a–c’ to

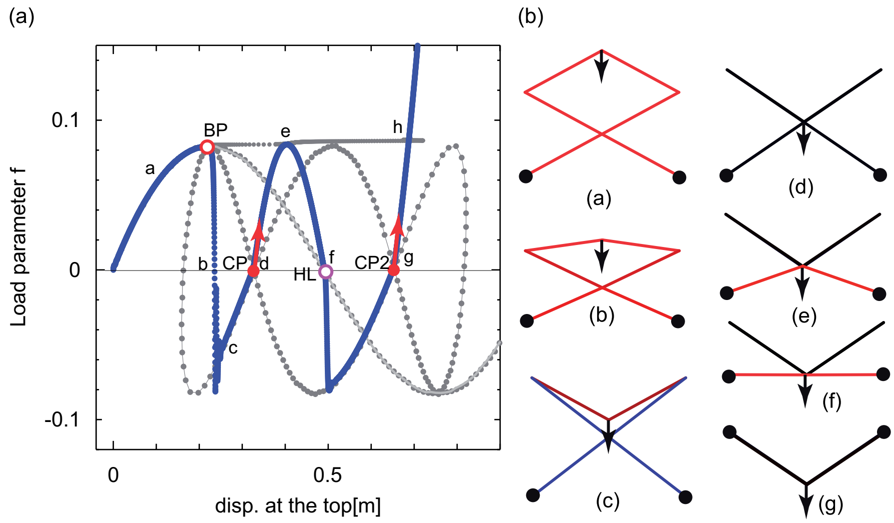

Fig. 5. Equilibrium curves for folding pattern allowing for contact and folding process showing contact: (a) dynamic methods comparing with static problem; (b) the process of each deformation.

'd–f'. After this we see that as the system passes through the ground position labelled 'f' of Fig. 5(b), the system becomes unstable and there is a loss of stiffness as every member in the system has undergone snap-through behaviour. In the folding process from 'b' to 'c'; both of the side support nodes become higher than the position at the loaded node when the top two elements *snap-back* during the folding process. Finally, we see that the shape for the model has only two elements, which is shown in the process of 'g' in Fig. 5(b) which corresponds to all members of the system being in contact. The dynamic path just described is a very different path to the no contact and static problem. The analysis was carried out by using an FEM model composed of elastic material.

4. Conclusion

Using a numerical approach we have established the folding mechanism for the dynamic nonlinear folding process of a pantographic truss system which compares well with a folding experiment, based on the multi-folding concept. The nonlinear equilibrium paths for the folding smart passive structures have been successfully simulated and it has been possible to calculate the capacity of the maximum impact energy and/or strain energy in these systems. Although this model is simple, the solution to the behaviour near BPs has required the finding of the equilibrium paths.

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