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Mathematical simulation of oil fields probing using shock impulse loading

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Abstract

Capabilities of approximate mathematical modeling of the sounding oil fields with use of shock impulses are studied. To this goal, a multilayered column-waveguide extracted from a solid half-space is considered. The analysis is carried out with reference to plane waves propagating in the multilayered medium with arbitrary located layers of various rheological properties. As a source of signals both surface impulses and underwater or underground explosions, single and periodic, and also harmonious excitation are considered, and displacements of a free surface—as the response. The process of wave propagation under repeated reflections from the layer borders is computed numerically with use of the authors' wave finite element method. The amplitude–frequency spectrum of the reflected signals is analyzed both qualitatively and using an original method of "focusing". It is shown, that the analysis of the response for a shock impulse loading allows receiving sufficiently full information about layers' location and thickness. The considered approaches can be used at processing and analyzing the reflected signals obtained experimentally during investigations of oil fields.

1. Introduction

In connection with the accelerated exhaustion of the Earth's oil resources, improvement of methods for new oil fields exploration, including ones situated under a seabed, gets the increasing value. One of such methods is sounding oil fields with use of shock impulses, actuating deformation waves, and the analysis of the acoustic signals reflected from various layers [1,2]. The similar problems demanding a study of wave propagation in multilayered media are of interest for problems of seismology, acoustics, dynamics, composite materials, and in other areas of science and engineering. In a number of papers devoted to simulation of wave propagation in layered media, plane elastic waves caused by sources distributed over the surfaces are mostly considered. Of greater interest is the analysis of disturbances induced by a point and underwater (underground) sources. A study of wave processes in such systems on the basis of exact 3-D equations of the continuous medium, even in an axisymmetric formulation, appears rather complex [3], as it demands consideration of spatial waves

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under their numerous refractions and reflections at layer borders. Yet more complexities arise in studies of repeated reflections in media with non-elastic characteristics.

In this paper, opportunities of the approximate mathematical modeling of the specified processes with the practical purpose are investigated. The analysis is carried out with respect to propagation of deformation waves concentrated in a small site of the multilayered half-space. The layer may be of different thickness with various physical and mechanical properties. A laminate can be transversely isotropic. As a source of the directed signals, single and periodic shock impulses, and also harmonic exciting loads located on the surface site or explosions at some depth are considered; and as the response—displacements of a free surface at the center of the loaded site. Note that studying the response at such a site is of the greatest practical importance. As an approximate approach, a column-waveguide is extracted from the half-space normally to the free surface, and an analysis of 1-D wave propagation along the column is carried out. Repeated partial reflections from layer borders and an interaction with other part of the half-space in the axisymmetric formulation are accounted. The process of wave propagation is numerically calculated with use of a wave finite element method (WFEM), described in Refs. [4,5]. An amplitude–frequency spectrum of the reflected signals is investigated both analytically and by means of the special analysis system [6].

2. Problem's formulation

A circular cylindrical column-waveguide of small radius R and height $L \gg R$ is extracted from a solid half-space normally to the free surface $z = 0$ (Fig. 1). The half-space consists of n layers of various thickness L_k with different physical and mechanical properties assumed constant within each layer. The beginning of the layer k has coordinate $z_k = \sum_{l=1}^{l=k-1} L_l$, and the last layer $k = n$ bases on a rigid or viscous-elastic ground at $z = L$. The column's deformed state is considered plane and axisymmetric. The effect of the column interaction with the medium on the lateral surface is described using a generalized elastic foundation assuming independent resistance of environment at each section z to column's lateral deformation. Mechanical properties of the column and environment can be identical or different, for example, elastic transversely isotropic, and also simulate behavior of ideal or viscous liquids. At the time $t = 0$ a longitudinal load $F_z(0) = \pi R^2 q_z(0)$ that changes further according to any law $F_z(t)$ is suddenly applied to a free surface of the column or at any depth z_{imp}. The response of the system (longitudinal displacement u_z at the free surface) under propagation of plane waves of longitudinal stresses σ_z and strains ε_z is studied with regard to their repeated reflections from column ends and layer borders.

For numerical analysis the WFEM [4] in the form, most convenient for modeling 1-D wave fronts and conditionally named in work [5] "direct mathematical modeling", is used. In the present paper, the analysis was carried out for an elastic medium, but the approach can be also developed for non-elastic media with complex rheological properties [6].

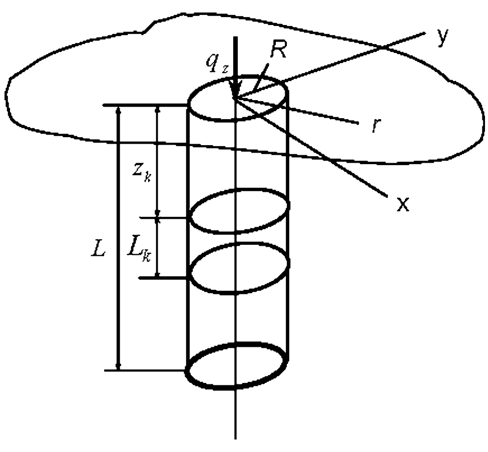

Fig. 1. Scheme of a column-waveguide extracted from elastic half-space.

3. Simulating a wave process in an elastic column-waveguide extracted from transversely isotropic medium

3.1. Approximate model of a column-waveguide with medium interaction

Under axial symmetry, tangential and radial components of stresses and strains at the column axis (at $r = 0$) coincide with each other, $\sigma_\varphi = \sigma_r$, $\varepsilon_\varphi = \varepsilon_r$. These equalities are assumed to be approximately true also within a small area adjoining the axis. In this case the general equations of elasticity for the column yield:

$$\varepsilon_r = \varepsilon_\varphi = \frac{(1-\mu)\sigma_r - \mu\sigma_z}{E}, \quad \varepsilon_z = \frac{\sigma_z - 2\mu\sigma_r}{E}, \tag{1}$$

where E is the Young's modulus, μ is the Poisson's ratio.

Radial (transverse) displacements of the column's lateral surface

$$\mu_r = R\varepsilon_\varphi = \frac{R}{E}[(1-\mu)\sigma_r - \mu\sigma_z] \tag{2}$$

should be equal to radial displacements of an environment at $r = R$ which can be approximately found from Lame's formulae for a long pipe with a small hole of the radius R exposed to internal pressure $p = -\sigma_r$:

$$u_r^* \approx (1+\mu^*)\frac{pR}{E^*}, \tag{3}$$

where the superscript (*) refers to the medium terms in a radial direction.

From equality $u_r = u_r^*$, we find

$$\sigma_r = K_r\sigma_z, \tag{4}$$

where

$$K_r = \frac{\mu}{1-\mu+(1+\mu^*)E/E^*}. \tag{5}$$

Inserting Eq. (4) into the second Eq. (1), we obtain the governing equation of elasticity for a 1-D problem of longitudinal deformation of column-waveguide layers in a transversely isotropic medium

$$\sigma_z = E_g\varepsilon_r, \tag{6}$$

where the reduced modulus of elasticity is

$$E_g = \frac{E}{1-2\mu K_r}. \tag{7}$$

In particular cases:

$E/E^* \ll 1$ (rigid medium), $K_r \to \mu/(1-\mu)$, $\varepsilon_r \to 0$, and

$$E_g = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}, \tag{8}$$

that coincides with the usual bulk modulus of an elastic body; $E/E^* \gg 1$ (free column without lateral constraints), $K_r \to 0$, $\sigma_r \to 0$, and

$$E_g = E; \tag{9}$$

$E/E^* = 1$ and $\mu/\mu^* = 1$ (materials of the column and medium are isotropic and identical), $K_r = 0.5\mu$ and

$$E_g = \frac{E}{1-\mu^2}. \tag{10}$$

Eqs. (5)–(7) allow taking into account the influence of anisotropy of the medium on longitudinal rigidity of a column.

Equality of medium's and column's axial displacements $u_z^*(R) = u_z$ at their common border $r = R$ causes shear $\gamma_{rz}^*(R)$. If a column's foundation is firm, displacements $u_z^*(r)$ and shears $\gamma_{rz}^*(r)$ must fade at distance from the axis. Under the assumption that the medium's response is described by generalized elastic foundation and displacements $u_z^*(r)$ for a first approximation are $u_z^*(r) = u_z^*(R)(R/r)$, tangential stresses are

$$\tau_{rz}^*(R) = G_{rz}^*\gamma_{rz}^*(R) = -G_{rz}^* u_z/R, \tag{11}$$

where

$$\gamma_{rz}^*(r) \approx \frac{\partial u_z^*(r)}{\partial r} \tag{12}$$

and G_{rz}^* is the medium's shear modulus in the plane rz.

In a case of interaction of the column with a liquid medium, in which no tangential stresses arise, $G_{rz}^* = 0$.

3.2. Calculation method using WFEM approach

Index z at longitudinal forms is further omitted.

Due to WFEM [4,5], the stress-inertial state of a small element j of finite length Δz_j at the discrete time instants $t_i = (i-1)\Delta t$, where $i = 1, 2, \ldots$ is a number of time steps and Δt is a small time interval, is assumed homogeneous with values of stress $\sigma_{j,0}$, strain $\varepsilon_{j,0} = \sigma_{j,0}/E_{jr}$ and velocity $v_{j,0}$. At these moments, strong discontinuities of the specified parameters arise at borders between neighboring elements. As such discontinuities at any point of the solid must immediately decay, node parameters receive new values that are denoted as $\sigma_j^{\pm}$, $\varepsilon_j^{\pm} = \sigma_j^{\pm}/E_{jr}$ and $v_j^{\pm}$ for "left (−)" and "right (+)" element borders. Disturbances $\sigma_j^{\pm} - \sigma_{j,0}$ and others propagate into depth of the element with a speed $c_j = (E_{jr}/\rho_j)^{0.5}$, where ρ_j is material's density for element j, and will reach its opposite borders in time $\Delta t_j = \Delta z_j/c_j$ that assumes the same magnitude $\Delta t_j = \Delta t$ for all elements. Then, the length of elements j of each layer should fulfill a condition $\Delta z_j = c_j\Delta t$ that can be sufficiently well achieved by means of the increase in the number of elements. At the instant $t_{i+1} = t_i + \Delta t$ the stress-inertial state of the element becomes again homogeneous with new values of the parameters defined by equations

$$\sigma_j = \sigma_j^- + \sigma_j^+ - \sigma_{j0}, \quad v_j = v_j^- + v_j^+ - V_{j0}. \tag{13}$$

For the given time step, the border forms are connected with initial ones by equations

$$\sigma_j^{\pm} = \sigma_{j,0} \pm \beta_j(v_j^{\pm} - v_{j,0}), \tag{14}$$

where $\beta_j = \rho_j c_j = \sqrt{\rho_j E_j}$ is an element impedance.

For a column of a constant area, conditions of continuity and equilibrium at borders of the neighboring elements, for example, (j−1) and j, yield

$$v_j = v_{j-1}^+ = v_j^-, \quad \sigma_j^- - \sigma_{j-1}^+ + q_j = 0. \tag{15}$$

Here $q_j = q_{j,f} + q_{j,g}$, where $q_{j,f}$ is external longitudinal force, referred to a border between elements j−1 and j, averaged in time Δt and carried to the area of a column $A = \pi R^2$, and $q_{j,g}$ is a force from tangential stress. According to Eq. (11),

$$q_{j,g} = 2\pi R\Delta z_{jm}\tau_{rz}^*(R)/A = -2\Delta z_{jm}G_{rz}^* u_{jm}/R^2, \tag{16}$$

where an average length of adjacent elements is $\Delta z_{jm} = 0.5(c_{j-1} + c_j)\Delta t$ and averaged for a time Δt displacement of the border between them is $u_{jm} \approx u_{j,0} + 0.5v_j\Delta t$ with initial value $u_{j,0}$ for the given step.

Stresses and velocities at borders between elements are uniquely determined by Eqs. (14)–(16).

Further, the following dimensionless quantities are introduced:

$$\begin{gathered}\bar{\sigma} = \sigma/q_0, \quad \bar{\varepsilon} = \varepsilon E_{r1}/q_0, \quad \bar{v} = v\rho_1 c_1/q_0, \quad \bar{u} = uE_1/L_1 q_0, \quad \bar{t} = tc_1/L_1, \quad \bar{z} = z/L_1, \\ \bar{E} = E/E_1, \quad \bar{G} = G/E_1, \quad \bar{\rho} = \rho/\rho_1, \quad \bar{c} = c/c_1, \quad \bar{\beta} = \beta/\beta_1, \quad \bar{R} = R/L_1,\end{gathered} \tag{17}$$

where q_0 is a value of pressure applied to the column.

Later, the over-bars in most cases are omitted.

It follows from Eqs. (14) and (16)

$$\begin{aligned} \sigma_{j-1}^{+} &= \sigma_{j-1,0} + \beta_{j-1}(v_{j-1}^{+} - v_{j-1,0}), \\ \sigma_{j}^{-} &= \sigma_{j,0} - \beta_{j}(v_{j}^{-} - v_{j,0}), \\ q_{j,g} &= q_{j,g0} - \beta_{j,g} v_{j} \end{aligned} \tag{18}$$

and

$$q_{j,g0} = -\frac{(c_{j-1} + c_j)G_{rz}^{*} u_{j,0}}{R^2}, \quad \beta_{j,g} = -\frac{(c_{j-1} + c_j)G_{rz}^{*}}{2R^2}. \tag{19}$$

Substituting Eq. (18) into (15), we obtain

$$v_{j-1}^{+} = v_{j}^{-} = \frac{\beta_{j-1} v_{j-1,0} + \beta_j v_{j,0} - \sigma_{j-1,0} + \sigma_{J,0} + q_{j,f} + q_{j,g0}\Delta t}{\beta_{J-1} + \beta_j + \beta_{j,g}\Delta t^2} \tag{20}$$

and then, using Eq. (18), we find stresses σ_{j-1}^{+}, σ_{j}^{-}, and also new value $u_j = u_{j,0} + v_j \Delta t$.

Boundary conditions at the column ends at $z = 0$ ($j = 1$), and also at $z = 1$ ($j = n$) are defined according to Eq. (14) as

$$v_1^{-} = v_{1,0} - \sigma_1^{-} + \sigma_{1,0}, \quad \text{with } \sigma_1^{-} = -q \text{ (as } \beta_1 = 1); \tag{21}$$

$$\sigma_n^{+} = \sigma_{n,0} - \beta_n v_{n,0}, \quad \text{with } v_n^{+} = 0 \text{ (for a firm ground).} \tag{22}$$

Calculations are carried out by means of a step-by-step procedure with the step number n_t corresponding to the time $t_{fin} = n_t \Delta t$. By increasing the number of elements and relevant decreasing time steps, the described system of recurrent equations leads to the "exact" wave solution, as a classical method of characteristics [4]. For a multilayered medium each layer is divided into n_k equal elements of the length $\Delta z_k = L_k/n_k$, approximately satisfying condition $\Delta z_k = c_k \Delta t$ with $\Delta t = \text{const.}$, or

$$n_k = n_1 L_k / c_k. \tag{23}$$

4. Numerical results

Three variants of the column-waveguide with characteristics presented in Tables 1 and 2 are studied below. In all cases $\rho_k = 1$, $\Delta z_1 = \Delta t = 1/n_1$, $T_k = L_k/c_k$, $G_{rz}^{*} = 0$. On the subsequent figures, patterns of reflected signals perceived at the free surface of column-waveguides are shown. The response is presented as a dependence of dimensionless longitudinal displacement u upon dimensionless time t with the changed sign of u (a surface displacement directed upwards against the z-axis is considered as positive).

4.1. Response induced by a single impulse

According to the WFEM approach, minimal duration t_{imp} of a single impulse (explosion) can be equaled to one time step Δt, and impulse magnitude related to the area of a column is $q_0 \Delta t$. For imitation of the explosion at some depth $z_{\text{imp}} = (j_{\text{imp}} - 0.5)\Delta z_1$ of the first layer, we accept that during the first step Δt external longitudinal forces $q_{j,f} = -q_0$ and $q_{j+1,f} = q_0$ affect the left and right borders of element j_{imp}, respectively.

Table 1
Characteristics of two-layered systems A and B

System	A		B	
k	1	2	1	2
L_k	1	0.6	1	1.2
C_k	1	2	1	5
T_k	1	0.3	1	0.24
n_k	10	3	100	24

Table 2
Characteristics of a four-layered system C

k	1	2	3	4
L_k	1.0	0.5	0.5	0.2
C_k	1	5	2	5
T_k	1	0.1	0.25	0.04
n_k	100	10	25	4

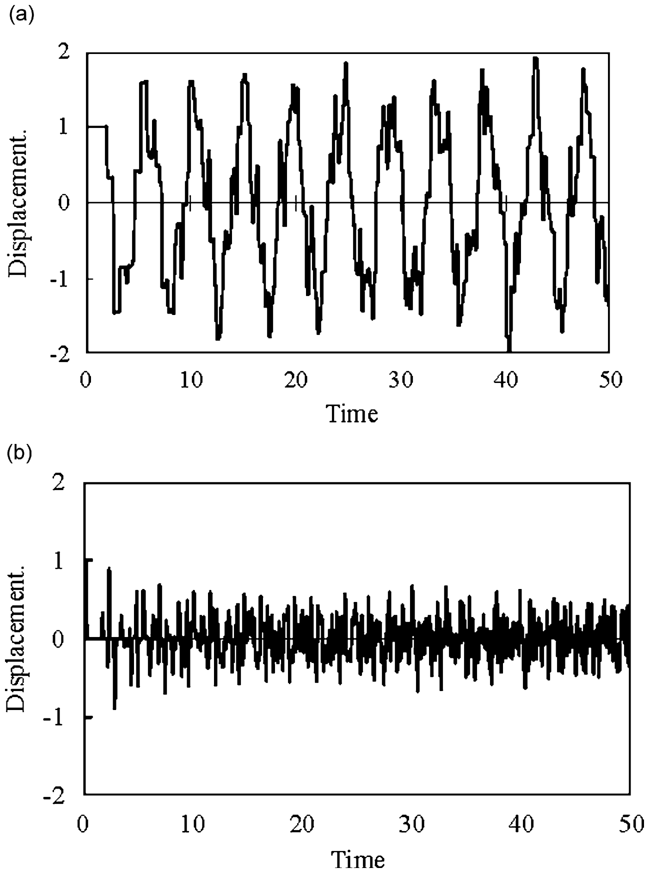

Fig. 2. Signals sequences at a free surface of two-layered system A: (a) surface impulses and (b) inner explosions.

Disturbances induced by them in the form of compression waves will go to the opposite directions—"upward", to the free surface and "downward", to the boundary with the subsequent layer.

In Fig. 2 responses of two-layered system A depending on forms of the applied impulse are presented. In the response to a surface unidirectional impulse ($z_{\text{imp}} = 0$, Fig. 2a) oscillations with the period $t \approx 0.47$ corresponding to the lowest natural frequency of the whole column prevail. Compared to them, other signals are almost not detectable. In the response to an inner explosion ($z_{\text{imp}} = 0.25$, Fig. 2b), when the full impulse applied to the system vanishes, oscillations due to the first natural form are not seen, but the numerous signals reflected from layer borders become distinctly apparent. Below, we shall basically consider responses of this kind. For imitation of a short impulse, a number of elements in each layer must be $n_k \gg 1$, as the system B demonstrates (see Table 1). Here is accepted $z_{\text{imp}} = 0.195$ ($j_{\text{imp}} = 20$). The pattern displayed in Fig. 3 describes wave process in detail, namely: $t_1 = 0.19$—output of the "upward" compression wave to the surface, $t_2 = 1.81$—arrival of the "downward" compression wave reflected from the layers border, $t_3 = 2.19$—arrival of the first expansion wave induced by repeated reflection of the "upward" wave from the layers border,

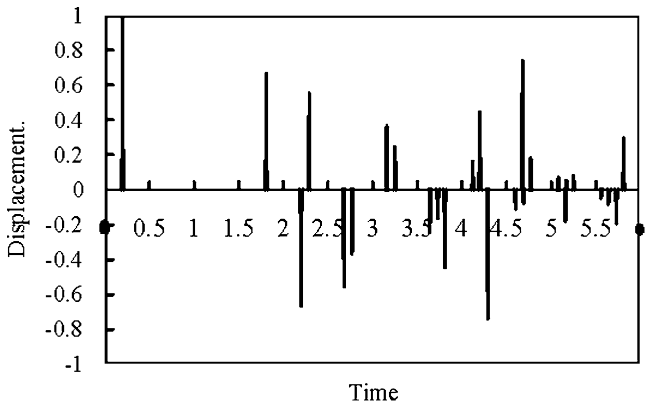

Fig. 3. Surface signals for two-layered system B after inner explosions.

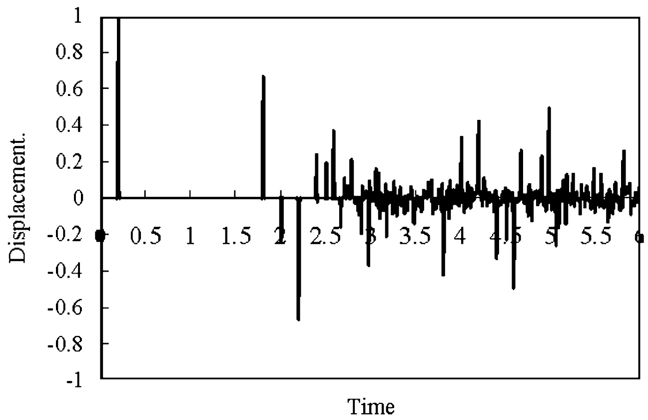

Fig. 4. Surface signals for four-layered system C.

$t_4 = 2.29$—arrival of the "downward" compression wave reflected from the ground, $t_5 = 2.67$—arrival of the second expansion wave generated by "upward" one after its reflection from the ground, $t_6 = 2.77$—arrival of the secondary wave caused by wave reflections within the second layer, etc. Considering the pattern in Fig. 3 as a result of experimental measuring of reflected signals with known layers properties, we can determine positions and thickness of layers as $L_1 = c_1(t_3 - t_1)/2 = 1$ and $L_2 = c_2(t_4 - t_2)/2 = 1.2$. On the other hand, knowing a depth at which an explosion has been made, it is possible to find speed of wave propagation in a corresponding layer. In our case $c_1 = (z_{\text{imp}} - 0.5\Delta z_1)/t_1 = 1$.

In Fig. 4 calculation results for the four-layered system C (see Table 2) are presented. The signal source has remained at the same place. The signal pattern becomes more complex. Already at $t_3 = 2.01$ a new peak appears connected with an expansion wave, reflected from a border between the second and third, softer layer. Careful analysis also allows establishing sources of other peaks. Using the WFEM approach for calculations of wave propagation and reflection, laws of momentum and energy conservation are strictly satisfied. Under the accepted conditions $G^*_{rz} = 0$ and a rigid ground the system of multilayered column-waveguide is conservative; then the reflected signals should not fade with time that is verified by the aforesaid results.

4.2. Response induced by periodical impulses

Results of calculations for the two-layered system B affected by series of consecutive internal explosions with different intervals t_{int}, equal to 0.47, 0.48, 0.49 and 0.50, are presented in Figs. 5a–d, respectively. In Fig. 6 peak values of the response with a level exceeding $u = 1$ are depicted for explosions intervals 0.47, 0.48

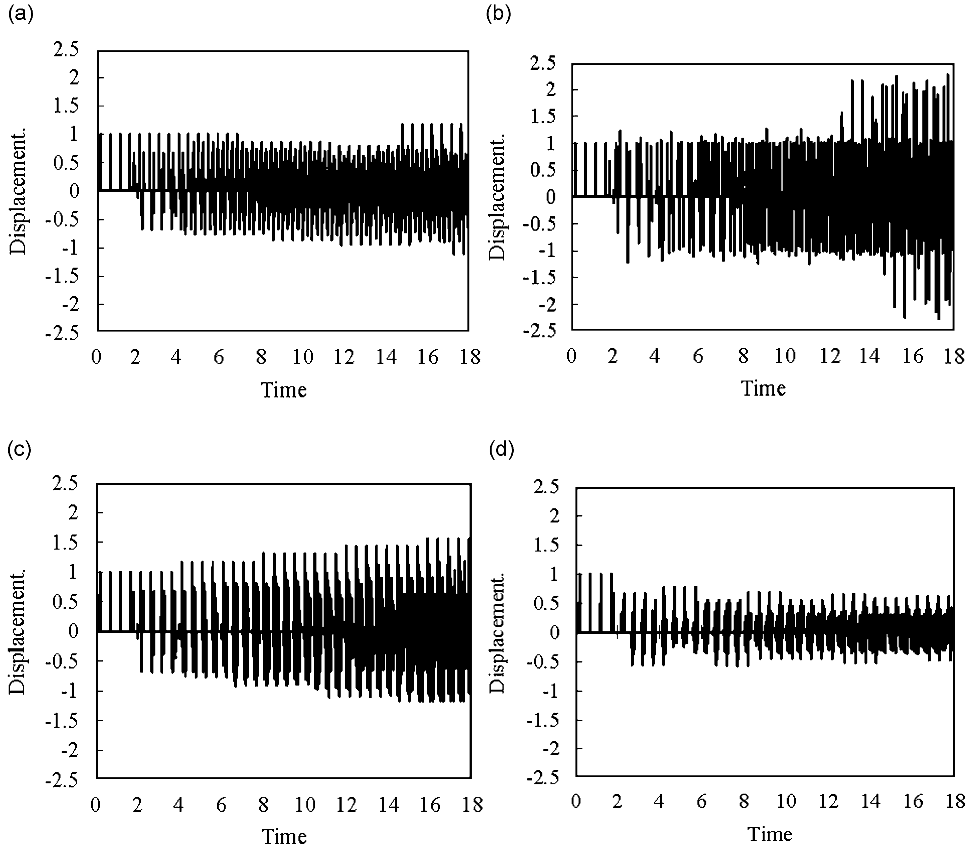

Fig. 5. Surface signals for system B after periodical inner explosions with intervals t_{int}: (a) $t_{int} = 0.47$, (b) $t_{int} = 0.48$, (c) $t_{int} = 0.49$, and (d) $t_{int} = 0.50$.

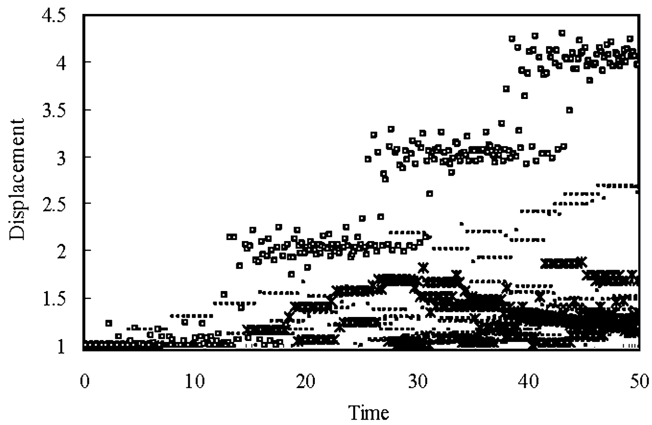

Fig. 6. Same as in Fig. 5 for signal peak values: (∗) $t_{int} = 0.47$, (□) $t_{int} = 0.48$, and (●) $t_{int} = 0.49$.

and 0.49. Here, the process is traced up to essential great time values. The interval $t_{int} = t_4 - t_2 = t_5 - t_3 = 0.48$ is equal to a double time that an elastic wave requires passing through the second layer. One can see, that at such an interval unlimited (though stepwise) amplification of the response is observed, as well as under a condition of a usual resonance in a linear conservative system. Some response amplification occurs also at $t_{int} = 0.49$. But at $t_{int} = 0.47$ and 0.50 such phenomenon is not marked. At the explosions interval $t_{int} = 4.17$

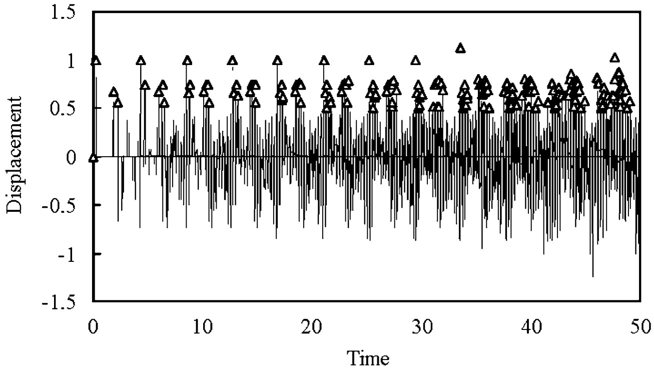

Fig. 7. Response of system B to periodical impulses with interval $t_{int} = 1.47$; vertical lines—full response, dots—local maxima.

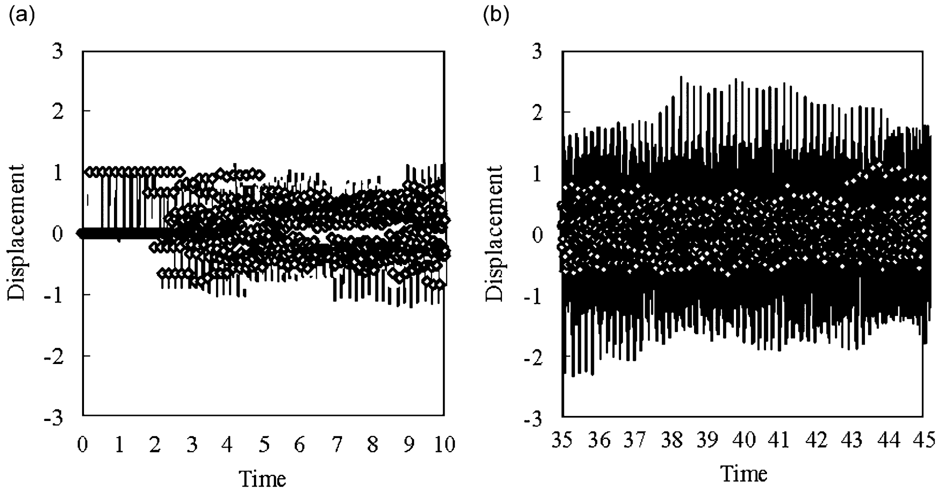

Fig. 8. Surface signals for system C after periodical inner explosions with intervals t_{int}: dots—$t_{int} = 0.18$ and $t_{int} = 0.21$; vertical lines—$t_{int} = 0.19$ and 0.20.

corresponding to the first own oscillation period of the considered two-layered elastic column, periodic impulses caused by internal explosions do not result in a resonance, as it is visible from Fig. 7 where vertical lines show the full calculated response, and dots—local maxima.

The process becomes, naturally, complicated in multilayered system. It is seen from Table 2 that for the four-layer system C dimensionless times of elastic waves passage through the second and third layers are 0.10 and 0.25, respectively. A pattern of the surface response to waves induced by internal explosions repeating with intervals 0.18, 0.19, 0.20 and 0.21 is presented in Fig. 8. The interval $t_{int} = 0.20$ answers the double value of the wave passage time through the second layer. Fig. 8a concerns with the first thousand steps (i.e., $t = 0.01$–10), Fig. 8b, constructed in the same scale, with a range of values $t = 35$–45. The response at $t_{int} \approx 0.19$–0.20 significantly increases with time.

Fig. 9 presents the response to waves' passage through the third layer. The strongest responses are marked near interval $t_{int} \approx 2T_3 \approx 0.50$, but at exact value $t_{int} = 0.50$ the response sharply drops, that, seemingly, is caused by opposite phases of waves reflected from layer borders.

4.3. Harmonic excitation of multilayered column-waveguide oscillations

For harmonic excitation by loads applied to the free surface, it is naturally to await the resonant response of the system to its own frequencies f_m. In Fig. 10a surface displacements of two-layered column-waveguide A are shown under harmonic excitation with the period $t_1 = 1/f_1 = 0.47$, and in Fig. 10b the response under periodic

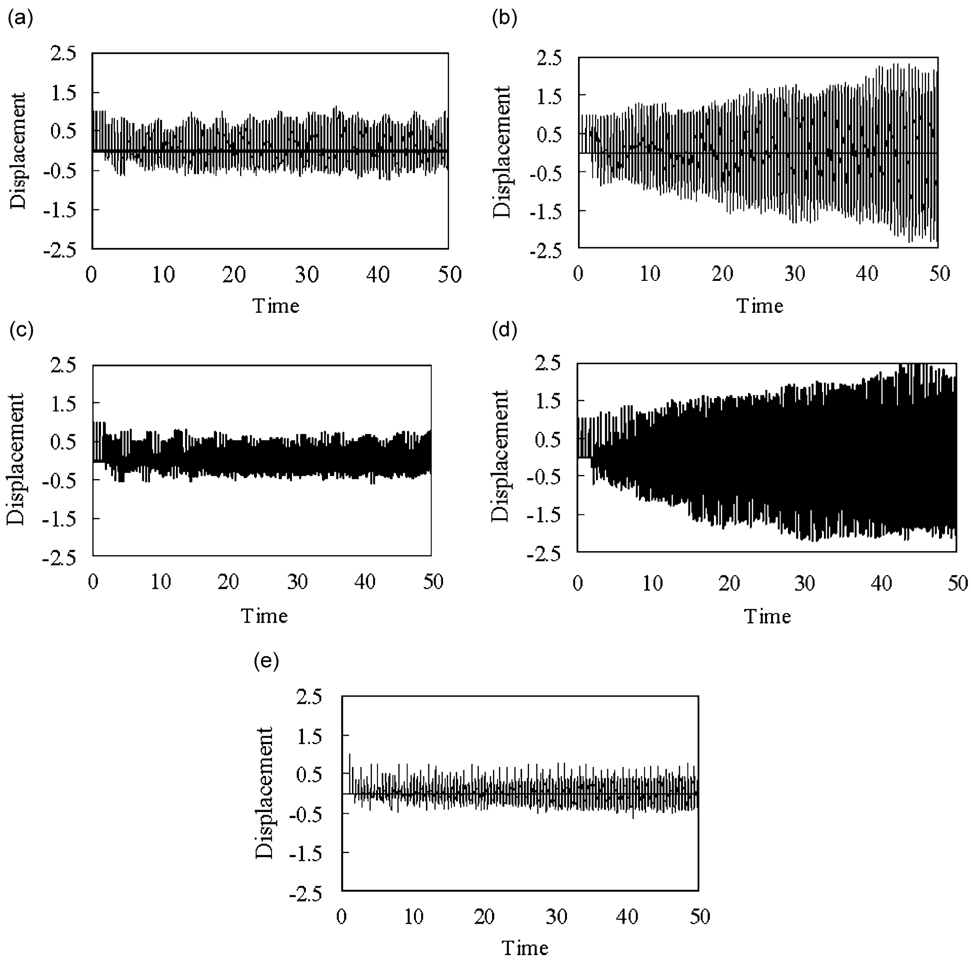

Fig. 9. Same as in Fig. 8 for intervals t_{int}: (a) $t_{\text{int}} = 0.25$, (b) $t_{\text{int}} = 0.49$, (c) $t_{\text{int}} = 0.50$, (d) $t_{\text{int}} = 0.51$, and (e) $t_{\text{int}} = 1.00$.

impulses with the same time interval. In both cases a resonant amplification is seen, but of various intensity. Diagnostic attributes of layers parameters, as well as in the case of a single impulse applied to the surface (cf. Fig. 2), become absolutely imperceptible against a background of strong increase of resonant displacement.

5. Spectral analysis of reflected signals

5.1. Computational procedure

Although a transient wave process for multilayered systems (especially under resonant amplification without damping forces) is not periodic in time, some repeated frequencies of signals are clearly traced at displayed figures. Regarding a calculated response domain within some steps n_t as a part of any periodical process consisting of N sequential discrete values, a procedure of spectral analysis becomes available. Obviously, $N \leqslant n_t$. Due to the discrete Fourier transform, the mth spectral component S_m of the response is determined as

$$S_m = \frac{1}{N}\sum_{n=0}^{n-1} s_n \exp(-\mathrm{i} \times 2\pi mn/N), \quad m = 0, 1, \ldots, N-1, \tag{24}$$

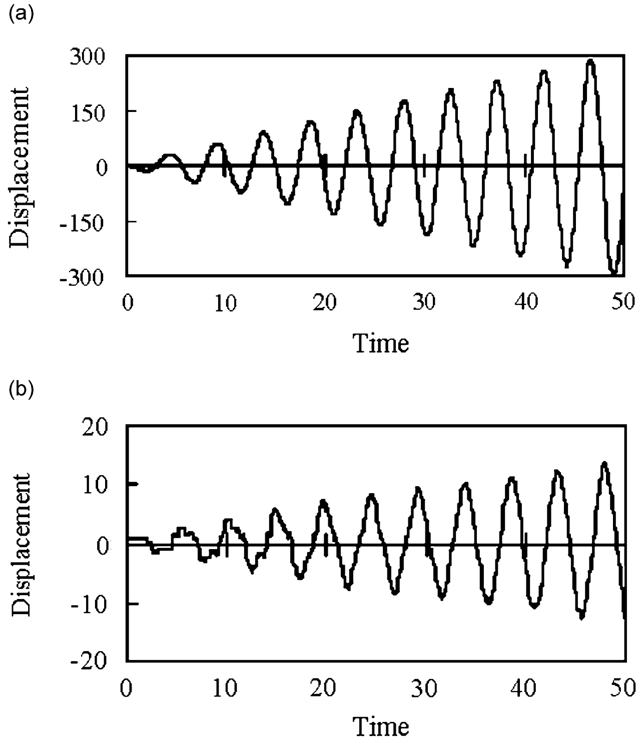

Fig. 10. Surface displacements of system A as a response to applied variable pressure with interval $t_{int} = 1.47$: (a) harmonic excitation, (b) periodical impulses.

where s_n are specified values of displacement $u(t)$ with quantization period Δt, e.g.

$$s_n = u(t), \quad t = n\Delta t, \quad n = 0, 1, \ldots, N-1. \tag{25}$$

The fast Fourier transform algorithm was used to calculate spectrum amplitudes

$$A_m = 2|S_m|; \quad m = 1, 2, \ldots, N/2, \tag{26}$$

with realization length $N = 2^9$–2^{11} (512, 1024, 2048).

To compare the previous calculation data with results of the spectrum analysis, the later are presented in forms $A_m = f(N/m)$ or $A_m = f(i_m)$, where i_m is the nearest integer to the fraction N/m. The value of i_m is equal to the number of response points s_n related to the spectral component with the amplitude A_m. As, on the other hand, $i = t/\Delta t = tn_1$, we obtain a definite connection of spectral amplitude A_m with any dimensionless signal period $T_m = i_m/n_1$. With the purpose to reduce spectral analysis distortions connected with the structure of the Fourier transform limited within its fragment's period, the special method of "focusing" offered in [7] has been used.

For analysis of responses induced by a single impulse, the method is applied to assess harmonics' amplitudes by means of the local spectrum maxima. If at some value m a distinct maximum in the amplitude spectrum is observed, it means that a harmonic constituent of the reflected signal with a period approximately equal to $T_m \approx N/mn_1$ is located in the vicinity $m \pm 0.5$. For periodic impulses the maxima of spectral amplitudes for the same realization length are calculated in dependence on the period of impulse application.

5.2. Examples on spectral analysis of reflected signals

As an elementary example, reflected signals for the column-waveguide A induced by a surface impulse are studied. Fig. 11 shows spectrograms $A_m = f(N/m)$ with a realization length $N = 512$. The first spectrogram corresponds to the excitation due to a single impulse, the subsequent ones to the excitation provoked by

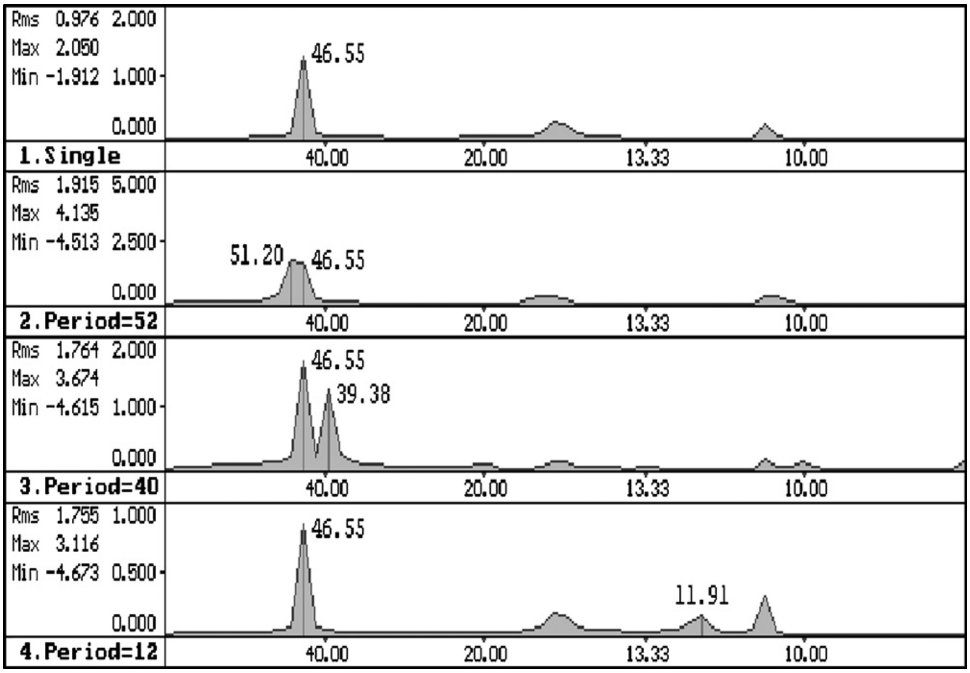

Fig. 11. Spectrogram of reflected signals for system A.

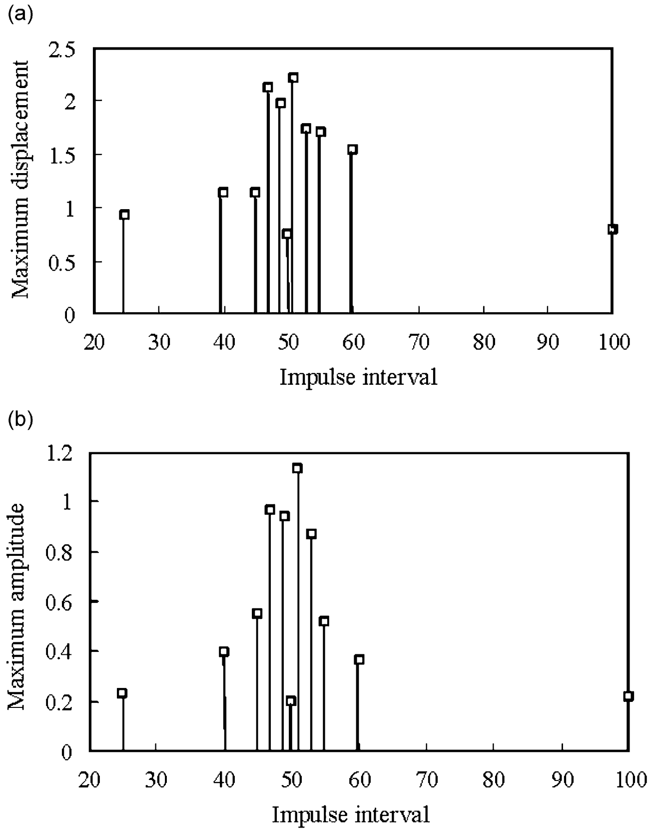

Fig. 12. Response of system C to periodical pulse exciting with interval $n_1 t_{int}$: (a) maximum of calculated displacements u_{max} and (b) maximum of spectral amplitude A_{max}.

repeated impulses with periods multiple to time of wave passage through the corresponding layers: the second with period $4(n_1 + n_2) = 52$, the third with $4n_1 = 40$, the fourth with $4n_2 = 12$. The component $N/m = 46.55$ ($m = 11$, $i_{11} = 47$) which is distinctly determined in all the spectrograms, leads to close correlation of time $T_{11} = i_{11}/n_1 = 4.7$ with the first period of own oscillations $T \approx 4.7$ for the whole console column. Along with this, responses to periodic excitation in the form of local maxima (accurate to the spectral resolution) clearly appear in corresponding spectrograms Nos. 2–4 in Fig. 11, despite the lack of periodicity for the process as a whole.

For the column-waveguide C the spectral analysis of responses to excitation with the periods $t_{\text{int}} < 0.50$, ≈ 0.50 and > 0.50 has been carried out for realization length $N = 2048$. In Fig. 12 the maximum of primary calculated displacements $u_{\max}$ and the maximum of spectral amplitude $A_{\max}$ for each impulse interval $i_{\text{int}} = n_1 t_{\text{int}}$ at $n_1 = 100$ are obtained. In conformity with the qualitative estimate of the curves in Fig. 9, the response sharply drops at value $t_{\text{int}} = 0.50$. Curves of spectral amplitude maxima have smoother character and their magnitude is smaller than the primary calculated maxima depending on various responses superposition. It makes the characteristic $A_{\max}$ more objective for diagnostics of the reflected signals source.

6. Conclusion

The possibility of mathematical simulation of wave propagation in a multilayered medium using a model of column-waveguide affected by surface or internal impulses that approximately imitates a process of oil field sounding is demonstrated. A source of signals can be single or periodic surface impulses and internal explosions. Propagation of 1-D waves under repeated reflections from layer borders is effectively calculated numerically with use of a WFEM. In spite of the fact that the pattern of reflected responses as a whole is not periodic, presence and magnitude of periodical components in their amplitude–frequency spectrum can be revealed by means of the special spectral analysis.

The numerical examples show that calculation sensitively responds to varying loading conditions. The analysis of arrival time of reflected signals caused by internal explosion allows receiving the certain information on layers' location and thickness, including the wave propagation speed. Periodic impact-pulse loading reveals frequencies at which the response noticeably amplifies or weakens, thus allowing to its "resonant" values and associated times required for wave fronts to pass through each layer. However, approaches to practical diagnostic use of calculation results for periodic pulse loading demand further studies.

The proposed way for approximate investigation of wave propagation in a multilayered medium using a single or periodic pulse loading of a column-waveguide model can find applications at processing and analysis of the reflected signals experimentally recorded at sounding of oil fields.

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