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On parameters identification of computational models of vibrations during quiet standing of humans

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Abstract

Vibration of the center of pressure (COP) of human body on the base of support during quiet standing is a very popular clinical research, which provides useful information about the physical and health condition of an individual. In this work, vibrations of COP of a human body in forward–backward direction during still standing are generated using controlled inverted pendulum (CIP) model with a single degree of freedom (dof) supplied with proportional, integral and differential (PID) controller, which represents the behavior of the central neural system of a human body and excited by cumulative disturbance vibration, generated within the body due to breathing or any other physical condition. The identification of the model and disturbance parameters is an important stage while creating a close-to-reality computational model able to evaluate features of disturbance. The aim of this study is to present the CIP model parameters identification approach based on the information captured by time series of the COP signal.

The identification procedure is based on an error function minimization. Error function is formulated in terms of time laws of computed and experimentally measured COP vibrations. As an alternative, error function is formulated in terms of the stabilogram diffusion function (SDF). The minimization of error functions is carried out by employing methods based on sensitivity functions of the error with respect to model and excitation parameters. The sensitivity functions are obtained by using the variational techniques.

The inverse dynamic problem approach has been employed in order to establish the properties of the disturbance time laws ensuring the satisfactory coincidence of measured and computed COP vibration laws. The main difficulty of the investigated problem is encountered during the model validation stage. Generally, neither the PID controller parameter set nor the disturbance time law are known in advance. In this work, an error function formulated in terms of time derivative of disturbance torque has been proposed in order to obtain PID controller parameters, as well as the reference time law of the disturbance. The disturbance torque is calculated from experimental data using the inverse dynamic approach.

Experiments presented in this study revealed that vibrations of disturbance torque and PID controller parameters identified by the method may be qualified as feasible in humans. Presented approach may be easily extended to structural models with any number of dof or higher structural complexity.

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Nomenclature

A, B	first-order low-pass filter coefficients
c	damping of the dynamic system
g	acceleration of gravity
h	distance of COM from the ankle joint
I	mass moment of inertia of the body about the ankle joint
J	error function
J_u	error function based on time laws of computed and experimentally measured COP vibrations
J_S	error function based on SDF
J_T	error function based on time derivative of T_d
k	stiffness of the dynamic system
K_P, K_I, K_D	PID coefficients
m	body mass
$\mathbf{p} = \{K_P, K_I, K_D\}$	vector of CIP model parameters
t	time
T	total time of experiment
T_c	corrective torque
T_d	cumulative disturbance torque
u	sway angle (COM position)
u_{COP}	COP position
u_{ref}	COP values recorded during physical experiment
$u_{\mathrm{ref\_SDF}}$	SDF of COP signal recorded during physical experiment
u_{SDF}	SDF of COP signal generated by CIP model
w	excitation of the dynamic system
x	Gaussian noise
λ, μ, η	conjugate variables
τ	maximum value of Δt
Δt	time lag
COM	center of mass; COM is a point where the body behaves as if its mass is concentrated in that single point
COP	center of pressure; COP is a point in the base of support where ground reaction force acts
CIP	controlled inverted pendulum
PID	proportional, integral, differential (controller)
SDF	stabilogram diffusion function

1. Introduction

The time law of small vibrations of the center of pressure (COP) of humans in standing position (human posture) may provide a lot of useful information about the physical and health condition of an individual. One of the most popular ways to measure standing stability is to register movements of COP on a base of support. The resulting figure is referred to as a *stabilogram* [1,2]. The stabilogram is a collective outcome of all systems that are responsible for maintaining the body in upright position.

One of the most popular models used in order to explain this signal is a model of inverted pendulum [2–9]. It was the model of a pendulum supported in upright position by an elastic spring and linear damper and has been primarily reported in early publications [3,4]. Presented in a form of a very simple mechanical structure, the model is obviously controversial in the context of its ability to represent the features of biomechanical behavior of the human posture. The real signals produced by the neural "control system" of a human in order to provide ability of still standing are obviously much more complex than could be represented by linear feedback present in the model. On the other hand, many authors believed it was still possible to extract the physiologically meaningful information by investigating the behavior of the model and performed corresponding attempts. The investigations on the topic continued to appear over the last three decades [2–6,8–11]. The complexity of the investigated biomechanical model has been increased essentially only by providing the "integrating" feedback, which allowed the overall control to be referred to as proportional, integral and differential (PID) controller, and/or the time delay of the control signal, which could represent the delay of sensory systems of a human body. The inverted pendulum model supplied with a linear PID controller is referred to as a *controlled inverted pendulum* (CIP) *model*, Fig. 1. As real stabilograms contain vibrations in a wide frequency range (Fig. 2a), the stabilogram diffusion function (SDF) was introduced by Collins and De Luca [2] in order to evaluate integral characteristic properties of the signal. The simple linear CIP model is known as able to "maintain features of experimental SDFs" [8]. CIP model represents

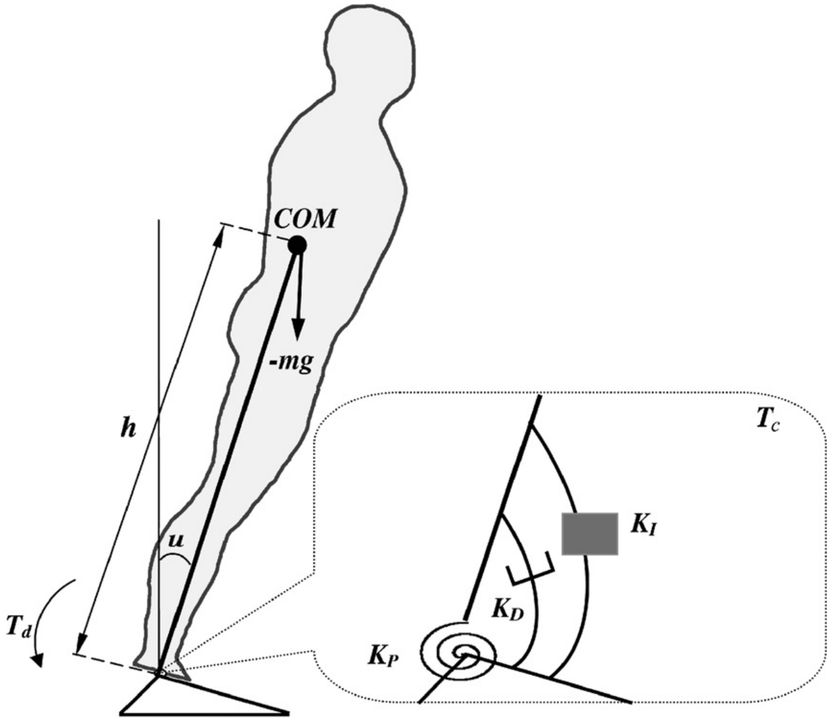

Fig. 1. CIP model of a human posture in forward–backward direction. Center of body mass m (COM) is placed at the height h above the ankle joint and sways away by an angle u from the equilibrium position ($u = 0$) due to the gravity acceleration g and the disturbance torque T_d. The standing stability is maintained due to the corrective torque T_c, which is implemented as linear combination of proportional (K_P), integral (K_I) and differential (K_D) controller.

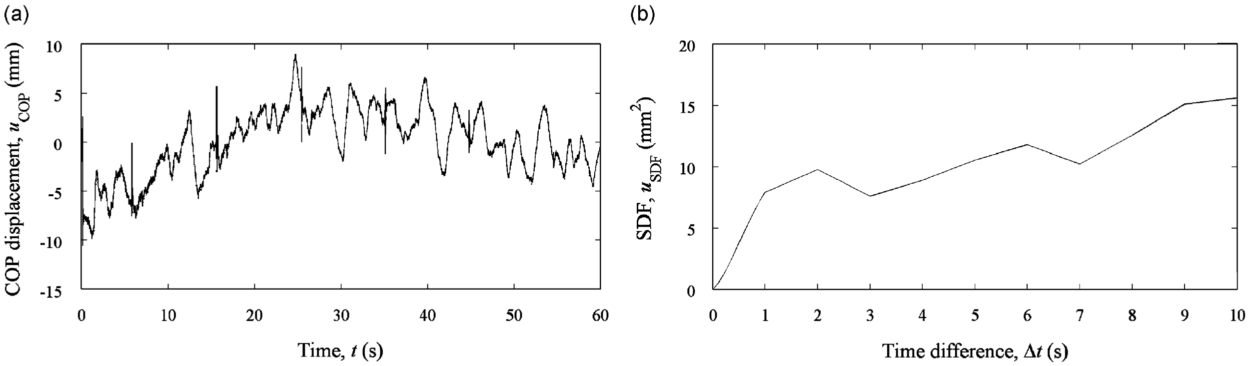

Fig. 2. (a) Stabilogram in forward–backward direction and (b) stabilogram diffusion function.

movements of COP in forward–backward direction during still standing. According to Peterka [8], the advantage of CIP model is the ability to explain the movement of COP in physiologically meaningful terms. The equilibrium of a standing person is maintained by means of forces appearing due to the displacement of the center of mass (COM) of the body (proportional component of PID controller), speed of the COP (differential component of PID controller) and integral of past displacements of COM (integral component of PID controller).

There were recent attempts to identify parameters of neural control system (PID controller) of the CIP model based on parameters derived from COP signal [3] (but not from the time series of the recorded COP signal) or from the time series of COM position [7]. In the latter research, the CIP model was not excited by any kind of disturbance torque, which many researchers [2,5–8] suppose is constantly generated by the central nervous system of a human body. There were no attempts reported so far which intended to identify the parameters of CIP model from time series of COP signal of a particular person recorded during specific physical experiment and the time series of disturbance torque at the same time.

The aim of this study is to identify the set of CIP model control parameters of a particular person from the recorded COP signal (stabilogram) and the time series of the disturbance torque based on the data from real experiments. Parameter identification technique uses sensitivity functions, which serve as search directions during the minimization of the error function. Error functions based on differences of displacement time laws, as well as based on SDF are employed. As novel features of the approach, improved evaluation of the disturbance torque, the inverse dynamic problem solution and parameter identification by using the data of conducted experiment could be mentioned. The identification of the disturbance torque time law together with the parameters of PID controller has been performed applying an iterative procedure proposed in this work.

2. Methods

2.1. Controlled inverted pendulum model and error functions formulation

2.1.1. Controlled inverted pendulum model

The CIP model represents the human body as a rigid body with COM oscillating around the ankle joint. An assumption is made that the posture deviates away from upright position due to some physiological factors (e.g. aging or loss of vision) [1,13,15] and the torque generated by the gravity force and that "neural control system senses a deviation of the body away from an upright reference position" [8]. The influence of such factors in the CIP model is represented as the *cumulative disturbance torque* T_d.

According to [8], the cumulative disturbance torque represented as low-filtered Gaussian noise is feasible in humans. In order to counteract the disturbance torque the central nervous system of a human body produces the *corrective torque* $T_c(t)$. The corrective torque is assumed to be a linear function of the sway angle u, the time integral of the sway angle u and the angular velocity $\dot{u}$. In Ref. [8] it has been referred to as *a PID controller*.

The dynamic equation of such a system is obtained on the base of the angular momentum principle as

$$I\ddot{u}(t) - mghu(t) = T_d(t) - T_c(t), \tag{1}$$

where cumulative disturbance torque T_d is obtained from the filtered Gausian noise: $T_d(t) + B\dot{T}_d(t) = Ax(t)$; corrective torque T_c is organized as PID controller: $T_c(t) = K_P u(t) + K_I \int_0^t u(t)\,\mathrm{d}t + K_D \dot{u}(t)$.

Proper set of values of CIP model corrective torque T_c parameters $\mathbf{p} = \{K_P, K_I, K_D\}$ have to be determined in order to make the response of the model close to time law of vibrations of a COP signal registered during physical experiment.

2.1.2. Error function formulation in terms of time laws of computed and experimentally measured COP vibrations

A quantitative measure of the deviation of the model behavior from the available experimental record can be introduced by means of the error function J as

$$J(u) = \int_0^T \psi(u)\,\mathrm{d}t, \tag{2}$$

where

$$\psi(u) = \tfrac{1}{2}(u_{\mathrm{COP}}(t) - u_{\mathrm{ref}}(t))^2, \tag{3}$$

$$u_{\mathrm{COP}}(t) = hu(t) - \frac{I\ddot{u}(t)}{mg} - \text{COP values calculated from CIP model (taken from [8])}. \tag{4}$$

The error function J formulated in terms of time laws of computed and experimentally measured COP vibrations (Eq. (3)) will be denoted as error function J_u.

2.1.3. Error function formulation in terms of stabilogram diffusion functions of computed and experimentally measured COP vibrations

An SDF was introduced by Collins and De Luca [2]. It is an average estimate of difference of two COP positions separated by the time Δt:

$$u_{\mathrm{SDF}}(\Delta t)=\frac{\int_0^{T-\Delta t}(u_{\mathrm{COP}}(t+\Delta t)-u_{\mathrm{COP}}(t))^2\,\mathrm{d}t}{T-\Delta t}. \tag{5}$$

For the healthy humans SDF has a specific two part form [2,8,14] (Fig. 2b): the first part is the steeper and the second—flatter. SDF shows how the current displacement of COP depends of past displacements of COP. Two regions of control are distinguished: the short term region (Δt changes from 0 to 1 s), where the future displacements increments of COP are positively correlated with the present position of COP (steeper part of SDF), and the long-term region (Δt changes from 1 to 10 s), where the future increments of displacements of COP are negatively correlated with the present position of COP (flatter part of SDF). SDF is often used in studies of human posture stability [5,8,13–16]. Besides that, Peterka [8] demonstrated that COP signals generated by CIP model may be treated as realistic and feasible in humans and reported margin values of **p**, which are able to maintain SDF in a physiological shape. **p** values recalculated in SI units are shown in Table 1.

As an alternative to the error function J_u based on time laws of computed and experimentally measured COP vibrations, error function J_S is constructed by using the difference of the model SDF and SDF obtained from the physical experiment:

$$J_S(\Delta t)=\int_0^{\tau}\frac{1}{2}\left(u_{\mathrm{SDF}}(\Delta t)-u_{\mathrm{ref\_SDF}}(\Delta t)\right)^2 d(\Delta t). \tag{6}$$

Substitution of Eqs. (4) and (5) into Eq. (6) gives

$$J_S(\Delta t)=\int_0^{\tau}\psi(u,\ddot{u},\Delta t)d(\Delta t), \tag{7}$$

where

$$\psi(u,\ddot{u},\Delta t)=\frac{1}{2}\left(\frac{\int_0^{T-\Delta t}\left(hu(t+\Delta t)-(I/mg)\ddot{u}(t+\Delta t)-hu(t)+(I/mg)\ddot{u}(t)\right)^2\mathrm{d}t}{T-\Delta t}-u_{\mathrm{ref\_SDF}}(\Delta t)\right)^2. \tag{8}$$

2.2. Error function minimization

2.2.1. The basic relations of the method

The error function J minimization is performed by using the steepest descent method [19], which is based on the sensitivity functions [17]. The application of the method to CIP model parameters identification was presented by Barauskas and Krušinskienė in Ref. [18].

Consider the dynamic system as

$$m\ddot{u}+c\dot{u}+ku=w(u,\dot{u},\mathbf{p}),$$

where vector **p** contains the parameters of the model.

Table 1
CIP models' parameters sets **p** (K_P—proportional component coefficient, K_I—integral component coefficient, K_D—differential component coefficient), which are able to maintain physiological stabilogram diffusion function (SDF)

p	Minimal value	"Center normal" value	Maximal value
K_P (N m rad^{-1})	1005.54	1117.27	1228.99
K_I (N m rad^{-1} s^{-1})	0	14.32	114.59
K_D (N m rad^{-1} s)	232.05	257.83	283.61

The sensitivity function vector of the error function reads as $\partial J/\partial \mathbf{p}$. The vector represents the search direction in the space of parameters $\mathbf{p}$. In order to obtain $\partial J/\partial \mathbf{p}$, we introduce time derivatives of conjugate variables and express the variation δJ of the error function in terms of $\delta \mathbf{p}$. The basic variation relation reads as

$$\delta J = \int_0^T (\lambda + \dot{\mu} + \ddot{\eta})\left(\frac{\partial w(u_T, \dot{u}_T, \mathbf{p})}{\partial \mathbf{p}}\delta \mathbf{p}\right) \mathrm{d}t. \tag{9}$$

Time laws of conjugate variables and their time derivatives $\lambda(t), \dot{\mu}(t), \ddot{\eta}(t)$ are obtained by time integration of the conjugate differential equations

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = \dfrac{\partial \psi}{\partial u}, \\ \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = -\dfrac{\partial \psi}{\partial \dot{u}}, \\ \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = \dfrac{\partial \psi}{\partial \ddot{u}}. \end{cases} \tag{10}$$

With the following boundary conditions

$$\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = \mu_T = \eta_T, \tag{11}$$

where

$$\tilde{c} = c - \frac{\partial w(u, \dot{u}, \mathbf{p})}{\partial \dot{u}}, \quad \tilde{k} = k - \frac{\partial w(u, \dot{u}, \mathbf{p})}{\partial u}.$$

Differential equation of CIP model equation (1) is considered together with error function J_u (based on Eqs. (2) and (3)).The partial derivatives $(\partial\psi/\partial u)$, $(\partial\psi/\partial\dot{u})$, $(\partial\psi/\partial\ddot{u})$ of under-integral of error function J_u read as

$$\begin{aligned} &\frac{\partial \psi}{\partial u} = \frac{\partial}{\partial u}\left(\frac{1}{2}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right)^2\right) = h\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right), \\ &\frac{\partial \psi}{\partial \dot{u}} = 0, \\ &\frac{\partial \psi}{\partial \ddot{u}} = \frac{\partial}{\partial \ddot{u}}\left(\frac{1}{2}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right)^2\right) = -\frac{I}{mg}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right). \end{aligned} \tag{12}$$

Taking into account Eq. (12), Eq. (10) reads as

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = h\left(hu(t) - \dfrac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right), \\ \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = 0, \\ \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = -\dfrac{I}{mg}\left(hu(t) - \dfrac{I\ddot{u}(t)}{mg} - u_{\mathrm{ref}}(t)\right) \end{cases} \tag{13}$$

with initial conditions derived from Eq. (11).

The sensitivity function vector $\partial J_u/\partial \mathbf{p}$ is calculated from Eq. (9):

$$\frac{\partial J_u}{\partial \mathbf{p}} = \begin{Bmatrix} \dfrac{\partial J_u}{\partial K_P} \\ \dfrac{\partial J_u}{\partial K_I} \\ \dfrac{\partial J_u}{\partial K_D} \end{Bmatrix} = \begin{Bmatrix} \int_0^T (-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))u(t))\,\mathrm{d}t \\ \int_0^T \left(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) \int_0^t u(\tau)d\tau\right) \mathrm{d}t \\ \int_0^T (-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\dot{u}(t))\,\mathrm{d}t \end{Bmatrix}. \tag{14}$$

In order to minimize the error function J_u and solve Eq. (14) the *steepest descent* optimization method was used [19].

2.2.2. Minimization of the error function based on SDF

Differential equation of CIP model equation (1) is considered together with the error function J_S (Eq. (6)) used as a target function. The partial derivatives $(\partial\psi/\partial u)$, $(\partial\psi/\partial\dot{u})$, $(\partial\psi/\partial\ddot{u})$ of under-integral of error function J_S read as

$$\begin{aligned}
\frac{\partial\psi}{\partial u} &= 2h\int_0^{T-\Delta t}\left(hz(t)-\frac{I}{mg}\ddot{z}(t)\right)\mathrm{d}t\left(\frac{\int_0^{T-\Delta t}\left(hz(t)-(I/mg)\ddot{z}(t)\right)^2 dt}{T-\Delta t}-u_{\mathrm{ref_SDF}}(\Delta t)\right),\\
\frac{\partial\psi}{\partial\dot{u}} &= 0,\\
\frac{\partial\psi}{\partial\ddot{u}} &= 2\frac{I}{mg}\int_0^{T-\Delta t}\left(hz(t)-\frac{I}{mg}\ddot{z}(t)\right)\mathrm{d}t\left(\frac{\int_0^{T-\Delta t}\left(hz(t)-(I/mg)\ddot{z}(t)\right)^2 \mathrm{d}t}{T-\Delta t}-u_{\mathrm{ref_SDF}}(\Delta t)\right),
\end{aligned}\tag{15}$$

where $z(t)=u(t+\Delta t)-u(t)$.

Therefore, Eq. (10) takes the form

$$\begin{cases}
\ddot{\lambda}m-\dot{\lambda}\tilde{c}+\lambda(\tilde{k}-\dot{\tilde{c}})-\mu\dot{\tilde{k}}-\dot{\eta}\dot{\tilde{k}} = 2h\int_0^{T-\Delta t}\left(hz(t)-\frac{I}{mg}\ddot{z}(t)\right)\mathrm{d}t\left(\frac{\int_0^{T-\Delta t}\left(hz(t)-(I/mg)\ddot{z}(t)\right)^2 \mathrm{d}t}{T-\Delta t}-u_{\mathrm{ref_SDF}}(\Delta t)\right),\\
\ddot{\mu}m-\dot{\mu}\tilde{c}+\mu\tilde{k}-\dot{\eta}\dot{\tilde{c}}+\eta\dot{\tilde{k}}=0,\\
\ddot{\eta}m-\dot{\eta}\tilde{c}+\eta\tilde{k} = 2\frac{I}{mg}\int_0^{T-\Delta t}\left(hz(t)-\frac{I}{mg}\ddot{z}(t)\right)\mathrm{d}t\left(\frac{\int_0^{T-\Delta t}\left(hz(t)-(I/mg)\ddot{z}(t)\right)^2 \mathrm{d}t}{T-\Delta t}-u_{\mathrm{ref_SDF}}(\Delta t)\right)
\end{cases}\tag{16}$$

with initial conditions derived from Eq. (11).

The sensitivity function vector $\partial J_S/\partial\mathbf{p}$ is calculated from Eq. (9):

$$\frac{\partial J_S}{\partial\mathbf{p}}=\begin{Bmatrix}\dfrac{\partial J_S}{\partial K_P}\\ \dfrac{\partial J_S}{\partial K_I}\\ \dfrac{\partial J_S}{\partial K_D}\end{Bmatrix}=\begin{Bmatrix}\int_0^T(-(\lambda(t)+\dot{\mu}(t)+\ddot{\eta}(t))z(t))\,\mathrm{d}t\\ \int_0^T\left(-(\lambda(t)+\dot{\mu}(t)+\ddot{\eta}(t))\int_0^t z(\tau)\,\mathrm{d}\tau\right)\mathrm{d}t\\ \int_0^T(-(\lambda(t)+\dot{\mu}(t)+\ddot{\eta}(t))\dot{z}(t))\,\mathrm{d}t\end{Bmatrix}.\tag{17}$$

2.3. Disturbance torque identification from experimental COP signal

Analysis of the data obtained by Peterka [8] and the results presented in Section 4 of this paper revealed that the COP signal shape generated by the CIP model is much more dependent on the time law of cumulative disturbance torque T_d than on changes of the model parameter set $\mathbf{p}$. The disturbance torque generation as low-filtered Gaussian noise as presented in Ref. [8] may appear unduly general when a particular person is investigated. On the other hand, the disturbance torque generated of a particular investigated person may be easily calculated employing the inverse dynamic problem approach by substituting the data of physical experiments (from COP signal) into Eq. (1). The procedure proposed in this work enables to identify both the disturbance torque signal T_d, which was present within the human body during particular physical experiment, and the set of the CIP models' parameters $\mathbf{p}$.

From Eq. (1) T_d reads as

$$T_d(t)=w(u,\dot{u},\ddot{u},\mathbf{p}),\quad w(u,\dot{u},\ddot{u},\mathbf{p})=I\ddot{u}(t)+K_D\dot{u}(t)+(K_P-mgh)u(t)+K_I\int_0^t u(t)\,\mathrm{d}t.\tag{18}$$

The COM signal u which is used for T_d reconstruction is calculated from the experimental COP signal passed through 4th order low-pass Butterworth filter as proposed by Benda et al. [20]. The time derivatives of u are calculated by using numerical differentiation and substituted into (18).

The obtained result may be considered as an approximation of disturbance torque T_d of a particular person provided that the values of CIP model parameters **p** are known. Moreover, T_d can be reconstructed for any set of parameters **p**.

In order to find the most realistic parameter set **p**, further restrictions are imposed on signal T_d. Let us formulate a target function J_T in terms of the disturbance torque time derivative as

$$J_T(u) = \int_0^T \psi(u, \dot{u}, \ddot{u}, p)\,\mathrm{d}t = \frac{1}{2}\int_0^T \left(\dot{T}_d(t)\right)^2 \mathrm{d}t, \tag{19}$$

which should be minimized by obtaining appropriate values of parameter set **p**. The objective to minimize the speed of disturbance torque $\dot{T}_d$ was chosen due to assumption that central nervous system tries to eliminate the disturbance torque and the general intension of a body is to stand still.

The minimization technique is based on sensitivity functions as described in Section 2.2.1.

From the basic variation relation (Eq. (9)) error function derivative $\partial J_T/\partial \mathbf{p}$ reads as

$$\frac{\partial J_T}{\partial \mathbf{p}} = \begin{Bmatrix} \dfrac{\partial J_T}{\partial K_P} \\ \dfrac{\partial J_T}{\partial K_I} \\ \dfrac{\partial J_T}{\partial K_D} \end{Bmatrix} = \begin{Bmatrix} \int_0^T ((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))u(t))\,\mathrm{d}t \\ \int_0^T \left((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) \int_0^t u(\tau)d\tau\right) \mathrm{d}t \\ \int_0^T ((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\dot{u}(t))\,\mathrm{d}t \end{Bmatrix}. \tag{20}$$

Partial derivatives $(\partial\psi/\partial u)$, $(\partial\psi/\partial\dot{u})$, $(\partial\psi/\partial\ddot{u})$ of under-integral of error function J_T calculated from Eq. (19) read as

$$\frac{\partial\psi}{\partial u} = K_I, \quad \frac{\partial\psi}{\partial\dot{u}} = -(K_P - mgh), \quad \frac{\partial\psi}{\partial\ddot{u}} = K_D. \tag{21}$$

The time laws of conjugate variables time derivatives $\lambda(t), \dot{\mu}(t), \ddot{\eta}(t)$ are obtained from Eqs. (10) and (21):

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = K_I, \\ \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = -(K_P - mgh), \\ \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = K_D. \end{cases}$$

They are solved together with initial conditions $\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = \mu_T = \eta_T$.

In order to minimize the error function J_T and solve Eq. (20) the *steepest descent* optimization method was used [19].

The whole procedure is performed iteratively: the signal of disturbance torque T_d is calculated employing inverse dynamic approach (Eq. (18)), then the error function minimization step is performed (Eqs. (19)–(21)). The algorithm stops when error function J_T changes less than 0.05%. Such an approach allows identification of models' disturbance torque T_d and parameters set **p** at the same time using only one COP signal, which was recorded during physical experiment.

3. Implementation

One healthy adult male whose physical data may be considered as a "typical" (according to [8]) for an average adult male (mass 60 kg, height 1.70 m) took part in physical experiments. His stabilograms were recorded. Anthropometric data were taken from Ref. [8]: the mass moment of inertia $I = 76\,\mathrm{kg\,m^2}$, mass $m = 60\,\mathrm{kg}$ and distance of COM from the ankle joint $h = 0.87\,\mathrm{m}$.

Experimental COP signal was recorded during physical experiments by using sample rate of 100 Hz. The duration of the experiment was 60 s. The CIP model parameters identification algorithms were implemented in Matlab7. Disturbance torque T_d signal used in parameters identification procedures which are presented in Sections 2.2.1 and 2.2.2 was produced by Matlab function "randn" and then low-filtered (Fig. 3) with parameters set to $A = 1000$, $B = 80$, proposed in Ref. [8] as able to construct disturbance torque feasible in human body during still standing.

The initial model parameters set **p** was chosen from Peterka [8] and recalculated in SI units. The initial set **p** is called "Center normal" (Table 1) and according to [8] is able to produce a physiological SDF.

4. Results

4.1. Identification of CIP model parameters when T_d is given as filtered white noise

Numerical experiment employing error function J_u has been performed in order to identify the CIP model's parameters set **p** presented in left column of Table 2. Comparison of obtained data with values of **p** presented in Table 1 shows that the identified set **p** slightly shifted towards maximal values. This means that the CIP model with identified set **p** is able to produce realistic SDFs and the most important gain in PID controller is the proportional K_P gain (it shifted most of all).

The fragment of COP signals computed from the model with the initial and identified set of parameters and experimental COP signal taken from 15 to 25th second of experiment are presented in Fig. 4.

Fig. 4 shows that after parameter identification modeled COP signal is closer to the experimental COP signal, but it cannot be stated that the identified and experimental signals are coincident. The error function J_u minimization took 29 iterations. During identification error function J_u decreased from the initial value 2.1513 to 2.0543 mm^2 s.

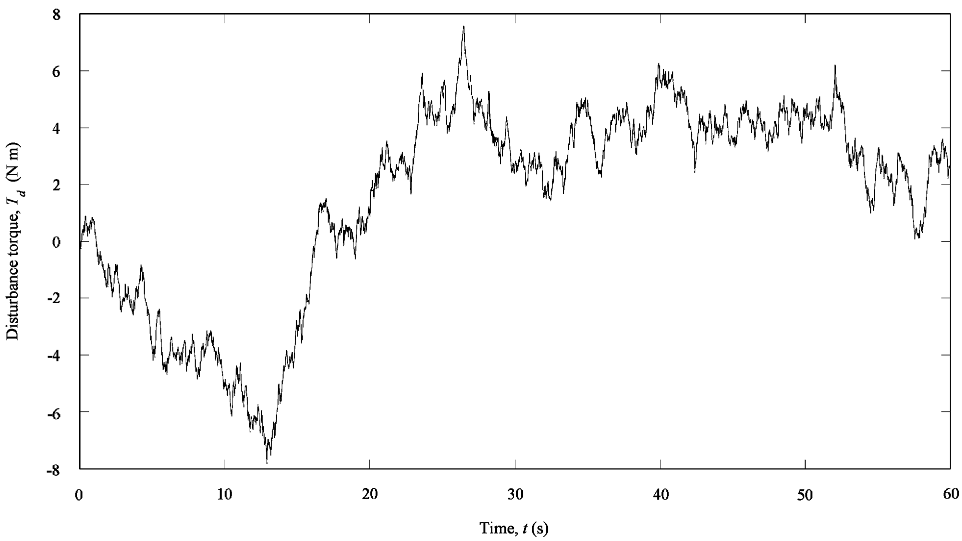

Fig. 3. Disturbance torque when generated as a low-filtered white noise.

Table 2
Identified CIP models' parameters sets **p** obtained using different identification techniques: J_u—error function based on time laws of computed and experimentally measured COP vibrations; J_S—error function based on COP SDFs; J_T—error function based on time derivative of T_d and comparison of J_u values calculated with identified sets **p**

Identification technique	Based on J_u minimization	Based on J_S minimization	Based on J_T minimization
Identified set **p**			
K_P (N m rad^{-1})	1131.62	1117.27	1045.75
K_I (N m rad^{-1} s^{-1})	14.70	14.32	16.6996
K_D (N m rad^{-1} s)	260.33	265.35	237.588
J_u (mm^2 s)	2.0543	2.14525	0.0345

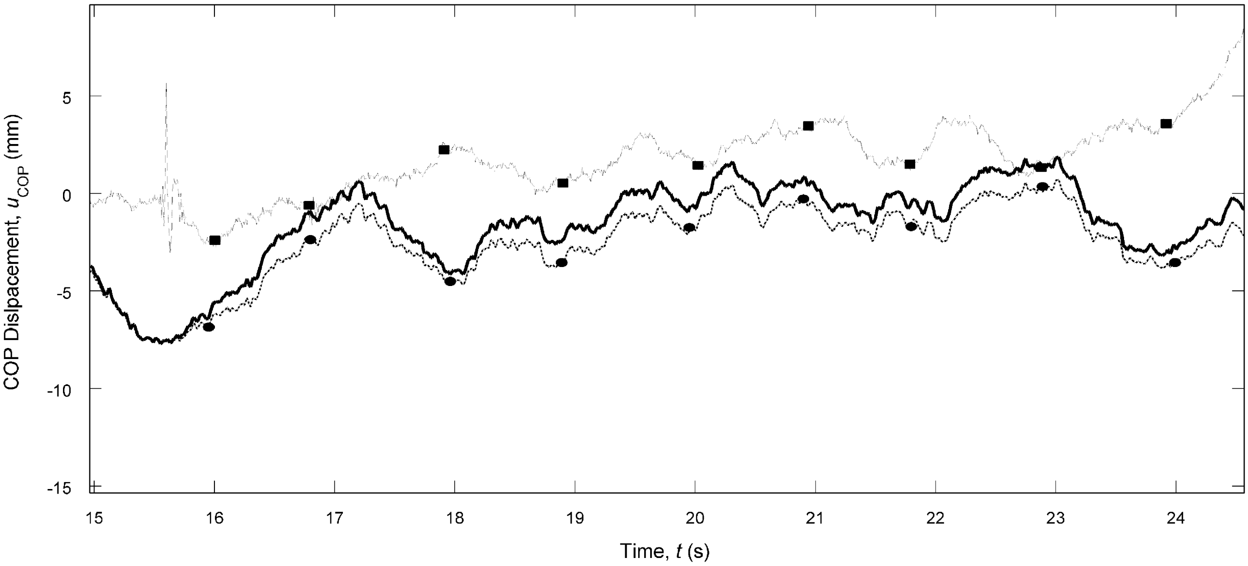

Fig. 4. COP signals. · · ■ · ·, u_{ref}; —, identified u_{COP}; - -●- -, u_{COP} with the initial set of parameters **p**.

The center column of Table 2 presents the parameters set **p** identified during numerical experiment employing error function J_S based on SDF. The error function J_S decreased from the initial value 1.3120 to $1.2990\,e^{-2}\,mm^2$. The SDFs of COP signals computed from the experimental COP signal, COP signal of CIP model with the initial and identified set of parameters **p** are presented in Fig. 5. It may be visually seen that "identified" SDF only slightly shifted towards SDF computed from the experimental COP signal.

In order to compare two experiments with error functions J_u and J_S, the error function J_u was calculated with the set **p** identified using J_S. The error function J_u was equal to $2.14525\,mm^2\,s$ (see central column of Table 2). This implies that the parameters identification algorithm employing error function J_u constructed from the difference of COP signals computed from the model and recorded during physical experiment was able to identify the parameters set **p** more accurately. It should be noted that results provided by both identification techniques were able to produce physiological SDFs (because identified sets **p** lie within the margin values provided in Table 1) and both techniques were not able to produce COP signal which have a fairly good coincidence with COP signal recorded during physical experiment.

Both experiments confirmed the fact that the shape of COP signal produced by CIP model is much more dependant on the time law of the disturbance torque T_d (Fig. 6) than on the slight changes in parameters set **p**.

4.2. Disturbance torque and PID parameters identification of a particular person

The identification of the disturbance torque and PID parameters was performed as described in Section 2.3. The disturbance torque T_d and parameters set **p** identification procedure took 46 iterations. During the procedure the error function J_T (Eq. (19)) decreased by 12% from the initial value of 130.75 to $114.76\,N\,m^2\,s^{-1}$. The identified parameters set **p** is presented in the right column of Table 2.

The identified set of parameters according to Peterka [8] may be treated as capable to generate realistic stabilograms. In order to evaluate accuracy of identification, the error function J_u was calculated with the set **p** obtained using T_d and PID parameters of a particular person. The error function J_u was equal to $0.0345\,mm^2\,s$. From comparison of J_u values calculated with sets of **p** identified using different techniques (see Table 2) it is obvious that the identified disturbance torque T_d had huge impact on the accuracy of the obtained results.

The identified disturbance torque T_d, COM signal computed using CIP model with parameters set **p** from right column of Table 2 and COM signal computed using "center normal" parameters set **p** from Table 1 are presented in Fig. 7. It is visually seen that shapes of disturbance torque T_d, COM signal with an identified set **p**, and the COM signal with "central normal" are very similar.

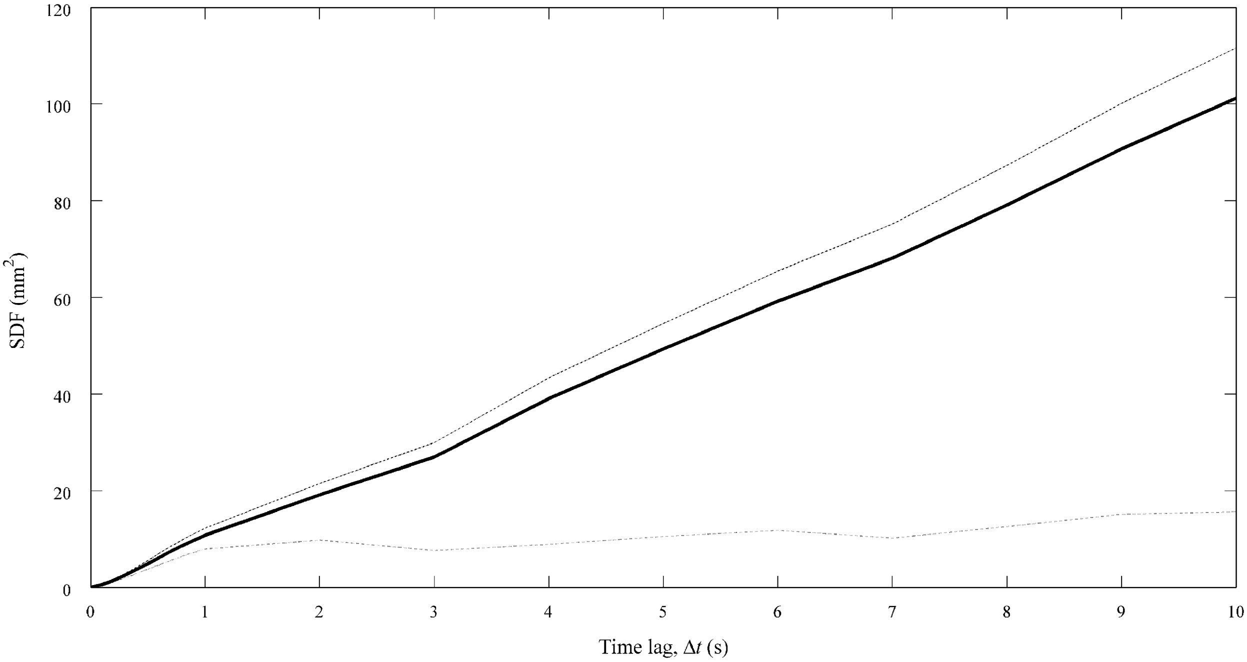

Fig. 5. SDFs of COP signals. · · · · · · · · ·, computed from u_{ref}; —, computed from identified u_{COP}; - · - · - ·, computed from u_{COP} with the initial set of parameters $\mathbf{p} = \{K_P = 1117.27\,\mathrm{N\,m\,rad^{-1}},\ K_I = 14.32\,\mathrm{N\,m\,rad^{-1}\,s^{-1}},\ K_D = 257.83\,\mathrm{N\,m\,rad^{-1}\,s}\}$.

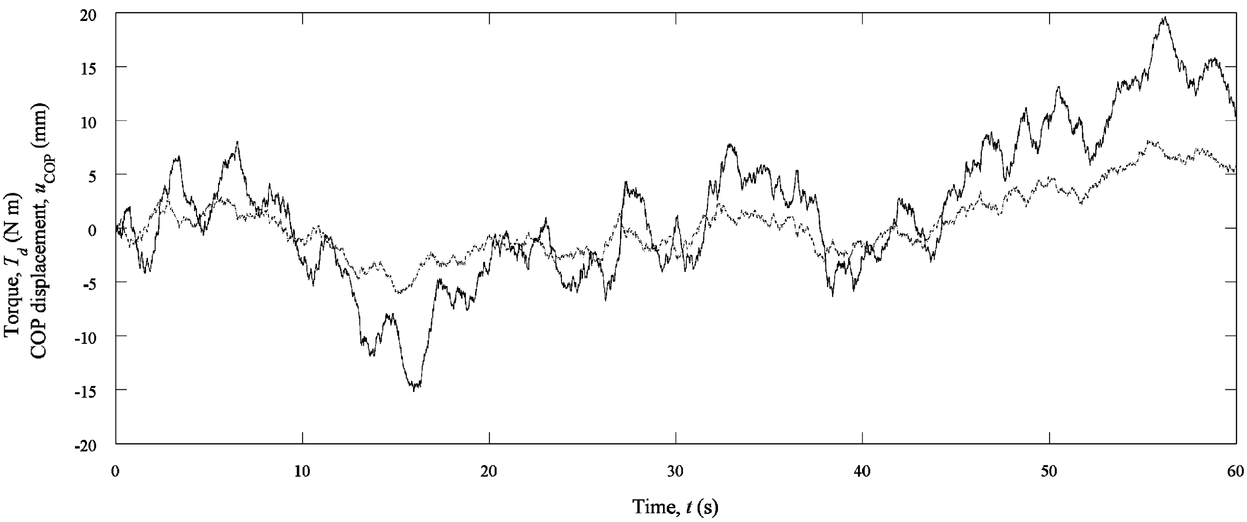

Fig. 6. Signal shape comparison. · · · · · · · · ·, disturbance torque T_d; —, COP signal produced by CIP model with "center normal" parameters set $\mathbf{p}$.

This similarity confirms the notion that in order to identify the set $\mathbf{p}$ of a particular person the disturbance torque of the CIP model must be identified as well.

5. Conclusions

The human posture model represented as the CIP with a single dof has been investigated in order to obtain parameter values, which provide satisfactory coincidence between simulation and experimental results. Three

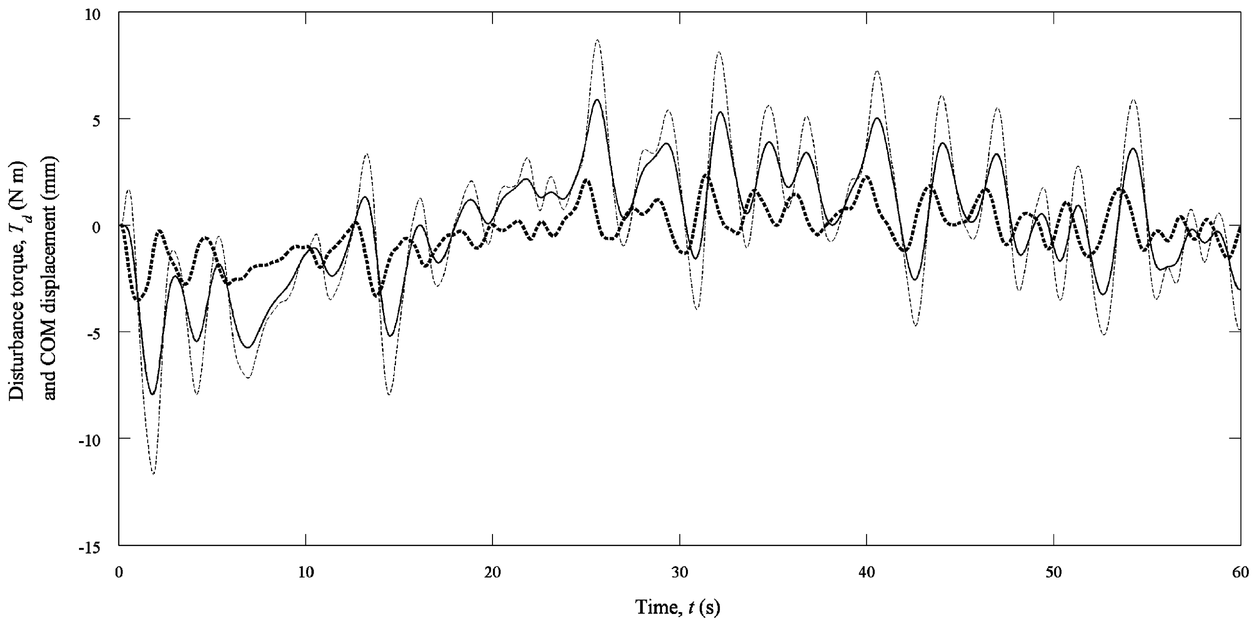

Fig. 7. Comparison of signal shapes. ·········, disturbance torque T_d; –––, COM signal produced by CIP model with "center normal" parameters set **p**; —, COM signal produced by CIP model with identified (right column of Table 2) parameters set **p**.

different error functions have been minimized where sensitivity functions were employed as search directions. Minimization of different error functions produced different parameter sets, however, values of each of them remained within the limits enabling to produce realistic stabilograms. The experiments revealed that COP signal produced by CIP model is highly dependent on the cumulative disturbance torque of the model.

In order to identify the parameter set of the model representing a real person, the reference time law of the disturbance torque signal which acted within a human body during the physical experiment has been also identified. We introduced a new approach for cumulative disturbance torque generation based on inverse dynamic problem solution and error function minimization.

The results of the work suggest the idea that the combination of the parameters of the CIP model can be identified for a particular person and may serve for characterization of the state of a person by using physiologically meaningful terms rather than by parametric description of COP time law. At this stage of the research, we may conclude that the identification of CIP model parameters may provide better insight into the resultant action of the control performed by central nervous system in order to maintain the body in upright posture.

Further experiments are to be conducted to ascertain that parameters set and disturbance torque identification procedure provides the same set of values for the same particular person when error function is constructed using time derivative of disturbance torque signal. It appears reasonable to expect that the identified CIP model parameters enable to classify health condition of humans in a plausible way. Obviously, there is still a wide uncovered research field for investigation of more complex models (e.g. nonlinear or with more dof), which could be able to increase the level of classification.

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