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Vibroimpact regimes and stability of system "Rotor—Sealing Ring"

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Abstract

Vibroimpact interaction between rotor and floating sealing ring is studied. The two-mass model "high-speed rotor—sealing ring" is considered. The model includes an unbalanced flexible rotor with elastic bearings. The rotor rotates inside the floating sealing ring. The ring is able to contact with the casing. The hydrodynamic forces in the clearance between the rotor and the ring as well as the dry friction between the ring and the casing are taken into account. The investigation of flow-coupled vibroimpact oscillations of a rotor and ring are presented. For these regimes, the analytical solution as well as numerical results are obtained. The main dynamic features of these behaviour and stability domains are discussed.
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1. Introduction

Floating sealing rings are widely used in high-speed rotary machines (for example in pumps) because they have no regular contact with the rotor and are rather effective for leakage restriction. The absence of leakages is based on hydraulic resistance arising in a very small clearance (0.1 mm) between the rotor and seal. Some designs of floating seals were cited in the book by Childs [1]. The liquid flow through the clearance excites hydrodynamic forces; their determination is performed, for example, by Nelson [2]. There are publications devoted to oscillation stability of rotor in seals under action of the non-conservative component of the hydrodynamic forces (see, for example, the paper of Black [3] and the book by Lalanne and Ferreris [4]. The present paper is concerned with the problems of vibroimpact regimes. These regimes often occur between rotor and ring. In a previous paper by Akhmetkhanov et al. [5] various types of vibroimpact regimes are considered. In this paper, the existence and stability domains for these regimes are defined.

2. Dynamical model

In this model (Fig. 1), the flexible shaft supported by elastic bearings has a massive disc m_1 in the middle of the shaft. An equivalent stiffness of the shaft with bearings is k. The disc is encased inside the floating ring m_2.

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Nomenclature

a	rotor eccentricity
d	bearings damping
f	coefficient of Coulomb's friction
h_1	flow stiffness
h_2	flow damping
k_z	turbulence factor ($k_z \approx 0.005Re_z$)
L	ring width
q_1	radial rotor displacement ($q_1 = x_1 + iy_1$)
q_2	radial ring displacement ($q_2 = x_2 + iy_2$)
R	internal ring radius
Re_z	Reynolds number for axial flow ($Re_z = 2\rho\delta w/\mu'$)
S_{fr}	ring friction surface
w	speed of turbulent axial flow ($w \approx 10\delta\sqrt{(p_1 - p_2)/(\rho L)}$)
δ	value of annular clearance
δ_0	minimal value of annular clearance ensuring non-impact regime at critical speed
η	hydraulic resistance in annular clearance ($\eta \approx 75\,\delta/L$)
μ'	dynamic viscosity
ρ	density of liquid (gas)
ω	rotor speed

The disc is unbalanced and is loaded by centrifugal force $F_c = m_1 a\omega^2$. The pressure in front of the ring is high and equals to p_1. Behind the ring the pressure is low and equals to p_2. Pressure difference sets the floating ring in motion relative to the casing. The dry friction force between the ring and the casing equals to $F_{\mathrm{fr}} = fS_{\mathrm{fr}}(p_1 - p_2)$. Pressure distribution in the radial clearance between rotor and ring generates hydrodynamic forces which include three components [5].

Elastic force:

$$F_e = \frac{\pi LR(p_1 - p_2)\eta}{2\delta(1+\eta)^2}|q_1 - q_2| = h_1|q_1 - q_2|.$$

Damping force:

$$F_d = \frac{\pi\mu' k_z L^3 R}{12\delta^3}|q_1 - q_2| = h_2|q_1 - q_2|$$

and non-conservative force $F_n = \frac{1}{2}\omega h_2|q_1 - q_2|$. This two-mass model may describe the coupled system "rotor–ring" if its critical speeds are widely separated and if up to nominal rotor speed the ring may be considered as a solid. Then the flow-coupled oscillations are described by the following equations:

$$\begin{aligned} &m_1\ddot{q}_1 + d\dot{q}_1 + kq_1 + h_1(q_1 - q_2) + h_2(\dot{q}_1 - \dot{q}_2) - \mathrm{i}0.5\omega h_2(q_1 - q_2) = m_1 a\omega^2 \mathrm{e}^{\mathrm{i}\omega t}, \\ &m_2\ddot{q}_2 + h_1(q_2 - q_1) + h_2(\dot{q}_2 - \dot{q}_1) - \mathrm{i}0.5\omega h_2(q_2 - q_1) + F_{\mathrm{fr}}\left[\mathrm{sgn}\,\dot{x}_2 + \mathrm{i}\,\mathrm{sgn}\,\dot{y}_2\right] = 0. \end{aligned} \quad (1)$$

Owing to the design constraints the ring can move only along the x- and y-axis and cannot rotate. It is assumed that the ring is completely balanced. Its motion for non-impact interaction is determined by ratio between dry friction force and elastic hydrodynamic force $c = F_{\mathrm{fr}}/F_e$. There are three various ring motions

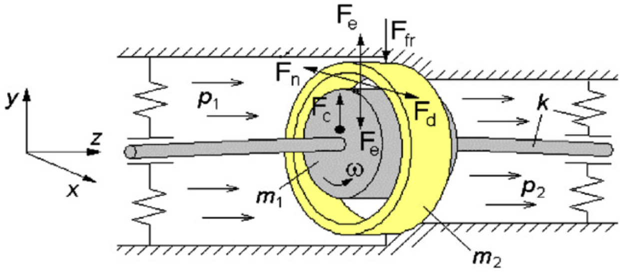

Fig. 1. Dynamical model of "rotor–ring".

depending on value c: if $c = 0$ vibration amplitude of the ring is maximum and the ring is self-centered around the rotor, if $c \geqslant 1$ the ring is fixed, and if $0.5 < c < 1$ motion of the ring is accompanied by periodic sticking.

Vibroimpact interaction occurs in systems when the impact condition is fulfilled, i.e. $|q_1 - q_2| > \delta$.

The change of vibration velocities due to impact is described by restitution coefficient $e \in [0, 1)$, see Harris and Crede's book [6]. For radial velocities of the rotor and the ring after impacts $\dot{q}_1^+$, $\dot{q}_2^+$:

$$\dot{q}_1^+ = \frac{(m_1 - em_2)\dot{q}_1^- + m_2(1+e)\dot{q}_2^-}{m_1 + m_2}, \quad \dot{q}_2^+ = \frac{(m_2 - em_1)\dot{q}_2^- + m_1(1+e)\dot{q}_1^-}{m_1 + m_2}.$$

3. Analytical solution

During analytical investigation the damping and the non-conservative hydrodynamic forces are not taken into account because their influence on system dynamics is very small in comparison with the strong influence of impacts.

Really, for the system under study the damping and the non-conservative components of real hydrodynamic forces are small in comparison with the elastic component. For example, at critical speed of a non-rotating rotor with $\omega_0 = \sqrt{k/m_1}$, the dimensionless coefficients of the hydrodynamic forces are equal:

$$\frac{h_1}{m_1\,\omega_0^2} = 0.138, \quad \frac{h_2}{m_1\,\omega_0} = 0.0178, \quad \frac{0.5h_2\omega_0}{m_1\,\omega_0^2} = 0.0107.$$

Therefore, the influence of damping and non-conservative forces will not be considered at impacts. For analytical consideration we shall take into account only the elastic component of hydrodynamic forces $h_1 x$. Under these conditions the motion equations along axes x and y as well as conditions of impact are not coupled. Therefore, it is possible to consider the oscillations along the x- and y-axis independently. The oscillations of the rotor and the ring in mutually perpendicular directions occur similarly but with a shift of phases equal to $\pi/2$ between the next impacts due to design symmetry. Thus, in a plane (x, y) there will be a 4-impact regime.

So the motion equations of system "rotor–ring" along the x-axis take the form

$$\begin{aligned} &m_1\ddot{x}_1 + kx_1 + h_1\,(x_1 - x_2) = m_1 a\omega^2 \cos(\omega\, t + \Psi), \\ &m_2\ddot{x}_2 + h_1(x_2 - x_1) + F_{\text{fr}}\,\text{sgn}\,(\dot{x}_2) = 0, \end{aligned} \tag{2}$$

angle Ψ is phase shift of impact moment with respect to disturbing centrifugal force.

The impact succession between rotor and ring for this model is shown in Fig. 2. The rotor moves from a position of equilibrium under action of force $F_c = m_1 a\omega^2$ and strikes the ring ($t = 0$). The ring bounces off in a direction of rotor motion ($t = T/8$ where $T = 2\pi/\omega$) and sticks under action of dry friction force ($t = T/4$). At the same time, the rotor reaches maximal displacement and the elastic hydrodynamic force in a clearance between the rotor and the non-moving ring reaches its greatest value. As the elastic hydrodynamic force becomes more than dry friction force the ring starts to move in the initial direction. But it is delayed in phase with respect to the rotor, which changes a motion direction to the opposite one ($t = 3T/8$). Then moving towards each other the rotor again collides with the ring ($t = T/2$). After this the motion of the rotor and the ring repeats itself similarly.

Conditions of periodicity and stitching for these oscillations of the rotor and the ring are written as

$$\begin{aligned} &x_1(0) = x_0, \quad x_2(0) = x_0 + \delta, \quad \dot{x}_1 = \dot{x}_1^+, \quad \dot{x}_2 = \dot{x}_2^+, \\ &x_1(T/2) = -x_0, \quad x_2(T/2) = -(x_0 + \delta), \quad \dot{x}_1 = \dot{x}_1^-, \quad \dot{x}_2 = \dot{x}_2^-, \quad F_{\text{fr}} = +F_{\text{fr}}, \end{aligned}$$

where x_0—impact coordinate, index + denotes velocities after impacts, index − denotes velocities before impacts (for half-period $t = T/2/T$ all signs are opposite).

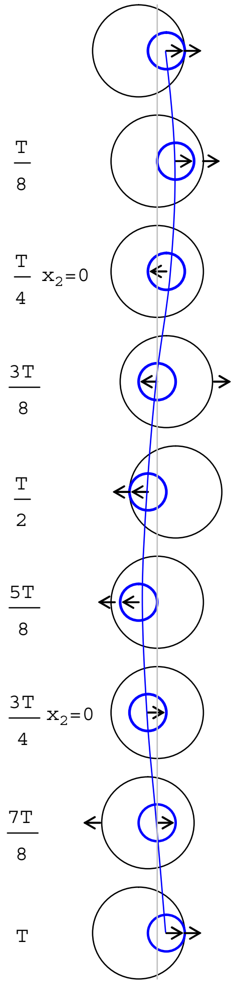

Fig. 2. Planar model of vibroimpact system "rotor–ring".

Using these conditions we obtained a solution of Eq. (2) corresponding to vibroimpact oscillations of the rotor and the ring:

$$x_1(t) = \frac{F_c(h_1 - m_2\omega^2)\cos(\omega t + \Psi)}{\Delta} + C_2\frac{\cos\,\lambda_1(t - T/4)}{\sin\,\lambda_1 T/4} + C_4\frac{\cos\,\lambda_2(t - T/4)}{\sin\,\,\lambda_2 T/4},$$
$$x_2(t) = \frac{F_{\rm fr}}{h_1} + \frac{F_c h_1\,\cos(\omega t + \Psi)}{\Delta} + a_1 C_2\frac{\cos\,\lambda_1(t - T/4)}{\sin\,\lambda_1 T/4} + a_2 C_4\frac{\cos\,\lambda_2(t - T/4)}{\sin\,\lambda_2 T/4}. \quad (3)$$

Here, $\lambda_{1,2}$ are natural frequencies of system "rotor–ring" determined from the characteristic equation

$$\Delta = (k + h_1 - m_1\omega^2)(h_1 - m_2\omega^2) - h_1^2, \quad C_2 = -\frac{\dot{x}_2^-}{\lambda_1 u}\frac{a_2 + u}{a_1 - a_2}, \quad C_4 = \frac{\dot{x}_2^-}{\lambda_2 u}\frac{a_1 + u}{a_1 - a_2}.$$

The values

$$a_{1,2} = \sqrt{u}\frac{1 \mp \sqrt{1 + s^2}}{2s}$$

are coefficients of oscillation modes calculated by ratios

$$u = \frac{m_1}{m_2}, \quad s = \gamma\frac{2p_1 p_2}{|p_1^2 - p_2^2|}, \quad \gamma = \frac{h_1}{\sqrt{h_1(k + h_1)}},$$

where $p_2 = \sqrt{h_1/m_2}$, $p_1 = \sqrt{(k+h_1)/m_1}$, are partial frequencies of rotor and ring. Value $\dot{x}_2^-$ is determined by formulas

$$\cos^2 \Psi + \sin^2 \Psi = 1, \quad \sin \Psi = \frac{\Delta}{F_c(h_1 - m_2\omega^2)} \frac{1+u}{u} \frac{1-e}{1+e}\dot{x}_2^-,$$

$$\cos \Psi = - \frac{\Delta}{F_c m_2 \omega^2}\left[\frac{F_{\mathrm{fr}}}{h_1} - \delta - \dot{x}_2^-\left(\frac{1-a_2}{\lambda_2 u} \frac{a_1+u}{a_1-a_2} ctg\frac{\lambda_2 T}{4} - \frac{1-a_1}{\lambda_1 u} \frac{a_2+u}{a_1-a_2} ctg\frac{\lambda_1 T}{4}\right)\right].$$

Solution (3) shows that vibroimpact system "rotor–ring" is unstable at the critical speeds $\omega = \lambda_i$ and at the subharmonic frequencies $\omega = \lambda_i/2n$, $n = 1, 2, 3, \ldots$. Really, at the frequencies λ_1 and λ_2, a determinant $\Delta = 0$, on the frequencies $\omega = \lambda_i/2n$, $n = 1, 2, 3, \ldots$ denominators $\sin\ \lambda_i T/4 = 0$.

4. Numerical solution

The vibroimpact oscillations of system (1) "rotor–ring" with account of the damping and non-conservative forces were investigated numerically. It was found that vibroimpact oscillations of this system have some specific features listed below.

A vibroimpact trajectory of the rotor always has a square form (Fig. 3a). The sharpness of the ring trajectories are essentially dependent on the value of radial clearance δ. When $0 < \delta < 0.75\delta_0$ the ring motion may be a square (Fig. 3b, line 1), when they are commensurable ($0.75\delta_0 < \delta < \delta_0$) it is a star (Fig. 3b, line 2). In Fig. 4, the small time interval of rotor and ring vibration is shown. It is seen that the ring motion is interrupted by impacts (point I) and by periods of sticking (line S). There is the anti phase oscillations of rotor and ring as well. Such vibroimpact motion of system "rotor–ring" along one axis agrees with planar model (see Fig. 2).

The analysis of rotor amplitudes near the critical speed of a non-rotating rotor ω_0 has shown that impacts intensify the amplitude of vibration if restitution coefficient is decreased and ring mass is increased (Table 1). For comparison the rotor amplitude at the frequency ω_0 for non-impact interaction with a ring mass $m_2 = 0.1m_1$ equals 30 m cm.

Spectral analysis shows (Fig. 5) that with vibroimpact interaction there are oscillations with high frequencies, multiple of the rotor speed and equal to 3ω, 5ω, $7\omega \cdots (2n-1)\omega$. Generation of harmonics with odd frequencies agrees with the analytical solution too.

Domains of non-impact, stable and unstable vibroimpact regimes for the "rotor–ring" system are determined dependent on the rotor speed, value of a radial clearance and dry friction force between ring and casing. They were obtained from the analysis of rotor and ring trajectories (see Fig. 3). The widest domain of vibroimpact regimes is close to a critical speed (Fig. 6a). Rotor instability due to impacts with ring occurs at frequencies $\omega = \omega_0$, $\omega = \omega_0/2$. But for all rotor speeds the domain of vibroimpact interaction is minimum for ratio $c = F_{\mathrm{fr}}/F_e = 0.55$. So the non-impact regime at $\omega = \omega_0$ takes place already when a radial clearance is $\delta = 0.32\delta_0$ (Fig. 6b).

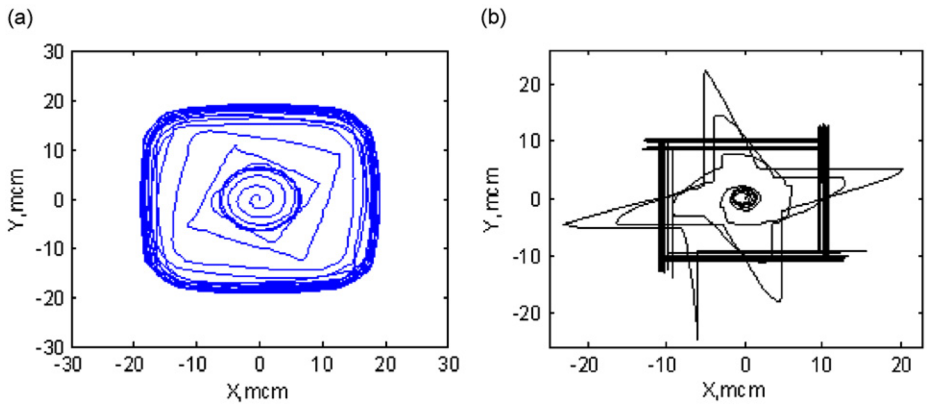

Fig. 3. Trajectories of rotor (a) and ring (b) at stable vibroimpact regime.

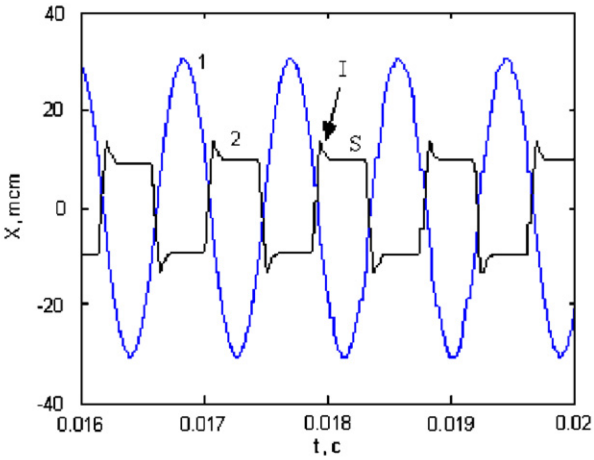

Fig. 4. Vibroimpact oscillations of rotor-1 and ring-2.

Table 1

e	0.5	0.4	0.3	0.13	0.3	0.13
μ	0.1	0.1	0.1	0.06	0.1	0.14
A, m cm	34	49	56	43	56	60

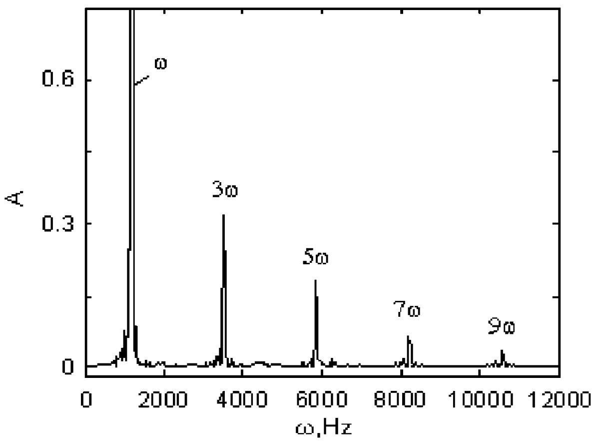

Fig. 5. Frequency spectrum for vibroimpact oscillations of ring (rotor).

Also in Fig. 6b, there are the additional points. They correspond to the boundary, which separates non-impact and vibroimpact regimes for planar model (Section 3). Thus, there is a good correlation between domains of planar and orbital models.

5. Vibroimpact regimes with account for fiction force during impact

Let us write the momentum equations (the theorem of impulses), which connect translational velocities before and after impact for the rotor inertia centre G and a geometrical centre of the ring O_2. They will have more simple presentation in coordinates (u,v) adhered to a point of impact C, laying on lines of centres O_1O_2.

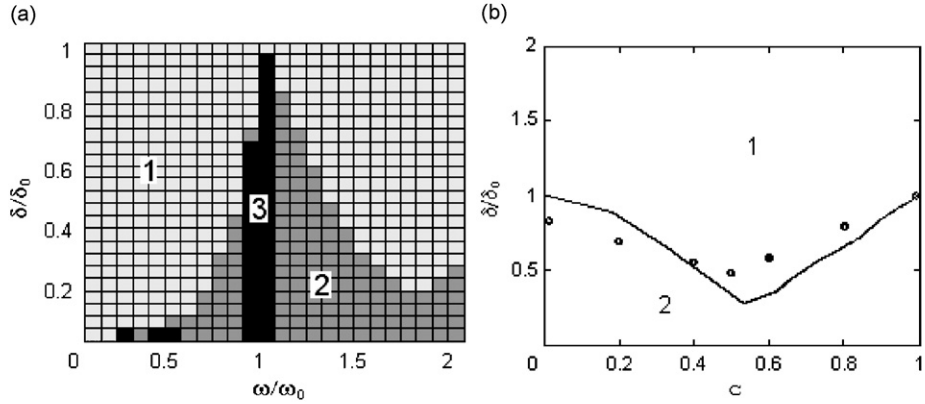

Fig. 6. Domains of non-impact-1, stable vibroimpact-2 and unstable vibroimpact-3 regimes of full system ''rotor–ring''.

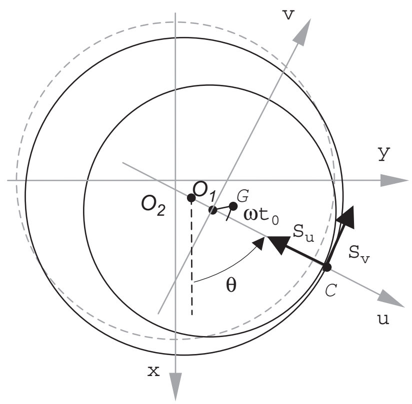

Fig. 7. Projections of impact force impulse.

The rotation angle of this coordinate system is determined by an angle of impact θ (Fig. 7):

$$
\begin{aligned}
m_1\left(\dot{u}_G^+ - \dot{u}_G^-\right) &= S_u, \quad m_2\left(\dot{u}_{O_2}^+ - \dot{u}_{O_2}^-\right) = -S_u, \\
m_1\left(\dot{v}_G^+ - \dot{v}_G^-\right) &= S_v, \quad m_2\left(\dot{v}_{O_2}^+ - \dot{v}_{O_2}^-\right) = -S_v,
\end{aligned} \tag{4}
$$

where $(\dot{u}_G^-, \dot{u}_G^+)$, $(\dot{v}_G^-, \dot{v}_G^+)$—velocity projections of the inertia centre of the rotor to the normal and tangent directions before and after the impact, the appropriate velocity projections of the ring geometrical centre have index O_2.

Transition from velocities $\dot{x}, \dot{y}$ to $\dot{u}, \dot{v}$ is carried out under relations:

$$
\dot{u} = \dot{x}\cos\theta + \dot{y}\sin\theta, \quad \dot{v} = -\dot{x}\sin\theta + \dot{y}\cos\theta, \quad tg\theta = -\frac{y_1}{x_1}.
$$

Relations of velocities of the rotor's inertia centre G and its geometrical centre O_1 are defined from the equations

$$
\dot{u}_G = \dot{u}_{O_1} + a\omega\cos\omega t_0 \quad \dot{v}_G = \dot{v}_{O_1} + a\omega\sin\omega t_0, \tag{5}
$$

where $a = O_1G$—rotor eccentricity, ωt_0—impact phase.

From Eqs. (4) and (5), it is possible to find the equations connecting velocities of the geometrical centres of the rotor and rings before and after impact

$$m_1\left(\dot{u}_{O_1}^{+}-\dot{u}_{O_1}^{-}\right)=S_u,\quad m_2\left(\dot{u}_{O_2}^{+}-\dot{u}_{O_2}^{-}\right)=-S_u,$$
$$m_1\left(\dot{v}_{O_1}^{+}-\dot{v}_{O_1}^{-}\right)=S_v,\quad m_2\left(\dot{v}_{O_2}^{+}-\dot{v}_{O_2}^{-}\right)=-S_v, \tag{6}$$

where $(\dot{u}_{O_1}^{-},\dot{u}_{O_1}^{+})$, $(\dot{v}_{O_1}^{-},\dot{v}_{O_1}^{+})$—velocity projections of the rotor geometrical centre on a normal and tangent before and after impact. Further instead of an index "O_1" it is applied to simplify the index "1" as above.

Direct non-elastic impact is described by a hypothesis of Newton

$$\dot{u}_2^{+}-\dot{u}_1^{+}=-e(\dot{u}_2^{-}-\dot{u}_1^{-}),\quad e\in[0,1). \tag{7}$$

That is carried out only in limited range of approach velocities—from 10 cm up to several metres per second. And velocities of the rotor and ring are in such a range.

From Eq. (6) for a normal shock impulse S_u it is possible to find normal components of velocities for the rotor and the ring geometrical centres after impact based on the equality (7)

$$\dot{u}_1^{+}=\frac{(1-\mu e)\dot{u}_1^{-}+\mu(1+e)\dot{u}_2^{-}}{1+\mu},\quad \dot{u}_2^{+}=\frac{(\mu-e)\dot{u}_2^{-}+(1+e)\dot{u}_1^{-}}{1+\mu}, \tag{8}$$

where $\mu=m_2/m_1$—the ratio of the ring mass to the rotor mass.

Impact of rough bodies is accompanied by friction that derives from a tangent component of an impact impulse S_v. Using a hypothesis of friction by Coulomb, we shall write

$$S_v=-fS_u, \tag{9}$$

where f-coefficient of a sliding friction between the rotor and the ring.

From Eq. (9) we shall find tangential components of velocities after impact for the rotor's and the ring's centres:

$$\dot{v}_1^{+}=\dot{v}_1^{-}+f\mu\frac{(1+e)(\dot{u}_1^{-}-\dot{u}_2^{-})}{1+\mu},\quad \dot{v}_2^{+}=\dot{v}_2^{-}-f\frac{(1+e)(\dot{u}_1^{-}-\dot{u}_2^{-})}{1+\mu}. \tag{10}$$

Each of expressions (10) is true only in the case when the value $\dot{v}_1^{+}$ has the same sign, as $\dot{v}_1^{-}$; otherwise it is necessary to consider $\dot{v}_1^{+}=0$, i.e. the tangential component disappears; it is the same for ring's velocities.

So relations (8) and (10) for the rotor's and the ring's velocities before and after impact allow to solve the problem of impact movements with account to friction force during impact.

As calculations are shown, consideration of impact friction does not bring basic qualitative changes in the motion picture and the four-impact regime is still possible. There are the following insignificant differences:

- due to the tangential component of impact force impulse, influence of impacts on the rotor oscillations weakens (Fig. 8a) and for the ring amplifies: when $0<\delta<0.75\delta_0$, the trajectory of the ring is a concave one inside a rhomb (Fig. 8b). When $0.75\delta_0<\delta<\delta_0$ it is like a cross (Fig. 8c);

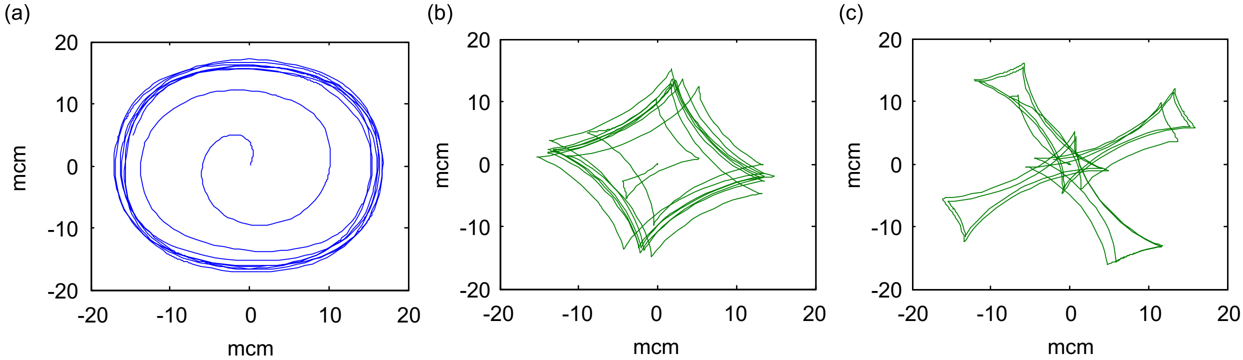

Fig. 8. Trajectories of rotor (a) and ring and (b,c) with account for friction at the moment of impacts.

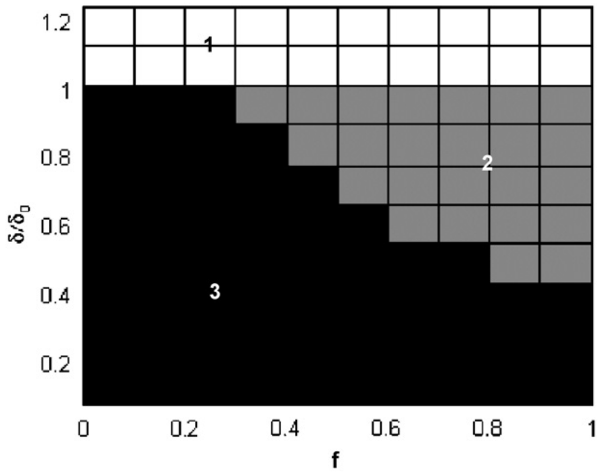

Fig. 9. Domains of non-impact-1, stable vibroimpact-2 and unstable vibroimpact-3 regimes system ‘‘rotor–ring’’.

- domain of existence of the non-impact regimes does not change, but friction increases stability of system ‘‘rotor–ring’’ at impact regimes (Fig. 9).

Oscillations of the rotor due to friction force at impact in the motionless bearing (corresponding to the motionless or heavy enough ring) were considered in Banakh [7].

6. Conclusions

- Vibroimpact regimes in a system ‘‘rotor–ring’’ produce only destabilising action. They excite also superharmonic oscillations of a rotor and ring. This can be the reason for subharmonic resonances and instability of a rotor.
- The stabilizing effect (the impact damping of rotor vibration by a floating ring) was not detected.
- Domain of non-impact interaction at small radial clearance as well as dynamic stability domain of the rotor with impact regimes are optimal when $c = F_{\text{fr}}/F_e = 0.5$, $m_2 \to 0$, $e \to 0.5$ and $f \to 1$.

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