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Short Communication

Effect of enclosed fluid on the dynamic response of inflated torus

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Abstract

Large inflatable structures have been the subject of renewed interest for scientists/engineers in recent years due to their potential space applications such as communication antennas, solar thermal propulsion and space solar power. The major advantages of using inflatable structures in space are their extremely low-weight, on-orbit deployability and inherent low launch volume. An inflated torus is a key component of many inflated space structures such as a thin membrane reflector. In view of their importance, structural static and dynamic behavior of inflated torus need to be investigated. In order to develop a more realistic model, dynamic interaction between the enclosed fluid and the torus has been included in the present work. An appreciable decrease in the modal frequencies is observed when fluid–structure interaction is taken into account. Some additional modes are also obtained. It is concluded that fluid–structure interaction significantly affects the dynamic behavior of inflatable space structures.

1. Introduction

The development of large, deployable, efficient structures is an enabling technology for many future spacecraft missions. These missions include large deployable antennas, sunshields, solar arrays and solar sails. A large inflated structure can be stowed in a compact configuration and deployed on a single launch. Therefore, they are receiving a great deal of attention due to the need for low-cost space missions and the potential to greatly improve capabilities in a variety of applications. Some of the potential applications of inflatable structures are cited in Cassapakis and Thomas [1], Fang and Lou [2], Freeland [3] and Veldman and Vermeeren [4].

However, inflatable space structures are more susceptible to shape distortion and vibration problems because of low structural stiffness and material damping. This makes the structure dysfunctional for a period of time and leads to a drastic reduction in accuracy and precision of operation. Thus, it is highly desirable to control shape distortion and vibration of the structure during operation. The control action in space structures is exerted by a small number of actuators. Hence the vibration modes of the structure should be known to a high degree of accuracy in order to provide better control authority.

It is very helpful to take a building-block approach to modeling and understanding inflatable assemblies by first investigating in detail the behavior of components such as the inflatable torus. Inflatable tori have

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considerable practical application and potential for use as components of inflatable concentrator assemblies, antenna structures, space power systems, etc. Therefore, free and forced vibration response of inflated torus form important aspects of the dynamic response investigations.

Many researchers have investigated the vibration behavior of inflated torus. Liepins [5] presented an extensive study of the free vibration analysis of a toroidal membrane subjected to an internal pressure using finite difference method. He demonstrated that only lower frequencies are affected by the prestress due to internal pressure. Jordan [6] predicted a few lower vibratory frequencies using the Rayleigh quotient. Later, he presented experimental results for the vibration testing of inflated tori of free and fixed boundary conditions [7]. He also noticed the acoustic resonance due to the air inside the torus. Saigal et al. [8] found a closed-form solution for the natural frequencies and mode shapes of a prestressed toroidal membrane with fixed boundary conditions. Lewis and Inman [9] analyzed an inflated toroidal structure using the finite-element software package ANSYS. Jha et al. [10] and Jha and Inman [11] performed free vibration analysis using Galerkin's method and Fourier series. They demonstrated the importance of geometric non-linearity and direct action of pressure force and presented a comparison of the results from different approximate shell theories. Griffith and Main [12] and Park et al. [13,14] performed experimental investigation into the dynamics of an inflated torus. Park et al. [13,14] indicated the potential use of smart materials in the actuation and control of inflated structures. A literature review by Ruggiero et al. [15] describes many analytical as well as experimental works on the dynamic response of inflated torus. To the best of authors' knowledge, the dynamic coupling between the inflated torus and the fluid inside it has not been studied. Smalley and Tinker [16] have studied the dynamic behavior of an inflated beam. They have modeled the effect of the enclosed fluid by adding non-structural mass to the shell elements. However, they have not modeled the dynamic interaction between the fluid and the structure. The present work is an effort in the direction of understanding the effect of fluid–structure coupling on the dynamics of an inflated cylindrical torus. This work is highly significant because of the interest in inflatable structures for space applications and because of the difficulty in accurately modeling such systems.

2. Mathematical formulation

The main objective of this investigation is to understand the effect of enclosed fluid on the dynamic response of inflatable structures. Due to the vibration of the shell, the fluid inside the torus vibrates and a standing pressure wave is created in the fluid medium. The pressure wave in the fluid, in turn, affects the vibratory response of the structure.

The inflatable torus is modeled by shell elements. The governing equation for vibration of a shell under pressure has been presented in many earlier works. The authors follow the procedure adopted by Kraus [17] and Soedel [18] wherein, a non-linear static deflection problem of a shell is analyzed first to find out the prestress due to inflation pressure. The prestressed shell is assumed to have small-amplitude vibrations. The vibration of the shell causes small-amplitude pressure disturbance in the fluid inside the shell. Because of the assumed small-amplitude vibrations, a linear vibration analysis is carried out. The FEM formulation for the vibratory response of the shell is given by Reddy [19]

$$\mathbf{M}_s\ddot{\mathbf{U}}_s + \mathbf{K}_s\mathbf{U}_s = \mathbf{F}_s + \mathbf{F}_{sf}, \tag{1}$$

where $\mathbf{U}_s$ and $\ddot{\mathbf{U}}_s$ are the vector of nodal displacement and acceleration of the shell. $\mathbf{M}_s$ and $\mathbf{K}_s$ are the mass and stiffness matrices, respectively. It should be noted here that the stiffness $[\mathbf{K}_s]$ depends on the inflation pressure. $\mathbf{F}_s$ is due to externally applied loads and $\mathbf{F}_{sf}$ is due to dynamic pressure changes in the fluid.

Let the spatial variations of the pressure 'P' and displacement components '$\mathbf{U}$' be given by

$$P = \mathbf{N}_f^{\mathrm{T}}\mathbf{P}_f \quad \text{and} \quad \mathbf{U} = \mathbf{N}_s^{\mathrm{T}}\mathbf{U}_s, \tag{2}$$

where $\mathbf{N}_f$ and $\mathbf{N}_s$ are the interpolation function for pressure variation in the fluid medium and displacement for the shell respectively and $\mathbf{P}_f$ is the nodal pressure variation about the mean pressure inside the torus.

$\mathbf{F}_{sf}$ can be obtained by integrating the pressure over internal surface of the shell, i.e.

$$\mathbf{F}_{sf} = \int_S \mathbf{N}_s P\mathbf{n}\,\mathrm{d}S,$$

where $\mathbf{n}$ is the outward-directed unit vector normal to the shell surface, S is the fluid–structure interface area, $\mathbf{F}_{sf}$ can also be written as

$$\mathbf{F}_{sf} = \mathbf{R}_P \mathbf{P}_f, \text{ where } \mathbf{R}_P = \int_S \mathbf{N}_s \mathbf{N}_f^{\mathrm{T}} \mathbf{n} \, \mathrm{d}S. \tag{3}$$

$\mathbf{F}_{sf}$ in Eq. (1) can be obtained if the dynamic pressure variation in the fluid is known. It is assumed that the fluid inside the shell is inviscid and there is no mean flow of fluid. Under these conditions, pressure variations in the fluid is governed by

$$\frac{1}{c^2}\frac{\partial^2 \mathrm{P}}{\partial t^2} - \nabla^2 \mathrm{P} = 0, \tag{4}$$

where 'c' is the sonic speed in the fluid medium. Following the approach of Petyt and Jones [20], the FEM formulation of Eq. (4) results in

$$\mathbf{M}_f \ddot{\mathbf{P}}_f + \mathbf{K}_f \mathbf{P}_f + \rho_f \mathbf{R}_P^{\mathrm{T}} \ddot{\mathbf{U}}_s = \mathbf{0}, \tag{5}$$

where $\mathbf{M}_f$ and $\mathbf{K}_f$ are fluid mass matrix and stiffness matrix. ρ_f is the density of the enclosed fluid. The third term in Eq. (5) is obtained due to fluid–structure interface condition.

Eqs. (1) and (5) taken together give the coupled set of equations governing the response of fluid–shell system

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \rho_f \mathbf{R}_P^{\mathrm{T}} & \mathbf{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_s \\ \ddot{\mathbf{P}}_f \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_s & -\mathbf{R}_P \\ \mathbf{0} & \mathbf{K}_f \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{P}_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s \\ \mathbf{0} \end{Bmatrix}. \tag{6}$$

In the present study, finite-element software ANSYS is used to solve the fluid–structure coupled equations. Elements FLUID30 and SHELL181 are used to model the enclosed fluid and the structure, respectively. At first a non-linear static analysis is carried out to calculate the prestress in the structure element. Stiffness matrix of shell elements depends on the prestress, i.e. on the inflation pressure. Modal and harmonic analysis of the coupled problem is carried out for the torus–fluid system.

3. Results and discussion

In the present work an inflated torus has been modeled. The geometric and material properties of inflatable torus are taken from Lewis and Inman [9]. These properties are tabulated in Table 1. The torus is considered to be free, i.e. no constraint is provided to the torus for the modal analysis. Air is taken as the enclosed fluid within the structure. The finite-element mesh of the inflatable torus is shown in Fig. 1.

To highlight the fluid–structure interaction effect of the enclosed fluid, three different models have been studied and their results are compared. These models are

Model M1: structure modeled with shell element only. Effect of stress stiffening is included.

Model M2: structure modeled with shell elements with stress stiffening and mass of enclosed fluid added to the shell elements.

Table 1
Physical properties of the inflatable torus

Parameter	Value
Elastic modulus ($\mathrm{N\,m^{-2}}$)	2.55e9
Thickness (m)	76.2e-6
Poisson's ratio	0.34
Density ($\mathrm{kg\,m^{-3}}$)	1418
Diameter of ring (m)	15.24
Diameter of the tube (m)	0.61
Internal pressure ($\mathrm{N\,m^{-2}}$)	3447.38
Fluid density ($\mathrm{kg\,m^{-3}}$)	1.293
Sonic velocity ($\mathrm{m\,s^{-1}}$)	331.6

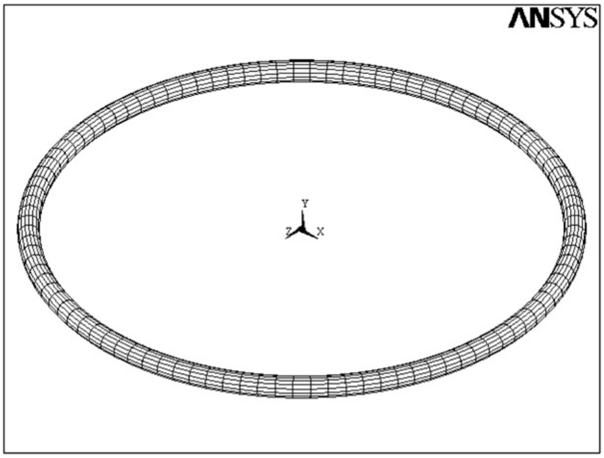

Fig. 1. Discretization of inflatable torus.

Table 2
Comparison of different modal frequencies for pressure = 3447.38 Pa

Model M1 (Hz)	Model M2 (Hz)	Model M3 (Hz)	Mode shapes
2.03	**1.21**	**2.04**	In plane rotation
2.22	1.33	1.33	Out of plane rotation
2.22	1.33	1.33	Out of plane rotation
3.37	2.01	2.02	In plane
3.37	2.01	2.02	In plane
4.96	2.96	2.98	Out of plane
4.96	2.96	2.98	Out of plane
7.58	4.53	4.56	In plane
7.58	4.53	4.56	In plane
8.69	5.19	5.24	Out of plane
8.69	5.19	5.24	Out of plane
		7.08	In plane translational
		7.08	In plane translational
12.91	7.71	7.79	In plane
12.91	7.71	7.79	In plane
13.67	8.17	8.28	Out of plane
13.67	8.17	8.28	Out of plane
19.41	11.59	11.76	In plane
19.41	11.59	11.76	In plane
19.93	11.90	12.12	Out of plane
19.93	11.90	12.12	Out of plane

Bold values highlight the importance of the analysis.

Model M3: structure modeled with shell elements with stress stiffening and enclosed fluid modeled by acoustic elements.

3.1. Modal analysis

The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. Modal analysis gives insight of these parameters of the structure. The natural frequencies and mode shapes of the inflated torus have been obtained for all the three models. For model M1, all the frequencies in the range of 0–20 Hz have been reported in column 1 of Table 2. By comparing the mode shapes, corresponding frequencies obtained from models M2 and M3 are reported in columns 2 and 3 of Table 2. One additional mode at frequency 7.08 Hz is obtained for model M3 which has been included in

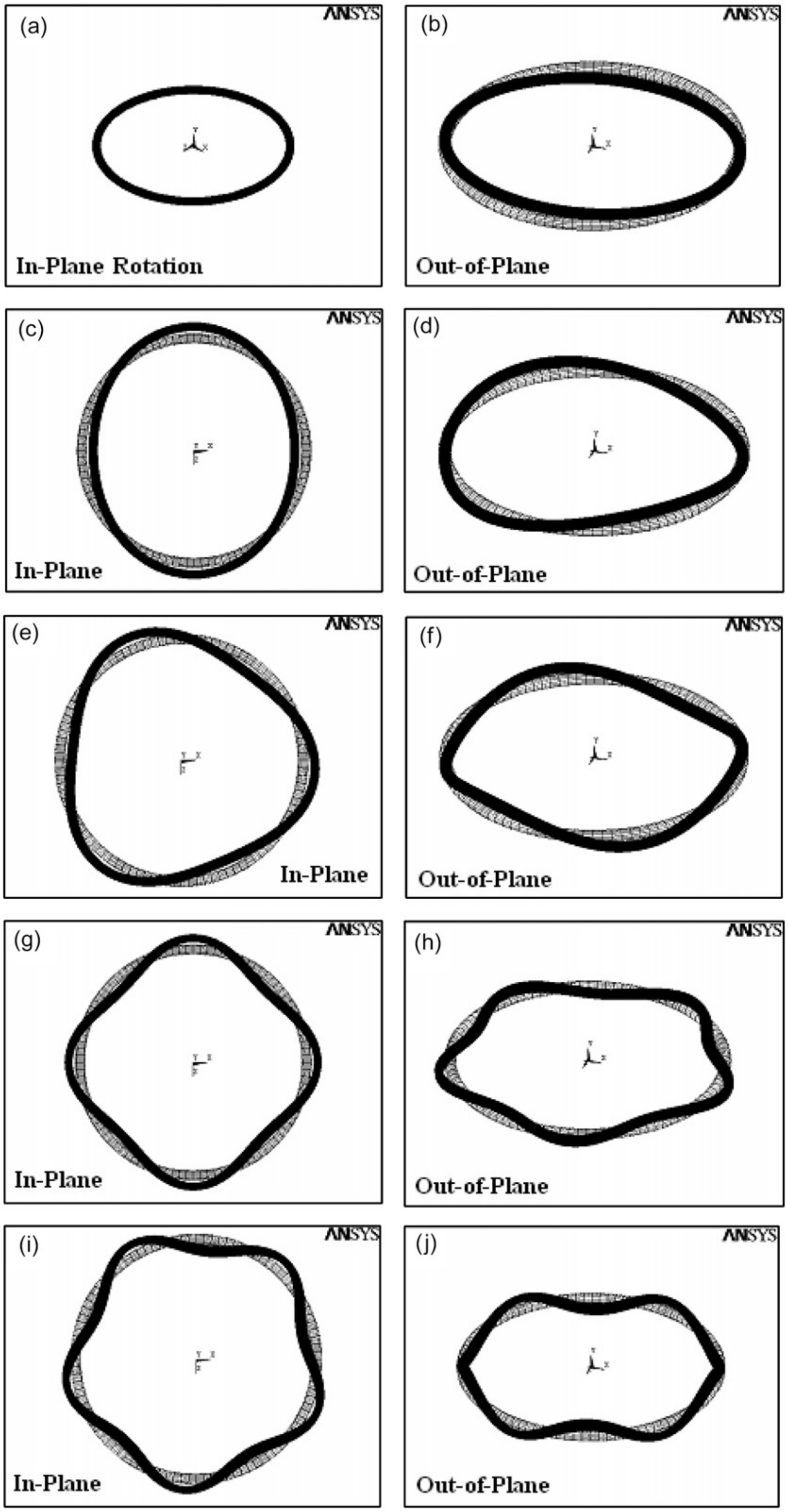

Fig. 2. Mode shapes obtained from all the three models.

Table 2. However, in the frequency range 0–20 Hz, some other natural frequencies of M2 and M3 are present, but they have not been listed in Table 2.

The first model M1 does not account for the mass of the enclosed fluid. This model is similar to the model described by Lewis and Inman [9]. The results of model M1 match with the results reported by Lewis and Inman [9]. Due to the symmetry of the structure, frequencies of model M1 appear in pair. The mode shapes corresponding to first few frequencies are shown in Fig. 2 in the ascending order. The mode shape corresponding to the first frequency, i.e. 2.03 Hz corresponds to in-plane rotation and hence the deformed configuration is not clearly shown in Fig. 2.

The second model M2 is a modification of model M1 wherein the mass of the enclosed fluid is distributed evenly on the structure. This model is similar to the model proposed by Smalley and Tinker [16] for inflated beam. The results of model M2 for inflated beam were compared with the results of Smalley and Tinker [16] and an excellent agreement was observed. The natural frequencies of model M2 are approximately 40% less than that of model M1. This decrease in natural frequencies can be attributed to the increased mass. It is concluded that the mass of enclosed fluid plays a crucial role in the analysis of inflatable structure and it must be included in the analysis of such lightweight inflatable structures. The mode shapes are similar to that of model M1 and can be shown by Fig. 2 itself.

The third model M3 accounts for the dynamic interaction between enclosed fluid and structure. The space inside the inflatable torus is meshed with acoustic elements and hence the stiffness and inertia properties of the fluid are directly incorporated in the model. Most of the frequencies obtained from model M3 are almost equal to the same obtained from model M2. For these frequencies, mode shapes obtained from analyzing either models M1, M2 or M3 are similar, i.e. these mode shapes may be represented by Fig. 2. However, there are two notable differences. The first natural frequency obtained from model M1 and M2 are 2.03 and 1.21 Hz, respectively. The corresponding vibration mode is in-plane rotation. However, analysis of model M3 suggests that in-plane rotation mode is the third natural frequency of the coupled problem. The frequency for this mode remains close to that of M1 and not to that of M2. The first in-plane rotation mode of vibration of torus is not affected by the fluid inside it. This is so because, an inviscid fluid has been assumed and the coupling between fluid and structure is through the normal component of displacement (acceleration) only. Also, one additional vibration mode has been noticed at around 7.08 Hz. The corresponding mode is in-plane translation. This additional mode is a direct consequence of strong fluid–structure interaction effect. Corresponding mode shape has been shown in Fig. 3.

From the discussion above, it is concluded that enclosed fluid greatly affects the natural frequency of the inflated torus. Modeling of the fluid–structure coupling is essential because modeling the effect of enclosed fluid by taking added mass, i.e. model M2, can some time be misleading, for example, the first in-plane rotation mode frequency is obtained as 1.21 Hz from model M2 and 2.04 Hz from model M3. Moreover, some

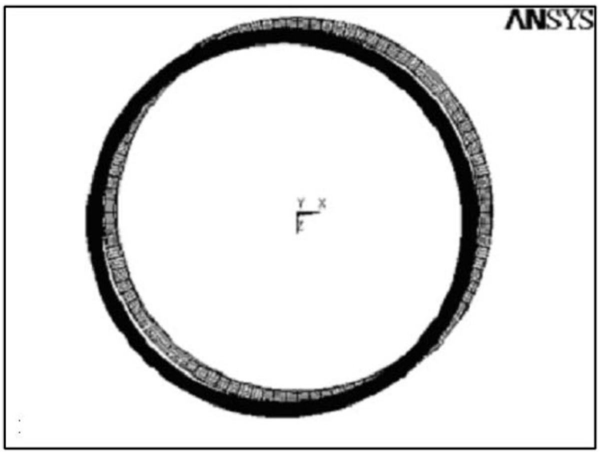

Fig. 3. Additional mode shapes (model M3) of the inflated torus.

new vibration modes (in-plane translation mode with frequency 7.08 Hz) are obtained when the fluid–structure interaction is taken into account.

In the next section, harmonic analysis is presented and the comparison is made between Model M1 and Model M3.

3.2. Harmonic response analysis

Harmonic response analysis gives us the ability to predict the sustained dynamic behavior of structures, thus enabling us to verify whether structure design will successfully overcome resonance, fatigue and other harmful effects of forced vibrations or not. In the present study, it is assumed that the harmonically varying stresses are much smaller than the prestress due to internal pressure. The damping ratio for each mode has been taken as 0.01.

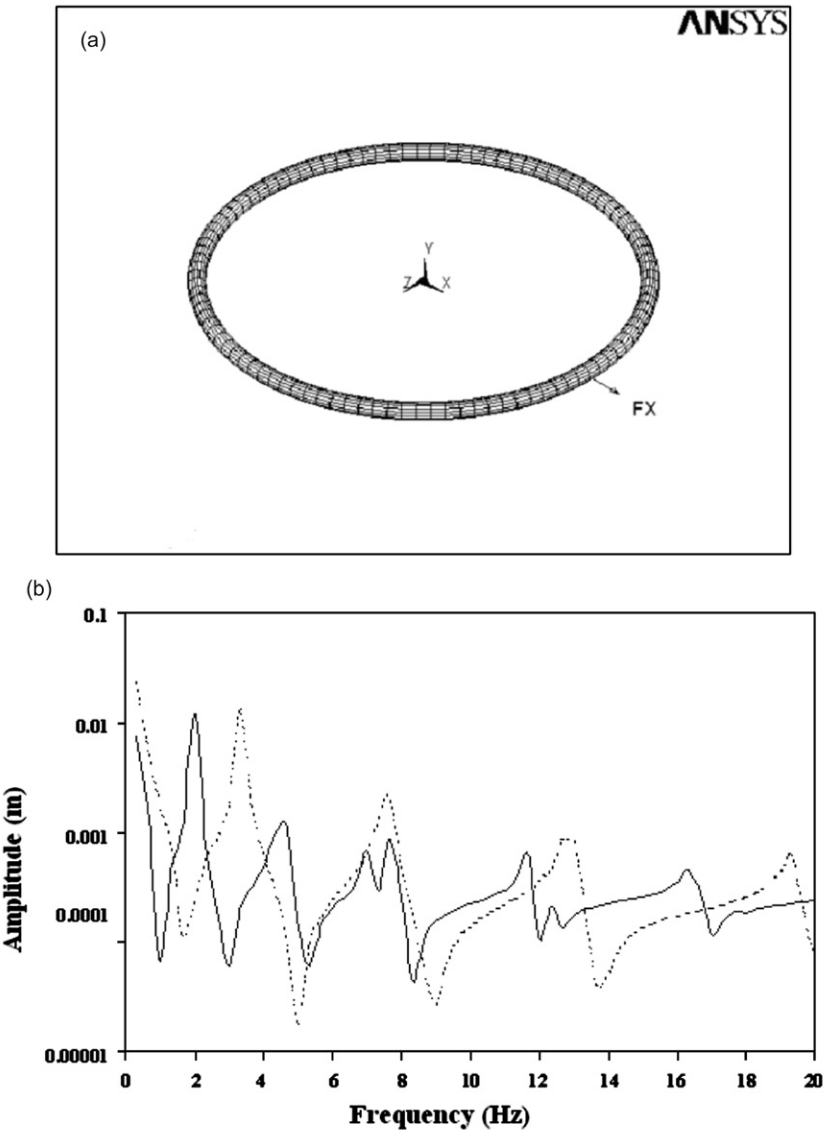

Fig. 4. In-plane harmonic analysis: (a) in-plane harmonic excitation; (b) in-plane harmonic response; (- - - -, model M1; ———, model M3).

Some of the natural frequencies for the inflated torus (Table 2) are so close that it is difficult to differentiate the resonance peak in the response due to harmonic loading. Therefore, three different harmonic analyses are carried out to excite all types of modes of vibration. In the first analysis, an in-plane harmonic load normal to the surface of the torus is applied (Fig. 4a). This loading is able to excite in-plane modes of vibration. In the second analysis, a harmonic load in the plane normal to the torus is applied (Fig. 5a) to excite out-of-plane vibration modes. Finally, third analysis is carried out by applying a harmonic load in the tangential direction in the plane of the torus to excite the in-plane rotational modes of the torus (Fig. 6a). Sinusoidal force of unit magnitude (arbitrarily taken) is applied for a frequency range of 0–20 Hz. The dynamic response of the point of application of the force is plotted between frequency and amplitude of vibration.

Fig. 4b shows the response of inflatable torus subjected to an in-plane harmonic loading as shown in Fig. 4a. When force is applied in in-plane direction, only the in-plane modes are excited. Within the range of excitation, model M1 has in-plane modes corresponding to 2.03, 3.4, 7.6, 12.9 and 19.4 Hz frequencies whereas

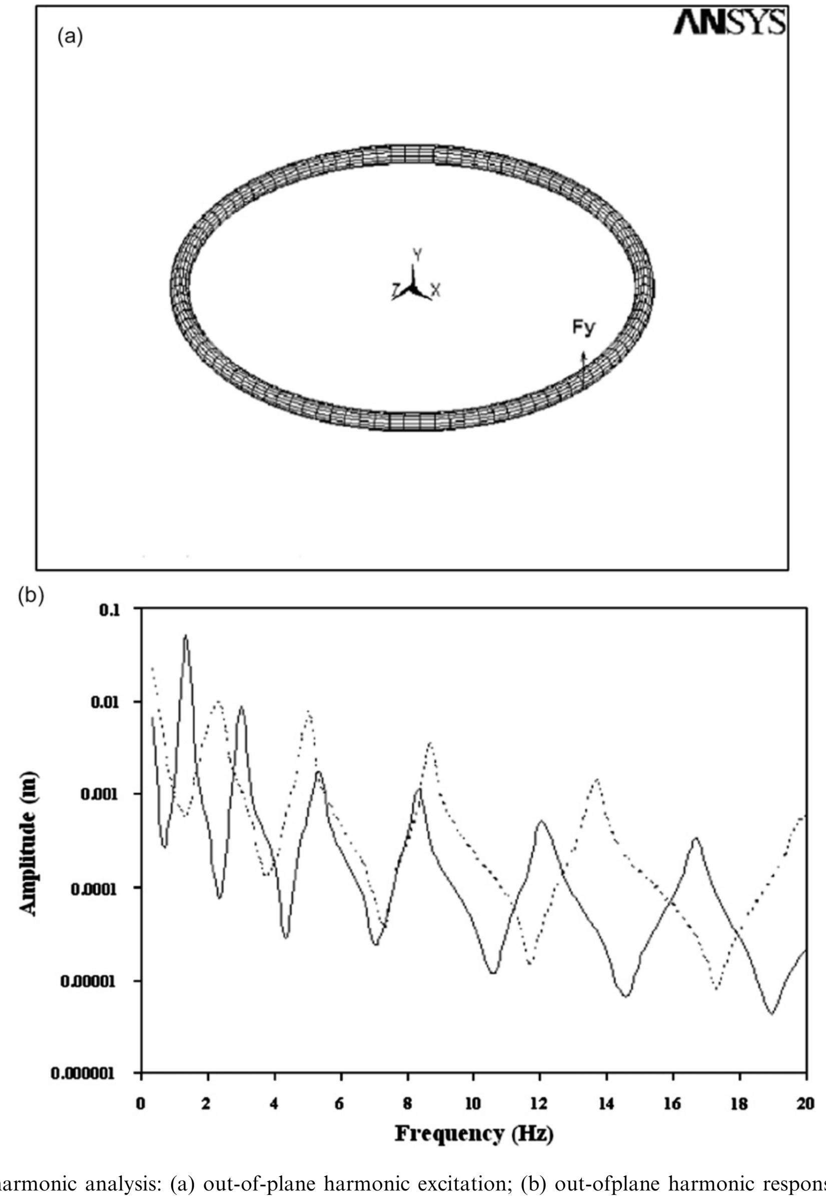

Fig. 5. Out-of-plane harmonic analysis: (a) out-of-plane harmonic excitation; (b) out-ofplane harmonic response; (- · · · · -, model M1; ———, model M3).

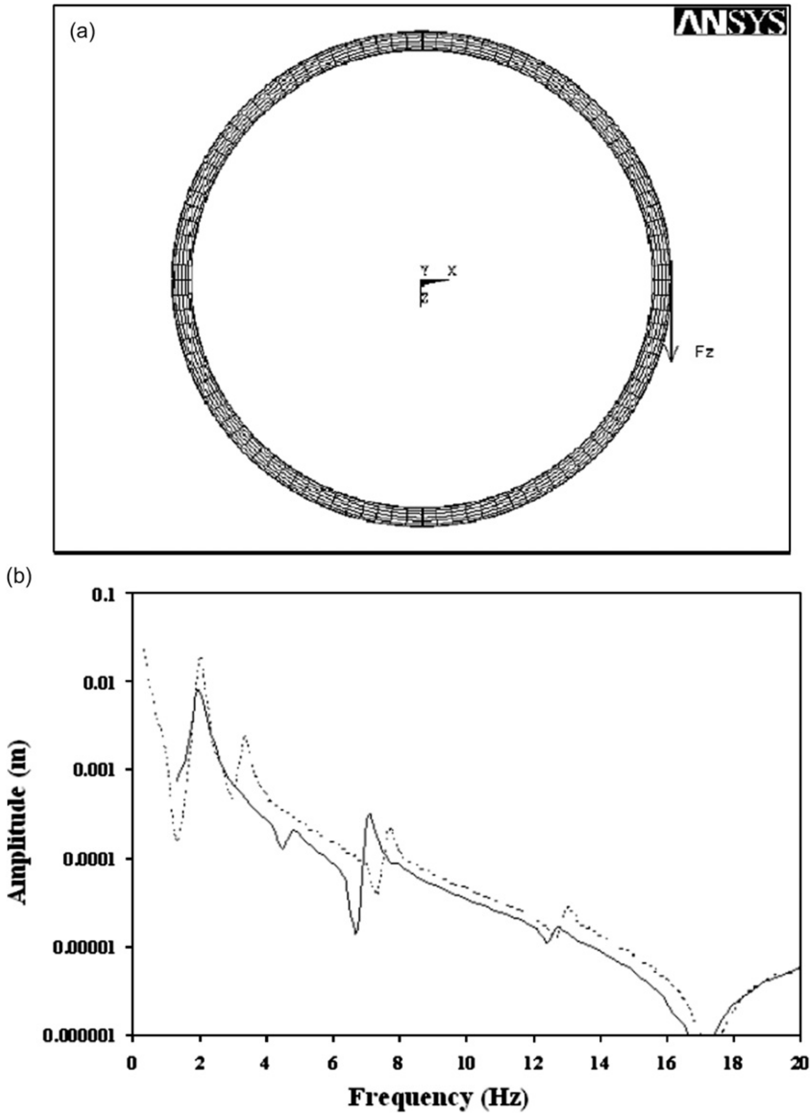

Fig. 6. In-plane tangential harmonic analysis: (a) in-plane tangential harmonic excitation; (b) in-plane tangential harmonic response; (- · · · -, model M1; ———, model M3).

model M3 has in-plane modes corresponding to 2.02, 2.04, 4.6, 7.1, 7.8, 11.8, 12.4, 16.4, 16.8 and 17.9 Hz. The peak corresponding to 2.03 Hz frequency for model M1 is not obtained since it is an in-plane rotational mode and it is not excited by this type of loading. Also, it is noticed that first peak for model M3 is observed around 2.0 Hz. Table 2 indicates that two frequencies exist at 2.0 Hz, i.e. 2.02 and 2.04 Hz. A close analysis reveals that the in-plane excitation considered is able to excite the in-plane vibration mode with frequency 2.02 Hz. Thus in-plane rotational mode with frequency 2.04 Hz is not excited by this force. The peaks corresponding to other frequencies such as 4.6, 7.1 Hz, etc. are duly observed. Fig. 4b indicates that the response is greatly altered by the consideration of dynamic interaction of the fluid inside.

When force is applied in the out of plane direction as shown in Fig. 5a, the out of plane modes are excited. Natural frequencies corresponding to out of plane modes are 2.2, 4.9, 8.7 and 13.7 Hz for model M1 and 1.3, 2.9, 5.2, 8.3, 12.1 and 16.7 Hz for model M3. Sharp peaks in the response are obtained at these frequencies (Fig. 5b). It is observed (Fig. 5b) that the response is significantly altered due to consideration of fluid–shell interaction. The reason for the change in response is mainly the changes in natural frequencies of the two models.

When force is applied in the tangential direction in the plane of torus (Fig. 6a), the in-plane rotational modes are excited. The response is plotted in Fig. 6b. The first peak obtained for both models M1 and M3 is at around 2.0 Hz frequencies. These frequencies are the natural frequencies corresponding to in-plane rotational modes of models M1 and M3. It verifies that the first in-plane rotational mode of vibration is unaffected by the consideration of enclosed fluids.

4. Concluding remarks

The inflatable torus is an integral part of many space inflatable structures. The present work investigates the dynamic coupling between the inflated torus and the enclosed fluid inside it. It is observed that the fluid–structure coupling significantly influences the dynamic behavior of inflated torus. Values of the natural frequencies change by as much as 40% and some additional modes are also observed. Incorporating the effect of fluid by simply adding the mass of the enclosed fluid may sometime lead to erroneous value of frequency. Moreover, some vibration modes may be completely missed out. Therefore, including the effect of coupling between fluid and structure in the analysis of inflatable structures is essential for an accurate description of the dynamic behavior of inflated structures.

References

[1] C. Cassapakis, M. Thomas, Inflatable structures technology development overview, *AIAA 95-3738*, 1995, pp. 1–10.
[2] H. Fang, M.C. Lou, Analytical characterization of space inflatable structures—an overview, *AIAA 99-1272*, 1999.
[3] R.E. Freeland, History of relevant inflatable high-precision space structures technology developments, gossamer spacecraft: membrane and inflatable structures technology for space applications, *AIAA 191*, 2000, pp. 33–48.
[4] S.L. Veldman, C.A.J.R. Vermeeren, Inflatable structures in aerospace engineering—an overview, *ESA SP-468*, 2001.
[5] A.A. Liepins, Free vibrations of prestressed toroidal membrane, *AIAA Journal* 3 (10) (1965) 1924–1933.
[6] P.F. Jordan, Vibration and buckling of pressurized torus shells, *4th Aerospace Science Meeting*, Los Angeles, CA, AIAA Paper No. 66-445, 1966.
[7] P.F. Jordan, Vibration study of a pressurized torus shell part I—experimental study, *NASA CR-884*, 1967.
[8] S. Saigal, T.Y. Yang, H.W. Kim, W. Soedel, Free vibration of a tire as a toroidal membrane, *Journal of Sound and Vibration* 107 (1) (1986) 71–82.
[9] J.A. Lewis, D.J. Inman, Finite element modeling and active control of an inflated torus using piezoelectric devices, *Journal of Intelligent Material Systems and Structures* 12 (2001) 819–833.
[10] A.K. Jha, D.J. Inman, R.H. Plaut, Free vibration analysis of an inflated toroidal shell, *Journal of Vibration and Acoustics* 124 (2002) 387–396.
[11] A.K. Jha, D.J. Inman, Importance of geometric non-linearity and follower pressure load in the dynamic analysis of a gossamer structure, *Journal of Sound and Vibration* 278 (1–2) (2004) 207–231.
[12] D.T. Griffith, J.A. Main, Modal testing of an inflated thin film polymide torus structure, *Proceedings of the 18th International Modal Analysis Conference*, San Antonio, TX, February 2000.
[13] G. Park, M.H. Kim, D.J. Inman, Integration of smart materials into dynamics and control of inflatable space structures, *Journal of Intelligent Materials Systems and Structures* 12 (6) (2001) 423–433.
[14] G. Park, E. Ruggiero, D.J. Inman, Dynamic testing of inflatable structures using smart materials, *Journal of Smart Materials and Structures* 11 (2002) 1–9.
[15] E.J. Ruggiero, A. Jha, G. Park, D.J. Inman, A literature review of ultra-light and inflated toroidal satellite components, *The Shock and Vibration Digest* 35 (3) (2003) 171–181.
[16] K.B. Smalley, M.L. Tinker, Nonlinear pressurization and modal analysis procedure for dynamic modeling of inflatable structures, *Journal of Spacecrafts and Rockets* 39 (5) (2002) 732–739.
[17] H. Kraus, *Thin Elastic Shells*, Wiley, New York, 1967.
[18] W. Soedel, *Vibrations of Shells and Plates*, third ed., Marcel Dekker, New York, 2004.
[19] J.N. Reddy, *An Introduction to the Finite Element Method*, second ed., Tata McGraw-Hill Publishing Company Limited, New Delhi, 2003.
[20] M. Petyt, C.J.C. Jones, Numerical methods in acoustics, in: F. Fahy, J. Walker (Eds.), *Advanced Applications in Acoustics, Noise and Vibration*, Spon Press, New York, 2004, pp. 53–99.