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ScienceDirect
Journal of Sound and Vibration 309 (2008) 778–793
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

An automated parameter selection procedure for finite-element model updating and its applications

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Received 8 May 2006; received in revised form 31 January 2007; accepted 28 July 2007
Available online 25 October 2007

Abstract

Finite-element model updating is an inverse problem to identify and correct uncertain modeling parameters, which leads to better predictions of the dynamic behavior of a target structure. Unlike other inverse problems, the restrictions on selecting parameters are very high since the updated model should maintain its physical meaning. That is, only the regions with modeling errors should be parameterized and the variations of the parameters should be kept small while the updated results give acceptable correlations with experimental data. To avoid an ill-conditioned numerical problem, the number of parameters should be kept as small as possible. Thus it is very difficult to select an adequate set of updating parameters which meet all these requirements. In this paper, the importance of updating parameter selection is illustrated through a case study, and an automated procedure to guide the parameter selection is suggested based on simple observations. The effectiveness of the suggested procedure is tested with two example problems, one is a simulated case study and the other is a real engineering structure.

1. Introduction

The predicted dynamic behavior of a finite-element (FE) model often differs from experimental results of a target structure. Thus, an FE model needs to be verified and, if necessary, updated for further applications. FE model updating is an inverse process to identify and correct uncertain modeling parameters, which leads to better predictions of the dynamic behavior of the structure.

The selection of parameters in model updating is an important problem and there have been some outstanding works in the past. Mottershead et al. [1] performed model updating experiments on an aluminum frame structure by many different sets of updating parameters, which show that all the updated parameters are not justified physically although all the updated models give improved predictions. Gladwell and Ahmadian [2] introduced generic element matrices which can replace the unmodeled effects in the initial model. Friswell

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.07.076

et al. [3] suggested a procedure to select a subset of parameters that can effectively reduce the residual errors between the measured data and the predicted ones using angles between subspaces. The special issue 17(1) of *Mechanical System and Signal Processing* is devoted to the model updating such as the work done by Link and Friswell [4].

All real structures have infinite numbers of degrees of freedom (dofs) and modes, but the data that can be obtained from modal tests are quite limited for practical reasons. Experimental modal analysis rarely uses more than a couple of hundred sensors. Thus, the number of measured dofs is very small, and also the available transducers and hardware limit the frequency range of measurements. On the other hand, FE models consist of many FEs, easily extending in many cases to several thousands. Thus, due to the inherent limitations of experimental data, the number of parameters that can be used to modify an FE model far exceeds that of the measured data of a target structure. There can be numerous modified or updated FE models that agree well with the incomplete test data [1,5]. But, if the aim of model updating is not simply to mimic the incomplete test results, there must be some restrictions on the selection of updating parameters and their allowable changes so that the updated model retains its physical foundation.

Updating parameters should be selected with the aim of correcting modeling errors. So, only the regions containing modeling errors should be parameterized and allowed to change in correction process [6]. The criteria to be minimized for model improvement should be sensitive to chosen parameters. Otherwise, the updating parameters easily deviate far from their initial values and lose their physical meaning [6]. If only the sensitivity is concerned, the best way of parameter selection is to assign an updating parameter to each of the FEs having modeling errors. But, usually an FE model for a real structure has modeling errors in so many FEs, it is impractical to allocate an updating parameter for each of the FEs. This is because the updated parameter values of neighboring elements can be oscillatory, which are physically meaningless [7]. Moreover, in numerical point of view, too many updating parameters cause ill-conditioned problems or trapping in many local minima [6]. This paper suggests an idea to select suitable FE model updating parameters among the many candidates set.

In the first part of this paper, the importance of updating parameter selection is illustrated through a case study and then an automated procedure to guide the parameter selection is suggested based on simple observations. After assigning an updating parameter to each of the FEs with modeling errors, this method iteratively reduce the number of the parameters by grouping neighboring parameters at the cost of minimum sacrifice of total sensitivity. Finally, the effectiveness of the suggested procedure is tested with two example problems, one is a simulated case study and the other is a real engineering structure.

2. Importance of updating parameter selection

In the followings, key issues related to the selection of updating parameters are discussed and its importance is illustrated through a case study.

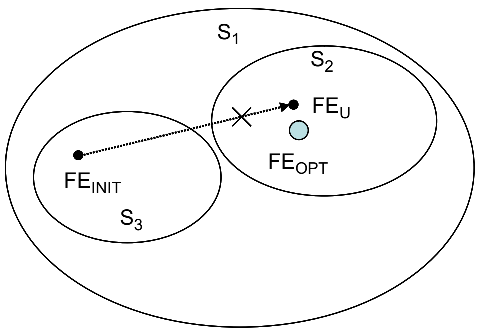

Fig. 1. Importance of the updating parameter selection.

2.1. Key issues

The selection of updating parameters is a very important step in model updating. Fig. 1 shows its importance schematically.

- Space $\mathbb{S}_1$ contains all the possible FE models of a structure.
- Space $\mathbb{S}_2$ contains all the FE models that correlate well with experimental results. One of these models, FE_{opt}, gives the best possible description of dynamic behavior of the structure.
- Space $\mathbb{S}_3$ is a set of models that can be obtained from the initial FE model, FE_{init}, by varying the selected updating parameters. Both the initial FE model, FE_{init}, and the updated model, FE_u, are the members of $\mathbb{S}_3$.

It should be emphasized that the dimension of $\mathbb{S}_3$ is determined by the selected updating parameters and their constraints. Thus, an inappropriate selection of updating parameters cause $\mathbb{S}_3$ does not have a common space with $\mathbb{S}_2$ (Fig. 1). As a consequence, the updated model having satisfactory correlation with experimental results can never be obtained whatever updating techniques are used.

For a successful model updating, it is known that updating parameters should satisfy the three requirements [6]:

- The regions containing modeling errors should be parameterized.
- The criteria or objective functions that designate the differences between analytical and experimental results should be sensitive to selected parameters.
- The number of updating parameters should be as small as possible to avoid numerical difficulties.

The first step of updating parameter selection is to locate the regions with modeling errors. Examples of obvious candidates can be boundaries and joints. Also, systematic approaches can be used for this purpose [8]. Error location is an important topic of its own right in model updating study, so it is not further dealt on this topic. In this paper, it is assumed that the regions or FEs with modeling errors are correctly located and the modeling parameters (such as mass density, Young's modulus, thickness, etc.) associated with the errors are identified. Thus, parameter selection procedure suggested in this paper is focused on the last two requirements.

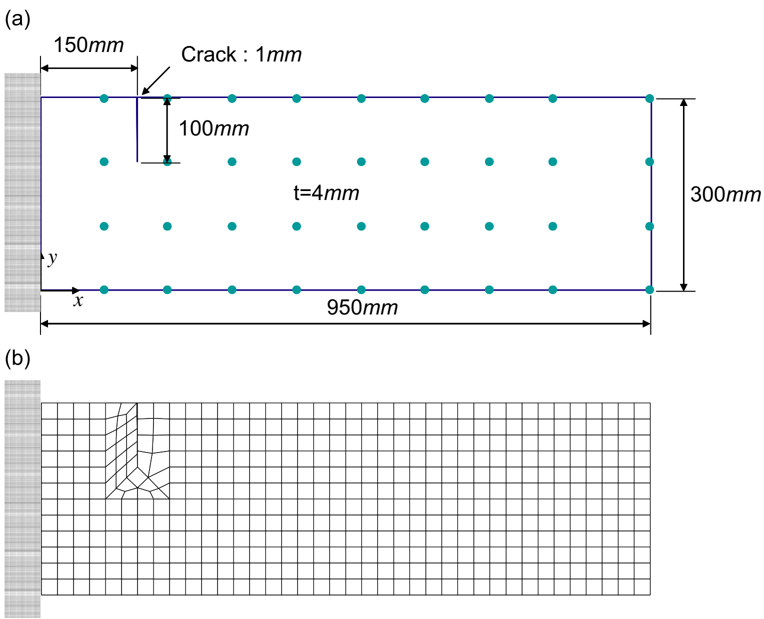

Fig. 2. A cracked clamped plate: (a) geometric dimensions and simulated vibration measurement points and (b) fine FE model (3126dofs).

2.2. Case study

A clamped plate having a crack is provided to demonstrate the effects of updating parameter selections on updated results (Fig. 2(a)). To simulate experimental data, a fine FE model with 3126dofs, is constructed (Fig. 2(b)). It is assumed that out-of-plane (z-direction) vibrations are measured at 36 points as marked in Fig. 2(a). The simulated experimental mode shapes are plotted in Fig 3. Fig. 4 shows an initial FE model with 840dofs. The corresponding mode shapes are plotted in Fig. 5. Due to the crack of the test plate, the modal properties from the initial FE model show deviations from those of the test model as summarized in Table 1. Here, the experimental and analytical modes are paired using the modal assurance criteria (MAC). The MAC between a measured mode ψ_i and an analytical mode ϕ_j is defined as

$$\mathrm{MAC}_{ij} = \frac{|\psi_i^{\mathrm{T}} \phi_j|^2}{(\phi_j^{\mathrm{T}} \phi_j)(\psi_i^{\mathrm{T}} \psi_i)}.$$

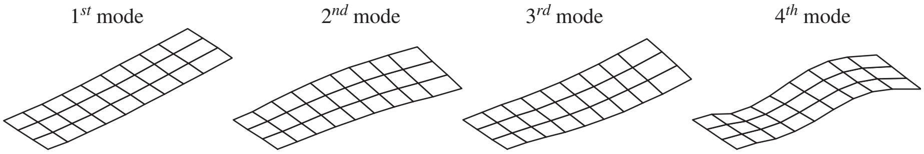

Fig. 3. Mode shapes of the plate from the fine FE model.

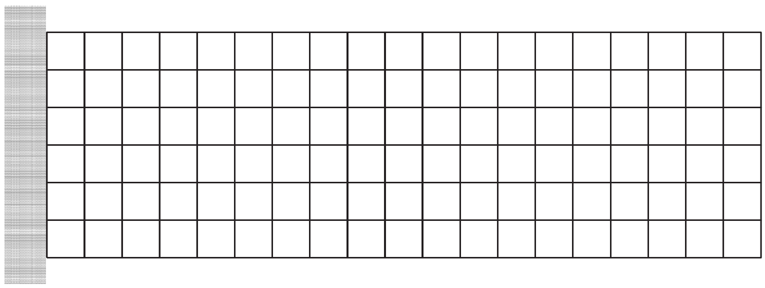

Fig. 4. Initial FE model (840dofs).

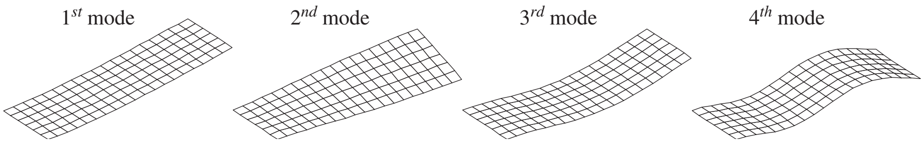

Fig. 5. Mode shapes of the plate from the initial FE model.

Table 1
Comparison of modal properties of the cracked plate

Mode	Natural frequency (Hz)			MAC
	Simulated experiment	Initial FE model	Error (%)	
1	3.6011	3.7721	4.7488	1.0000
2	22.7184	24.5476	8.0520	0.6531
3	23.7103	23.5895	0.5095	0.4894
4	65.0973	66.3571	1.9352	0.9501

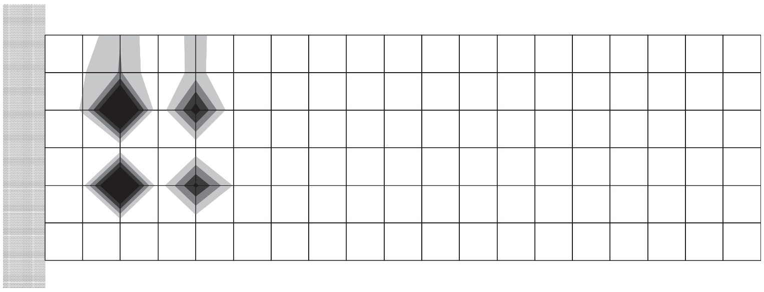

Fig. 6. Error location of the initial FE model utilizing a force balance method.

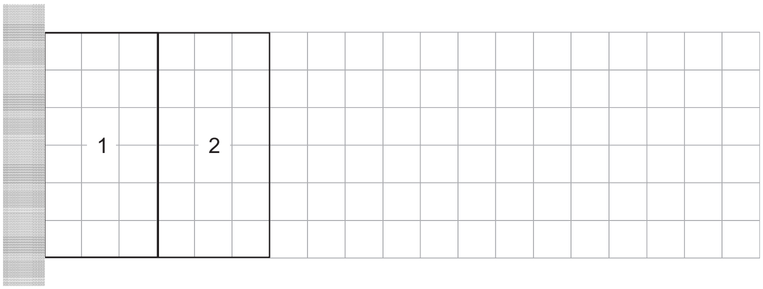

Fig. 7. Case study—setting two updating parameters.

The 2nd and 3rd mode pairs are poorly correlated and the initial FE model needs to be updated for a better correlation. Using an error location technique [8], the region with dominant modeling errors are checked as shown in Fig. 6. The plot shows dominant errors in the initial model around the cracked area. In this case study, the FEs around the dominant error region are grouped into two as in Fig. 7. It is assumed that the mass matrix of the initial FE model is correct and only the stiffness matrix needs to be updated. Thus, the stiffness correction matrix is expressed when we setting two updating parameters p_{k_1} and p_{k_2}.

$$\Delta\mathbf{K} = \sum_{i=1}^{2} p_{k_i}\mathbf{K}_i, \tag{1}$$

where $\mathbf{K}_i$ is the stiffness matrix of the ith region, and the coefficient p_{k_i} is the updating parameter. Among the correlations shown in Table 1, the natural frequency error of the 2nd mode pair and the MAC values of 2nd and 3rd mode pairs, which show the most undesirable correlations, are set as the multiobjective function to be minimized:

$$\{F_1, F_2, F_3\} = \{((f_{a_2} - f_{x_2})/f_{x_2})^2, \quad 1 - \mathrm{MAC}_{22}, \quad 1 - \mathrm{MAC}_{33}\}, \tag{2}$$

where MAC_{ii} is the MAC value of ith mode pair, and f_{x_i} and f_{a_i} are the ith experimental and analytical natural frequencies, respectively. To prevent the other values moving to poor optimization results, they are bounded with constraints:

$$\begin{aligned} &\left(\frac{f_{a_i} - f_{x_i}}{f_{x_i}}\right)^2 \leqslant 0.0025, \quad i = 1,3,4, \\ &\mathrm{MAC}_{ii} \geqslant 0.9, \quad i = 1,4. \end{aligned} \tag{3}$$

The maximum allowable change of the updating parameters, p_{k_1} and p_{k_2}, are set as 0.7. To evaluate the effectiveness of the selected parameters, the multiobjective optimization problem defined by Eqs. (2) and (3) is solved using a multiobjective evolutionary algorithm [9]. The resulting Pareto front is plotted as in Fig. 8. It should be noted that the objective functions, F_2 and F_3, seldom vary compared to their initial values, although

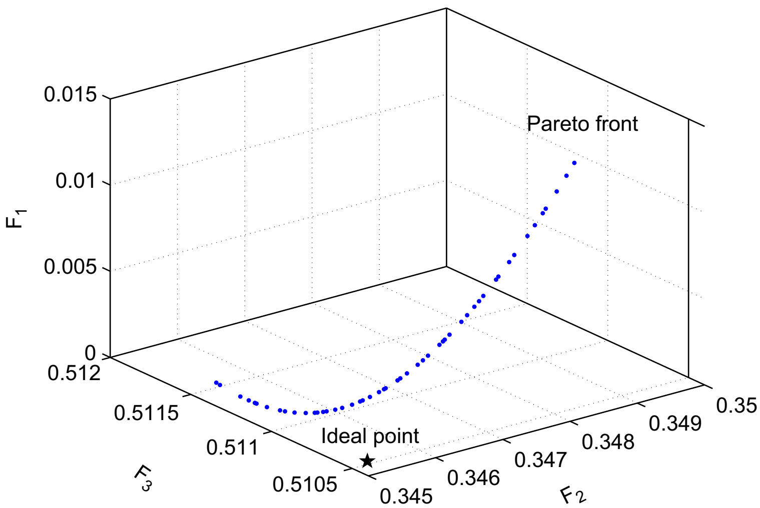

Fig. 8. Case study—Pareto front and the ideal point.

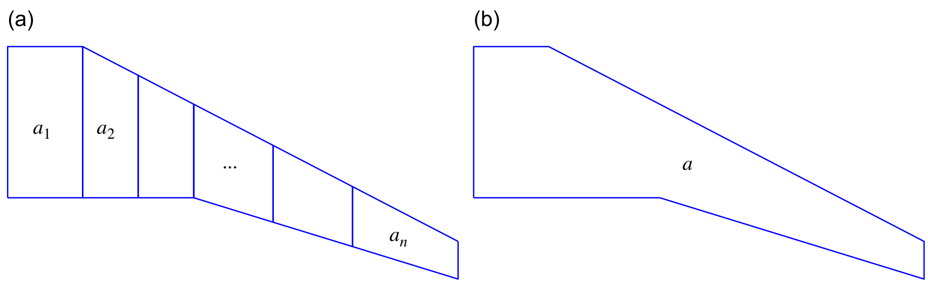

Fig. 9. A substructure with modeling errors : (a) n updating parameter and (b) one updating parameters.

F_1 changes drastically. The *ideal point* of Eq. (2) is calculated as

$$\{F_1, F_2, F_3\} = \{0.0000, 0.3453, 0.5105\}. \tag{4}$$

The ideal point [10] is obtained by minimizing each of the objective functions in Eq. (2) individually subject to the constraints (3). Note that the ideal point corresponds to the lower bound of the Pareto front, which is not realizable.

Although the parameters are selected in the regions of large modeling errors, even the lower bound of the Pareto front is not satisfactory. Thus it can be concluded that the parameter selection is not appropriate. Then, how can we obtain an appropriate parameter set? This usually requires a considerable physical insight into the target structure, and trial-and-error approaches are commonly used. But in this work, an idea to get appropriate parameters will be suggested in the following section.

3. Updating parameter selection procedure

3.1. Basic observations

Consider a substructure with n updating parameters $(a_1, a_2, \ldots, a_n)$ as in Fig. 9(a). An objective function or criterion F which defines a difference between analytical and experimental results is modified utilizing the updating parameters. In a linear approximation, the maximum possible variation of F is given by

$$\sum_{i=1}^{n} \left| \frac{\partial F}{\partial a_i} \right| \Delta a, \tag{5}$$

where Δa is the maximum allowable change of the updating parameters. Thus, the absolute sum of the sensitivities, $\sum_{i=1}^{n}|\partial F/\partial a_i|$, represents the effectiveness of the selected updating parameters in modifying the objective function and is defined as *total sensitivity*.

Now, suppose the updating parameters, $a_1, a_2, \ldots, a_n$, are merged into one updating parameter a as in Fig. 9(b). The total sensitivity of the objective function F with respect to the merged parameter a is simply $|\partial F/\partial a|$. It is easy to see that

$$\frac{\partial F}{\partial a} = \frac{\partial F}{\partial a_1} + \frac{\partial F}{\partial a_2} + \cdots + \frac{\partial F}{\partial a_n} = \sum_{i=1}^{n} \frac{\partial F}{\partial a_i}, \tag{6}$$

and hence

$$\left|\frac{\partial F}{\partial a}\right| = \left|\frac{\partial F}{\partial a_1} + \frac{\partial F}{\partial a_2} + \cdots + \frac{\partial F}{\partial a_n}\right| \leqslant \sum_{i=1}^{n} \left|\frac{\partial F}{\partial a_i}\right|. \tag{7}$$

The equality in Eq. (7) holds only when the signs of $(\partial F/\partial a_1), (\partial F/\partial a_2), \ldots, (\partial F/\partial a_n)$ are the same. Thus, it can be said that the two requirements of updating parameters in section 2.1, number of parameters and their sensitivities, are competitive. That is, by grouping updating parameters into larger parameters, the number of updating parameters can be reduced, but the total sensitivity decreases in general.

From this basic observations, we can construct a set of updating parameters such that the objective functions of primary concern are most sensitive to the selected updating parameters. The parameter selection procedure suggested in this study is accomplished by a sequence of two different selection phases. After the 1st phase of parameter selection procedure, the analyst can stop the parameter selection procedure if the resulting number of parameters are acceptable. Otherwise, he or she can proceeds to the 2nd phase so that the number of parameters can be further reduced.

3.2. Updating parameter selection

3.2.1. 1st phase of parameter selection

Among the two requirements of updating parameters that are dealt with in this study, if only the sensitivities of updating parameters are concerned, the best way of selecting updating parameters is to assign an updating parameter to each of the FE with modeling errors. By grouping the individual elements into several substructures and assigning an updating parameter to each substructure, the number of parameters can be reduced at the cost of total sensitivity decrease. But, examining Eq. (7), the number of parameters can be lowered without sacrificing the total sensitivity by merging the neighboring elements as long as the signs of the sensitivities of the merging elements are the same. Based on this fact, the 1st phase of updating parameter selection for a single-objective function F is stated as

Step 1: Assign an updating parameter a to each FE having modeling errors and calculate $\partial F/\partial a_i$ for each parameter,

Step 2: Merge two neighboring parameters a_i and a_j into one parameter if $\text{sign}(\partial F/\partial a_i) = \text{sign}(\partial F/\partial a_j)$. Note the sensitivity of the objective function with respect to the merged parameter is simply equal to $\partial F/\partial a_i + \partial F/\partial a_j$ from Eq. (6). Repeat this until no neighboring parameters have the same sensitivity sign.

Note that the total sensitivity of F with respect to the resulting parameters remains unchanged. The updating parameter selection procedure is schematically shown in Fig. 10(a).

When multiple objective functions $\mathbf{F} = \{F_p, F_q, \ldots, F_r\}$ are considered simultaneously, the procedure can simply be extended:

Step 1: Assign an updating parameter a to each FE with modeling errors and calculate $\partial \mathbf{F}/\partial a_i = (\partial/\partial a_i)\{F_p, F_q, \ldots, F_r\}$ for each parameter,

Step 2: Merge two neighboring parameters a_i and a_j into one parameter if $\text{sign}(\partial \mathbf{F}/\partial a_i) = \text{sign}(\partial \mathbf{F}/\partial a_j)$, where $\text{sign}(\partial \mathbf{F}/\partial a_i) \equiv \{\text{sign}(\partial F_p/\partial a_i), \ldots, \text{sign}(\partial F_r/\partial a_i)\}$. Note the sensitivity of the objective functions with respect to the merged parameter is simply equal to $\partial \mathbf{F}/\partial a_i + \partial \mathbf{F}/\partial a_j$. Repeat this until no neighboring parameters have the same sensitivity sign.

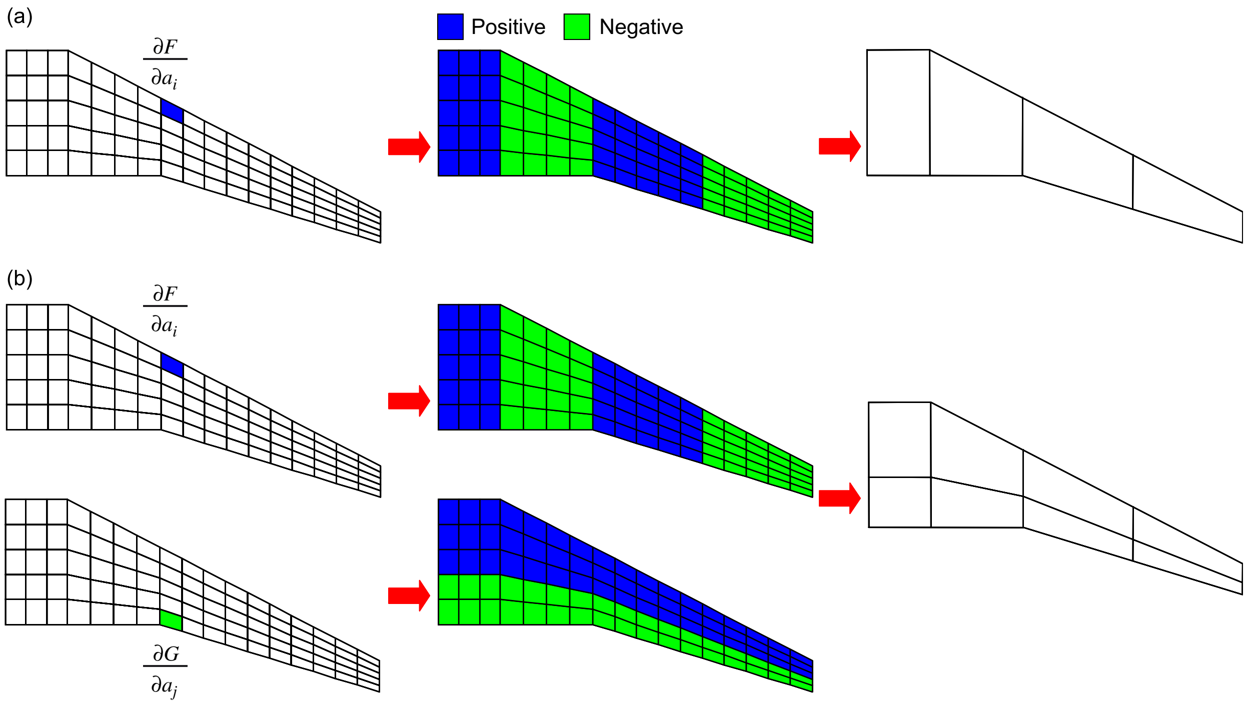

Fig. 10. Schematic of the 1st phase of updating parameter selection procedure: (a) for a single-objective function and (b) for multiple objective functions.

As with the single-objective function case, the total sensitivity of each objective function is not changed by the parameter reduction. Fig. 10(b) shows the parameter selection procedure when objective function F and G are considered simultaneously.

As readily seen in the above, the outstanding feature of the 1st phase of the parameter selection procedure is that the objective functions of primary concern are kept sensitive to the resulting parameters while the number of the parameters are reduced as small as possible.

3.2.2. 2nd phase of parameter selection

As a result of the 1st phase of the parameter selection procedure, a list of updating parameters are obtained. Obviously, none of the neighboring parameters have the same sign of the sensitivities. When the number of the resulting parameters are still large and unacceptable, the 2nd phase can be processed. In this case, sacrifice of total sensitivity should be accepted to some extend.

Consider two neighboring parameters a_i and a_j. By merging the two parameters, the total sensitivity is changed from

$$\sum_{\substack{k=1\\k\neq i,j}}^{n}\left|\frac{\partial \mathbf{F}}{\partial a_k}\right|+\left|\frac{\partial \mathbf{F}}{\partial a_i}\right|+\left|\frac{\partial \mathbf{F}}{\partial a_j}\right| \tag{8}$$

to

$$\sum_{\substack{k=1\\k\neq i,j}}^{n}\left|\frac{\partial \mathbf{F}}{\partial a_k}\right|+\left|\frac{\partial \mathbf{F}}{\partial a_i}+\frac{\partial \mathbf{F}}{\partial a_j}\right|, \tag{9}$$

where $|\mathbf{F}| \equiv \{|F_p|, |F_q|, \ldots, |F_r|\}$ and n is the total number of the updating parameters. Thus, the decrement of the total sensitivity by this grouping is expressed as

$$\left|\frac{\partial \mathbf{F}}{\partial a_i}\right|+\left|\frac{\partial \mathbf{F}}{\partial a_j}\right|-\left|\frac{\partial \mathbf{F}}{\partial a_i}+\frac{\partial \mathbf{F}}{\partial a_j}\right|. \tag{10}$$

In other words, the required sacrifice for reducing one parameter is equal to Eq. (10). Thus, it is quite reasonable to search two neighboring parameters that minimize Eq. (10) and merge them as one parameter. This results in one parameter reduction at the minimal cost. Note that $\mathbf{F} = \{F_p, F_q, \ldots, F_r\}$ is a vector quantity. Thus, there can be various methods to evaluate the vector sacrifice (Eq. (10)). The nondominance concept of multiobjective optimization can be an example. In this study, a scalar index, assuming that every objective function is equally important, is presented as

$$\sum_{k=p,q,\ldots,r} \Delta F'_k, \tag{11}$$

where the normalized sacrifice $\Delta F'_k$ is defined as the sacrifice of the objective function F_k divided by its total sensitivity at the beginning of the 2nd phase (or before any decrement). From these observations, the 2nd phase of the updating parameter selection procedure is suggested:

Step: Find two neighboring substructures which minimize Eq. (11) and merge them as one parameter. Repeat this procedure until some ending criteria, such as the finial number of parameters, the maximum allowed sacrifice or both, are met.

3.2.3. Program implementation

For simple FE models, the parameter selection procedure can be performed manually as illustrated in Fig. 10. But for complex structures, this can be a tedious or difficult work. For a program implementation, the computer needs to know whether two FEs or substructures are neighboring or not. As a FE consists of a group of nodes, two neighboring elements must share some nodes. For example, an 1D element must share one node with the other element if they are neighboring. For each combination of three different kinds of elements, the number of sharing nodes of two neighboring elements is summarized in Fig. 11. Pseudocodes implementing the parameter selection procedure as well as the neighborhood test are provided in Ref. [11].

3.3. Discussions

In this section, the importance of updating parameters are demonstrated using a cracked plate. Although the regions having modeling errors were correctly located, the concerning error criteria, F_2 and F_3 in Eq. (2), can be insensitive to such selected parameters. To evaluate the effectiveness of selected updating parameters, the concept of total sensitivity was used. When we group the FEs into several substructures inappropriately, the resulting total sensitivity can be useless because the sacrifice of the total sensitivity is too large. In the cracked plate example, the potential sensitivity of $\{F_1, F_2, F_3\}$ has changed from $\{0.0422, 1.5004, 2.2371\}$ to $\{0.0422, 0.0088, 0.0014\}$ when we group the elements as Fig. 7. From simple observations, a parameter

	1D element	2D element	3D element
1D element	1	1	1
2D element	1	2	2
3D element	1	2	3 or 4

Fig. 11. Number of sharing nodes of two neighboring elements.

selection procedure was proposed which is accomplished by a sequence of two different selection phases. The noticeable advantage of the 1st phase of the parameter selection method is that the objective functions of primary interest remain sensitive to the resulting parameters while the number of parameters are reduced considerably. When the resulting number of the updating parameters after the 1st phase is still unacceptable, then the parameters are grouped at the sacrifice of total sensitivity in the 2nd phase. Using the suggested parameter selection method, it is guaranteed that the concerning objective functions remain most sensitive to the resulting parameters. But it should be emphasized that the suggested parameter selection method must be followed by correct error location. Unless, the updated model cannot describe the real dynamic characteristics of the target structure and the updated parameters lose their physical foundation.

4. Examples

In this section, the proposed parameter selection procedure is applied to two example problems, one is the plate example given in Section 2.2 and the other is a cover structure of hard disk drive (HDD).

4.1. Cracked clamped plate

The cracked plate given in section 2.2 is taken as an example problem. From Table 1, it can be noticed that the natural frequency error of the 2nd mode pair and the MAC values of 2nd and 3rd mode pairs show most undesirable correlations. Thus the updating parameter selection procedure is applied considering the following three criteria:

$$\{F_1, F_2, F_3\} = \{((f_{a_2} - f_{x_2})/f_{x_2})^2, \quad 1 - \mathrm{MAC}_{22}, \quad 1 - \mathrm{MAC}_{33}\}. \tag{12}$$

As in Section 2.2, it is assumed that only the stiffness matrix need to be updated. For each FE in the region with the modeling error (see Fig. 6), the sensitivities of the criteria (Eq. (12)) with respect to the chosen stiffness parameter are calculated and the resulting signs of the sensitivities are plotted in Fig. 12. In this case, the sensitivities of F_2 and F_3 have the same sign. Using this information, the 1st phase of the parameter selection procedure is applied so that the criteria given in Eq. (12) remain sensitive to the resulting parameters.

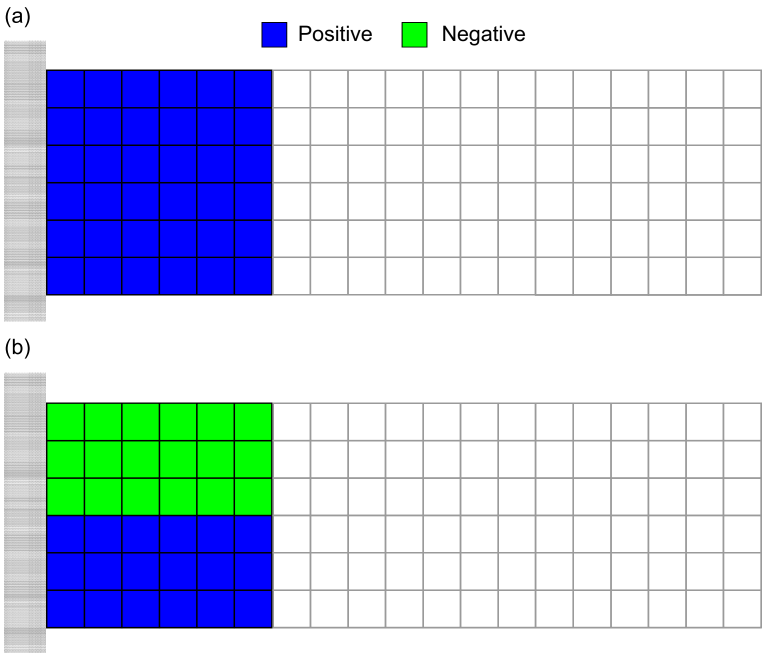

Fig. 12. Sign maps of the sensitivities of $\{F_1, F_2, F_3\}$: (a) $\partial F_1/\partial p_{k_i}$; (b) $\partial F_2/\partial p_{k_i}$ and $\partial F_3/\partial p_{k_i}$.

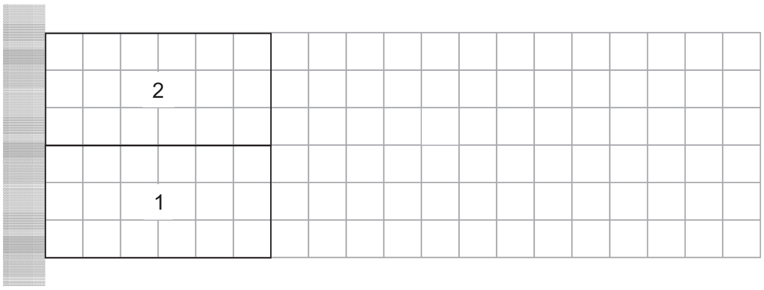

Fig. 13. Two updating parameters after the 1st phase of the parameter selection procedure.

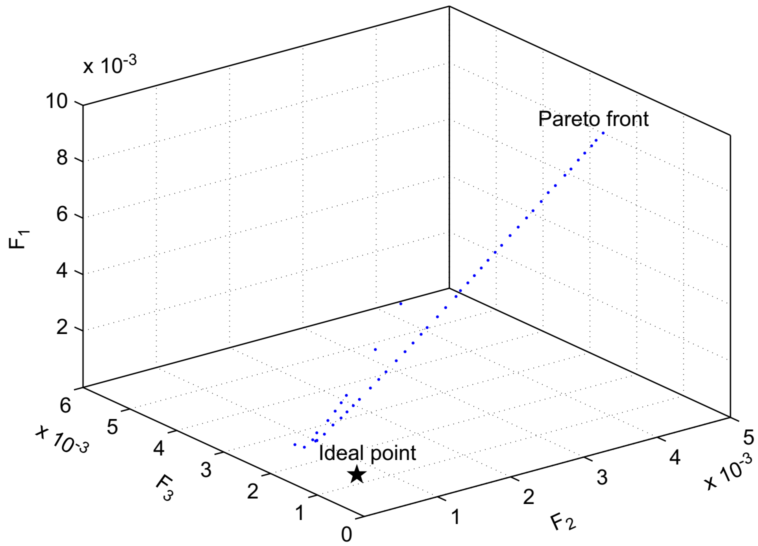

Fig. 14. Cracked plate—Pareto front and the ideal point.

Fig. 13 shows the selected parameters after the 1st phase. The stiffness correction matrix is written as

$$\Delta \mathbf{K} = \sum_{i=1}^{2} p_{k_i} \mathbf{K}_i, \tag{13}$$

where p_{k_i} and $\mathbf{K}_i$ are the updating parameter and stiffness matrix associated with the ith substructure. Obviously, the total sensitivity of the criteria (Eq. (12)) remains unchanged and is evaluated as

$$\left\{ \sum_{i=1}^{2} \left| \frac{\partial F_1}{\partial p_{k_i}} \right|, \sum_{i=1}^{2} \left| \frac{\partial F_2}{\partial p_{k_i}} \right|, \sum_{i=1}^{2} \left| \frac{\partial F_3}{\partial p_{k_i}} \right| \right\} = \{0.0422, 1.5004, 2.2371\}. \tag{14}$$

Since the number of the updating parameters is acceptable, the 2nd phase of the updating parameter selection procedure is not necessary. Thus the parameter selection procedure stops here.

The initial FE model is updated using the selected parameters. Fig. 14 shows the Pareto front of the multiobjective function of Eq. (12) under the same constraints as in Section 2.2. The ideal point is obtained as

$$\{F_1, F_2, F_3\} = \{0.0000, 0.0007, 0.0013\}. \tag{15}$$

Compared to the results in Section 2.2, it can be noticed that the initial FE model is improved drastically. Table 2 summarizes modal properties of an updated FE model. The updated parameter have physical meaning because, due to the crack, p_{k_2} is negative and p_{k_1} is close to zero as shown in Table 2. From these observations, it can be said that the model updating was successful.

Table 2
Comparison of modal properties of the cracked plate and the updated FE model

Mode	Natural frequency (Hz)			MAC
	Simulated experiment	Updated model[a]	Error (%)	
1	3.6011	3.4526	−4.1231	0.9999
2	22.7184	22.0003	−3.1606	0.9694
3	23.7103	23.7655	0.2327	0.9694
4	65.0973	62.4133	−4.1231	0.9912

[a] $p_{k_1} = 0.1772$, $p_{k_2} = -0.5495$.

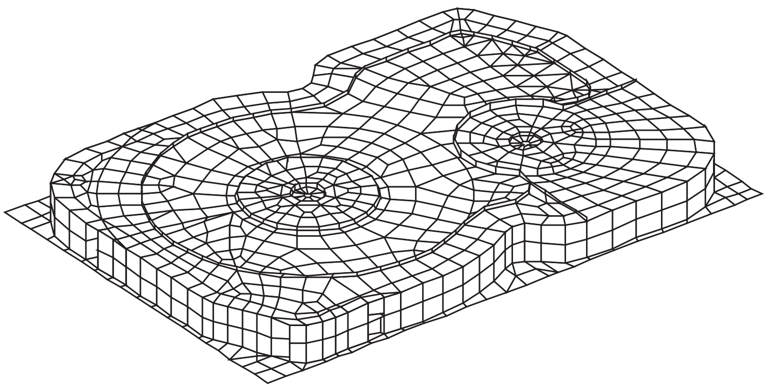

Fig. 15. Finite element model of hard disk drive (HDD) cover structure.

4.2. Hard disk drive cover structure

The suggested parameter selection procedure is applied to an FE model of an HDD cover structure. The HDD cover is a rather complex 3D structure. In the FE model development, approximations are made in thickness because the actual HDD cover shell does not have uniform thickness but has tapered or continuous variations in thickness. The thicknesses are measured at some representative points to build an FE model. The resulting FE model is shown in Fig. 15, which consists of solid, shell and beam elements (total 1115 elements, 6732dofs).

To validate the initial FE model, a modal test of the target structure is conducted. The frequency range of interest is from 0 to 3 kHz where high vibration and noise levels were observed during an operational test of the HDD. To simulate free–free boundary condition, the cover is supported using soft rubbers. The structure is excited by an impact hammer and responses are measured at 66 points by a laser doppler vibrometer. A CADA-X system is used to measure frequency-response functions and extract natural frequencies and mode shapes. The modal properties are compared in Table 3. It tells that the natural frequency errors of the 6th, 7th, 8th, and 9th mode pairs are larger than 3%. Also the MAC values of 3rd, 4th, 5th, and 10th mode pairs are below 0.9. To improve these unsatisfactory correlations, an FE model updating is performed.

First, using an error location technique [8], the region with dominant errors are located as in Fig. 16. According to the error location results, 628 shell elements out of 1115 elements turn out to contain the dominant modeling errors. Now the suggested parameter selection procedure is applied to these shell elements. From Table 3, it can be noticed that the natural frequency error of the 7th mode pair is much larger than 3%, and the 3rd, 4th and 5th mode pairs show undesirable correlations as their MAC values designate. Thus, the updating parameters are selected considering the following four criteria:

$$\{F_1, F_2, F_3, F_4\} = \left\{ \left(\frac{f_{a_7} - f_{x_7}}{f_{x_7}} \right)^2, 1 - \mathrm{MAC}_{33}, 1 - \mathrm{MAC}_{44}, 1 - \mathrm{MAC}_{55} \right\}. \tag{16}$$

Table 3
Comparison of the experimental and analytical modal properties before updating

Mode	Natural frequency (Hz)			MAC
	Experiment	Initial FE model	Error (%)	
1	409.68	404.13	−1.3507	0.9847
2	908.15	931.94	2.6206	0.9831
3	1707.65	1669.00	−2.2633	**0.8326**
4	1748.86	1709.13	−2.2717	**0.7754**
5	1793.23	1757.94	−1.9681	**0.8382**
6	2474.99	2399.10	**−3.0663**	0.9496
7	2843.29	2723.27	**−4.2213**	0.9469
8	2976.06	2878.29	**−3.2853**	0.9360
9	3113.84	3016.39	**−3.1298**	0.9582
10	3268.98	3182.76	−2.6374	**0.8905**

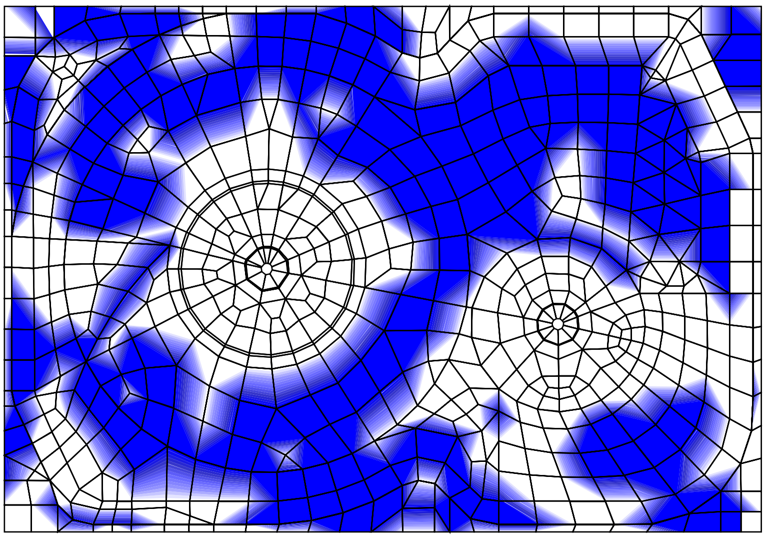

Fig. 16. Error location of the initial FE model.

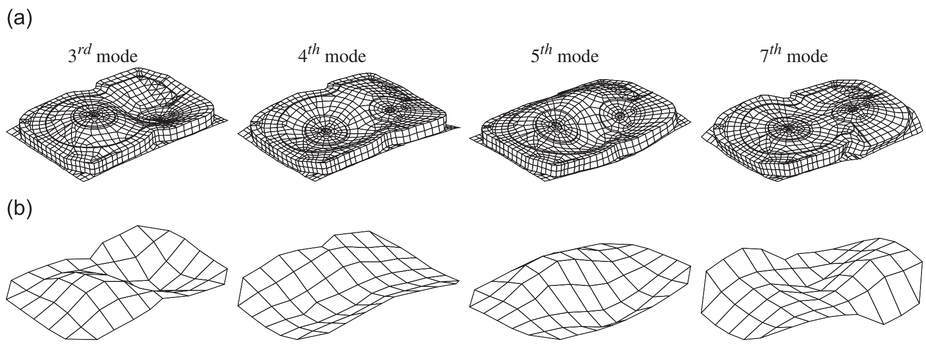

Fig. 17. Mode shapes of the cover structure: (a) experimental mode shapes and (b) analytical mode shapes.

Some important mode shapes are directly compared in Fig. 17. For simple structures, mode shapes can provide a useful information for selecting updating parameters. But, it is not for complicated structures like this HDD cover structure.

Among various physical quantities for updating, thickness is selected because approximation of thickness is made in the initial model development as stated in the above. For each of the shell elements having modeling errors, the sensitivities of the criteria (Eq. (16)) with respect to thickness parameter are calculated and the resulting signs of the sensitivities are shown in Fig. 18. The total sensitivities of the criteria are calculated as

$$\left\{\sum_{i=1}^{628}\left|\frac{\partial F_1}{\partial t_i}\right|,\sum_{i=1}^{628}\left|\frac{\partial F_2}{\partial t_i}\right|,\sum_{i=1}^{628}\left|\frac{\partial F_3}{\partial t_i}\right|,\sum_{i=1}^{628}\left|\frac{\partial F_4}{\partial t_i}\right|\right\} = \{0.0598, 8.1355, 16.4920, 5.2681\}. \tag{17}$$

From these information, the 1st phase of the parameter selection procedure is applied. Fig. 19 shows the resulting updating parameters after the 1st phase. The number of the parameters can be reduced from 628 to 150. Obviously the total sensitivities are not changed:

$$\left\{\sum_{i=1}^{150}\left|\frac{\partial F_1}{\partial t_i}\right|,\sum_{i=1}^{150}\left|\frac{\partial F_2}{\partial t_i}\right|,\sum_{i=1}^{150}\left|\frac{\partial F_3}{\partial t_i}\right|,\sum_{i=1}^{150}\left|\frac{\partial F_4}{\partial t_i}\right|\right\} = \{0.0598, 8.1355, 16.4920, 5.2681\}. \tag{18}$$

Although the number of the updating parameters are reduced to 150 without any sacrifice of the total sensitivities, it is still too many. Thus numerical difficulties are expected in the optimization process. Thus, the 2nd phase of the parameter selection procedure is applied to further reduce the number of updating parameters. The substructures are grouped until the number of the updating parameters become 20 with the minimal sacrifice of the potential sensitivity at each step. Although the total sensitivities are decreased slightly from Eq. (18) to

$$\left\{\sum_{i=1}^{20}\left|\frac{\partial F_1}{\partial t_i}\right|,\sum_{i=1}^{20}\left|\frac{\partial F_2}{\partial t_i}\right|,\sum_{i=1}^{20}\left|\frac{\partial F_3}{\partial t_i}\right|,\sum_{i=1}^{20}\left|\frac{\partial F_4}{\partial t_i}\right|\right\} = \{0.0568, 7.7484, 15.6068, 4.9034\}, \tag{19}$$

but the number of the updating parameter is drastically reduced from 150 to 20. The finial updating parameter are shown in Fig. 20. The initial FE model is updated by varying the selected 20 thickness parameters. The allowed maximum change of the parameters is set to 5% (about 50 μm) considering both the actual variation in thickness of the cover and measurement error (or resolution) of the measuring device. Modal properties of

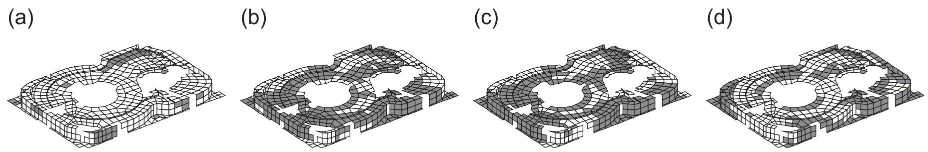

Fig. 18. Signs of sensitivities: (a) $\partial F_1/\partial t_i^e$ (b) $\partial F_2/\partial t_i^e$ (c) $\partial F_3/\partial t_i^e$ (d) $\partial F_4/\partial t_i^e$; □ positive; ■ negative.

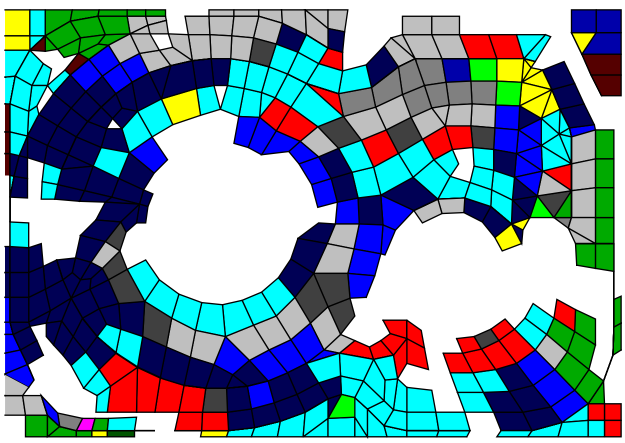

Fig. 19. Updating parameters after the 1st phase of the parameter selection procedure.

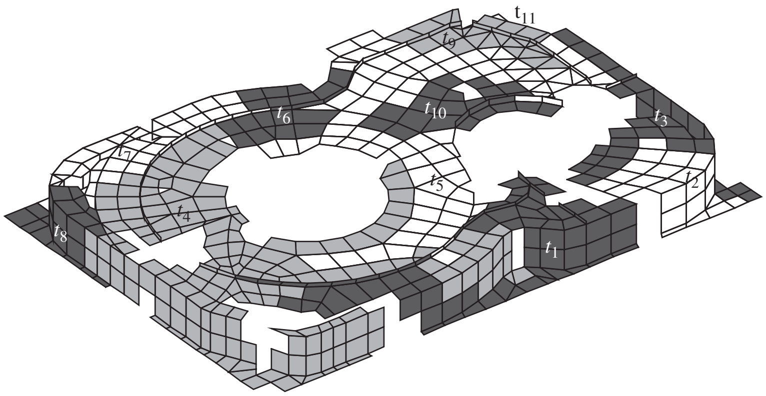

Fig. 20. Updating parameters after the 2nd phase of the parameter selection procedure.

Table 4
Comparison of the experimental and analytical modal properties after updating

Mode	Natural frequency (Hz)			MAC
	Experiment	Updated FE model	Error (%)	
1	409.68	403.77	−1.4389	0.9846
2	908.15	934.95	2.9520	0.9834
3	1707.65	1682.01	−1.5016	0.9560
4	1748.86	1724.98	−1.3658	0.9356
5	1793.23	1760.03	−1.8513	0.9356
6	2474.99	2427.18	−1.9318	0.9547
7	2843.29	2761.50	−2.8766	0.9547
8	2976.06	2890.45	−2.8766	0.9525
9	3113.84	3035.52	−2.5154	0.9672
10	3268.98	3197.55	−2.1849	0.9356

an updated model are compared with experimental results in Table 4. The updated results give quite acceptable correlations. For all the mode pairs, the natural frequency errors are less than 3%, and the MAC values are larger than 0.93.

5. Conclusion

The problem of updating parameter selection was addressed in this work. The importance of updating parameter was demonstrated through case studies. By introducing the concept of total sensitivity, an updating parameter selection procedure was suggested. The suggested procedure is accomplished by a sequence of two different selection phases. The outstanding feature of the 1st phase of the parameter selection procedure is that the objective functions of concern are kept sensitive to the resulting parameters while the number of the parameters are reduced as small as possible. After the 1st phase, the parameter selection procedure can stop if the number of the resulting parameter is acceptable. Otherwise, the parameter selection procedure proceeds to the 2nd phase. In this phase, the updating parameters are grouped at the sacrifice of sensitivities. But a procedure was provided to minimize such sacrifice. Using the suggested parameter selection method, objective functions of interest remain most sensitive to the resulting parameters.

It should be emphasized that the suggested parameter selection procedure does not guarantee the physical significance of the updated model. Only the physical quantities with errors should be allowed to change in the updating process for the physical meaning of the updated model, which requires engineering deep insight into the target structure. When these requirements are met, the suggested parameter selection can lead to an updated model with physical meaning by minimizing deviations from the initial model built upon physical foundation. The effectiveness of the suggested method is proved by both a simulated case study and a real engineering problem.

Acknowledgments

This work was supported by the Post-doctoral Fellowship Program of Korea Science and Engineering Foundation (KOSEF). This support is gratefully acknowledged.

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