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Moving pressure running over a plate coupled with a liquid: The analytical stationary response in the one-dimensional case

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Abstract

This paper presents an analytical study of the stationary response of a long plate in contact with a liquid and subjected to a plane pressure front propagating along the plate at a constant velocity. The plate is assumed to be long enough to disregard reflections from the boundaries in the analysis of the stationary response. The velocity of the pressure front, by virtue of its magnitude relative to the characteristic velocities of the system, determines the nature of the response: vibrating, decaying or both. The solution is found by applying the integral Fourier transforms which enable to formulate a synthesis of results, both for subsonic and supersonic cases. The possible occurrence of critical velocities of loading is discussed.

1. Introduction

The mechanical problem of moving loads on long flexible systems has motivated numerous studies, the most of which is probably the case of a beam on an elastic foundation traversed by a moving load widely studied in the 1960s and yet revisited more recently [1,2]. The response depends upon the velocity of the load which significantly depends on practical application under consideration. It may range from about 10 m/s, in case of trains moving on railway lines, to several thousands meters per second in the case of projectiles moving in gun barrels. This range of high velocity of charge gave rise to new studies to account for new possible kinds of waves in the response [3].

A more complicated problem is obtained with an elastic medium support instead of an elastic foundation [4]. In that case, the elastic waves can propagate in the whole medium supporting the beam and the coupling is of high significance between the beam and its support.

The first particularity of the present study is to use a liquid as medium to support a plate. The second one is to consider all load speeds provided that these are not equal to the characteristic speeds in the system. That is why the results can be applied both to supersonic loads (such as those produced by explosions) and to subsonic loads.

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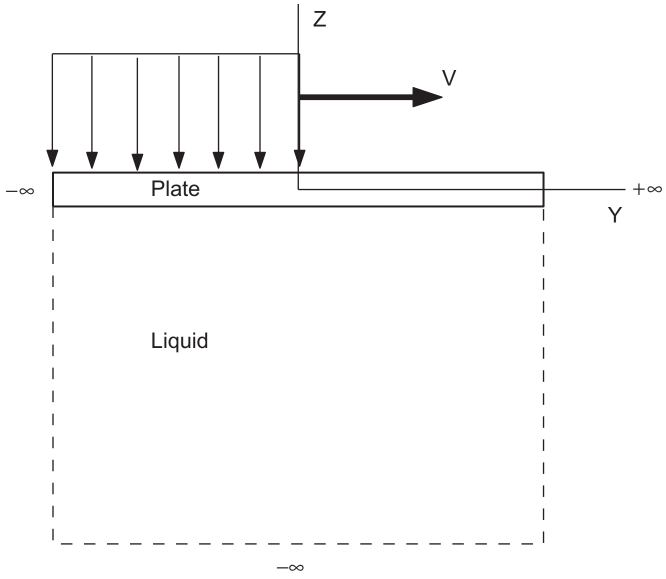

Fig. 1. The coupled plate–liquid system and its stationary loading.

The realistic problem encountered in the industrial field is the case of plates subjected to explosions. The general case requires a sophisticated numerical study and the result often remains uncertain. To be able to give a theoretical analytical contribution to the problem, the study presented here is reduced to the stationary response in the one-dimensional case. So, the plate is considered as a strip coupled with a liquid and run over, at a constant velocity, by an ideal pressure step (Fig. 1). Finally, it is underlined that such a restricted contribution is directly applicable for guided detonations, but is also of interest for coupled plates subjected to aerial detonations.

2. Recalled equations

Previous works [5,6] have widely detailed the equation setting. The assumption of Mindlin Reissner is retained for the plate behaviour; this improved plate theory is necessary for wavelengths shorter than six times the plate thickness and sufficient down to two times the thickness.

The liquid in contact with the plate is supposed inviscid and compressible. The plate is infinite and always in contact with an infinite half-space of liquid. The response is sought for in the elastic and acoustic ranges and concerns only the small perturbations around the equilibrium position, which is realistic because, for very short times, the liquid cannot move significantly.

From a practical aspect, the useful information in the coupled system concerns the stresses in the plate and the pressure in the liquid. As usual, these values are deduced from more fundamental functions, namely the displacements in the plate and the velocity potential in the liquid.

These fundamental functions are ψ, the cross section rotation of the plate, w, its out of plane displacement, and φ, the velocity potential in the liquid. Dividing all lengths by $r_0 = h/\sqrt{12}$ and the time by r_0/v_p (h being the plate thickness and v_p the velocity of sound waves in the plate), all the values are made non-dimensional. A non-dimensional velocity takes the form $V = v/v_p$.

In this study, the variables are taken in their non-dimensional form, noted in capital letters.

The system is studied in the one-dimensional case; for this reason, the plate is restricted to a strip of unit width.

X being the longitudinal coordinate and Z the normal one, the following system is obtained:

$$\ddot{W} = \theta^2(\partial_{,XX} W - \partial_{,X} \Psi) + \mu(\dot{\Phi})_{Z=0} + P_e(X, T), \tag{1}$$

$$\ddot{\Psi} = \partial_{,XX}\Psi + \theta^2(\partial_{,X}W - \Psi), \tag{2}$$

$$\ddot{\Phi} = \delta^2(\partial_{,XX}\Phi + \partial_{,ZZ}\Phi), \tag{3}$$

$$\dot{W} = -(\partial_{,Z}\Phi)_{Z=0}, \tag{4}$$

with the notations $(\dot{\ }) \equiv \partial/\partial T$, $\partial_{,X} \equiv \partial/\partial X$ and $\partial_{,Z} \equiv \partial/\partial Z$.

The first two equations describe the dynamics of the plate; the third one governs the liquid motion and the fourth one is the coupling condition.

Three non-dimensional parameters are sufficient to represent the coupling between the plate and the liquid, $\delta = v_l/v_p$, $\theta = v_s/v_p$, and $\mu = \rho_l/\rho\sqrt{}12$, where v_l is the sound velocity in the liquid, v_s that of shear waves, ρ the mass density of the plate and ρ_l that of the liquid.

The stresses of interest in the plate are σ-the maximum flexural stress on the upper side of the plate, τ is the average shear stress in the cross section, p is the pressure in the liquid. They correspond, respectively, to the non-dimensional forms by

$$\Sigma = \frac{\sigma}{\rho V_p^2\sqrt{12}}, \quad \Gamma = \frac{\tau}{\rho V_p^2\sqrt{12}}, \quad P = \frac{p}{\rho V_p^2\sqrt{12}}. \tag{5}$$

They are deduced from the fundamental functions by the relations:

$$\Sigma = -\frac{1}{2}\frac{\partial \Psi}{\partial X}, \quad \Gamma = \frac{\theta^2}{\sqrt{12}}\left(\frac{\partial W}{\partial X} - \Psi\right), \quad P = \mu\frac{\partial \Phi}{\partial T}. \tag{6}$$

3. The steady-state case

In the steady-state regime, the solution is frozen in a translation at the velocity V and all the functions depend only on $Y = X - VT$. The partial derivative operator $\partial/\partial X$ can be replaced by $\mathrm{d}/\mathrm{d}Y$ and $\partial/\partial T$ by $-V\mathrm{d}/\mathrm{d}Y$. With the notation $d_{,Y} \equiv \mathrm{d}/\mathrm{d}Y$, one obtains:

$$(V^2 - \theta^2)d_{,YY}W + \theta^2 d_{,Y}\Psi + \mu V(\partial_{,Y}\Phi)_{Z=0} = P_0 H(-Y), \tag{7}$$

$$(1 - V^2)d_{,YY}\Psi + \theta^2(d_{,Y}W - \Psi) = 0, \tag{8}$$

$$(\delta^2 - V^2)\partial_{,YY}\Phi + \delta^2\partial_{,ZZ}\Phi = 0, \tag{9}$$

$$Vd_{,Y}W = (\partial_{,Z}\Phi)_{Z=0}. \tag{10}$$

The external loading pressure has been chosen to be the Heaviside step-function of amplitude P_0.

To introduce symmetry facility, the following expression will be preferred:

$$P_0 H(-Y) = \frac{P_0}{2} + P_0\left(H(-Y) - \frac{1}{2}\right) = P_s + P_a. \tag{11}$$

So, the solution will be the response to the dynamical antisymmetrical excitation P_a added to a uniform static pressure P_s in the whole liquid domain. For this reason, in the beginning, the only considered forcing pressure will be P_a (The effect of P_s will be added at the end).

To find a solution, the Fourier transform is used, according to the following notations and definitions:

$$\bar{f}(\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(Y)\mathrm{e}^{\mathrm{i}\xi Y}\,\mathrm{d}Y, \tag{12}$$

$$f(Y) = \int_{-\infty}^{+\infty} \bar{f}(\xi)\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi. \tag{13}$$

Eqs. (7) and (8) governing the plate dynamics are always hyperbolic and correspond to propagating waves.

On the contrary, Eq. (9) can be hyperbolic or elliptic depending on the value of V. For $V>\delta$, the load is supersonic as compared to the proper velocity of acoustic waves in the liquid; the equation describes wave propagation in the liquid. For $V<\delta$, the load is subsonic relative to the liquid; the equation is of Laplace type and describes a permanent solution valid in the whole liquid, depending only on the boundary conditions, that is to say on the stationary coupled shape of the plate.

From this discussion, the fundamental difference between the responses appears clearly, depending on the value of the velocity V. So, two classes of responses are expected according to the sign of (δ^2-V^2).

To proceed, a new parameter is introduced:

$$\Omega = \sqrt{\left|\delta^2 - V^2\right|}/\delta. \tag{14}$$

3.1. Supersonic case

The hyperbolic type of Eq. (9) imposes for the potential the following form:

$$\Phi(Y,Z) = f(Y-\Omega Z) + g(Y+\Omega Z). \tag{15}$$

For a pressure front going forward, the stationary response cannot contain upward propagating waves, so the potential will result simply in:

$$\Phi(Y,Z) = f(Y-\Omega Z) \tag{16}$$

which imposes that

$$\partial_{,Z}\Phi = -\Omega\partial_{,Y}\Phi. \tag{17}$$

At the interface, according to Eq. (10), one gets:

$$(\partial_{,Y}\Phi)_{(Y,0)} = -1/\Omega(\partial_{,Z}\Phi)_{(Y,0)} = -V/\Omega(\partial_{,Y}W)_{(Y)}. \tag{18}$$

Replacing the derivative $(\partial_{,Y}\Phi)_{(Y,0)}$ in Eq. (7) by its expression (18), the following system is obtained:

$$(V^2-\theta^2)d_{,YY}W + \theta^2 d_{,Y}\Psi - \frac{\mu V^2}{\Omega}d_{,Y}W = P_a, \tag{19}$$

$$(1-V^2)d_{,YY}\Psi + \theta^2(d_{,Y}W-\Psi) = 0. \tag{20}$$

These two equations are sufficient to describe the coupled plate behaviour.

Applying to Eqs. (19) and (20) the Fourier transform, and retaining only the dynamical part of the excitation, the following system is obtained:

$$\begin{cases} \left[\xi^2(V^2-\theta^2) - \mathrm{i}\xi\dfrac{\mu V^2}{\Omega}\right]\bar{W} + [\mathrm{i}\xi\theta^2]\bar{\psi} = \dfrac{P_0}{2\pi}\dfrac{1}{\mathrm{i}\xi}, \\ [\mathrm{i}\xi\theta^2]\bar{W} + \left[\theta^2 + \xi^2(1-V^2)\right]\bar{\psi} = 0 \end{cases} \tag{21,22}$$

with the characteristic polynomial:

$$\xi\left[\xi^3(\theta^2-V^2)(V^2-1) + \xi\theta^2V^2 + \mu\frac{V^2}{\mathrm{i}\Omega}\left(\xi^2(1-V^2)+\theta^2\right)\right]. \tag{23}$$

3.2. Subsonic case

Applying the Fourier transform to Eq. (9), one obtains:

$$-\Omega^2\xi^2\bar{\Phi} + \partial_{,ZZ}\bar{\Phi} = 0. \tag{24}$$

Z being non-positive (the liquid is in the half-plane $Z<0$), the solution is

$$\bar{\Phi}(\xi,Z) = \left|\begin{array}{ll} a(\xi)\,\mathrm{e}^{\xi\Omega Z} & \text{for } \xi\geqslant 0, \\ b(\xi)\,\mathrm{e}^{-\xi\Omega Z} & \text{for } \xi<0. \end{array}\right. \tag{25}$$

To sum up

$$\bar{\Phi}(\xi, Z) = C(\xi)\,\mathrm{e}^{|\xi|\Omega Z}, \tag{26}$$

(the explanation can be found in Haberman [7]).

$$\bar{\Phi}(\xi, 0) = C(\xi), \tag{27}$$

$$d_{,Z}\bar{\Phi}(\xi, 0) = \Omega|\xi|C(\xi). \tag{28}$$

Since the coupling condition, Eq. (10), gives

$$d_{,Z}\bar{\Phi}(\xi, 0) = -\mathrm{i}\xi V \bar{W} \tag{29}$$

it follows that

$$C(\xi) = -\mathrm{i}\,\mathrm{Sgn}\,(\xi)\tfrac{V}{\Omega}\bar{W}. \tag{30}$$

Finally, the system is obtained with $\bar{W}$ and $\bar{\Psi}$ only

$$\begin{cases} \left[\xi^2(V^2-\theta^2) + |\xi|\dfrac{\mu V^2}{\Omega}\right]\bar{W} + [\mathrm{i}\xi\theta^2]\bar{\psi} = \dfrac{P_0}{2\pi}\dfrac{1}{\mathrm{i}\xi}, \\ [\mathrm{i}\xi\theta^2]\bar{W} + \left[\theta^2 + \xi^2(1-V^2)\right]\bar{\psi} = 0. \end{cases} \tag{31}$$

The associated characteristic function looks like the pseudo-polynomial:

$$F(\xi) = \xi\left[\xi^3(\theta^2 - V^2)(V^2-1) + \xi\theta^2 V^2 + \mathrm{Sgn}(\xi)\mu\frac{V^2}{\Omega}\left(\xi^2(1-V^2)+\theta^2\right)\right]. \tag{33}$$

Returning to the original functions, one resumes, for the subsonic case:

$$W(Y) = \int_{-\infty}^{+\infty} \bar{W}(\xi)\,\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi = \frac{P_0}{2\pi}\int_{-\infty}^{+\infty}\frac{\theta^2 + \xi^2(V^2-1)}{\mathrm{i}\xi F(\xi)}\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi, \tag{34}$$

$$\Psi(Y) = \int_{-\infty}^{+\infty} \bar{\Psi}(\xi)\,\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi = -\frac{P_0}{2\pi}\theta^2\int_{-\infty}^{+\infty}\frac{1}{F(\xi)}\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi. \tag{35}$$

By composing these functions according to Eq. (6), the stresses and the pressure can be obtained:

$$\Sigma(Y) = P_0\frac{\theta^2}{2}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{-\mathrm{i}\xi}{F(\xi)}\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi \tag{36}$$

$$\Gamma(Y) = P_0\frac{\theta^2(V^2-1)}{\sqrt{12}}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\xi^2}{F(\xi)}\mathrm{e}^{-\mathrm{i}\xi Y}\,\mathrm{d}\xi, \tag{37}$$

$$P(Y,Z) = -P_0\frac{\mu V^2}{\Omega}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{i}\,\mathrm{Sgn}\,(\xi)\frac{\xi^2(1-V^2)+\theta^2}{F(\xi)}\mathrm{e}^{-\mathrm{i}\xi Y}\mathrm{e}^{|\xi|\Omega Z}\,\mathrm{d}\xi. \tag{38}$$

If one designate with $F_+(\xi)$ the polynomial corresponding to the positive values of ξ,

$$F_+(\xi) = \xi\left[\xi^3(\theta^2 - V^2)(V^2-1) + \xi\theta^2 V^2 + \mu\frac{V^2}{\Omega}\left(\xi^2(1-V^2)+\theta^2\right)\right], \tag{39}$$

the result for the inverse functions, accounting for parity, takes the following form of inverse Sine and Cosine Fourier transforms:

$$\Sigma(Y) = -P_0\frac{\theta^2}{2}\frac{1}{\pi}\int_0^{+\infty}\frac{\xi}{F_+(\xi)}\sin(\xi Y)\,\mathrm{d}\xi, \tag{40}$$

$$\Gamma(Y) = P_0\frac{\theta^2(V^2-1)}{\sqrt{12}}\frac{1}{\pi}\int_0^{+\infty}\frac{\xi^2}{F_+(\xi)}\cos(\xi Y)\,\mathrm{d}\xi, \tag{41}$$

$$P(Y,Z) = -P_0\frac{\mu V^2}{\Omega}\frac{1}{\pi}\int_0^{+\infty}\frac{\xi^2(1-V^2)+\theta^2}{F_+(\xi)}\sin(\xi Y)\,\exp(\xi\Omega Z)\,\mathrm{d}\xi. \tag{42}$$

For the supersonic case, one obtains:

$$\Sigma(Y) = P_0 \frac{\theta^2}{2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{-\mathrm{i}\xi}{F(\xi)} \exp(-\mathrm{i}\xi Y)\,\mathrm{d}\xi, \tag{43}$$

$$\Gamma(Y) = P_0 \frac{\theta^2(V^2-1)}{\sqrt{12}} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\xi^2}{F(\xi)} \exp(-\mathrm{i}\xi Y)\,\mathrm{d}\xi, \tag{44}$$

$$P(Y,0) = -P_0 \frac{\mu V^2}{\Omega} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\xi^2(1-V^2)+\theta^2}{F(\xi)} \exp(-\mathrm{i}\xi Y)\,\mathrm{d}\xi, \tag{45}$$

$$P(Y,Z) = P(Y-\Omega Z, 0). \tag{46}$$

By setting

$$\omega = \frac{(\delta^2 - V^2)^{1/2}}{\delta}, \tag{47}$$

that is to say $\omega = \Omega$ for $V < \delta$ and $\omega = \mathrm{i}\Omega$ for $V > \delta$, one obtains the following unique definition of polynomial:

$$F(\xi) = F_+(\xi) = \xi\left[\xi^3(\theta^2 - V^2)(V^2-1) + \xi\theta^2 V^2 + \mu\frac{V^2}{\omega}\left(\xi^2(1-V^2)+\theta^2\right)\right]. \tag{48}$$

In both cases, subsonic and supersonic, to carry out the inverse transform, the above polynomial ratios have to be converted into a partial-fraction expansion.

After factorization,

$$F(\xi) = (\theta^2 - V^2)(V^2-1)\xi(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3) \tag{49}$$

and

$$\frac{1}{\xi(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)} = \frac{C_0}{\xi} + \frac{C_1}{(\xi-\xi_1)} + \frac{C_2}{(\xi-\xi_2)} + \frac{C_3}{(\xi-\xi_3)}. \tag{50}$$

The C_i and ξ_i may be complex and a rearranging could lead to the real form. The complexity of the spectrum of roots, as visible in the example presented in Fig. 2, with numerous changes in the sign and the nature of roots, deters from going on in this direction. The simplest way to generate the solution, avoiding difficulties, is to keep the complex form. The omitted elaborations developments can be found in Appendix A. Finally one obtains:

for the subsonic case:

$$\Sigma(Y) = P_0 \frac{\theta^2}{2} \frac{1}{(\theta^2 - V^2)(V^2-1)} \sum_{j=1}^{j=3} C_j \xi_j Fs_{\xi_j}(Y), \tag{51}$$

$$\Gamma(Y) = P_0 \frac{\theta^2}{\sqrt{12}} \frac{1}{\theta^2 - V^2} \sum_{j=1}^{j=3} C_j \xi_j^2 Fc_{\xi_j}(Y), \tag{52}$$

$$P(Y,0) = -P_0\left(Fs_0(Y) + \frac{\mu V^2}{\Omega} \frac{1}{(\theta^2 - V^2)(V^2-1)} \sum_{j=1}^{j=3} C_j\left(\theta^2 + \xi_j^2(1-V^2)\right) Fs_{\xi_j}(Y)\right) \tag{53}$$

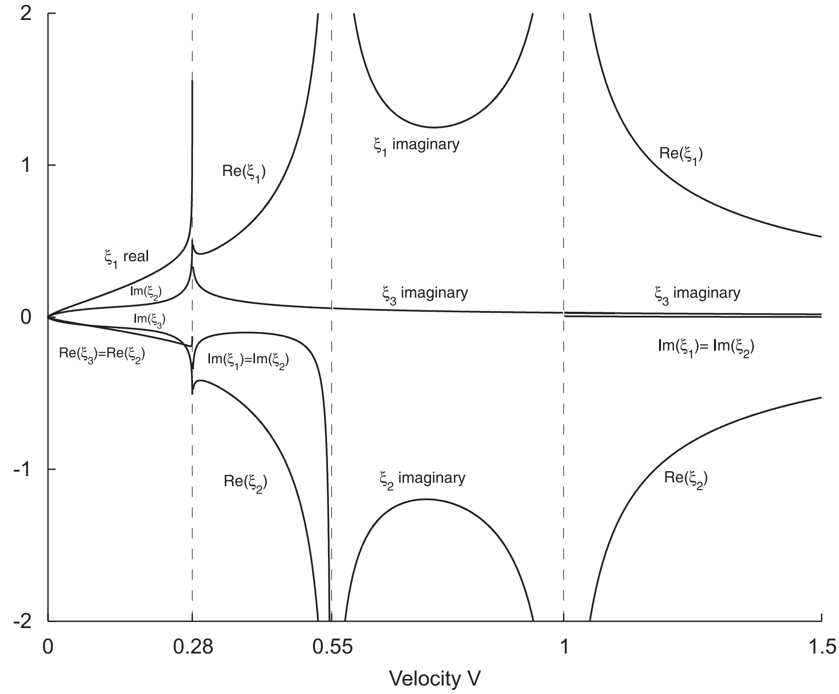

Fig. 2. Root spectrum ($\delta = 0.28$, $\mu = 0.1$, $\theta = 0.55$).

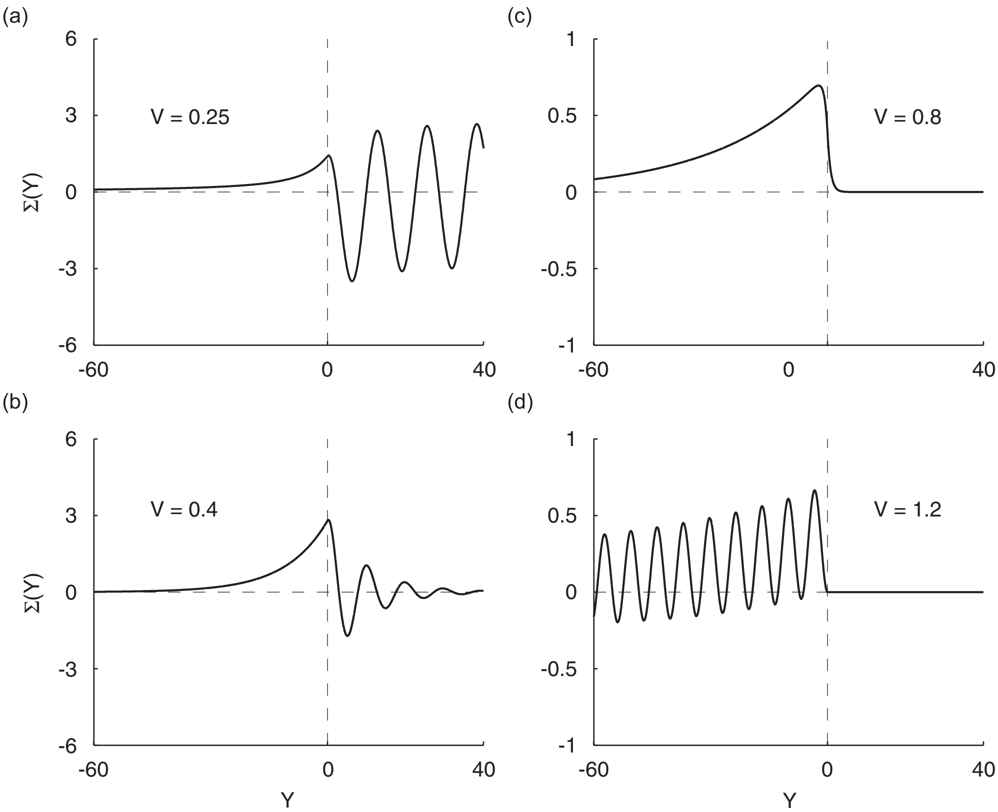

Fig. 3. Flexural stress on the non-wetted face of the plate.

and, more generally, in the liquid,

$$P(Y,Z) = -P_0\left[\frac{1}{\pi}\arctan\left(\frac{Y}{\Omega Z}\right) - \frac{\mu V^2}{\Omega}\frac{1}{(\theta^2 - V^2)(V^2-1)}\right.$$
$$\times\sum_{j=1}^{j=3} C_j\left(\theta^2 + \xi_j^2(1-V^2)\right)\left(\frac{1}{2}\left(Fs_{\xi_{j\leftarrow}}(Y+\mathrm{i}\Omega Z) + Fs_{\xi_j}(Y-\mathrm{i}\Omega Z)\right)\right.$$
$$\left.\left.+\frac{\mathrm{i}}{2}\left(Fc_{\xi_{j\leftarrow}}(Y+\mathrm{i}\Omega Z) - Fc_{\xi_j}(Y-\mathrm{i}\Omega Z)\right)\right)\right]. \tag{54}$$

for the supersonic case:

$$\Sigma(Y) = P_0\frac{\theta^2}{2}\frac{1}{(\theta^2-V^2)(V^2-1)}\sum_{j=1}^{j=3} C_j\xi_j F_{\xi_j}(Y), \tag{55}$$

$$\Gamma(Y) = P_0\frac{\theta^2}{\sqrt{12}}\frac{1}{\theta^2 - V^2}\sum_{j=1}^{j=3} C_j\xi_j^2\left(\mathrm{i}F_{\xi_j}(Y)\right), \tag{56}$$

$$P(Y,0) = -P_0\left(\frac{\mathrm{Sgn}(Y)}{2} + \frac{\mu V^2}{\Omega}\frac{1}{(\theta^2-V^2)(V^2-1)}\sum_{j=1}^{j=3} C_j\left((\theta^2+\xi_j^2(1-V^2))\mathrm{i}F_{\xi_j}(Y)\right)\right), \tag{57}$$

$$P(Y,Z) = P(Y-\Omega Z, 0). \tag{58}$$

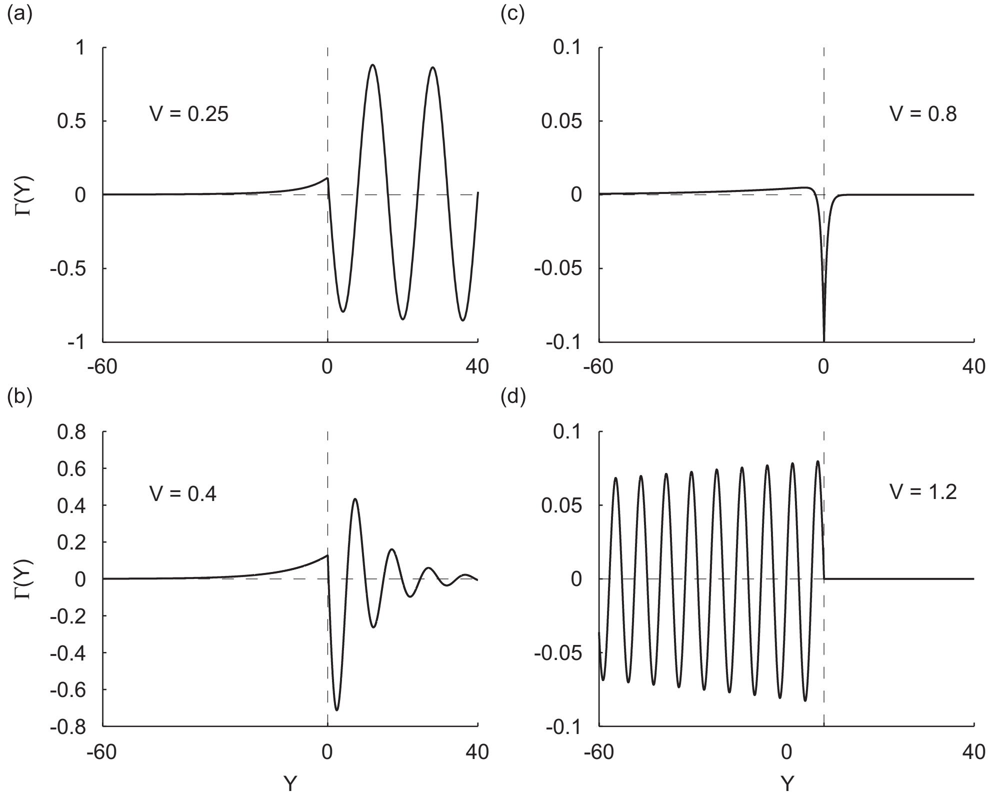

Fig. 4. Average shear stress in the plate.

With

$$F_{\xi_j}(Y) = \frac{-\mathrm{i}}{2\pi}\int_0^{\infty} \frac{\exp(-\mathrm{i}\xi Y)}{\xi - \xi_j}\,\mathrm{d}\xi$$
$$= \exp(-\mathrm{i}\xi_j Y)(Y<0)\big(\mathrm{Im}\,(\xi_j)>0\big) - \exp(-\mathrm{i}\xi_j Y)(Y>0)(\mathrm{Im}\,(\xi_j)<0), \tag{59}$$

$$Fs_{\xi_j}(Y) = \frac{1}{\pi}\int_0^{\infty} \frac{\sin(\xi Y)}{\xi - \xi_j}\,\mathrm{d}\xi = \frac{1}{\pi}\left(\cos(\xi_j Y)\left(\mathrm{Si}(\xi_j Y) + \frac{\pi}{2}\right) - \sin(\xi_j Y)\mathrm{Ci}(\mathrm{Sgn}\big(\mathrm{Re}(\xi_j)\big)\xi_j Y)\right),$$
for $Y>0$

$$Fs_{\xi_j}(Y) = -Fs_{\xi_j}(-Y) \quad \text{for } Y<0, \tag{60}$$

$$Fc_{\xi_j}(Y) = \frac{1}{\pi}\int_0^{\infty} \frac{\cos(\xi Y)}{\xi - \xi_j}\,\mathrm{d}\xi = \frac{1}{\pi}\left(-\sin(\xi_j Y)\left(\mathrm{Si}(\xi_j Y) + \frac{\pi}{2}\right) - \cos(\xi_j Y)\mathrm{Ci}\big(\mathrm{Sgn}(\mathrm{Re}(\xi_j))\xi_j Y\big)\right),$$
for $Y>0$

$$Fc_{\xi_j}(Y) = Fc_{\xi_j}(-Y) \quad \text{for } Y<0. \tag{61}$$

4. Numerical results and discussion

The above equations, Eqs. (51)–(61), are valid in any case of coupling respecting the earlier defined conditions . They are applicable whatever the value of the velocity of the load among all the possible arrangements of the characteristic velocities. In the presentation of the results, only one case will be retained,

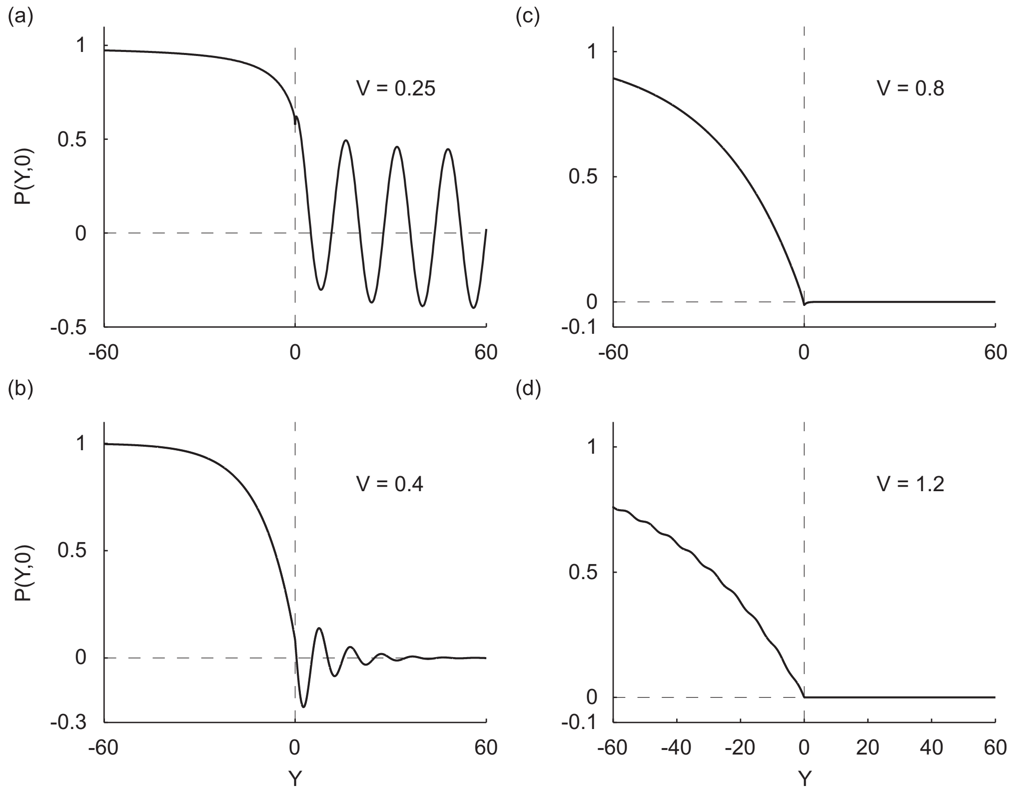

Fig. 5. Fluid pressure at the interface.

corresponding to the following characteristic velocities: $V_l < V_s < V_p$. When a practical example is presented, it refers to a water–aluminium coupling for which the fundamental dimensionless parameters take the following numerical values: $\delta = 0.28$, $\theta = 0.55$, $\mu = 0.1$.

4.1. Overview on results

The results are presented by means of several figures displaying the shape of stresses and pressure, for any location of the charge velocity.

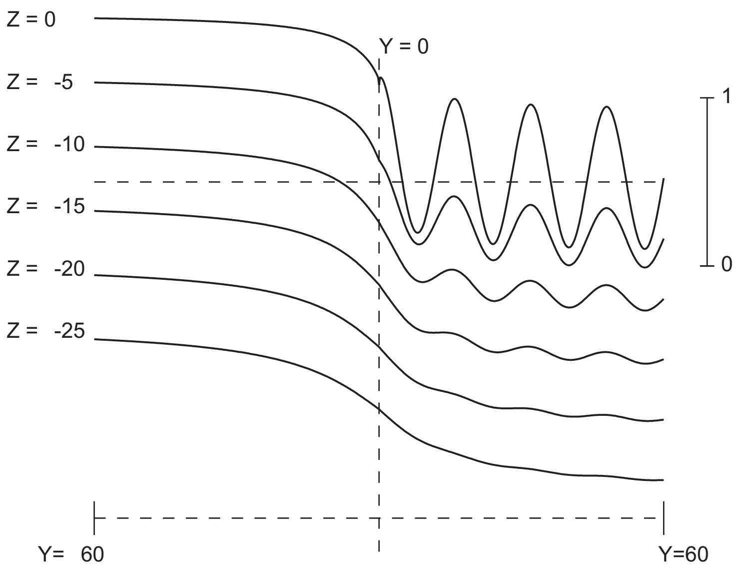

Fig. 6. Evolution of the pressure with the depth in the fluid (subsonic case: $V = 0.25$).

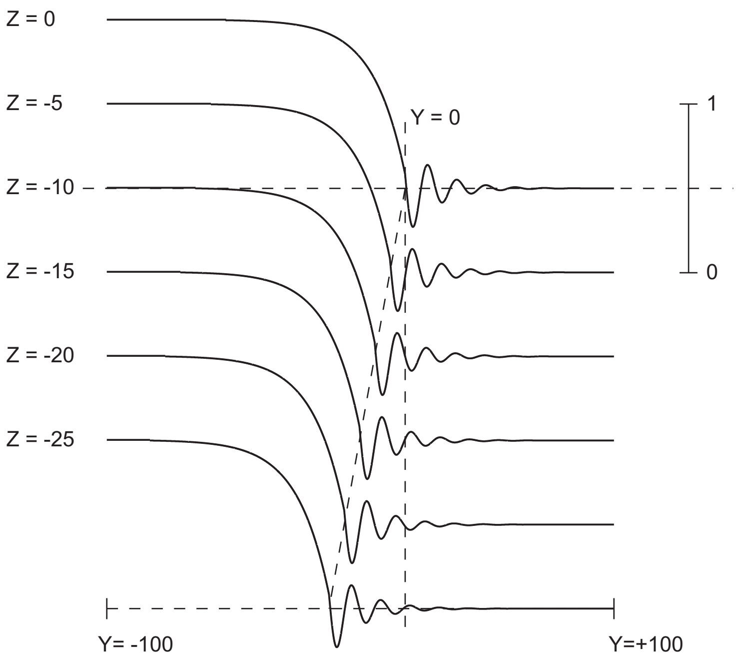

Fig. 7. Evolution of the pressure with the depth in the fluid (supersonic case: $V = 0.4$).

Fig. 3 presents the flexural stress, on the upper face of the plate, as a function of the running coordinate Y. Fig. 4 presents the shear stresses and Fig. 5 the pressure in the liquid immediately under the plate.

Each figure contains four graphs corresponding to the different regions of the spectrum presented Fig. 2.

Region (a) corresponds to the subsonic case. That is the only case containing a purely harmonic component in the response. This harmonic wave spreads all over the plate but only ahead of the load front. The other components of the response are made of sine and cosine integrals. For this response in region (a), a free wave has been added to the result contained in Eqs. (51)–(54), as explained in a previous work [6].

The other regions correspond to the supersonic case. The solution is much more simple and its components are made of exponential or decaying sinusoidal branches. For region (b), the rear part of the response is exponential and the front part is a decaying sinusoid. For region (c), the two parts are exponential and for region (d), the rear part is a decaying sinusoid superposed to an exponential component. For this region, the response ahead of the pressure front remains zero because no information can overtake the characteristic plate velocity.

Concerning the pressure in the liquid, the two cases, subsonic and supersonic, are very different. For the subsonic case (Fig. 6), a vertical line, exactly under the pressure front, separates the solutions in the liquid. The harmonic vibration of the pressure can propagate undisturbed ahead of the charge, while it cannot penetrate the depth of the liquid, decaying exponentially.

For the supersonic case, whatever the region concerned, the pressure wave penetrates the liquid, identical to the pressure distribution immediately under the plate (Fig. 7). An oblique front takes place in the liquid at the sonic velocity. In that case, the pressure disturbance can never spread far from the front line.

4.2. On the critical nature of the characteristic velocities

The spectrum of Fig. 2 highlights the discontinuity of roots at the border of different regions. For these values of V, called characteristic values, the responses cannot be calculated by the same general equations that those valid in the general case of the stationary theory. Perhaps some of them could become of infinite amplitude and look as critical, as in the problem of the beam on an elastic foundation. An in-depth theoretical study of these cases is beyond the scope of this work. Nevertheless, it is possible, using the stationary theory,

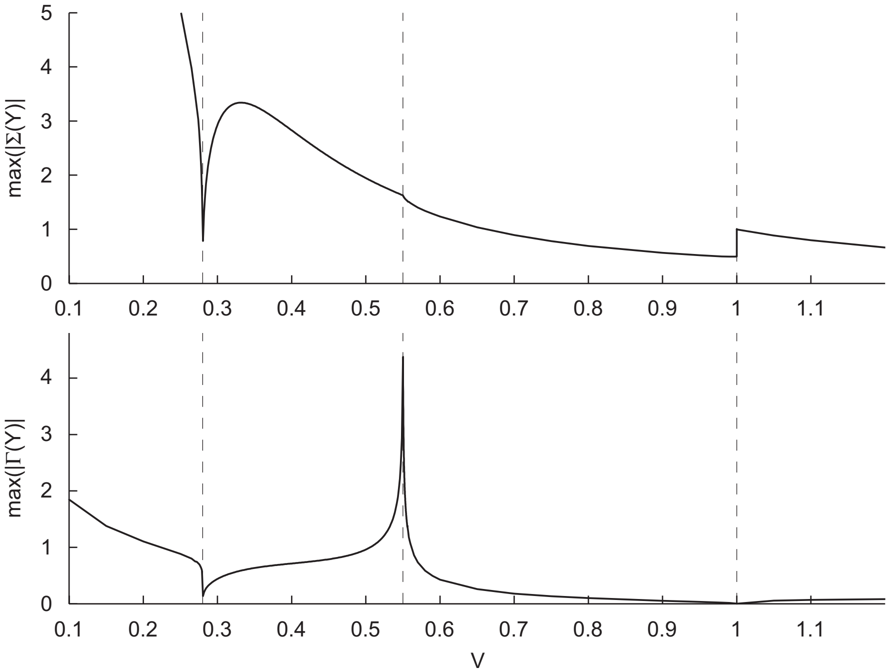

Fig. 8. Maximum absolute values of the stresses in the plate vs. the velocity of loading.

to foresee the amplification of each displacement on the whole range of V and, specifically, in the neighbourhood of the characteristic values. This work has been done for the stresses (Fig. 8), using the numerical maximum values of the functions and displaying their dynamic amplification. The curves reveal that no velocity of charge can be considered as critical in this problem. The greatest amplification peak is observed for Γ, around θ, the characteristic velocity of shear waves. As close as it is possible to numerically approach θ, the amplification remains finite. Moreover, a theoretical reformulation of the problem,[1] accounting for the exact value of V equal to θ, is possible and leads to the theoretical maximum of Γ equal to 4.84215.

It is not really surprising that *no* velocity appears as *critical*. The system couples a plate and a liquid and each medium has its proper characteristic velocities, which are not the same. It is known that the velocity of shear waves is critical for a plate alone, but, taking in account the coupling, it does not remain the same.

5. Conclusion

The presented study addressed both the problem of long flexible structures subject to moving loads and those of fluid/structure coupling. The actual possibility of loading by explosion calls to explore the solution for wide-ranging velocities of charge, containing all the natural characteristic velocities of the system, i.e. the velocity of acoustic waves in the liquid and the velocities of shear waves and plane waves in the plate.

This study was focused on the derivation of a theoretical analytical response, in the one-dimensional case, for any plate coupled with any liquid and run over at any constant velocity by an ideal pressure step.

This analytical stationary solution, possible only for an infinitely long plate, can also be used in practical cases to approximate the response of a finite plate subjected to a loading that is not truly stationary. In fact, the wavelengths of vibration are so short and the number of oscillations so high that, even for a short time, the response in the neighbourhood of the pressure front is not far from a stationary response calculated with the same conditions of amplitude and velocity of loading.

The result of the study shows that the form of the response corresponds to each situation of the charge velocity among the characteristic velocities of the coupled system.

The most important change is observed when the velocity of charge crosses the proper velocity of the liquid, that is to say between the subsonic and supersonic cases.

The subsonic case, for which the evolution in the liquid is of elliptic type, induces a lot of difficulties in the resolution. Nevertheless, by means of Fourier transforms, a synthesis has been possible in the presentation of whole results.

As it was made for beams on elastic foundation, the cases of velocity of charge corresponding to one of the proper velocities of the system have been examined carefully. This could be a study in itself, which has not been developed theoretically in this context; nevertheless, the result of amplification of the response vs. the charge velocity highlights the non-existence of critical velocities.

Appendix A. Useful relations in partial fraction expanding of simple polynomial ratios

$$\frac{1}{\xi(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)} = \frac{C_0}{\xi} + \frac{C_1}{(\xi-\xi_1)} + \frac{C_2}{(\xi-\xi_2)} + \frac{C_3}{(\xi-\xi_3)}, \tag{62}$$

$$\frac{1}{(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)} = \frac{C_1\xi_1}{(\xi-\xi_1)} + \frac{C_2\xi_2}{(\xi-\xi_2)} + \frac{C_3\xi_3}{(\xi-\xi_3)}, \tag{63}$$

$$\frac{\xi}{(\xi-\xi_1)(\xi-\xi_2)(\xi-\xi_3)} = \frac{C_1\xi_1^2}{(\xi-\xi_1)} + \frac{C_2\xi_2^2}{(\xi-\xi_2)} + \frac{C_3\xi_3^2}{(\xi-\xi_3)} \tag{64}$$

[1] By setting $V=\delta$ in Eq. (19) one obtains a new system, whose characteristic polynomial, of degree 3 only, no longer contains the term $\theta^2 - V^2$. From this new starting point, a development, similar to the one presented in this article, leads to functions $\Sigma(Y)$, $\Gamma(Y)$, $P(Y,Z)$ for which $V=\theta$ is not a singular value, thus allowing the exact calculation of such functions.

with the values of coefficients

$$C_0 = -\frac{1}{\xi_1\xi_2\xi_3}, \quad C_1 = \frac{1}{\xi_1(\xi_1-\xi_2)(\xi_1-\xi_3)}, \quad C_2 = \frac{1}{\xi_2(\xi_2-\xi_3)(\xi_2-\xi_1)}, \quad C_3 = \frac{1}{\xi_3(\xi_3-\xi_1)(\xi_3-\xi_2)}. \tag{65}$$

And in the present case, the relations between roots and polynomial coefficients give:

$$C_0 = \frac{(\theta^2 - V^2)(V^2-1)\omega}{\mu V^2\theta^2}. \tag{66}$$

Recall of Sine and Cosine Fourier transforms:

$$\int_0^\infty \frac{\sin(\xi Y)}{\xi}\,\mathrm{d}\xi = \frac{\pi}{2}\,\mathrm{sgn}(Y), \tag{67}$$

$$\int_0^\infty \frac{\sin(\xi Y)\,\exp(\xi Z)}{\xi}\,\mathrm{d}\xi = \arctan\left(\frac{Y}{Z}\right). \tag{68}$$

Other useful relations can be found in specialized books [8,9].

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