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A spectral simulation approach to evaluate probabilistic measurement precision of a reactor coolant pump torsional vibration shaft crack monitoring system

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Abstract

A prototype torsional vibration monitoring system has been installed on two reactor coolant pumps at Tennessee Valley Authority Sequoyah Power Plant Unit 1 nuclear reactor for shaft crack monitoring. The system uses a change in torsional natural frequencies as a diagnostic feature since as a crack grows the shaft line torsional dynamics change commensurately. For effective diagnostic monitoring, it is critical to know the inherent feature variation resulting from the instrumentation and data processing. The integrated nature of the prototype hardware/software system and the inaccessibility of the equipment inside the reactor containment building make it impossible to evaluate the measurement capability in a traditional instrumentation chain approach. Hence, a simulation-based evaluation was performed to determine the cumulative effect of the measurement system on the uncertainty in the torsional natural frequency estimation. The identification algorithm operates on the measured torsional spectrum to estimate a natural frequency. All of the inherent measurement and data processing uncertainties ultimately affect the spectrum in some manner. Therefore, if the characteristics of the reactor coolant pump torsional spectrum can be replicated via a model; it can be used to assess the overall prototype system measurement uncertainty. The underlying premise is that the line shaft dynamics remain constant, in the absence of a fault, and hence any natural frequency variation is due strictly to the measurement system uncertainty. To this end, a single degree of freedom simulation model was first developed which matched the natural frequency and damping of a crack sensitive torsional mode in the reactor coolant pump. The model input was carefully selected such that the averaged spectral amplitude and variance from the ensembles matched the measured reactor coolant pump torsional vibration. The model was then used to create one hundred sample responses that replicated pump spectral response in a statistical sense. The same data processing routines used in the prototype system were applied to the simulation data thus creating sample populations of the natural frequency and damping. Statistical tolerance limits containing 99.9% of the sample population with a 99.9% probability were used to quantify the system's measurement uncertainty. The limits for the crack sensitive mode 3 of the reactor coolant pump were (1) natural frequency: ±0.04740 Hz and (2) damping ratio: ±0.00036.

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1. Introduction

The importance of shaft crack detection in nuclear power plants is apparent when considering the impact of past failures. Primary coolant pumps in both boiling water reactors and pressurized water reactors have experienced significant shaft cracking; often with little or no warning from state-of-the-art crack detection systems. The United States Nuclear Regulatory Commission recently recommended that shafts in all reactor recirculating pumps in boiling water reactors with over 80,000 h of operation be removed and inspected because of cracking [1]. Also, pre-1974 Westinghouse reactor coolant pumps in pressurized water reactors have recently come under particular scrutiny, as at least five have experienced significant cracking. A root cause analysis indicated that Model 93A pumps, which operate in counterclockwise flow loops, are especially susceptible to developing shaft cracks [2]. In late 2000, Tennessee Valley Authority Sequoyah Unit 1, reactor coolant pump 4 experienced severe cracking that resulted in an extended forced outage. The crack detection system on the reactor coolant pump provided no warning of crack propagation [3]. After shutdown, inspection revealed an elliptical front crack of 252° around the shaft circumference with only one-third of the cross-sectional area remaining. These mechanical failures have motivated efforts to develop condition-based monitoring methods for the structural integrity of rotating equipment components. One such approach utilizes torsional vibration signature analysis. As the integrity of rotating components degrades due to the initiation and propagation of a fatigue crack the torsional natural frequencies decrease. This method appears to be less sensitive to changes in the pump rotor context (e.g., seals, oil film and supports) than the existing translational-based crack detection systems. Therefore, the tracking of torsional natural frequencies can be used as a health diagnostic feature for rotating machinery. As a cautionary note, there are other factors besides crack formation that can cause changes in the shaft line torsional dynamics, such as degradation of couplers, thermal growth and impeller mass loading. Hence, a reduction in the torsional natural frequency is not necessarily indicative of crack growth.

The TVA Sequoyah Nuclear Power Plant operates pressurized water reactor designs with four reactor coolant pump loops. Two pumps operate in clockwise flow loops and two operate in counterclockwise flow loops. Prototype torsional vibration monitoring systems were installed on two reactor coolant pumps on Unit 1 in November 2004. Both instrumented pumps operate in counterclockwise loops and were selected because of the greater historical propensity for crack formation [1]. Reactor coolant pump 1–2 is an original pump installed when the reactor was commissioned in the 1970s and has over 25 years of service. Reactor coolant pump 1–4 is a replacement pump installed in 2000. The objectives of this prototype torsional vibration shaft crack monitoring system installation are: (1) evaluate the overall system suitability and performance; (2) identify installation and operational system design refinements; and (3) trend the shaft crack sensitive torsional vibration features over the 18 month fuel cycle. The scope of the work discussed herein is restricted to a discussion of measurement uncertainty aspects of the prototype system.

The severe cracks reported in the 93A reactor coolant pumps have all occurred at the same shaft location. A finite element analysis was performed on the line shaft assembly to identify the baseline torsional modal characteristics. Subsequently, a sequence of incrementally larger cracks was placed in the model and the resulting torsional modal parameters computed. The modeling analysis results were compiled to provide: (1) identification of crack sensitive torsional vibration modes; (2) correlation between the crack size and changes in the modal parameters; (3) identification of preferred torsional sensing locations to maximize the response measurement of the crack sensitive modes.

The finite element analysis identified the sensitive torsional features to track and their general relationship to crack growth. From a measurement point of view it is critical to know the smallest torsional modal feature change that can be detected. The greater the feature measurement resolution, the earlier and more accurately cracks can be detected and tracked. Hence, from a diagnostic trending perspective, torsional feature measurement precision is more important than accuracy. This is because changes in the torsional features (i.e., natural frequency) are the indicators of shaft degradation and not their actual values. Hence, it is critical to know the smallest statistically significant change in a crack sensitive features that can be detected given the inherent system measurement uncertainty.

The prototype torsional shaft crack monitoring system installed on the reactor coolant pumps has the following general characteristics:

1. The measurement system uses an integrated and inseparable set of hardware to sense the torsional vibration response.
2. A sequence of algorithms is used to estimate and improve the data quality of the torsional vibration time responses from the acquired data.
3. Fast Fourier transform spectral estimation algorithms are employed to estimate frequency domain quantities from the computed torsional time arrays.
4. A torsional feature identification algorithm is applied to the spectrum to estimate the desired trending features (e.g., modal parameters).

Each of these items has its own respective measurement errors and combines to affect the overall system precision and accuracy. Evaluation of system performance and errors is further complicated by pragmatic issues related to pump operation and installation on licensed nuclear reactor equipment.

1. A very limited number measurement positions are available on the pump for installation of the torsional sensing hardware. Therefore, non-ideal locations may have to be used, potentially resulting in low-level responses of the crack sensitive modes. Hence, the signal/noise (S/N) can be affected.
2. Most of the sensing hardware is located inside the reactor containment building making it physically inaccessible after the reactor is started.
3. Absolutely no changes are allowed to the pumps other than for strictly reactor operation reasons. Hence, all measurement system performance must be evaluated as installed and operated under nominal conditions.
4. Each pump mode has unique modal parameters. Since the modal features are used as the diagnostics, the measurement capabilities are not uniform and vary between vibration modes.
5. The torsional modal response is dependent on the inherent excitation in the pump. Therefore, the pump design and operation also play important roles in determining the torsional measurement detectability, precision and accuracy.

All of these realities and issues make it impossible to evaluate the system's measurement precision in a traditional instrumentation chain sense. Therefore, it is necessary to evaluate the system's measurement capabilities in an alternative fashion that considers all the hardware and data processing uncertainty errors in a cumulative manner. A model-based probabilistic method is proposed. The integrated hardware and software system is first viewed as an "instrument" to estimate the crack sensitive torsional vibration features. The overall "instrument" measurement uncertainty can be simulated and statistically quantified by the following steps:

1. A simulation model is developed which replicates the response for a torsional reactor coolant pump mode that has shown sensitivity to shaft crack growth. Since the simulation and pump spectral response statistics are similar, the model data is assumed to capture the all the inherent upstream errors and uncertainties.
2. One hundred sets of time data are generated to replicate the modal response with different excitation sequences.
3. The prototype system spectral processing algorithms are applied to the simulation time data to estimate the response spectrum.
4. The prototype system identification algorithm is applied to estimate the natural frequency and damping ratio from the simulated spectra. The result is a sample population of 100 estimates for each modal feature.
5. The statistics of the sample populations are computed.
6. Statistical tolerance limits are computed from the respective modal parameter sample distributions.

Ultimately, the statistical tolerance limits are viewed as the measurement precision achievable by the hardware–software system in a probabilistic sense and provides an indication of the overall capabilities. The application of the method is illustrated in the following sections by establishing the probabilistic measurement precision for one set of reactor coolant pump modal features. The work concludes with a discussion of the practical implications of the probabilistic analysis to crack monitoring via torsional vibration feature trending.

2. Prototype reactor coolant pump torsional vibration system

2.1. Torsional vibration sensing

The basic components of the prototype torsional measurement system are shown schematically in Fig. 1. Signal detection involves four main aspects: (1) shaft encoding; (2) transduction; (3) data discretization and (4) demodulation. The prototype reactor coolant pump system uses a specially designed split ring toothed encoder mounted on the motor side of the spool piece between the motor and pump. The toothed encoder ring was constructed of high magnetic permeability 411 stainless steel. Three Hall effect transducers were mounted at 120° increments around the circumference of the encoder to sense the passage of the teeth. Since all the equipment was located within the reactor containment building it had to be designed to operate completely self sufficiently without any human intervention once the reactor was started. Furthermore, all equipment had to be capable of withstanding the ambient radiation levels in the containment building. The wiring schematic is depicted in Fig. 2. Since two pumps were instrumented a total of six transducer leads had to be placed between the transducers and the data acquisition system. The data acquisition system was placed outside the containment building to enable data retrieval. All transducer wires had to exit the containment building through available penetrations. This dictated that (1) long cable runs, on the order of 300 m, be used; and

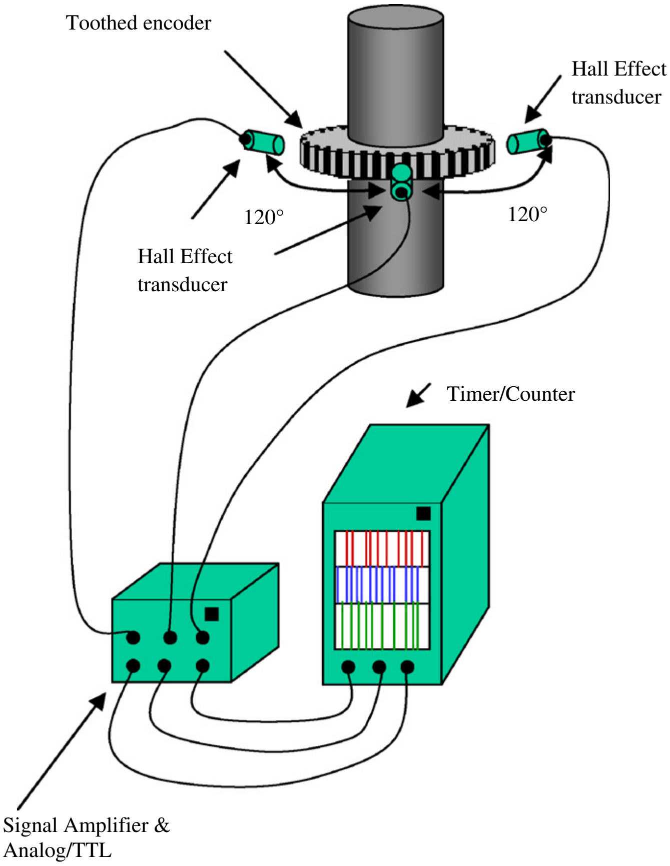


Fig. 1. Schematic of the torsional vibration measurement instrumentation.

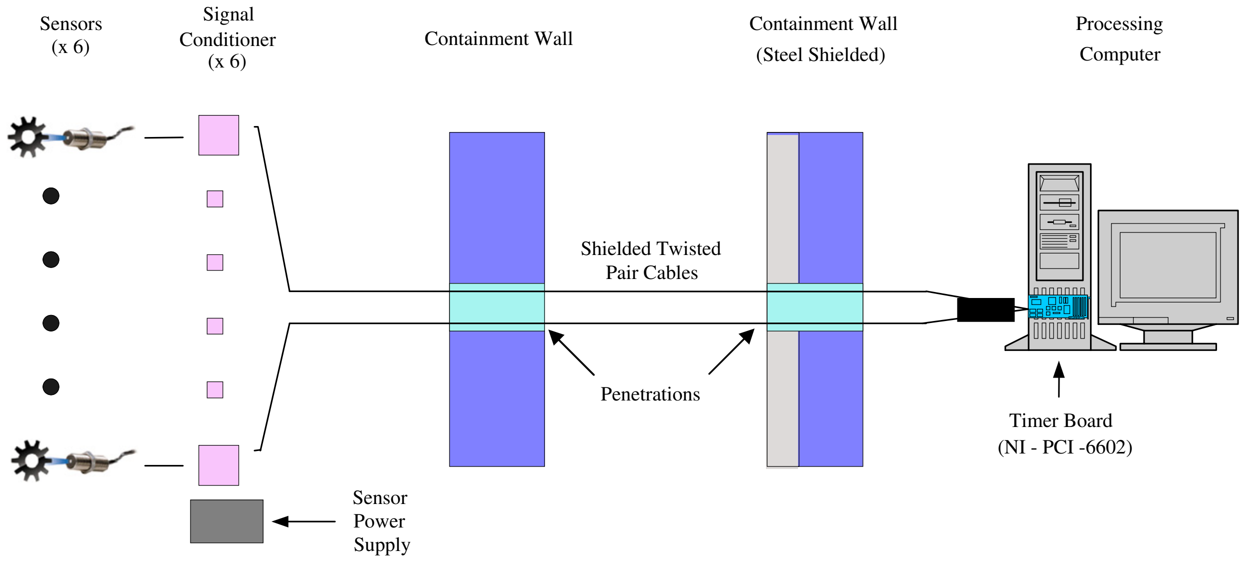


Fig. 2. Reactor coolant pump torsional vibration wiring and instrumentation chain.

(2) the cables would have to be modified and spliced at the penetrations. The long cable runs necessitated that line drivers be installed to ensure signal integrity and strength was maintained.

Once all the equipment was installed during the November 2004 reactor refueling outage no adjustments or intervention was permitted. Hence, all system operation and performance was dependent on installation judgments made during the system design phase.

2.2. *Data acquisition and processing*

The data acquisition system consists of a computer with a National Instruments PCI-6602 timer/counter board using an 80 MHz clock reference. The computer/timer board senses and records the passage times of the encoder teeth. The torsional vibration is then computed from the tooth passage time arrays using a digital demodulation approach based upon the time interval measurement system [4]. Various investigators have addressed a number of issues and developed enhancements to this approach [5–11]. The data processing algorithm options used in this effort include:

1. Simultaneous processing of signals from the three probes to separate two orthogonal shaft translation motions ($x(t)$, $y(t)$) and the rotation ($\theta(t)$) time arrays [8].
2. Removal of rotational order components [10].
3. Constant time resampling to compensate for changes in running speed [10].
4. Quantification and correction of encoder geometric errors [11].

At this point in the data processing, a time dependent torsional vibration array is produced. The spectrum can then be estimated using standard FFT methods with the parameters listed in Table 1. Of particular note is the high spectral resolution obtained by the large FFT block size and the steps taken to enhance the data quality. A typical torsional spectrum is shown in Fig. 3. Examination of the spectra, in conjunction with (1) shaft and motor/pump translational vibration; (2) torsional finite element modal analysis of the pump line shaft; (3) transducer probe mount impact test data; and (4) probe cross channel coherence functions, allowed the torsional modes to be identified. Fig. 4 shows the reactor coolant pump torsional responses for modes 1 and 3 from operational data recorded on 5 days over an 18-day period. Inspection of the spectra qualitatively indicates that the modal responses are consistent. Variation around the nominal response is also apparent and is due to the inherent nature of the data and the respective processing.

Table 1
Torsional spectral data processing specifications

Processing selection	Specification
Analysis frequency range, f_{analysis}	0–2048 Hz
FFT block size, N	131,072
Number of spectral lines	65,536
Spectral resolution, Δf	0.03125 Hz
Ensemble averages	120
FFT spectral estimation data window	Hanning
Order removal routine	Implemented [10]
Encoder correction routine	Implemented [10]
Constant time step resampling order-to-frequency domain routine	Implemented [10]
Simultaneous processing of 3 probe separation of translation and rotation routine	Implemented [8]

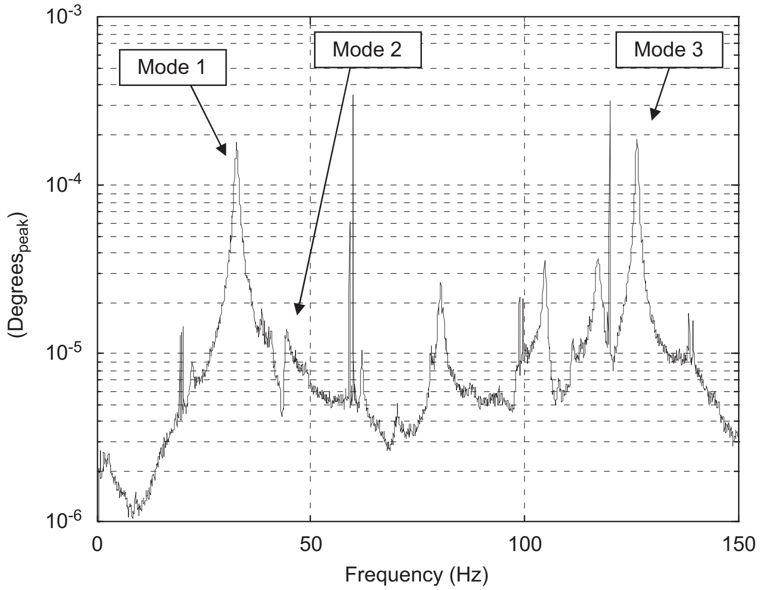


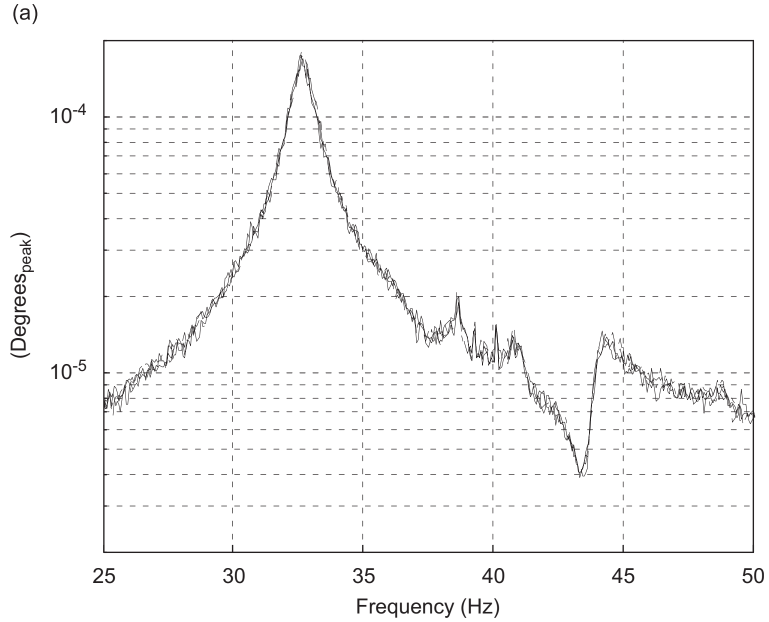
Fig. 3. Typical torsional vibration spectrum.

2.3. Torsional vibration modal feature identification

The prototype reactor coolant pump measurement and data processing system produces (1) the torsional vibration time array; and (2) torsional vibration spectrum of the pump responses. Hence, the modal identification algorithm must be capable of estimating the parameters from this "output only" data. An algorithm was developed to estimate the pump modal parameters based on fitting the response of a single degree of freedom system to the response spectrum. The underlying assumptions are (1) the reactor coolant pump modal responses are sufficiently well separated that their individual responses can be represented by a single degree of freedom model; (2) the reactor coolant pump torsional modal dynamics can be described by a linear viscously damped model; and (3) the reactor coolant pump torsional vibration excitation has primarily broadband random characteristics, at least over the frequency range surrounding a mode of interest.

The complex residue form of a linear viscous single degree of freedom system frequency response function is

$$H(f) = \frac{R}{2i(s-P)} - \frac{R^*}{2i(s-P^*)}, \tag{1}$$



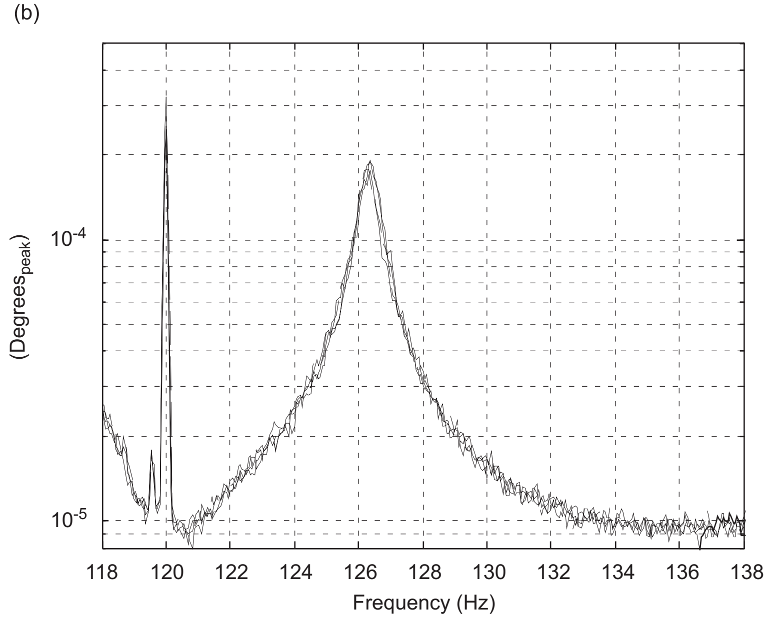


Fig. 4. (a) Closeup of spectra for modes 1 and 2 from 4 different days over a 3-week period; (b) closeup of spectra for mode 3 for 4 different days over a 3-week period.

where P is the system pole $P = -\xi 2\pi f_n \pm i2\pi f_d$; R is the residue; and f_n is the undamped natural frequency; ξ is the damping ratio, and f_d is the damped natural frequency. If the underlying excitation is assumed to be broadband and random in nature, the force spectrum ($G_{FF}(f)$) has a constant amplitude across the frequency range of interest. The system response spectrum ($G_{\theta\theta}(f)$) will then be a scaled version of the absolute value of the frequency response function in Eq. (1) squared:

$$G_{\theta\theta}(f) = |H(f)|^2 G_{FF}(f). \tag{2}$$

This relationship can be used to estimate the modal parameters from the reactor coolant pump response spectrum around each mode. First, the difference between the measured response spectrum and the frequency response function magnitude (Eq. (1)) at each spectral line is formed,

$$\varepsilon(f) = \left| \sqrt{G_{\theta\theta}(f)} - |H(f)| \right|. \tag{3}$$

The absolute values of the differences at each spectral line are then summed over a user specified frequency range surrounding the mode of interest. An optimization problem can be formulated with the objective function,

$$\text{Minimize} \left\| \sum_{f_{\text{low}}}^{f_{\text{high}}} \left| \sqrt{G_{\theta\theta}(f)} - |H(f)| \right| \right\|. \tag{4}$$

The optimization process solution produces estimates of the natural frequency, damping ratio and residue (R). The estimated R will not be strictly the modal residue, but will represent a combination of the modal response and excitation level. However, the estimated natural frequency and damping ratio will be representative of the actual pump dynamics. For the crack monitoring application these are the critical qualities of interest.

Many optimization algorithms can be utilized to estimate the parameters (f_n, ξ, R) that minimize the objective function as expressed in Eq. (4). The unconstrained Matlab optimization routine “fmin” was invoked due to its simplicity and it proved to yield accurate estimates in a computationally efficient fashion. The method showed some convergence sensitivity to initial values. However, once a parameter set reasonably close to the final values ($\pm 25\%$) is used, the optimization routine readily converged. The subject application benefits from the a priori knowledge of the baseline modal parameters, which was used as initial guesses for the optimization. Nominal frequency bands of ± 3–10 Hz surrounding the resonance peak proved effective.

To assess the identification outcome, the measured spectrum was compared by first placing the estimated modal parameters ($\hat{R}$, $\hat{f}_n$, $\hat{\zeta}$) into Eq. (1) and taking the absolute value,

$$|H(f)| = \left| \frac{\hat{R}}{2i(s-\hat{P})} - \frac{\hat{R}^*}{2i(s-\hat{P}^*)} \right|. \tag{5}$$

This result could then be compared directly to the square root of the measured response spectrum expressed in terms of peak amplitude to evaluate the identification accuracy. Fig. 5 shows a nominal comparison of the

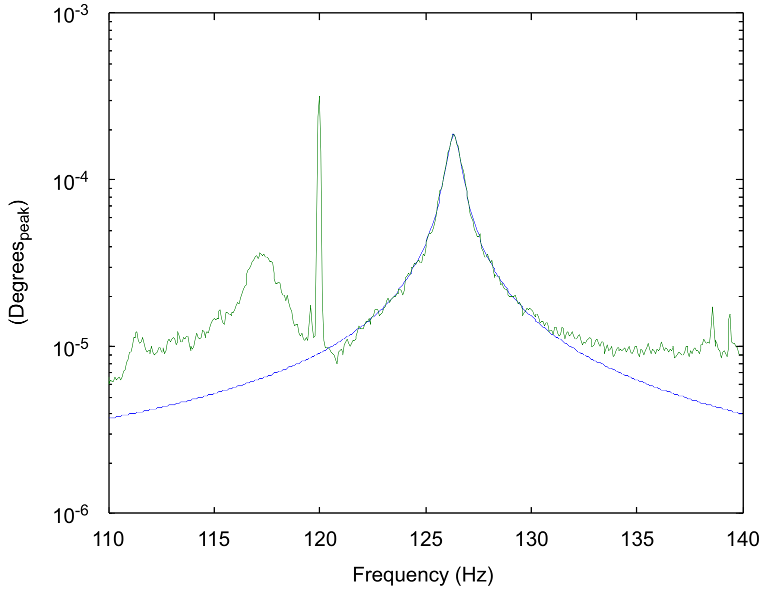


Fig. 5. A torsional spectrum around mode 3 for a nominal reactor coolant pump and the spectrum predicted from Eq. 5 using the modal parameters estimated from the single degree of freedom optimization algorithm (smooth line).

identification algorithm applied to the reactor coolant pump torsional spectral data around mode 3. The correlation is excellent and representative of typical results. Further evaluations showed that the method was accurate and repeatable. Since small changes in the modal parameters are of interest, estimation precision is paramount and other general features can be sacrificed without any adverse implications. Hence, this identification algorithm proved to be well suited and was adopted for utilization in the prototype torsional vibration crack monitoring system.

3. Torsional vibration feature uncertainty simulation

The modal responses shown in Fig. 4 illustrate the randomness of the spectral values around the nominal response. The spectral variation directly affects the identified modal parameters. Therefore, any variance in the natural frequency estimates can be traced directly to the spectrum. Since the spectrum is the end result of the sensing hardware and data processing stream it contains the errors and uncertainty, in a cumulative sense, for all the preceding operations. Hence, if the inherent nature of the reactor coolant pump spectral data can be replicated it can be used as a basis to quantify the measurement uncertainty.

It is unknown if any observed changes in the operational reactor coolant pump modal parameter estimates are from a change in the pump dynamics (e.g., crack) or artifacts of the data acquisition and processing. The overall variance in an estimated modal parameter (i.e., a torsional natural frequency) is a sum of the actual changes in the pump frequency and the torsional measurement system uncertainty,

$$\sigma^2_{\text{observed}} = \sigma^2_{\text{pump}} + \sigma^2_{\text{instrument}}. \tag{6}$$

The pump modal dynamics will remain constant in the absence of a line shaft fault, such as a crack. Hence, the pump shaft line dynamics may be represented by a constant parameter system and the modal parameter variance (σ^2_{pump}) will be zero. For a constant parameter system, Eq. (6) shows that any observed change is attributed to only the instrument variation.

To evaluate the overall uncertainty the spectral amplitude and variance need to be replicated. The spectral randomness can be controlled by the characteristics of the excitation signal. Since the model parameters are held constant, any resulting variation is due to data processing and can be statistically quantified. A reactor coolant pump finite element analysis showed the natural frequency of mode 3 possessed sensitivity to crack propagation. Therefore, the mode will be used to illustrate the evaluation of the statistical measurement precision.

3.1. Replication of reactor coolant pump spectral characteristics

The torsional spectrum was calculated by breaking the acquired time array into 120 segments $\theta_n(t,T)$ where n is an index. The segment array length, N, was 2^{17} or 131,072. Each time block was modified by a Hanning window to reduce leakage and a Fast Fourier Transform applied to produce a sample transform, $\Theta_n(f,T)$. A 50% overlap algorithm was used to increase the number of averages available from the 20 min reactor coolant pump record. The averaged spectrum is then produced by computing the mean of the magnitude squared across the respective 120 block transforms in each frequency bin,

$$G_{\theta\theta}(f,T) = \frac{2}{120(N^2)}\,\frac{8}{3}\sum_{n=1}^{120}\Theta_n^*(f,T)\Theta_n(f,T). \tag{7}$$

When the Matlab FFT algorithm is used in conjunction with Eq. (7), a power spectrum is produced in units of degrees rms squared (Θ^2_{rms}). The 8/3 factor is a broadband power correction factor to compensate for the use of the Hanning window.

Fig. 4(b) shows the torsional spectra around reactor coolant pump mode 3 on 4 different days with a logarithmic ordinate axis. The identical data is presented in Fig. 6(b) with a linear ordinate axis to allow closer examination of the spectral characteristics. The data shows the same general character with a certain degree of variation around the nominal response.

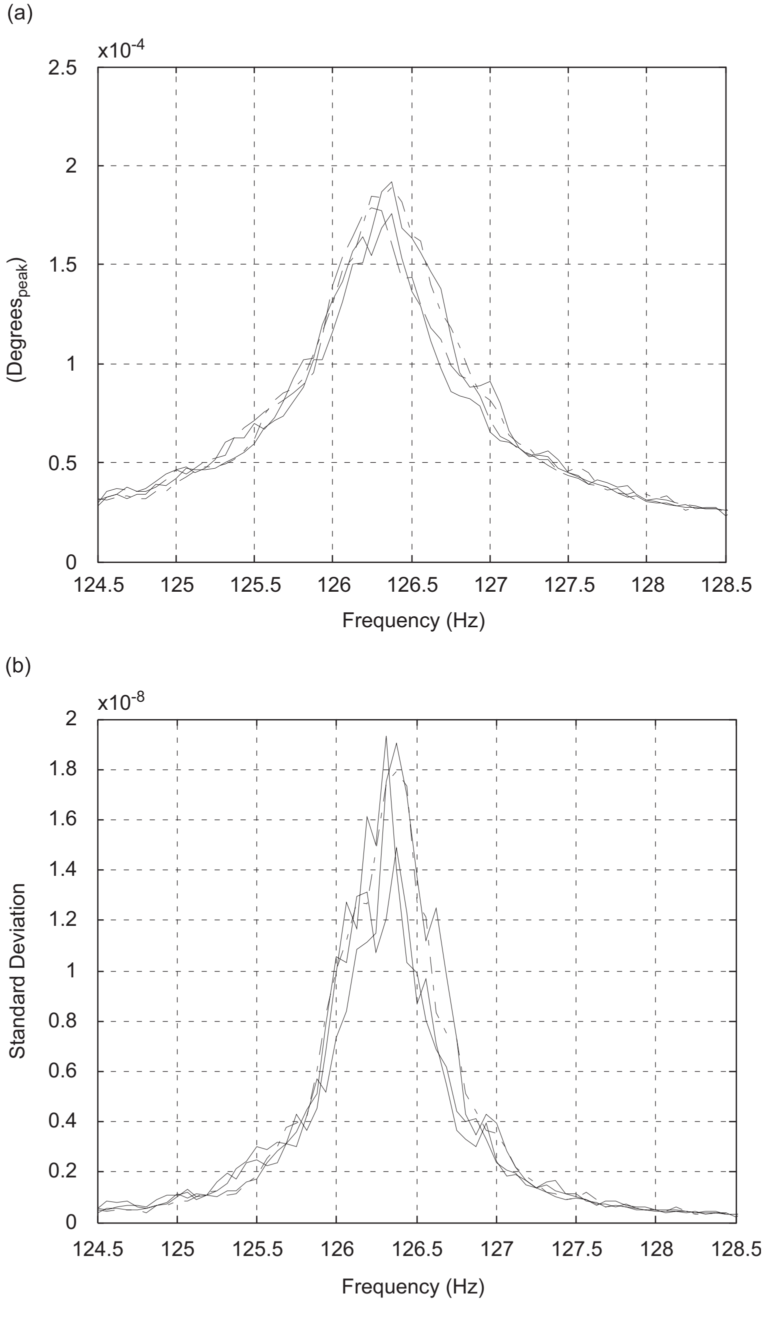


Fig. 6. (a) Reactor coolant pump torsional spectra around mode 3 for 4 different days computed with 120 averages; (b) spectral standard deviation of the 120 averages used to compute the spectra in Fig. 6(a).

The spectral characteristics can be further quantified by calculating the variance within each frequency bin from the 120 averages,

$$\sigma^2_{G_{\theta\theta}}(f,T) = \sum_{n=1}^{120}\left[\frac{8}{3}(\Theta_n^*(f,T)\Theta_n(f,T)) - G_{\theta\theta}(f,T)\right]^2. \tag{8}$$

The spectral standard deviation is computed by taking the square root of variance in Eq. (8). Fig. 6(b) shows the spectral standard deviation for the four spectra in Fig. 6(a). The standard deviations are (1) similar in nature showing; (2) frequency dependency, and (3) show an increase in the region surrounding the resonance. It is these spectral amplitude and variance characteristics that must be captured in order to simulate the natural frequency identification uncertainty.

Inspection of the torsional response around mode 3 shows it to be lightly damped and well separated from other modes. This indicates that a single degree of freedom system model may be suitable. The shape of the resonance region is representative of linear viscous damping. To capture the modal response a constant coefficient single degree of freedom mass-spring-damper model was developed. A Simulink model was formed to numerically solve the governing differential equation for an applied excitation signal. The model parameters were chosen by first applying the identification algorithm to a measured reactor coolant pump spectrum producing a natural frequency of 126.317 Hz and a damping ratio of 0.00241. The single degree of freedom mass was arbitrarily chosen as 0.1 kg and the corresponding damping and stiffness values computed to match the measured natural frequency and damping ratio. A fixed step 4th order Runge–Kutta numerical integration algorithm was selected with an identical time step to the reactor coolant pump torsional records. The spectral amplitude and standard deviation were then calculated using the identical processing steps and parameters as applied to the reactor coolant pump torsional time data.

The selection of the excitation to be used in the simulation is crucial. The objective is to use an excitation input signal that causes the simulation model to closely replicate the final reactor coolant pump spectral response shown in Fig. 6. The main source of torsional line shaft excitation in the reactor coolant pump is hydrodynamic turbulence acting on the pump impeller. Hence, it is this excitation frequency content that must be mimicked in the simulation. It is impossible to measure the hydrodynamic pressure on an operational reactor coolant pump due to a variety of issues. As an alternative, insight into the spectral forcing function characteristics was gained from measurements on a 41% reduced scale reactor coolant pump model acquired during prototype system development. Hydrodynamic pressure transducers were placed at the inlet and exit of the pump bowl. The pressure spectrum amplitude was broadband and decreased gradually as frequency increased with some tonal content related to impeller passage frequencies. Over a narrow frequency band (i.e., 50 Hz) the amplitude was reasonably flat. Therefore, the simulation excitation should be broadband, with possible candidate signals including white noise, colored noise and burst random. Initially, a white noise random input signal was applied to the single degree of freedom model to evaluate the feasibility.

The white noise excitation levels were iteratively adjusted until the spectral amplitude matched the reactor coolant pump data in Fig. 6. Fig. 7 shows the force spectra and standard deviation over the 10 Hz frequency band surrounding mode 3. The force amplitude was modified by a multiplicative constant applied to the force input time signal. The spectrum has the expected flat response. The spectral standard deviation shows an increase in both mean value and bin-to-bin variation as the level increases.

The corresponding model spectral response and standard deviation are shown in Fig. 8. For comparison, nominal results from the reactor coolant pump are also included. All model spectra parallel the reactor coolant pump, differing only by a multiplicative constant. The most important outcome is that the middle amplitude force in Fig. 7 closely captures the reactor coolant pump spectral response behavior. The spectral standard deviation of the response is shown in Fig. 8(b). The same trend observed in Fig. 7(b) is followed, in that the higher the response amplitude the higher the corresponding standard deviation. For comparison, the standard deviation from a nominal reactor coolant pump result is included and its characteristics are captured by the middle amplitude force level.

These results indicate that the combination of a single degree of freedom model and white noise excitation appears to closely mimic the reactor coolant pump spectral characteristics. The correlation was further examined by comparing the spectra from reactor coolant pump data records on 4 different days to four different simulation results and is shown in Fig. 9. No observable distinction is apparent between the data originating from either the reactor coolant pump or simulation data sets. All eight spectra and standard deviations fall within the respective nominal variation of each function. Hence, this builds further confidence in the ability of the simulation model to closely replicate reactor coolant pump spectral characteristics. The good correlation implies that the cumulative variability existing in data from the prototype hardware and data processing stream has been captured by the model and can be used to quantify measurement uncertainty (i.e., $\sigma^2_{\text{instrument}}$ in Eq. (6)).

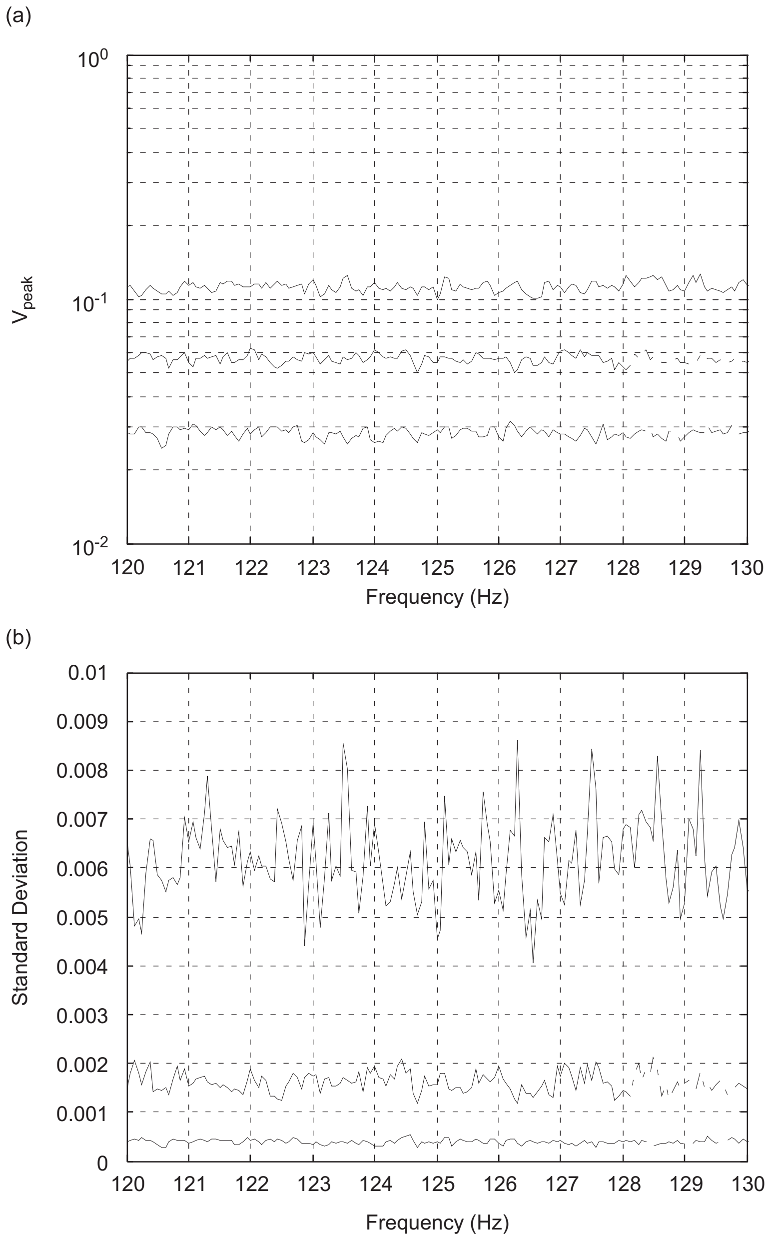


Fig. 7. (a) Trial model force input spectra; (b) spectral standard deviation for force spectra in 7(a).

As an aside for future applications, it is useful to note that it was possible to manipulate the excitation signal to produce the same spectra and with different standard deviations. This outcome is important for applications to other reactor coolant pump modes and rotating equipment since it is unrealistic to expect a white noise input will always replicate a mode's spectral response characteristics. Consider the excitation spectra and standard deviation in Fig. 10. The white noise excitation that matched the reactor coolant pump response is annotated in the figure. The time history for the alternative excitation was created by multiplying white noise signal by a weighting function comprised of an amplitude shifted cosine function,

$$f_{\text{shape}}(t) = A_{\text{offset}} + A_{\text{amplitude}} \cos\left(\frac{2\pi t}{T_{\text{shape}}}\right). \tag{9}$$

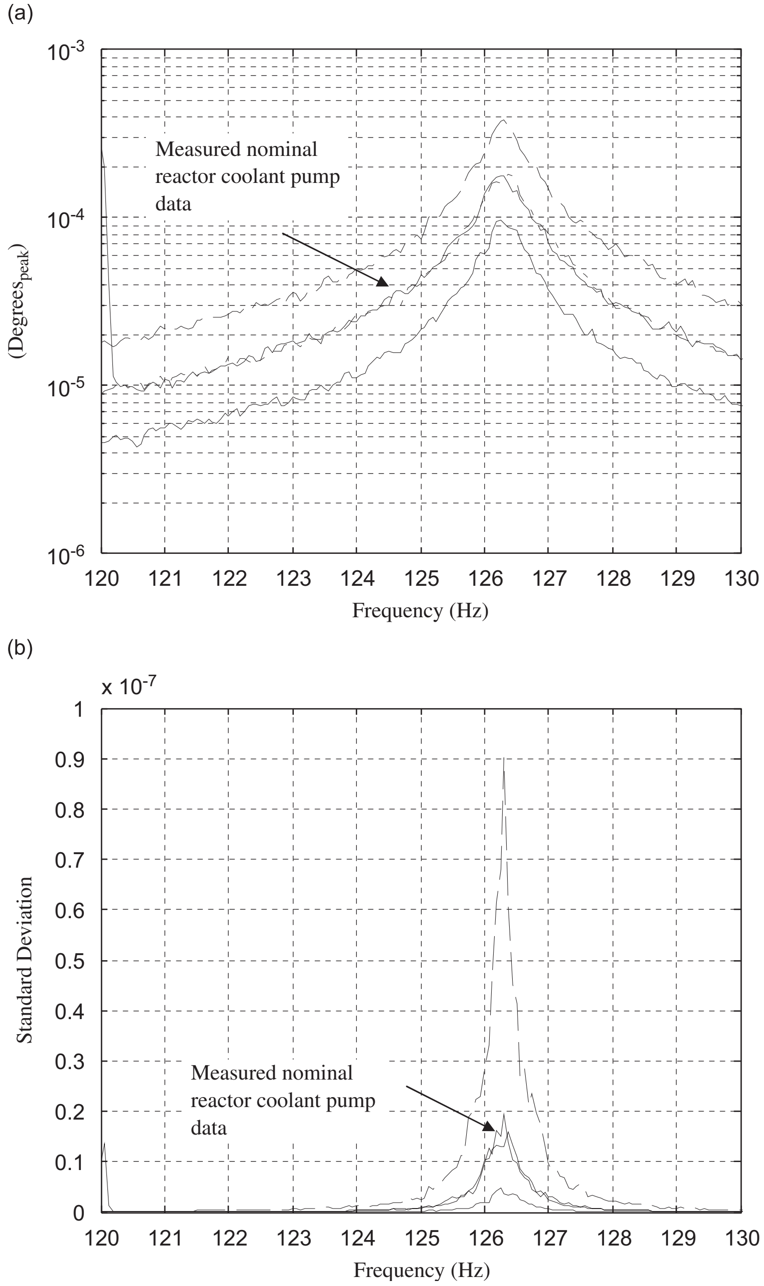
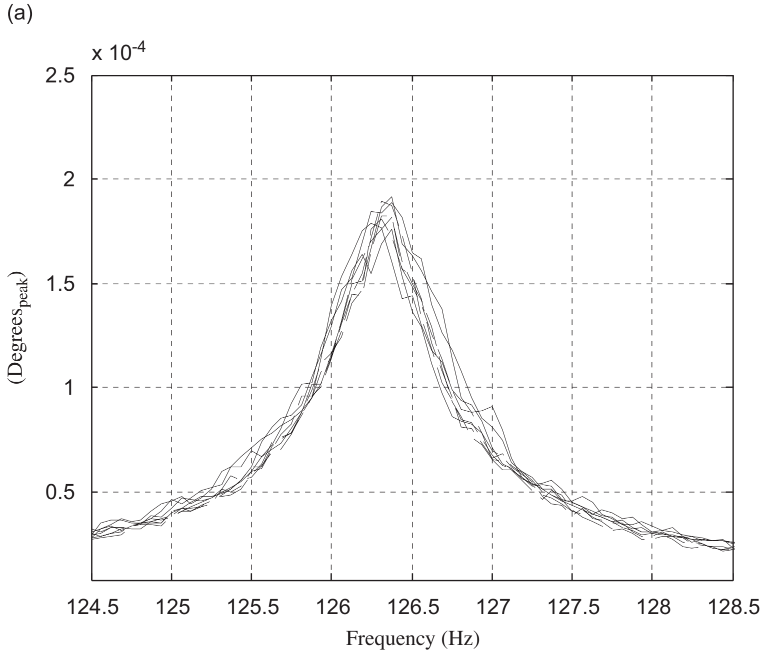


Fig. 8. (a) Model response for the force spectra in Fig. 7(a) compared to nominal reactor coolant pump torsional response spectrum for mode 3; (b) spectral standard deviation for response spectra in Fig. 8(a).

By adjusting the weighting function amplitude ($A_{\text{amplitude}}$), offset (A_{offset}) and period (T_{shape}) it was possible to create force spectra with similar ensemble averaged amplitudes but different standard deviations. With shape period (T_{shape}) of 40 s and $A_{\text{offset}} = A_{\text{amplitude}}/2$ a broadband excitation signal is produced where the amplitude gradually increases and decreases. Fig. 11 compares the model response for both excitation signals and a nominal reactor coolant pump spectrum. All three spectra are similar. Although subtle, the response for the shaped random excitation (dashed line) shows a higher variance. The spectral standard deviation (Fig. 11(b)) clearly shows the greater variance. The capability to create random excitation signals with



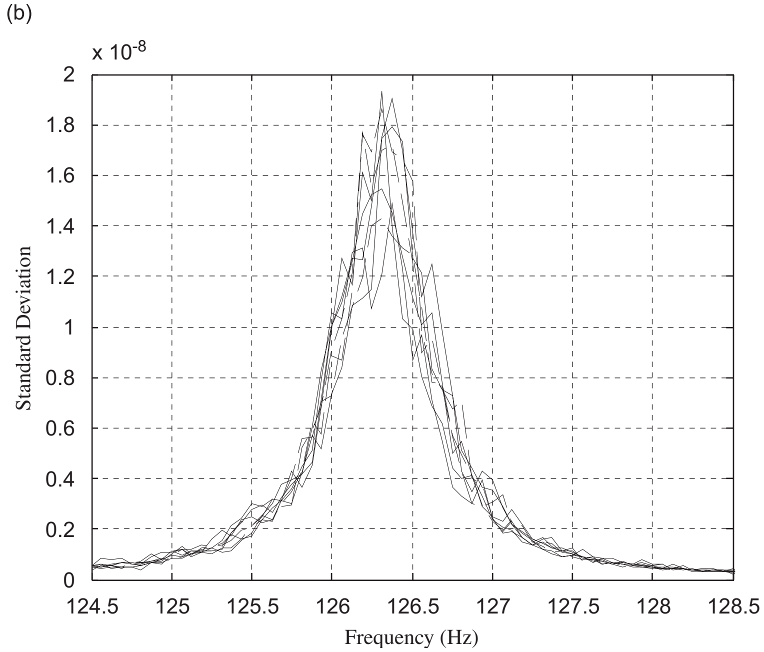


Fig. 9. (a) Mode 3 torsional reactor coolant pump spectra (___) on 4 days and four simulation data sets (_ _ _); (b) spectral standard deviation of the 120 averages used to compute the spectra in Fig. 9(a); measured (___), simulation (_ _ _).

different amplitude and variance characteristics is important for potential applications to other modes and rotating equipment.

3.2. Uncertainty simulation

The constant coefficient modal model and excitation signal characteristics that combine to replicate the reactor coolant pump response have now been established. Next, the simulation model was used to create one

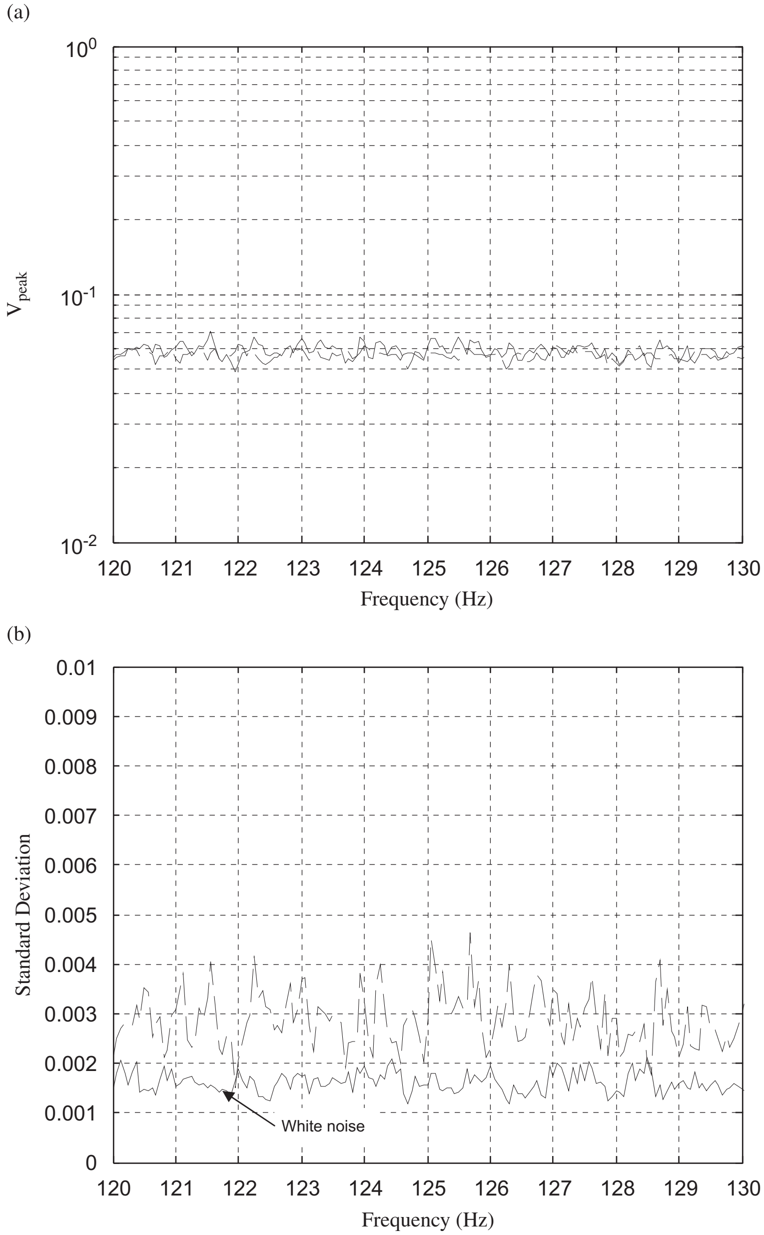


Fig. 10. (a) Excitation spectra from white noise and a white noise multiplied by an amplitude shifted low frequency sinusoidal function; (b) Spectral standard deviation of the 120 averages used to compute the spectra in Fig. 10(a).

hundred separate response signals, each with a different random excitation array seed value. Subsequently, the respective spectra were estimated and the modal identification algorithm applied. The objective function (Eq. (3)) frequency bands were set to 122 and 132 Hz. Fig. 12(a) shows the estimated natural frequencies from the sample population. The corresponding sample population damping ratios are depicted in Fig 12(b). The results show the general variation obtained from the simulations. The appearance of any bounding limits in either graphic is coincidence as none were imposed and is just the character of this sample population.

3.2.1. *Normal distribution evaluation of simulation population*

A Kolmogorov–Smirnov (K–S) goodness-of-fit test was applied to the natural frequency and damping ratio sample populations. The K–S test provides the ability to determine if the sample populations are described by

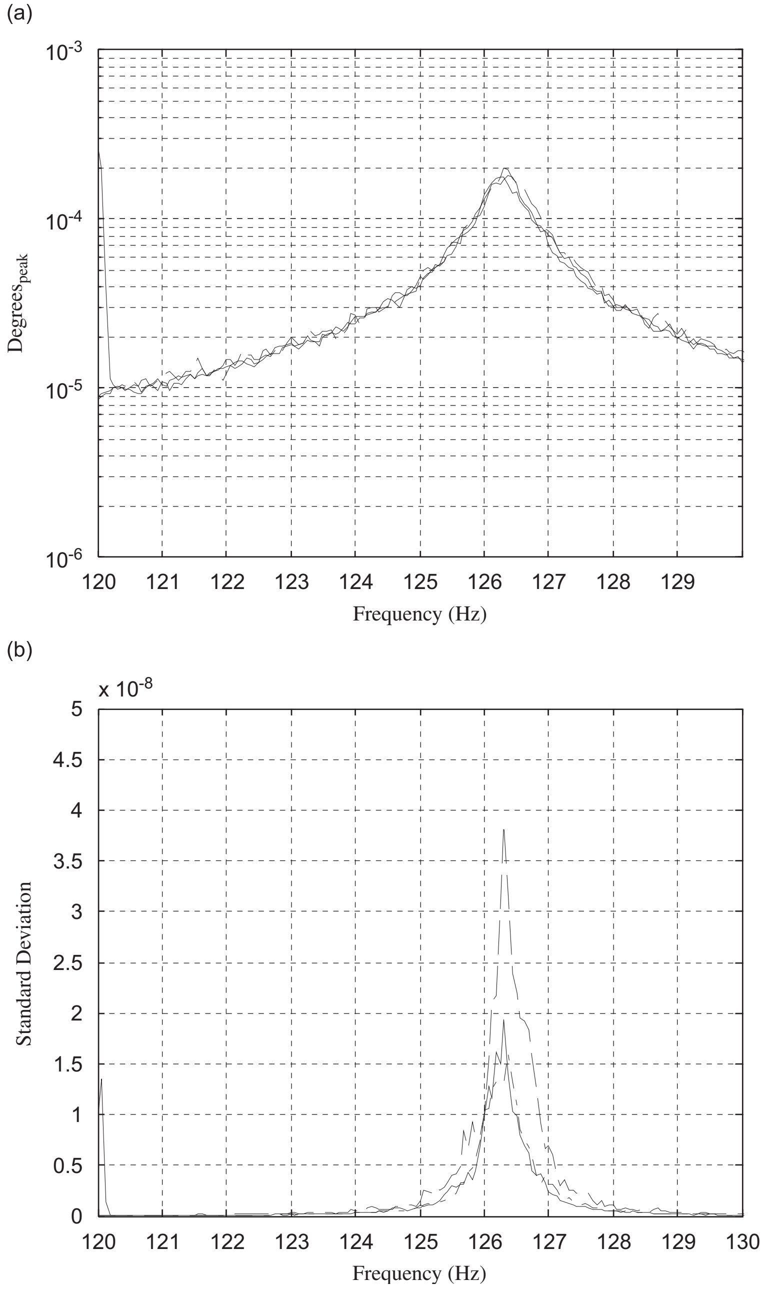


Fig. 11. (a) Simulation spectra from white noise and a white noise multiplied by an amplitude shifted low frequency sinusoidal function compared to nominal reactor coolant pump response; (b) spectral standard deviation of the 120 averages used to compute the spectra in Fig. 11(a).

a normal distribution. The test involved ordering the data, Y, in each population and defining the empirical distribution function for each value as

$$F(Y_i) = \frac{n(i)}{N}, \tag{10}$$

where $n(i)$ is the number of data points less than the ith value in the ordered set. The empirical cumulative distribution function was then compared to that of a theoretical normal distribution by defining a test

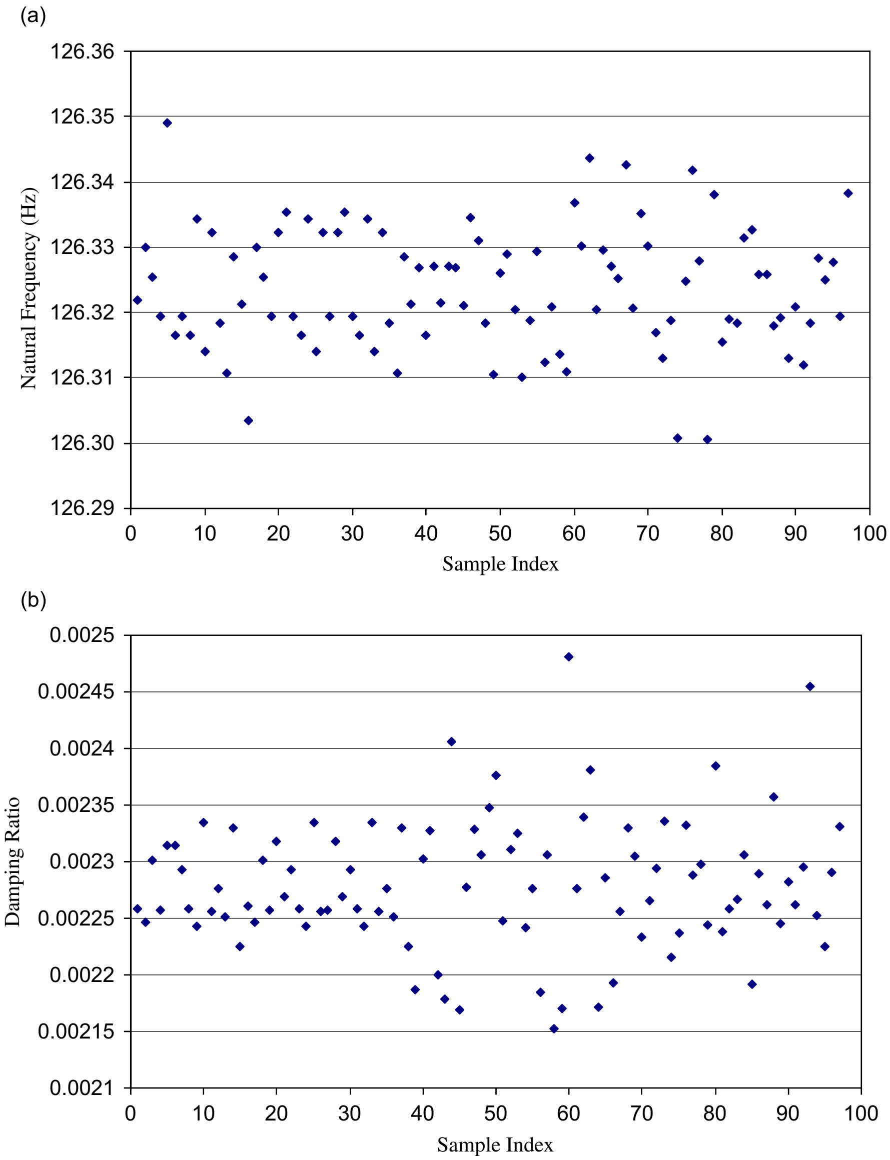


Fig. 12. (a) Natural frequency estimates from 100 simulation runs for mode 3 of a reactor coolant pump; (b) damping ratio estimates from 100 simulation runs for mode 3 for a reactor coolant pump.

statistic as

$$\mathrm{KS} = \max_{1 \leqslant i \leqslant N} \left[F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right]. \tag{11}$$

Fig. 13 shows the K–S test result for the frequency population of mode 3 on reactor coolant pump 1–2 calculated with Minitab statistical analysis package. The graphical results show the good fit of the empirical data to the theoretical. The close fit is representative of those obtained for all modes. The calculated K–S test statistic (0.065) for the natural frequency was less than the published critical value (0.159) for a significance level of 95%. Therefore, it was concluded that the distribution was in fact normal. These results were typical for all other modal parameters estimated from the simulations.

The implication of the normal distribution result was a necessary outcome as it permits further statistical manipulation of the simulation populations with the assumption of a normal distribution. It is critical that this

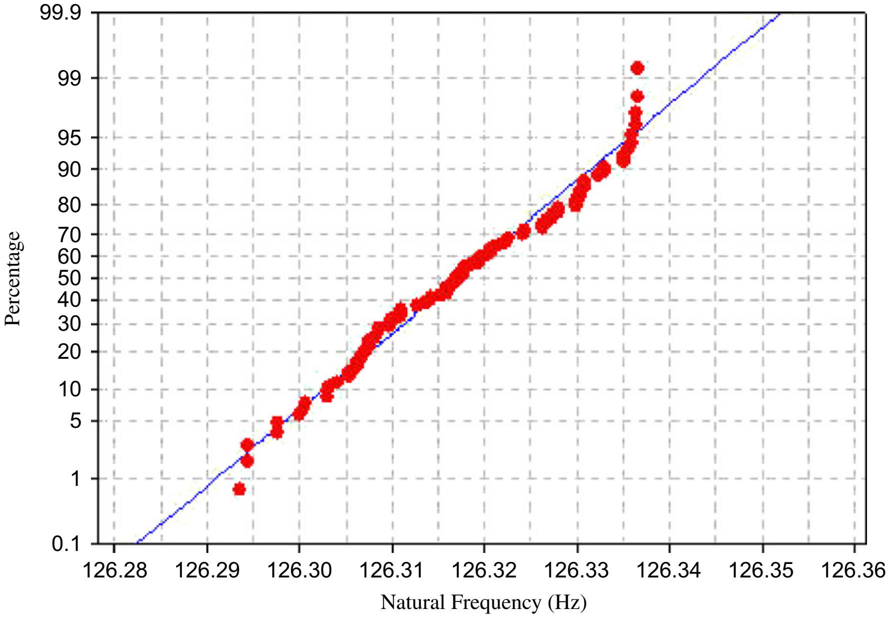


Fig. 13. K–S test results for mode 3 natural frequency depicting the comparison between empirical (●) and theoretical (___) cumulative distribution functions.

criterion be satisfied and it is up to the practitioner to make the verification in any application utilizing the approach presented herein.

3.3. Statistical confidence limits

Statistical tolerance limits were employed to quantify the overall system estimation precision. Statistical tolerance limits define value ranges within which an expected proportion of the population (p) will be contained, and can be used to judge the respective measurement uncertainty [12]. Hence the statistical tolerance limits can be used to define the expected modal parameter measurement resolution from a probabilistic viewpoint. The tolerance limit is defined as

$$\text{Tolerance limit} = k_2\sigma, \tag{12}$$

where k_2 is computed with a sample size (N), a chi-squared distribution value (χ^2) with $N-1$ degrees of freedom with the area in the tail region of the distribution equal to γ, a standard normal value (z) with the area in the tail region of the distribution equal to $(1-p)/2$ as shown in Eq. (13),

$$k_2 = \sqrt{\frac{(N-1)(1+(1/N))z^2_{(1-p)/2}}{X^2_{\gamma,N-1}}}. \tag{13}$$

The k_2 value, and subsequently the tolerance limit, is dependent on the selection of the population proportion and probability. The tolerance limits are assumed to be independent of the mean value, and depends on only the population standard deviation [13].

For this shaft crack monitoring application it is imperative that a very high level of confidence be achieved in the reactor coolant pump torsional modal parameters. This is due to the structural health implications and corresponding decisions that may be made from the trended data. Furthermore, the safety and high financial implications of decisions made from the monitoring system results dictate that there must be absolutely no false positive outcomes. To satisfy these goals, it is imperative to know if an observed change in the torsional modal parameters is due to the inherent measurement variation or is an actual change in the line shaft dynamics. Therefore, a conservative set of specifications is chosen to compute the tolerance limits. The limits are computed to contain 99.9% of the population with 99.9% confidence. The natural frequency and damping tolerance limits for the data in Fig. 12 are

$$\begin{aligned} f_3 &= 126.31722 \pm 0.04740\,\text{Hz}, \\ \xi_3 &= 0.00241 \pm 0.00036. \end{aligned} \tag{14}$$

The graphical limits for the sample populations in Fig. 12 reflect the respective tolerance limits. The conservative nature of the tolerance limits is qualitatively apparent in that none of the sample population values approach the limits.

The use of these statistical tolerance limits establishes the level of the system's measurement precision from a probabilistic viewpoint. The values can be loosely interpreted in a similar fashion as a traditional resolution would be in a deterministic instrument evaluation. The conditional caveat is that the tolerance limits are dependent on the probability and population percentage selected.

3.4. Statistical confidence limits for higher spectral variance

The results in Fig. 11 illustrate it is possible to replicate the modal response with excitation signals having different spectral standard deviations. To examine any potential effects of this difference on feature uncertainty, an additional study was performed. One hundred time records were calculated using the same simulation model, but with the shaped white noise excitation signal described by Eq. (9). The random sequence used a different starting seed value to insure each sequence was different. The identical computational, spectral and statistical procedures as used previously were applied. The natural frequency and damping tolerance limits which contain 99.9% of the population with 99.9% confidence are

$$\begin{aligned} f_3 &= 126.31722 \pm 0.0682\,\text{Hz}, \\ \xi_3 &= 0.00241 \pm 0.00053. \end{aligned} \tag{15}$$

The tolerance limits for both modal features are approximately 1.5 times higher than the previous results in Eq. (14). This indicates that the spectral variance of the excitation signal affects the modal feature identification uncertainty for the data processing used in this application. The implication of this outcome is the necessity to consider both the spectral response and spectral variance when assessing the uncertainty for each mode and/or machine. This result is important for future applications of this analysis approach.

4. Summary

The evaluation of the measurement uncertainty of a prototype torsional vibration monitoring system installed on a nuclear reactor coolant pump has been discussed. The desired features are the torsional natural frequencies and damping of the pump line shaft modes that are sensitive to crack growth. Since decreases in the torsional natural frequencies will occur as a crack grows the modal parameters may be used as diagnostics to monitor crack growth. The smaller the frequency change that can be sensed the more closely crack growth can be monitored. Hence, it is important to know the smallest statistically significant frequency change that can be sensed. The prototype hardware/software torsional system could not be evaluated in a traditional instrumentation chain sense due to pragmatic and regulatory issues related to licensed nuclear reactor operation. Therefore, a simulation-based probabilistic approach was adopted. A model was developed that replicated the spectral modal response of a crack sensitive mode. The model response contains all the inherent upstream measurement and data processing uncertainty in a cumulative fashion. Therefore, if the model accurately captures the form of the measured torsional data the overall system uncertainly can be simulated and quantified. Observations and conclusions from the modal modeling effort were:

1. If the crack sensitive mode was lightly damped and well separated from other resonances its dynamic response could be represented by a linear viscous damped single degree of freedom mass-spring-damper system.

2. The choice and characteristics of the excitation are crucial to replicating the actual torsional response. A broadband random signal worked well for the modal response of the pump under consideration.
3. Attention must be paid to both the spectral amplitude and standard deviation in each frequency bin from the ensemble averages to accurately replicate the modal response suitable for uncertainty analysis.
4. It was possible to modify the model excitation time signal to produce similar spectral response amplitudes with different standard deviations. This is important for future application to other rotating equipment.
5. The simulations were effective in capturing the pump's spectral characteristics such that the variation of the estimated modal parameters could be evaluated.

The developed model was subsequently used to create one hundred sample time arrays representative of the pump's modal response. The same data processing algorithms used in the prototype torsional monitoring system were applied to the simulation data resulting in sample populations of the modal features. The population statistics were calculated and subsequently used to quantify the overall measurement uncertainty. The measurement precision was quantified in a probabilistic sense using statistical tolerance limits. Observations and conclusions from the analysis showed:

1. Conservative limit specifications (99.9% of the population with 99.9% confidence) were used to minimize the possibility of false positive values being incurred.
2. The tolerance limits for mode 3 of the subject pump were (1) natural frequency: ± 0.04740 Hz; and (2) damping ratio: ± 0.00036.
3. The statistical tolerance limits assist in assessing the natural frequency trending data in that they define the expected level of process variation. Any observed trends that exceed the tolerance limits can be statistically significant and related to changes in the line shaft dynamics.
4. The modal response, and thus the tolerance limits, is expected to be unique for every mode and each rotating equipment platform. Therefore, the limits are not transportable and will need to be computed for each mode of interest independently.

Unlike the deterministic resolution in a traditional instrument sense, the statistical precision developed using this approach requires careful interpretation. The following must be considered:

1. Since the statistical limits are based upon simulation data, it is paramount that the model results replicate the form and variation achieved in the actual measurement system.
2. The computed limits are dependent on the user-selected specifications and would change accordingly. Therefore, appropriate parameters must be chosen to match the needs of the respective application.

This modeling/simulation approach allows the overall system measurement uncertainty to be evaluated for the selected data processing procedures. Moreover, the statistical simulation approach can be further used to enhance and improve the measurement system. Processing parameter studies can be performed and the effects quantified. The evaluation will help guide the identification of the optimal processing methods for the data available from the prototype system.

Acknowledgments

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References

[1] United States Nuclear Regulatory Commission, Monitoring vibration to detect circumferential cracking of reactor coolant pump and reactor recirculation pump shafts, NRC Information Notice 2008-08, Office of Nuclear Reactor Regulation, Washington, DC 20555, USA (2005).

[2] InfoGram IG-02-4, RCP Shaft Crack Investigation, Westinghouse Electric Company (2002).

[3] American Society of Mechanical Engineers Standard OM/G-2003 part 24, reactor coolant and recirculation pump condition monitoring (2003).

[4] J.M. Vance, *Rotordyamics of Turbomachinery*, Wiley, New York, 1988.

[5] K.P. Maynard, M.W. Trethewey, C.L. Groover, Application of torsional vibration measurement to shaft crack monitoring in power plants, *55th Meeting of the Society for Machinery Failure Prevention Technology,* Virginia Beach, VA, USA, 2–5 April 2001, pp. 217–226.

[6] K.P. Maynard, M.W. Trethewey, Blade and shaft crack detection using torsional vibration measurements part 3: field application demonstrations, *Noise and Vibration Worldwide* 32 (2001) 16–23.

[7] K.P. Maynard, M.W. Trethewey, On the feasibility of blade crack detection through torsional vibration measurements, *Noise and Vibration Worldwide* 31 (2000) 15–31.

[8] P. Wang, P. Davies, J.M. Starkey, R.L. Routson, A torsional vibration measurement system, *IEEE Transactions of Instrumentation and Measurement* 41 (1992) 803–807.

[9] J. Williams, Improved methods for digital measurement of torsional vibration, *Society of Automotive Engineers* 1203 (1996) 9–15 (Paper No. 962204).

[10] B.R. Resor, C.L. Groover, M.W. Trethewey, K.P. Maynard, Natural frequency identification in torsional vibration with high level order content, *XXIII IMAC: A Conference and Exposition on Structural Dynamics*, Dearborn, Michigan, USA, 26–29 January 2004.

[11] B.R. Resor, M.W. Trethewey, K.P. Maynard, Compensation for encoder geometry and shaft speed variation in time interval torsional vibration measurement, *Journal of Sound and Vibration* 286 (2005) 897–920.

[12] 〈http://www.itl.nist.gov/div898/handbook/prc/section2/prc263.htm〉.

[13] C. Basnet, K.E. Case, The effect of measurement error on accept/reject probabilities for homogeneous products, *Quality Engineering* 4 (1992).