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Acoustic length correction of duct extension into a cylindrical chamber

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Abstract

The one-dimensional (1D) analytical model for the accurate acoustic performance prediction of silencers and resonators with duct extensions require the introduction of acoustic length corrections in order to account for the multidimensional effects at junctions. In the present study, a two-dimensional (2D) axisymmetric analytical approach, and three-dimensional (3D) finite element method (FEM) are developed to determine the acoustic length correction of duct extension into cylindrical chamber. The effect of chamber geometry on the acoustic length correction is examined and an approximate expression for the acoustic length correction is provided. The corrected 1D analytical approach with acoustic length correction is then used to predict transmission loss of Helmholtz resonator with neck extension and expansion chamber with extended inlet/outlet. The 1D analytical solutions of transmission loss are compared with FEM predictions and experimental results.

1. Introduction

The sound fields inside silencers are clearly multidimensional. However, the one-dimensional (1D) analytical approach may be used to predict the acoustic attenuation performance of silencers at lower frequencies [1]. The multidimensional effects associated with the evanescent high-order modes at area sudden change can be considered by introducing an acoustic length correction to improve the accuracy of the 1D analytical approach.

Acoustic length correction depends on the structure geometry. Rayleigh [2] obtained an approximate value for the acoustic length correction of duct exposure to an infinite hemisphere space. Karal [3] investigated the acoustic inductance for sudden discontinuities in the infinite circular ducts and provided an approximated expression for the acoustic inductance in terms of correction factor. Ingard [4] obtained formulae for the acoustic length corrections of small circular and rectangular apertures radiated into circular and rectangular ducts based on the plane piston approximation. For the acoustic length correction of circular ducts in infinite circular waveguides, Karal and Ingard gave the same expressions. Kergomard and Garcia [5] provided a set of formulae for the case of the radiation of a plane piston into an infinite waveguide and the case of a change in

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cross-section in a circular guide. Davies [6] presented fitting expressions for the values of end corrections for unflanged pipes and ducts discontinuities. Peat [7] studied the effect of evanescent high-order modes for coaxial discontinuities using an analytical approach and FEM, and obtained an expression for the Karal correction factor in terms of a double Chebyshev series. The three-dimensional (3D) finite element method (FEM) was employed by Sahasrabudhe and Munjal [8] to determine the Karal correction factor for cases of coaxial and offset discontinuities, and a polynomial expression for the factors was also provided. Selamet and Ji [9] calculated acoustic length corrections of ducts with and without offsets in closed cylindrical chambers of different radius ratios and chamber lengths by 3D analytical approach. For the case of coaxial discontinuities of duct extension, Torregrosa et al. [10] studied the acoustic length corrections in extended-duct silencers with different diameter ratios by finite element approach. However, only the effect of diameter ratio on the acoustic length correction was studied, and the effect of chamber and duct extended length was not considered.

The objectives of the present study are (1) to develop a two-dimensional (2D) axisymmetric analytical approach and finite element approach to determine the acoustic length correction of duct extension into cylindrical chamber, (2) to examine the effect of chamber geometry on the acoustic length correction, and (3) to apply the corrected 1D analytical approach with acoustic length corrections to predict transmission loss of Helmholtz resonator with neck extension and expansion chamber with extended inlet/outlet and compare the predictions with the simple 1D analytical, 3D numerical and experimental results.

2. 2D axisymmetric analytical approach

Fig. 1 shows the geometry of cylindrical chamber with duct extension. The structure is divided into three parts: part 1—the inlet duct, part 2—the circular chamber, and part 3—the annular chamber.

The sound wave propagation is governed by Helmholtz equation [1]

$$\nabla^2 p + k^2 p = 0, \tag{1}$$

where p is the sound pressure, $k = \omega/c_0$ the wavenumber, c_0 the sound speed, $\omega = 2\pi f$ the angular frequency, and f the frequency.

In circular and annular ducts, the expressions for sound pressure and particle velocity can be obtained by solving Eq. (1) in 2D axisymmetric coordinates as [11–14]

$$p_i(r,x) = \sum_{n=0}^{\infty} \mathbf{\Psi}(\gamma_{i,n}, r)\left(P_{i,n}^{+}\mathrm{e}^{-\mathrm{j}k_{i,n}x} + P_{i,n}^{-}\mathrm{e}^{+\mathrm{j}k_{i,n}x}\right), \tag{2}$$

$$\rho c_0 U_i(r,x) = \sum_{n=0}^{\infty} \frac{k_{i,n}}{k} \mathbf{\Psi}(\gamma_{i,n}, r)\left(P_{i,n}^{+}\mathrm{e}^{-\mathrm{j}k_{i,n}x} - P_{i,n}^{-}\mathrm{e}^{+\mathrm{j}k_{i,n}x}\right), \tag{3}$$

where subscript i represents the ith structure, ρ is the air density, j the imaginary unit, n the radial mode number of sound waves, $P_{i,n}^{+}$ and $P_{i,n}^{-}$, the modal amplitudes corresponding to components traveling in the

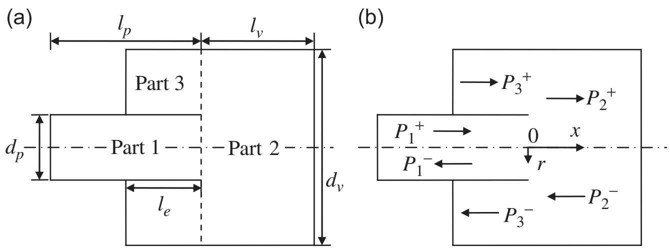

Fig. 1. Cylindrical chamber with duct extension.

positive and negative x directions, respectively, $k_{i,n}$ the axial wavenumber of the (0, n) mode, and

$$k_{i,n} = \begin{cases} \sqrt{k^2 - \gamma_{i,n}^2}, & k^2 - \gamma_{i,n}^2 \geqslant 0, \\ -\mathrm{j}\sqrt{\gamma_{i,n}^2 - k^2}, & k^2 - \gamma_{i,n}^2 < 0, \end{cases} \tag{4}$$

$$\gamma_{i,n} = \begin{cases} \alpha_n / r_p, & i = 1, \\ \alpha_n / r_v, & i = 2, 3. \end{cases} \tag{5}$$

The eigenfunctions in Eqs. (2) and (3) are [11,13]

$$\mathbf{\Psi}(\gamma_{i,n}, r) = \begin{cases} J_0(\alpha_n r/r_p), & i = 1, \\ J_0(\alpha_n r/r_v), & i = 2, \\ J_0(\alpha_n r/r_v) - [J_1(\alpha_n)/Y_1(\alpha_n)]Y_0(\alpha_n r/r_v), & i = 3, \end{cases} \tag{6}$$

where α_n is the root satisfying the radial boundary conditions of

$$\begin{cases} \mathbf{\Psi}'(\gamma_{i,n}, r_p) = 0, & i = 1, 3, \\ \mathbf{\Psi}'(\gamma_{i,n}, r_v) = 0, & i = 2. \end{cases} \tag{7}$$

A piston-driven face with oscillating velocity magnitude U_p is assumed at the inlet of the duct and the walls are assumed to be rigid, so it is required that $U_1|_{x=-l_p} = U_p$ and $U_2|_{x=l_v} = U_3|_{x=-l_e} = 0$. At the junction of duct extension and chamber ($x = 0$), the continuity of sound pressure requires $p_1 = p_2$ for $0 \leqslant r \leqslant r_p$ and $p_3 = p_2$ for $r_p \leqslant r \leqslant r_v$, while the continuity of the particle velocity requires $U_1 = U_2$ for $0 \leqslant r \leqslant r_p$ and $U_3 = U_2$ for $r_p \leqslant r \leqslant r_v$. Applying the orthogonality of eigenfunctions [15], the above boundary conditions result in a set of equations, as described in Refs. [13,14]. If the numbers of modes in ducts 1, 2, and 3 are truncated to a, b, and c, respectively, then the numbers of unknowns and independent equations are both $2(a+b+c+3)$, so the unknowns $P_{i,n}^+$ and $P_{i,n}^-$ can be solved from the set of equations by setting $\rho c_0 U_p = 1$.

Near the interface of parts 1 and 2, at $x = 0$, the continuity of sound pressure requires [3]

$$p_{1,0} + p_{1,h} = p_{2,0} + p_{2,h}, \tag{8}$$

where the subscripts 0 and h represent the (0, 0) mode and the higher-order modes, respectively. Below the cut-off frequency, only the (0, 0) mode is propagationable, and the discontinuity can be equivalent to the equation [5]

$$p_{1,0} = p_{2,0} + ZV, \tag{9}$$

where Z is lumped impedance, $V = S_1 U_{1,0}$ the volume velocity of the fundamental mode, S_1 the cross-section area of duct 1, $U_{1,0}$ the particle velocity in duct 1 at $x = 0$. The lumped impedance can be written as [1]

$$Z = \mathrm{j}\rho\omega\frac{\delta}{S_1} = \frac{p_{2,h} - p_{1,h}}{S_1 U_{1,0}} = \frac{p_{1,0} - p_{2,0}}{S_1 U_{1,0}}, \tag{10}$$

where δ is the acoustic length correction of the duct extension, and can be calculated using the following formula [9]:

$$\delta = \left|\frac{p_{1,0} - p_{2,0}}{\mathrm{j}\rho\omega U_{1,0}}\right| = \left|\frac{\left(P_{1,0}^+ + P_{1,0}^-\right) - \left(P_{2,0}^+ + P_{2,0}^-\right)}{\mathrm{j}k\left(P_{1,0}^+ - P_{1,0}^-\right)}\right|. \tag{11}$$

In the low-frequency range, the influence of frequency on the acoustic length correction is negligible [9]. The difference between results with mode numbers $a = b = c = 50$ and $a = b = c = 70$ is less than 1% for $d_p/d_v = 0.1$–0.9. Thus using $a = b = c = 50$, Eq. (11) may provide reasonable converged values for the range of $0.1 < d_p/d_v < 0.9$.

3. Finite element approach

To employ the FEM to determine the acoustic length correction of the duct extension into cylindrical chamber, the following boundary conditions are assumed:

$$\rho c_0 U = 1 \quad \text{(at the inlet)}, \tag{12}$$

$$\partial p/\partial n = 0 \quad \text{(on the rigid walls)}. \tag{13}$$

Using the FEM, the acoustic domain of the structure is discretized into a number of elements first, and then the matrix form of the equations of all elements may be obtained by making use of the variational formulation for the acoustic system [1]

$$\left([M] - k^2[K]\right)\{p_n\} = -\mathrm{j}\omega\rho\{F\}, \tag{14}$$

where $[M]$ and $[K]$ are the inertia and stiffness matrices of all the elements, respectively, $\{F\}$ the forcing vector determined by boundary conditions, vector $\{p_n\}$ the nodal acoustic pressures. Once Eq. (14) is solved, the acoustic pressure and particle velocity on the inlet may be combined to obtain the input acoustic impedance, and then the resonance frequency f_r of the structure may be obtained when the absolute value of the input acoustic impedance reaches the minimum.

To consider the effect of non-planar waves near the junction of duct extension and chamber, the length of duct extension l_e in the simple 1D analytical approach is replaced by $l'_e = l_e + \delta$, where δ is the acoustic length correction. Accordingly, the length l_p is replaced by $l'_\mathrm{p} = l_p + \delta$ and l_v by $l'_v = l_v - \delta$. So the input acoustic impedance is expressed as [1]

$$Z_p = \frac{Z_v + \mathrm{j}\rho c_0 \tan kl'_p}{\rho c_0 + \mathrm{j}Z_v \tan kl'_p}, \tag{15}$$

where $Z_v = -\mathrm{j}\rho c_0 S_p/[S_v \tan kl'_v + (S_v - S_p)\tan kl'_e]$ is the acoustic impedance on the junction of duct extension and chamber, S_p and S_v the cross-section areas of the ducts 1 and 2, respectively. The relation between the resonance frequency f_r and the acoustic length correction of the duct extension δ is

$$S_p - [S_v \tan k_r l'_v + (S_v - S_p)\tan k_r l'_e]\tan k_r l'_p = 0, \tag{16}$$

where $k_r = 2\pi f_r/c_0$. The acoustic length correction can be determined from Eq. (16), once the resonance frequency is obtained by FEM.

4. Results and discussion

For a given duct extension into a closed cylindrical chamber, the effect of chamber length on the acoustic length correction is examined first. The acoustic length correction predictions by 2D axisymmetric analytical approach and FEM are shown in Fig. 2 for different duct extended lengths. It is shown that the results of 2D analytical approach agree well with FEM predictions. The graphs also reveal that the acoustic length corrections are decreased with the increase of diameter ratio d_p/d_v. For a fixed d_p/d_v, the acoustic length corrections are almost independent on the chamber length for the long chambers with length-to-diameter ratio l_v/d_v larger than 0.3, while the acoustic length corrections are decreased with the increase of l_v/d_v for the short chambers with $l_v/d_v<0.3$.

The pressure contours for the short ($l_v/d_v = 0.1$) and long ($l_v/d_v = 1$) chambers with duct extensions are shown in Fig. 3. The figure illustrates that the sound waves in the short chambers propagate in the radial direction as the short chamber cannot decay the 3D waves sufficiently. So, it is impossible to introduce the acoustic length correction for the axial wave propagation in the short chamber, while the introduction of the acoustic length corrections is reasonable for the long chamber.

The acoustic length corrections for the finite versus semi-infinite cylindrical chambers are depicted in Fig. 4. The examinations of these graphs reveal that the differences in the acoustic length corrections due to finite versus infinite chamber are getting small as the increase of the length-to-diameter ratio l_v/d_v. Thus, for the

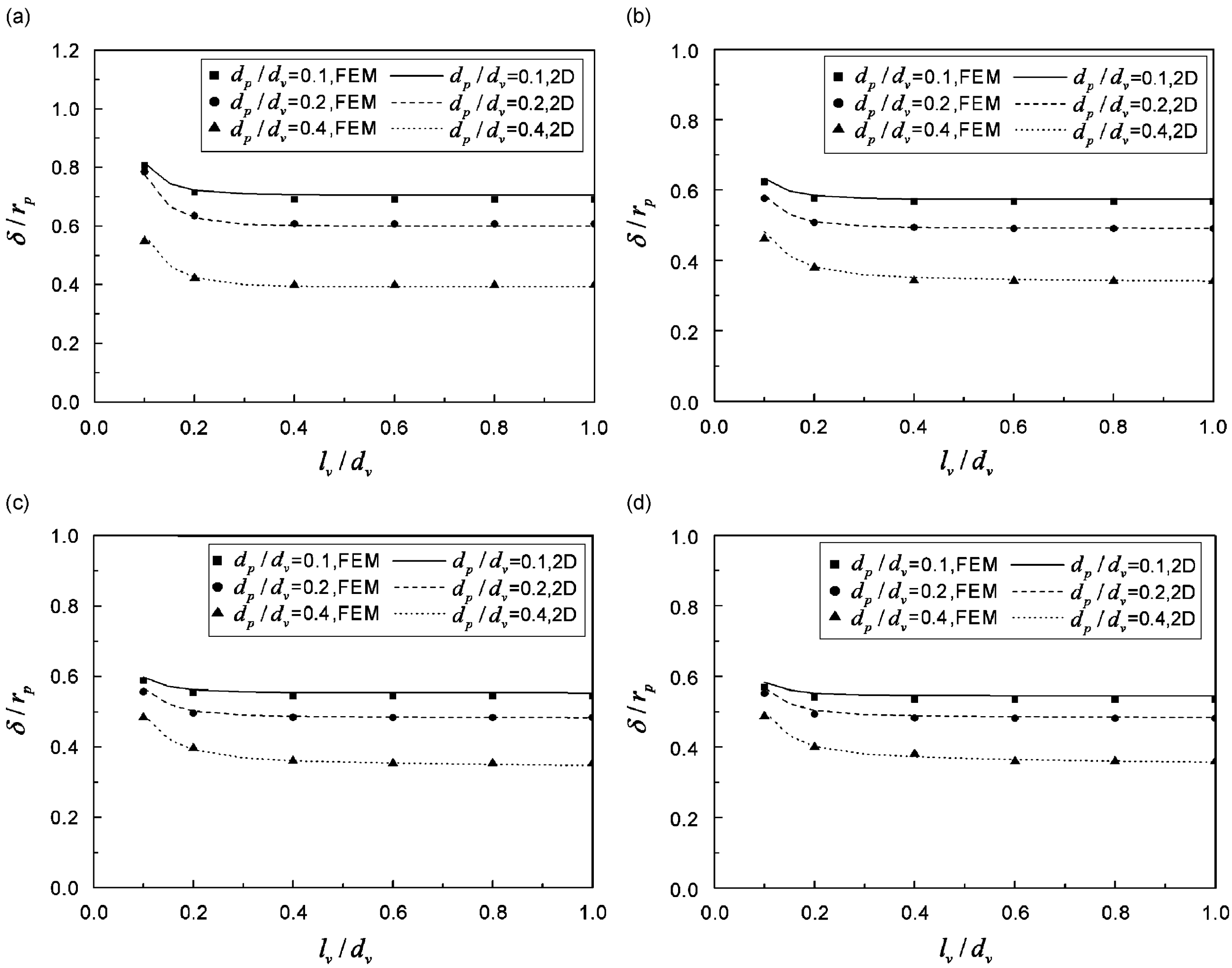

Fig. 2. Effect of chamber length to diameter ratio on acoustic length correction: (a) $l_e/d_p = 0$, (b) $l_e/d_p = 0.5$, (c) $l_e/d_p = 1$, and (d) $l_e/d_p = 2$.

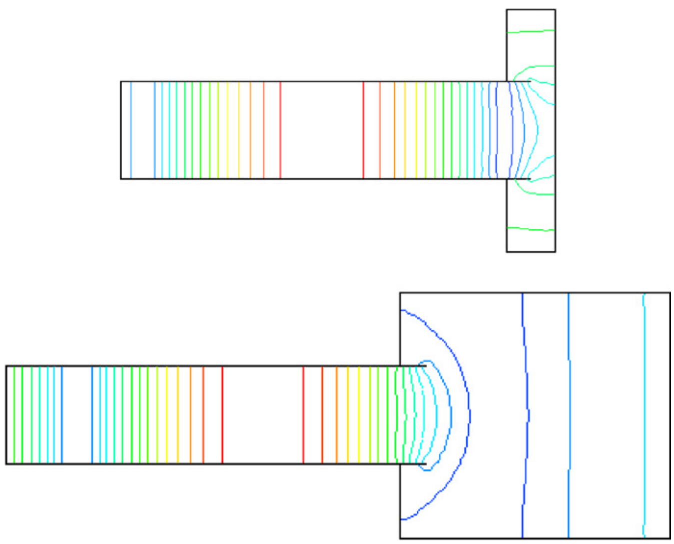

Fig. 3. Pressure contours in short and long chambers with duct extension at 900 Hz.

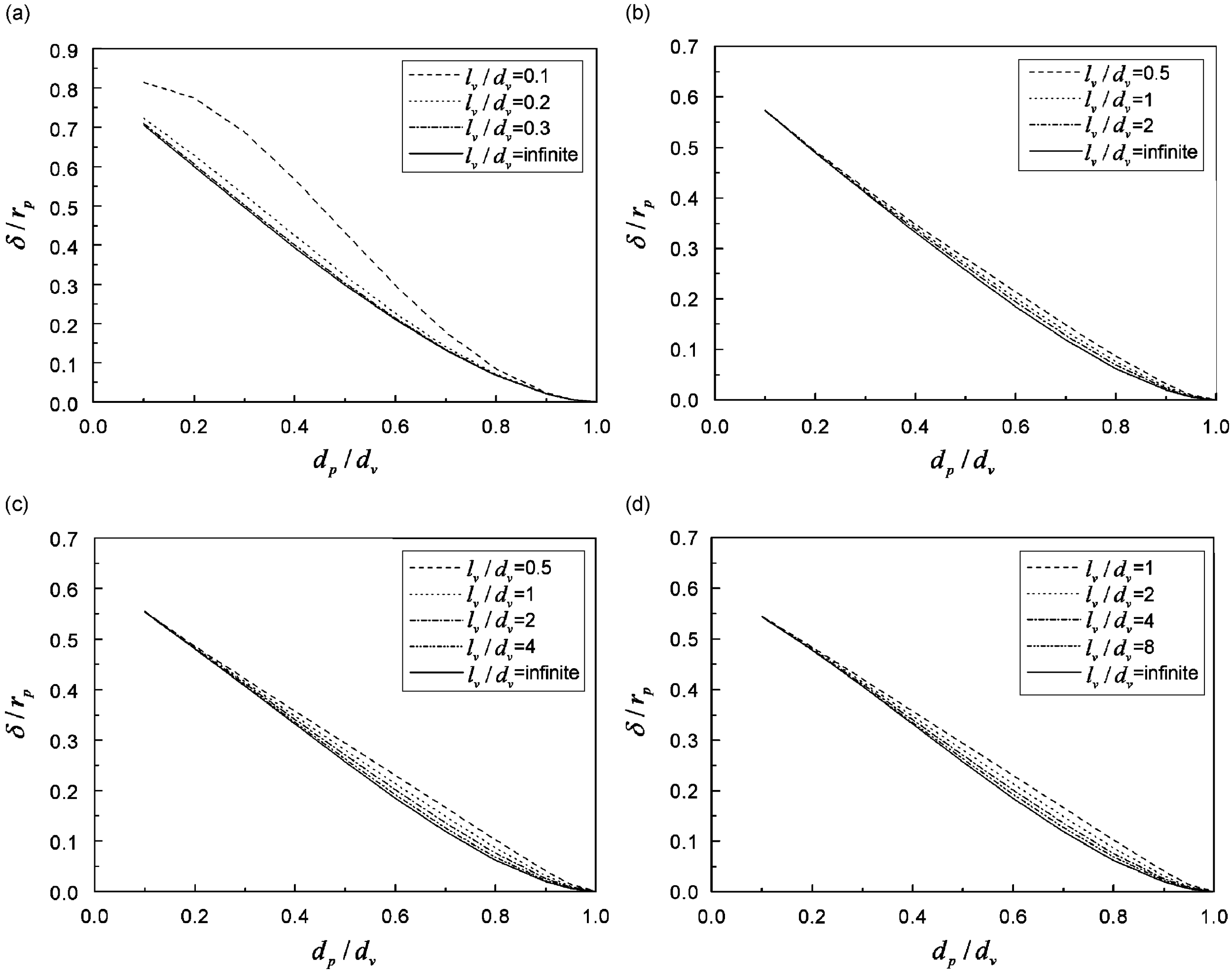

Fig. 4. Acoustic length correction of finite versus infinite cylindrical chambers: (a) $l_e/d_p = 0$, (b) $l_e/d_p = 0.5$, (c) $l_e/d_p = 1$, and (d) $l_e/d_p = 2$.

finite chamber with large l_v/d_v, the acoustic length correction for the semi-infinite chamber may be used as a reasonable approximation.

The effect of duct extension length on the acoustic length corrections is also examined. The acoustic length corrections for the semi-infinite chamber with the duct extension length l_e from zero to $2d_p$ are depicted in Fig. 5. The acoustic length corrections are decreased with the increase of the duct extension length, and the differences in the acoustic length corrections due to the duct extension are also getting small with the increase of the ratio l_e/d_p for the semi-infinite chamber. The difference between acoustic length corrections for $l_e/d_p = 1$ and 2 is small enough and may be ignored.

Based on the 2D analytical results, an approximate expression for acoustic length corrections of duct extension into semi-infinite cylindrical chamber is suggested as

$$\delta/r_p = 0.6165 - \frac{0.7046 d_p}{d_v} + 0.2051 \exp\left(-\frac{3.4453 l_e}{d_p}\right) - \frac{0.3749 d_p}{d_v} \exp\left(-\frac{2.6023 l_e}{d_p}\right) \tag{17}$$

and depicted in Fig. 6. It is clear that the results of expression (17) agree well with those of expression (11) for $d_p/d_v < 0.5$.

The acoustic length corrections may be introduced to improve the accuracy of the 1D analytical approach for prediction of the silencer acoustic attenuation performance. The Helmholtz resonator with neck extension, as shown in Fig. 7, is examined. The present study considers $d_m = 0.04859\,\text{m}$, $d_n = 0.04044\,\text{m}$, $d_v = 0.15319\,\text{m}$,

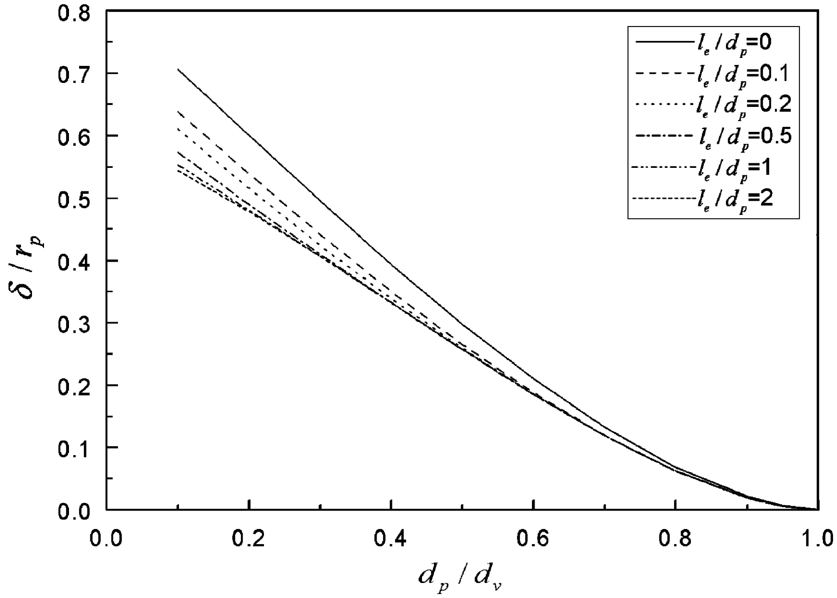

Fig. 5. Acoustic length correction of duct extension into infinite cylindrical chamber.

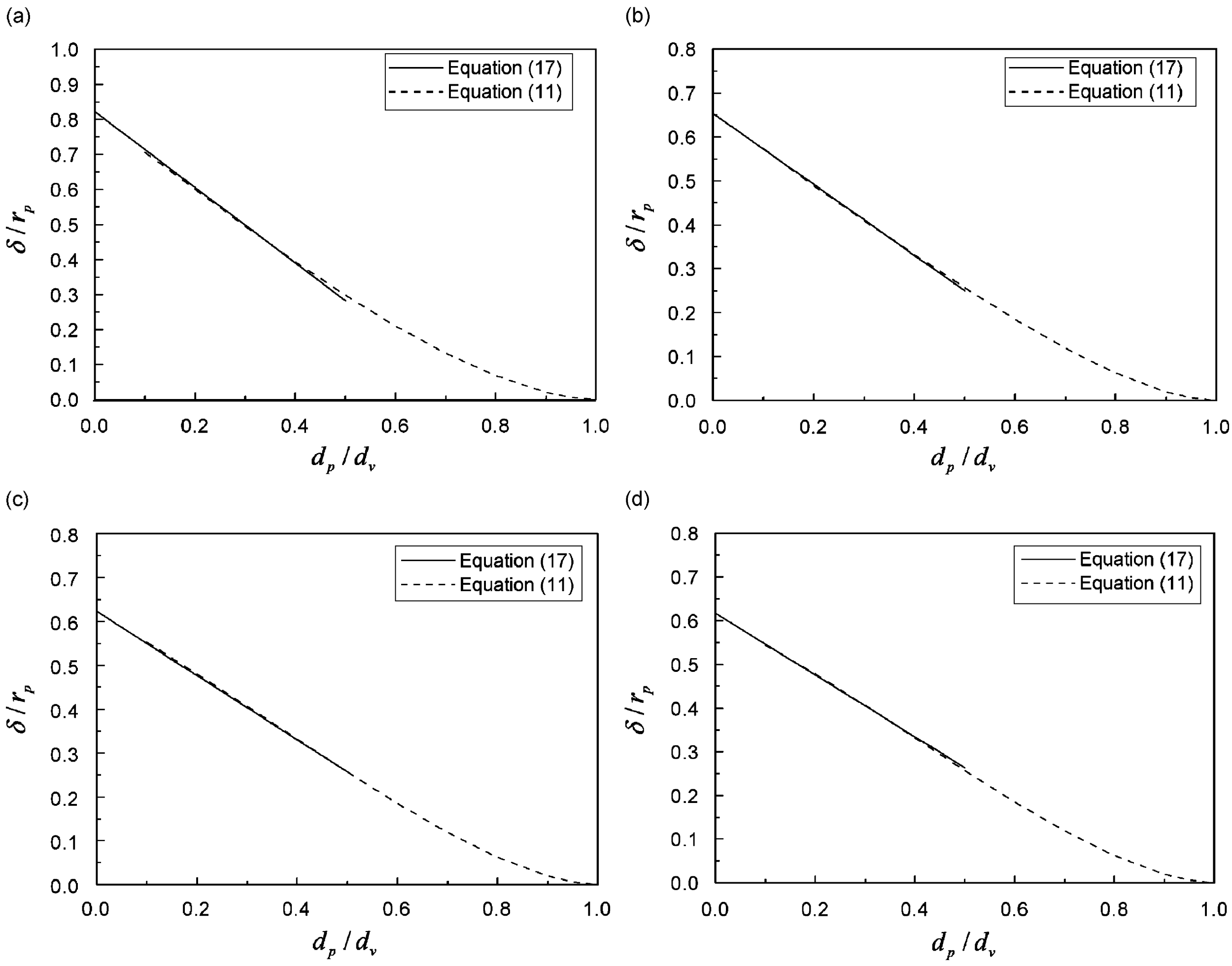

Fig. 6. Acoustic length correction of open chambers: (a) $l_e/d_p = 0$, (b) $l_e/d_p = 0.5$, (c) $l_e/d_p = 1$, and (d) $l_e/d_p = 2$.

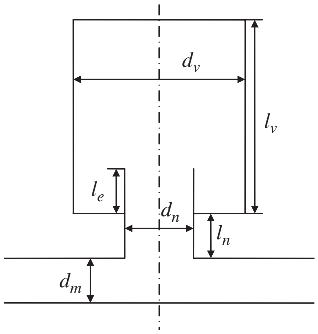

Fig. 7. Helmholtz resonator with neck extension.

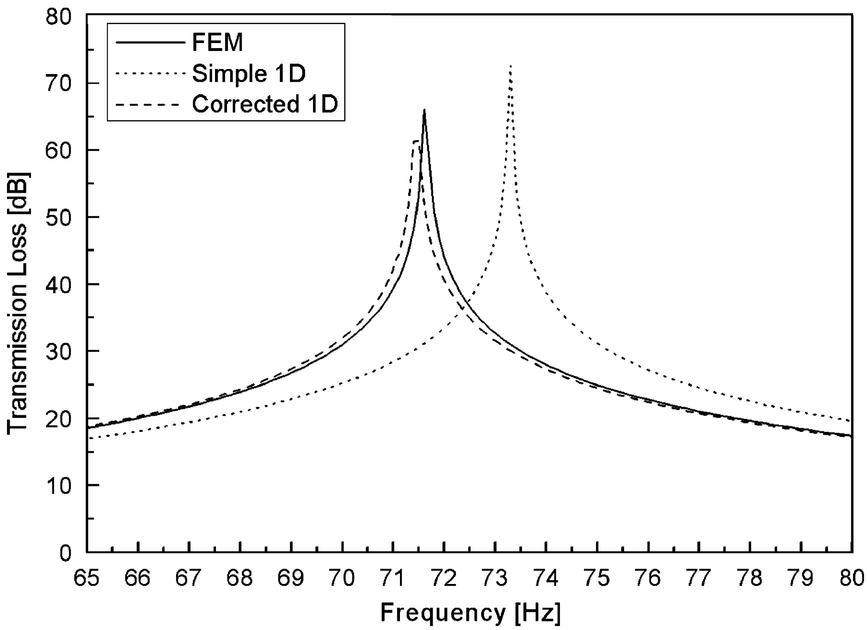

Fig. 8. Transmission loss of Helmholtz resonator with neck extension.

$l_n = 0.085\,\text{m}$, $l_e = 0.06\,\text{m}$, and $l_v = 0.2442\,\text{m}$. Fig. 8 compares the transmission loss predictions from the corrected 1D analytical approach with the present acoustic length correction, the simple 1D analytical approach and FEM. To consider the effect of non-planar waves near the junction of the main duct and the neck, expression (3) in Ref. [16] is used in the both 1D analytical approaches. The discrepancy between the resonance frequencies by the simple 1D analytical approach and FEM is 1.7 Hz, but the difference between the corrected 1D analytical approach and FEM is only 0.1 Hz. The transmission loss results by corrected 1D analytical approach agree well with FEM predictions.

The expansion chamber with extended inlet/outlet, as shown in Fig. 9, is examined next. The present study considers $d = 0.0486\,\text{m}$, $D = 0.1532\,\text{m}$, $L = 0.2823\,\text{m}$, $l_i = 0.08\,\text{m}$, and $l_o = 0.04\,\text{m}$. Fig. 10 compares the transmission loss predictions from the simple 1D analytical theory, corrected 1D analytical approach and FEM with experimental results [13]. It is shown that the transmission loss predictions of the corrected 1D approach agree well with those of FEM and experimental results in low-frequency range, while the discrepancy between the simple 1D analytical approach and FEM is obvious. The introduction of the acoustic length correction improves the prediction accuracy of the 1D analytical approach.

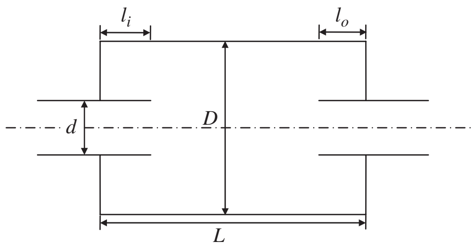

Fig. 9. Expansion chamber with extended inlet/outlet.

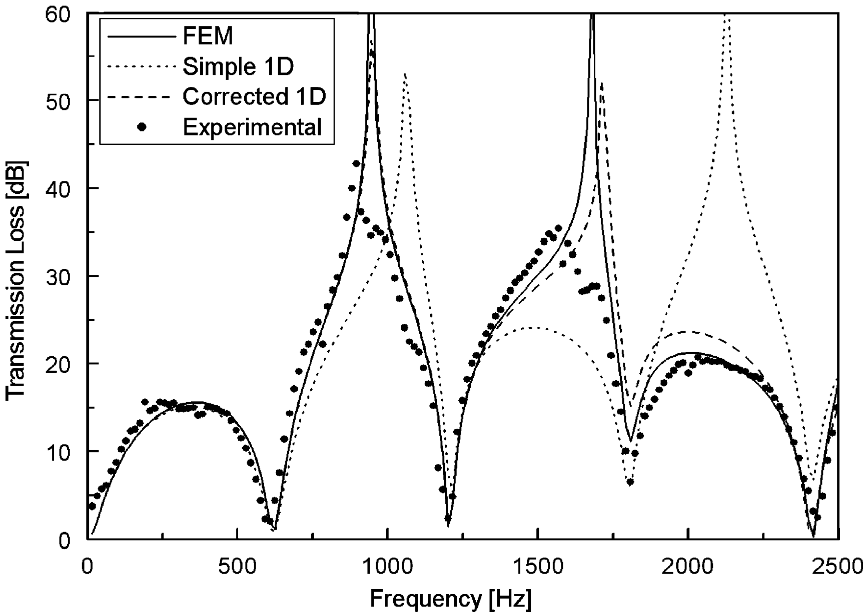

Fig. 10. Transmission loss of expansion chamber with extended inlet/outlet.

5. Conclusions

The acoustic length correction of duct extension into a cylindrical chamber is studied by 2D axisymmetric analytical approach and FEM. The effect of chamber geometry on the acoustic length correction is examined. The differences in the acoustic length correction due to the chamber length are decreased with increase of the ratio l_v/d_v. Thus for the finite chamber with large l_v/d_v, the acoustic length correction for the semi-infinite chamber may be used as an approximation. The acoustic length correction is decreased with increase of the duct extension, and the differences in the acoustic length correction are also getting small with increase of the ratio l_e/d_p. Based on the analytical and numerical study, an approximate expression for the acoustic length correction of duct extension into a cylindrical chamber is suggested.

The transmission loss of Helmholtz resonator with neck extension and expansion chamber with extended inlet/outlet is predicted by the present corrected 1D analytical approach. The predictions agree well with FEM predictions. It demonstrates that the introduction of the acoustic length correction improves the prediction accuracy of the 1D theory of silencers and resonators. The corrected 1D analytical approach may be used for the quick predictions of acoustic attenuation performance of silencers and resonators in low-frequency range.

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