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Suppression of stick-slip in drillstrings: A control approach based on modeling error compensation

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Abstract

An approach for the robust suppression of stick-slip oscillations in oil drillstrings is presented. Two control configurations are derived: a cascade control scheme, where a favorable choice of virtual input control variables is demonstrated, and a decentralized control scheme, where two control inputs are manipulated. The control approach is based on modeling error compensation techniques to provide robustness against uncertain parameters and friction terms. Numerical simulations are provided to illustrate the control performance.

1. Introduction

Complex dynamic behavior is often found in industrial oil drillstrings, which can be caused by the presence of friction, hysteresis, nonlinear damping, etc. [1–3]. Stick-slip oscillations is an example of such a complex behavior [4]. Stick-slip occurs when a section of the rotating drillstring is momentarily caught by friction against the borehole, then releases. Stick-slip can be severe enough to stop the rotation at the bit; when the friction is released, the collar rotation speeds up dramatically. This creates large centrifugal accelerations to occur. Stick-slip effects at the drill-bit result in complicated vibration behavior of the string and discontinuous rotation of the drill-bit [1]. The vibrations of the drillstring lead to fatigue and diminish the accuracy of the drilling process [5,6]. Thus, control actions are necessary in order to induce the suppression of this undesirable behavior. Moreover, since drilling is one of the most expensive operations in oil exploration and development, vibration control in the oil drilling process is required from an economical point of view.

In practice, the drilling operator typically controls the surface-controlled drilling parameters, such as the weight-on-bit, drilling fluid flow through the drill pipe, the drillstring rotational speed and the density and viscosity of the drilling fluid to optimize the drilling operations [7,8]. In particular, the only means of controlling vibration with current monitoring technology is to change either the rotary speed or the weight-on-bit [9]. Various control techniques have been devised to combat the stick-slip oscillations in drillstrings

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[5,8,10–15], including linear control [9], H_∞ control [6], optimal state feedback control [16], and nonlinear control designs [17]. In particular, Navarro-Lopez and Suarez [9] and Canudas-de-Wit, et al. [13], introduced the use of the weight-on-bit as an additional manipulable variable to suppress stick-slip oscillations of both the rotary table and bit velocities.

To deal with systems with friction, it is necessary to have a good characterization of the structure of the friction model and then to design appropriate compensation techniques [18,19]. As friction phenomena has not yet been completely understood, friction modeling is not an easy task. Indeed, uncertainty exists on most model that contain a friction component [18]. In this paper it is developed a robust and simple feedback control approach for the suppression of stick-slip oscillations at the bottom hole assembly in drillstrings. The proposed control is based on modeling error compensation techniques [20] and the application of both a recursive cascade control configuration that exploits the structure of simple drillstrings models, that accounts for stick-slip oscillations, and a decentralized control configuration, where two inputs are manipulated. The resulting feedback control approach leads to a robust feedback control schemes that deals with uncertainties in the friction model and drillstrings parameters.

This work is organized as follows. In the next section, a simple mathematical model of stick-slip oscillations in drillstrings is presented. In Section 3, both cascade and decentralized feedback control schemes are developed. In Section 4 numerical simulations shown the closed-loop performance of our control approach. Finally, in Section 5 we close this work with some concluding remarks.

2. Simple model for stick-slip oscillations in oil drillstrings

In order to design and implement an effective control system, an oil drillstring model that accounts for stick-slip oscillations is essential for identifying the critical parameters as well as predicting the behavior of oil drillstring system under realistic situations. We consider a generic simple model of an oil drillstring which displays stick-slip oscillations, where the drillstring is considered as a torsional pendulum with two degrees of freedom [6,9,13,14].

The model is derived from Fig. 1 and can be described as a simple torsional pendulum driven by an electrical motor. The model comprises two damped inertias mechanically coupled by an elastic shaft with stiffness k and damping c. The inertia J_1 represents the inertia of the rotary table augmented with the inertias of the electric motor and transmission box in the real system. J_2 corresponds to the inertia of the drillstring plus the inertia of the bottom hole assembly. Torques T_1 and T_2 are associated with inertias J_1 and J_2, respectively. The rotary table is driven by the torque T_m coming from the transmission box which is driven by a DC electric motor. The drillstring is modeled as a linear torsional spring with stiffness k. The model equations are

$$J_1\ddot{\theta}_1 + c(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = T_m - T_1, \quad (1)$$

$$J_2\ddot{\theta}_2 - c(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2) = -T_2, \quad (2)$$

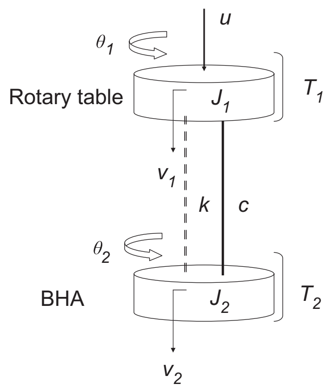

Fig. 1. Simplified model of an oilwell drillstring.

where θ_1 is the rotary table rotation angle θ_2 the bottom hole assembly rotation angle. In the above model, it is supposed that no lateral motion of the bit is present and the dynamics of the electric motor system in the rotary table is not considered.

The dynamics the fourth-order system, Eqs. (1) and (2) can be described by a third-order state-space system because its dynamics is independent of the angular positions of the rotary table and the bottom hole assembly, but depends on the difference between these two angular positions. Moreover, it is supposed that the rotary table and the bottom hole assembly are not allowed to rotate backwards, i.e., $\dot{\theta}_1 > 0$ and $\dot{\theta}_2 > 0$. Then, by defining the state $\varphi = \theta_1 - \theta_2$ and the angular velocities $v_l = \mathrm{d}\theta_1/\mathrm{d}t$ and $v_2 = \mathrm{d}\theta_2/\mathrm{d}t$, the state vector of the reduced model is given as $x = (v_2, \varphi, v_1)$. The reduced model can be written as

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = a_1(x_3 - x_1) + a_2 x_2 - a_3 T_2, \tag{3}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = x_3 - x_1, \tag{4}$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = -a_4(x_3 - x_1) - a_5 x_2 - a_6 T_1 + a_7 u, \tag{5}$$

where $u \in \mathbb{R}$ is the control input. Parameters are $a_1 = c/J_2$, $a_2 = k/J_2$, $a_3 = 1/J_2$, $a_4 = c/J_1$, $a_5 = k/J_1$, $a_6 = 1/J_1$ and $a_7 = k_m/J_2$. During the drilling process, the parameters J_2 and k, will change due to the increasing drillstring length [6]. Therefore, robustness towards variation in the drillstring parameters must be addressed. Torques T_1 and T_2 are nonlinear function that commonly are modeled as a decreasing and continuously differentiable for all $x_1 \neq 0$ and discontinuous otherwise due to the presence of the Coulomb friction. It is not hard to see that several published models of drillstrings that accounts for stick-slip oscillations can be described by Eqs. (3)–(5), for instance, the models of Navarro-Lopez and Suarez [9] (Case study 1), Serrarens et al. [6] (Case study 2) and Mihajlovic [14] (Case study 3).

2.1. Case study 1

The model of Navarro-Lopez and Suarez [9] is described by Eqs. (3)–(5) with Friction torques T_1 and T_2 given by

$$\begin{aligned} T_1 &= c_1 x_3 + T_{c_1}\,\mathrm{sign}(x_3), \\ T_2 &= c_2 x_1 + T_{f_2}, \end{aligned} \tag{6}$$

where c_1 and c_2 are the damping viscous coefficients associated with the rotary top system and the bit, respectively, and T_{c_1} is the Coulomb friction torque associated with J_1. Expression for T_{f_2} is a variation of the Stribeck friction with a dry friction model as proposed in Ref. [9],

$$\begin{aligned} T_{f_2} &= \begin{cases} T_{\mathrm{eb}} & \text{if } |x_1| < D_v \text{ and } |T_{\mathrm{eb}}| \leqslant T_{\mathrm{sb}}, \\ T_{\mathrm{sb}}\,\mathrm{sign}(T_{\mathrm{eb}}) & \text{if } |x_1| < D_v \text{ and } |T_{\mathrm{eb}}| > T_{\mathrm{sb}}, \\ R_b W_{\mathrm{ob}} \mu_b\,\mathrm{sign}(x_1) & \text{if } |x_1| > D_v, \end{cases} \\ \mu_b &= \mu_{\mathrm{cb}} + (\mu_{\mathrm{sb}} - \mu_{\mathrm{cb}})\exp(-\gamma_b |x_1|), \\ T_{\mathrm{eb}} &= c(x_3 - x_1) + k x_2 - c_b x_1, \end{aligned} \tag{7}$$

where μ_{sb} and μ_{cb} are the static and Coulomb friction coefficients associated with inertia J_1 and J_2, respectively, γ_b is a positive constant, T_{sb} is the static friction torque associated with J_1, R_b is the bit radius, W_{ob} is the weight-on-bit, which is directly related with the hook-on-load applied at the surface, T_{eb} is the applied external torque that must overcome the static friction torque T_{sb} to make the bit move, and $D_v > 0$ specifies a small neighborhood of $x_1 = 0$. Model formulation and corresponding assumptions are given in Ref. [9]. Fig. 2(a) shows the occurrence of stick-slip oscillations for model given by Eqs. (3)–(7) with the following parameter values: $J_1 = 0.0318\,\mathrm{kg\,m^2}$, $J_2 = 0.518\,\mathrm{kg\,m^2}$, $c_1 = 0.18\,\mathrm{Nm\,s/rad}$, $c_2 = 0.03\,\mathrm{Nm\,s/rad}$,

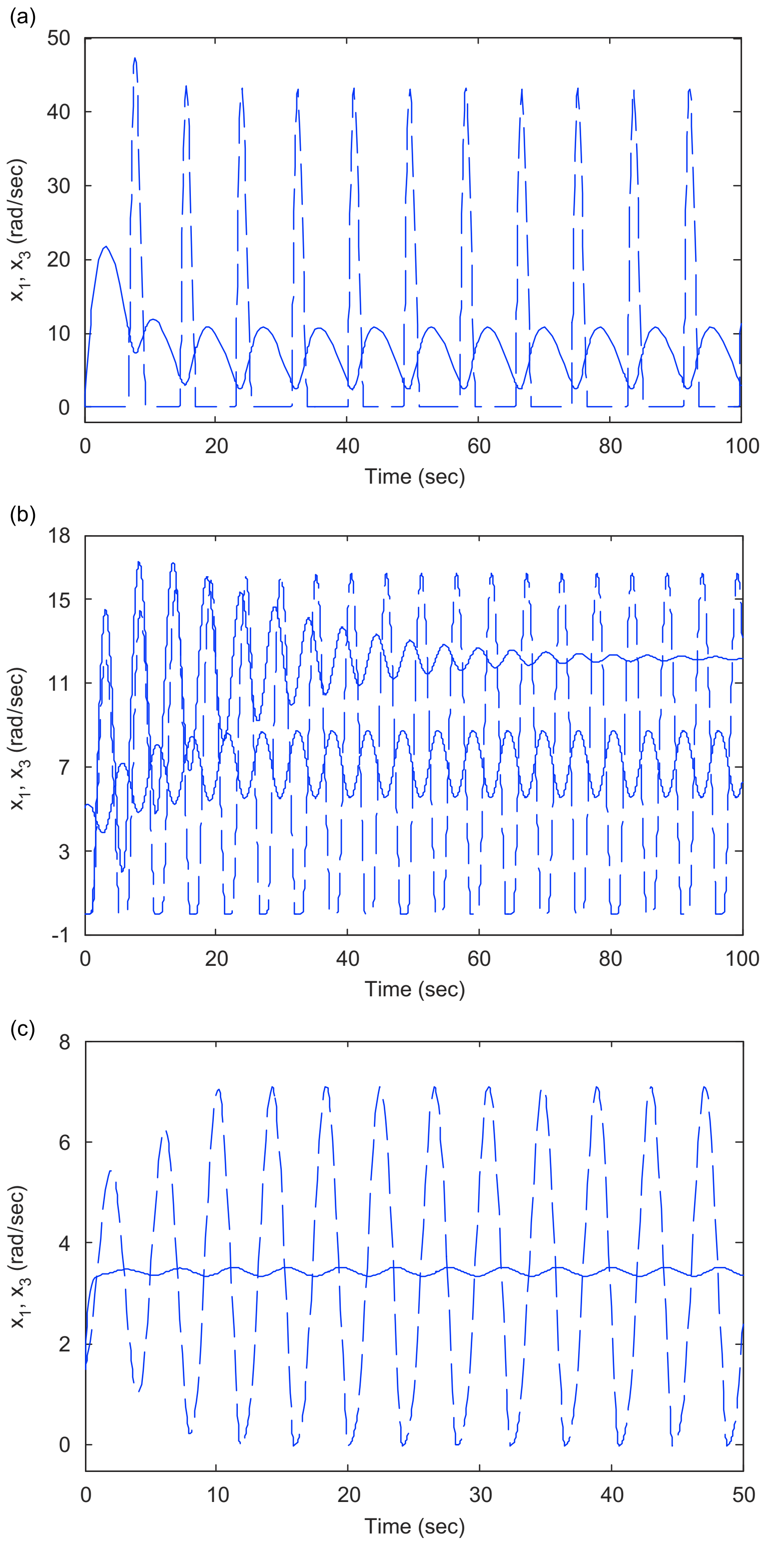

Fig. 2. Angular velocities at the rotary table (solid line) and the bottom hole assembly (dashed line): (a) case study 1; (b) case study 2, for $u = 6000\,\text{Nm}$ (dashed line) and $u = 8000\,\text{Nm}$ (dotted line); (c) case study 3.

$c = 0.0001\,\text{Nm s/rad}$, $k = 0.073\,\text{Nm/rad}$, $T_{cr} = 0.5\,\text{Nm}$, $T_{\text{sb}} = 8\,\text{Nm}$, $\gamma_b = 1.0$, $R_b = 0.1\,\text{m}$, $W_{\text{ob}} = 100\,\text{N}$, $\mu_{\text{sb}} = 0.8$, $\mu_{\text{cb}} = 0.5$, $D_v = 10^6$, $k_m = 1.0$ and initial conditions $x_1(0) = x_3(0) = 0\,\text{rad/s}$, and $x_2(0) = 0.0\,\text{rad}$.

2.2. Case study 2

Serreans et al. [6] proposed a model that models stick-slip oscillations in drillstrings. The proposed model was demonstrated to be quite realistic with respect to stick-slip vibrations, which were effectively eliminated through an H_∞ control scheme [6]. The model of Serreans et al. [6] can be represented with Eqs. (3)–(5). In this case the damping c is separated in two lumped dampings c_1 and c_2 associated with inertia J_1 and J_2, respectively, $T_1 = 0$, and T_2 is given by

$$T_2 = \frac{T_s - T_c}{1 + \delta|x_1|}\,\text{sign}(x_1) + T_c, \tag{8}$$

where δ is a positive parameter. Fig. 2(b) presents numerical simulations for two different values of the applied torque u, $u = 6000\,\text{Nm}$ (dashed line) and $u = 8000\,\text{Nm}$ (dotted line), with the following parameter values [6]: $J_1 = 374\,\text{kg m}^2$, $J_2 = 2122\,\text{kg m}^2$, $c_1 = 0 - 50\,\text{Nm s/rad}$, $c_2 = 425\,\text{Nm s/rad}$, $k = 473\,\text{Nm/rad}$, $T_s = 5000\,\text{Nm}$, $T_c = 2000\,\text{Nm}$, $\delta = 1.0$, $k_m = 1.0$ and initial conditions $x_1(0) = 0\,\text{rad/s}$, $x_2 = 5\,\text{rad}$, $x_3 = 5\,\text{rad/s}$. For $u = 6000\,\text{Nm} > T_s$, stick-slip oscillations are present, which displays alternate almost-zero bit velocities with sudden high velocities. For $u \gg T_s$ stick-slip oscillations are not longer displayed and the bit displays underdamped oscillations.

2.3. Case study 3

In Mihajlovic [14] an experimental drill-string setup is described and modeled as an electromechanical system. The model of the mechanical part of the setup can be described by Eqs. (3)–(5) with $c = 0$, since the torsional damping of the drillstring can be neglected under the experimental conditions, and the friction torques modeled as

$$T_1 = \begin{cases} T_u\,\text{sign}(x_3) & \text{for} \quad x_3 \neq 0, \\ [-T_u(0), T_u(0)) & \text{for} \quad x_3 = 0, \end{cases} \tag{9}$$

$$T_2 = \begin{cases} T_l\,\text{sign}(x_1) & \text{for} \quad x_1 \neq 0, \\ [-T_l(0), T_l(0)) & \text{for} \quad x_1 = 0, \end{cases} \tag{10}$$

$$T_u = T_{su} + \Delta T_{su}\,\text{sign}(x_3) + b_u x_3 + \Delta b_u x_3,$$

$$T_l = T_{sl} + T_a\left(1 - \frac{2}{1 + \exp(\beta_1|x_1|)}\right) + T_b\left(1 - \frac{2}{1 + \exp(\beta_2|x_1|)}\right) + b_l|x_1|,$$

where T_u and T_l represent friction torques present at the rotary table and the bottom hole assembly for nonzero angular velocities, and T_{su}, ΔT_{su}, b_u, b_l, Δb_u, T_{sl}, T_a, T_b, β_1 and β_2 are parameters of the friction model. Fig. 2(c) presents a numerical simulation of model given by Eqs. (3)–(5) with $c = 0$, subject to Eqs. (9)–(10) and parameter values: $J_1 = 0.4765\,\text{kg m}^2/\text{rad}$, $J_2 = 0.0326\,\text{kg m}^2/\text{rad}$, $k = 0.078\,\text{Nm/rad}$, $T_{su} = 0.3212\,\text{Nm}$, $\Delta T_{su} = 0.0095\,\text{Nm}$, $b_u = 1.9833\,\text{kg m}^2/\text{rad s}$, $b_l = 0.0042\,\text{kg m}^2/\text{rad s}$, $\Delta b_u = -0.0167\,\text{kg m}^2/\text{rad s}$, $T_{sl} = 0.0094\,\text{Nm}$, $T_a = 0.0826\,\text{Nm}$, $T_b = -0.291\,\text{Nm}$, $\beta_1 = 6.3598\,\text{s/rad}$, $\beta_2 = 0.0768\,\text{s/rad}$, $k_m = 3.5693\,\text{Nm/rad}$, $u = 2.0\,\text{V}$, and initial conditions $x_1(0) = x_3(0) = 1.5\,\text{rad/s}$, $x_2(0) = 1.5\,\text{rad}$.

2.4. The control problem

The control objective is to suppress the stick-slip oscillations at the bottom hole assembly displayed by system (3)–(5) about a given reference, i.e. $x_1 \to x_{1,\text{ref}}$ under the following assumptions:

A1. The full states are measured.
A2. Torques T_1 and T_2 are unknown and parameters a_j, $1 \leqslant j \leqslant 7$, are uncertain.

The following comments are in order:

- Different friction models are used in case studies 1–3. In case study 1, the bit–rock interaction is considered as a variation of the Stribeck friction with a dry friction model. A dry friction torque plus a viscous damping torque are also considered at the rotary table. In case study 2, friction model is composed essentially of a Coulomb friction model plus a Stribeck-type friction model with a decaying exponential law. For case study 3, a discontinuous static friction model is considered. Such a friction model is called a ''humped friction model'', where positive damping is present for very small angular velocities. At the upper disk, since the Stribeck effect is not present in the friction torque, the friction torque is modeled as the Coulomb friction with the viscous friction.
- Dynamical and design problems in drillstrings are analyzed in industry using large finite-element models, which give quantitative information and can help to give practical recommendations to circumvent drilling problems [4,7,11]. The finite-element models are however so complex (nonlinear large displacement, finite rotation, many degrees of freedom) that it is very difficult to obtain insight why certain vibrational phenomena occur. However, in practice, control and optimization techniques tend to be based on simple models. Indeed, for control systems design purposes, both low dimensional and less complex models can provide (to some degree) qualitative insight on the dominant complex phenomenon occurring in drillstrings.
- Assumption $A1$ is realistic. Indeed, typically, the information provided to the operator during drilling includes: (i) borehole pressure and temperature, (ii) drilling parameters, such as weight-on-bit, rotational speed of the drilling bit and/or the drillstring, and the drilling fluid flow rate [7,8]. Other possible data are about the bottom hole assembly parameters such as torque, bit bounce and whirl, etc. The bit speed measurement requires downhole equipment and may be the most challenging task. It is becoming a common practice, however, to use an instrumented bit, which makes this measurement possible [8]. That the terms T_1 and T_2, that contains friction effects are not well known is a realistic situation for practical applications [18].

3. Feedback control approach based on modeling error compensation

In this section, two control schemes are used to control stick-slip oscillations in drillstrings. The first control scheme is based on a cascade control configuration, where a single control input u, related to the electrical properties of the motor and the rotary system, and consequently, the torque supplied by the motor at the surface, is employed. In the second control scheme, a decentralized control configuration is designed, where the weight-on-bit is used a second control input, as suggested in the field [10] and literature [9,13]. As was mentioned above, since the drillstring model (3)–(5) has several terms that are in general uncertain or unknowns (Assumption $A2$), we will follow a robust feedback control approach based on modeling error compensation techniques to deal with such uncertain terms [20].

3.1. A cascade control scheme

We can exploit the structure of the model given by Eqs. (3)–(5) to design a recursive cascade procedure to control stick-slip oscillations at the bottom hole assembly in oil drillstrings. Cascade control is a common control configuration in process control, which can be thought of as partial state feedback [21,22]. A typical cascade control structure has two feedback controllers with the output of the primary (master) controller changing the set point of the secondary (slave) controller.

Fig. 3 shows the recursive cascade control configuration for the drillstring model given by Eqs. (3)–(5). Our cascade control configuration is based on the design of intermediate virtual control functions u_{vi}. The design is recursive because the computation of u_{vi+1} requires the computation of u_{vi} with $u_{vi+1} = u_{vn}$ for $i = n-1$. For the drillstring model, Eqs. (3)–(5), the master controller regulates the bit velocity to $x_{1,\mathrm{ref}}$ with the first virtual input $x_2 = u_{v1}$, the first (master) controller regulates the state variable to $x_{2,\mathrm{ref}} = u_{v1}$ with the second virtual input $x_3 = u_{v2}$, and the last loop regulates the rotary velocity to $x_{3,\mathrm{ref}} = u_{v2}$. In other words, the master controller provides reference values $x_{2,\mathrm{ref}}$ to the first slave controller, which in turn provides reference values $x_{3,\mathrm{ref}}$ to the last control loop, which is driven by the real control input u.

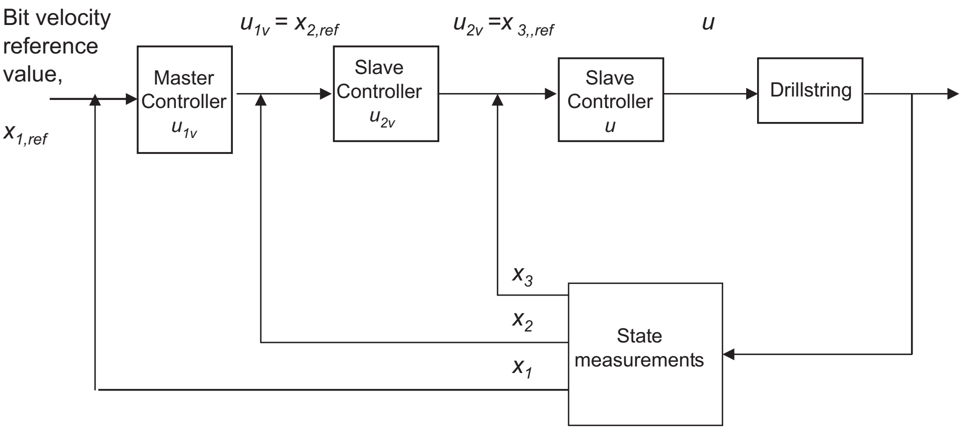

Fig. 3. Recursive cascade control scheme for the drillstring model.

In order to account for assumption $A2$ and by the fact that a rough estimate of parameters a_j can be available, we introduce modeling error functions η_i $(i = 1, 2)$ associated to both completely unknown torque functions T_1 and T_2 and estimated parameters $\widetilde{a}_j$. Then we introduce the modeling error functions as follows:

$$\eta_1 = [a_1 - \widetilde{a}_1](x_3 - x_1) - a_3 T_2 + [a_2 - \widetilde{a}_2]u_{v1}, \tag{11}$$

$$\eta_2 = [\widetilde{a}_4 - a_4](x_3 - x_1) + [\widetilde{a}_5 - a_5]x_2 - a_6 T_1 + [a_7 - \widetilde{a}_7]u_{v2}, \tag{12}$$

where $u_{v1} = x_2$ and $u_{v2} = x_3$ in the first and second states are the first and second virtual control inputs, respectively. System given by Eqs. (3)–(5) can be written as

$$\frac{dx_1}{dt} = \widetilde{a}_1(x_3 - x_1) + \eta_1 + \widetilde{a}_2 u_{v1}, \tag{13}$$

$$\frac{dx_2}{dt} = u_{v2} - x_1, \tag{14}$$

$$\frac{dx_3}{dt} = -\widetilde{a}_4(x_3 - x_1) - \widetilde{a}_5 x_2 + \eta_2 + \widetilde{a}_7 u. \tag{15}$$

The recursive cascade control configuration is given by stabilizing control functions for system 13–15 given by

$$u_{v1} = -\frac{1}{\widetilde{a}_2}[\widetilde{a}_1(x_3 - x_1) + \eta_1 + \omega_{c1}(x_1 - x_{1,\mathrm{ref}})], \tag{16}$$

$$u_{v2} = [x_1 - \omega_{c2}(x_2 - u_1)], \tag{17}$$

$$u = \frac{1}{\widetilde{a}_6}[\widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \eta_2 - \omega_{c3}(x_3 - u_2)], \tag{18}$$

where ω_{ci} are control design parameters. Under control actions (16)–(18) the closed-loop systems is given by,

$$\frac{dx_1}{dt} = -\omega_{c1}(x_1 - x_{1,\mathrm{ref}}). \tag{19}$$

It is noted that the dynamics (19) is stable and $x_1 \to x_{1,\mathrm{ref}}$ asymptotically with a convergence rate of the order of ω_{c1}.

Since the modeling error functions η_1 and η_2 contain uncertain terms, the following observer structure is introduced in order to use approximate modeling error functions:

$$\frac{d\widetilde{\eta}}{dt} = \omega_e(\eta - \widetilde{\eta}), \tag{20}$$

where $\omega_e > 0$ is an estimation frequency. From Eqs. (13) and (15) we have that

$$\eta_1 = \frac{dx_1}{dt} - \widetilde{a}_1(x_3 - x_1) - \widetilde{a}_2 u_{v1}, \tag{21}$$

$$\eta_2 = \frac{dx_3}{dt} + \widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{a}_7 u, \tag{22}$$

which allows to represent the modeling error functions in terms of measured signals, their derivatives and rough available estimates of parameters $\widetilde{a}_j$, of the uncertain parameters a_j. Then from Eq. (21) and Eq. (22), Eq. (20) can be rewritten as

$$\frac{d\widetilde{\eta}_1}{dt} = \omega_{e1}\left(\frac{dx_1}{dt} - \widetilde{a}_1(x_3 - x_1) - \widetilde{a}_2 u_{v1} - \widetilde{\eta}_1\right), \tag{23}$$

$$\frac{d\widetilde{\eta}_2}{dt} = \omega_{e2}\left(\frac{dx_3}{dt} + \widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{a}_7 u - \widetilde{\eta}_1\right). \tag{24}$$

A realizable version of the above estimators is obtained by introducing the variables $w_1 = \omega_{e1}^{-1}\widetilde{\eta}_1 - x_1$ and $w_2 = \omega_{e2}^{-1}\widetilde{\eta}_2 - x_3$ so that we have

$$\frac{dw_1}{dt} = -\widetilde{a}_1(x_3 - x_1) - \widetilde{a}_2 u_{v1} - \widetilde{\eta}_1, \qquad \widetilde{\eta}_1 = \omega_{e1}(w_1 + x_1), \tag{25}$$

$$\frac{dw_2}{dt} = \widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{a}_7 u - \widetilde{\eta}_2, \qquad \widetilde{\eta}_2 = \omega_{e2}(w_2 + x_3). \tag{26}$$

The resulting control approach is given by the following robust feedback functions:

$$u_{v1} = -\frac{1}{\widetilde{a}_2}[\widetilde{a}_1(x_3 - x_1) + \widetilde{\eta}_1 + \omega_{c1}(x_1 - x_{1,\mathrm{ref}})], \tag{27}$$

$$u_{v2} = [x_1 - \omega_{c2}(x_2 - u_{v1})], \tag{28}$$

$$u = \frac{1}{\widetilde{a}_6}[\widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{\eta}_2 - \omega_{c3}(x_3 - u_{v2})], \tag{29}$$

and the modeling estimators (23)–(24).

In order to consider the physical restrictions in the magnitude of the virtual inputs and the applied torque u we include a saturation function given by

$$U_{\mathrm{real}} = \mathrm{Sat}(U), \tag{30}$$

where $U = [u_{v1}, u_{v2}, u]$ and

$$\mathrm{Sat}(U) = \begin{cases} U_{\min} & \text{if } U \leqslant U_{\min}, \\ U & \text{if } U_{\min} < U < U_{\max}, \\ U_{\max} & \text{if } U \geqslant U_{\max}. \end{cases}$$

Thus, the control input u is limited by $u_{\min}$ for the minimum and $u_{\max}$ for the maximum applied input inputs.

3.2. A decentralized control scheme

Decentralized feedback controller is widely used in practice and, in general, is intended for modestly interacting processes. In this case, following the ideas proposed in Refs. [13,9], we consider two single decentralized control loops, where rotary table oscillations are regulated with the control input $u_2 = u$, and the bottom hole assembly oscillations are regulated with a drilling parameter, the weight-on-bit, $u_1 = W_{\mathrm{ob}}$. In this case the modeling error functions are defined as

$$\eta_1 = [a_1 - \widetilde{a}_1](x_3 - x_1) + [a_2 - \widetilde{a}_2]x_2 + [\widetilde{a}_3\widetilde{c}_2 - a_3 c_2]x_1 + [\widetilde{a}_3\widetilde{R}_b\widetilde{\mu}_b - a_3 R_b \mu_b]\,\mathrm{sign}(x_1)u_1, \tag{31}$$

$$\eta_2 = [\widetilde{a}_4 - a_4](x_3 - x_1) + [\widetilde{a}_5 - a_5]x_2 - a_6 T_1 + [a_7 - \widetilde{a}_7]u_2, \tag{32}$$

Thus, model Eqs. (3)–(5) with Eqs. (6)–(7) and (31)–(32) can be written as

$$\frac{dx_1}{dt} = \widetilde{a}_1(x_3 - x_1) + \widetilde{a}_2 x_2 - \widetilde{a}_3 \widetilde{c}_2 x_1 + \eta_1 - \widetilde{a}_3 \widetilde{R}_b \widetilde{\mu}_b \, \mathrm{sign}(x_1) u_1, \tag{33}$$

$$\frac{dx_2}{dt} = x_3 - x_1, \tag{34}$$

$$\frac{dx_3}{dt} = -\widetilde{a}_4(x_3 - x_1) - \widetilde{a}_5 x_2 + \eta_2 + \widetilde{a}_7 u_2. \tag{35}$$

Notice that we have relaxed Assumption $A2$, from an unknown model for torque T_2 to an uncertain torque model of T_2, in order to consider the weight-on-bit as a control input. In this case, the remaining parameters of torque T_2 are included in the modeling error because they are not exactly known.

Consider the following inverse-dynamics feedback functions:

$$u_1 = \frac{1}{\widetilde{a}_3 \widetilde{R}_b \widetilde{\mu}_b \, \mathrm{sign}(x_1)} [\widetilde{a}_1(x_3 - x_1) + \widetilde{a}_2 x_2 - \widetilde{a}_3 \widetilde{c}_2 x_1 + \eta_1 + \omega_{c1}(x_1 - x_{1,\mathrm{ref}})], \tag{36}$$

$$u_2 = \frac{1}{\widetilde{a}_7} [\widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \eta_2 - \omega_{c2}(x_3 - x_{3,\mathrm{ref}})]. \tag{37}$$

If $e_1 = x_1 - x_{1,\mathrm{ref}}$ and $e_3 = x_3 - x_{3,\mathrm{ref}}$, are the tracking errors, the closed-loop equations are given as

$$\frac{de_i}{dt} = -\omega_{ci} e_i \quad \text{for } i = 1, 3.$$

That is, $e_i(t) \to 0$ $(i = 1, 3)$ exponentially with convergence rates of the order of ω_{c1}^{-1} and ω_{c3}^{-1} for e_1 and e_3, respectively. To obtain a practical controller, as in the above section, the following observers are introduced in order to use approximate modeling error functions:

$$\frac{dw_1}{dt} = -\widetilde{a}_1(x_3 - x_1) - \widetilde{a}_2 x_2 + \widetilde{a}_3 \widetilde{c}_2 x_1 + \widetilde{a}_3 \widetilde{R}_b \widetilde{\mu}_b \, \mathrm{sign}(x_1) u_1 - \widetilde{\eta}_1, \tag{38}$$

$$\frac{dw_2}{dt} = \widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{a}_7 u_2 - \widetilde{\eta}_2, \tag{39}$$

where the variables $w_1 = \omega_{e1}^{-1} \widetilde{\eta}_1 - x_1$ and $w_2 = \omega_{e2}^{-1} \widetilde{\eta}_2 - x_3$ were introduced. In this way, the following practical feedback control functions are obtained:

$$u_1 = \frac{1}{\widetilde{a}_3 \widetilde{R}_b \widetilde{\mu}_b \, \mathrm{sign}(x_1)} [\widetilde{a}_1(x_3 - x_1) + \widetilde{a}_2 x_2 - \widetilde{a}_3 \widetilde{c}_2 x_1 + \widetilde{\eta}_1 + \omega_{c1}(x_1 - x_{1,\mathrm{ref}})], \tag{40}$$

$$u_2 = \frac{1}{\widetilde{a}_7} [\widetilde{a}_4(x_3 - x_1) + \widetilde{a}_5 x_2 - \widetilde{\eta}_2 - \omega_{c2}(x_3 - x_{3,\mathrm{ref}})]. \tag{41}$$

In this way, the decentralized control configuration based on modeling error compensation techniques is composed by the feedback functions given by Eqs. (38)–(39) and the modeling error estimators given by Eqs. (40)–(41).

Remark 1. The model-based control approach has only two control design parameters, i.e., ω_c and ω_e. For individual loops, the tuning of parameters ω_c and ω_e, can be set as follows. Determine the values of $\omega_c > 0$ up to a point where a satisfactory nominal response is attained. The estimation frequency constant $\omega_e > 0$, which determines the smoothness of the modeling error and the velocity of the time-derivative estimation respectively, can be chosen as ω_e about 2 to 5 times ω_c [20]. For the cascade control configuration, the tuning can be set as follows. The master closed-loop frequency constant ω_{c1} can be chosen as the order of the dominant open loop frequency [20]. The natural frequencies of the drillstring often fall in the range excited by typical drilling speeds, between 0.5 and 10 Hz depending on the bottom hole assembly and length of the drillstring [4]. Since, the response of the slave control loops should be sufficiently faster than the response of the master controller, the following criterion should be followed to guarantee a close tracking of the reference signal provided by the master controller [22], $\omega_{c1} < \omega_{c2} < \omega_{c3}$ and corresponding $\omega_{e1} < \omega_{e2}$.

Remark 2. We have assumed that the full state is available from measurements. However, in some cases the measurement problem is complicated by the absence of the real-time measurements of drill bit speed, since the measurement equipment is expensive and it is not available. Even in the absence of such measurements, a state estimator can be designed to estimate the bit speed from the other measurements since it is observable through the other states (see, for example Ref. [12]). For instance, in the cascade control scheme, this drawback can be overcome by extending estimators (20) and (25) to provide estimates of x_1 as well. In such a case, the proposed estimator is given by

$$\frac{\mathrm{d}w_v}{\mathrm{d}t} = \widetilde{a}_1 x_3 + \widetilde{a}_2 u_{v1} + \widetilde{\eta}_1 - 2\omega_e \widetilde{v}_2, \qquad \widetilde{v}_2 = w_v + (2\omega_e - \widetilde{a}_1)\theta_2,$$
$$\frac{\mathrm{d}w_\eta}{\mathrm{d}t} = -\omega_e^2 \widetilde{v}_2, \qquad \widetilde{\eta}_1 = w_\eta + \omega_e^2 \theta_2 \tag{42}$$

where $\widetilde{v}_2$ is an estimate of x_1 and θ_2 is the bottom hole assembly rotation angle. The resulting feedback function becomes

$$u_{v1} = -\frac{1}{\widetilde{a}_2}[\widetilde{a}_1(x_3 - \widetilde{v}_2) + \widetilde{\eta}_1 + \omega_{c1}(\widetilde{v}_2 - x_{1,\mathrm{ref}})]. \tag{43}$$

Remark 3. Stability must be preserved in the context of both structured uncertainties in the parameters as well as unstructured errors in modeling [23]. Although a rigorous stability analysis is beyond the scope of this study, several numerical simulations will show that the feedback controller is able of yielding suppression of stick-slip oscillations despite significant parameter departures from parameter nominal values. A stability analysis for the proposed control configurations should parallel the steps reported in Refs. [20,22].

4. Numerical simulations

In this section, simulation results are presented for the suppression of stick-slip oscillations in drillstrings for model (3)–(5) with the robust feedback control approach described above. Control objective is the suppression of stick-slip oscillations at the bit velocity, considering set point changes and typical disturbances to the drillstring operation, namely, drilling parameter uncertainties and changes in friction torques. Moreover, a parameter mismatch between $\pm 5\% - 15\%$ and in parameters a_1–a_7 for numerical simulations below. That is, while the system is simulated with the correct parameter values given in Section 2, the control design is based on about $\pm 5 - 15\%$ deviations from such values.

4.1. Controlling stick-slip oscillations via the applied torque

Numerical simulations considering a single input u, were carried out for case studies 2 and 3. For case study 2, the control input is the applied voltage to the power amplifier of the motor. For case study 3, the control input is the applied torque to the rotary system. In both cases, the desired bit velocity is 5.0 rad/s, which is within the common operating range for oilwell drilling. The control law is turn on at $t = 50$ s.

- *Regulation of bit velocity*. Figs. 4 and 5 shows the control performance for the bit regulation for case studies 2 and 3, respectively. To illustrate the fact that an arbitrary convergence rate of the regulation of bit velocity can be prescribed, Fig. 4(a) presents the bit velocity regulation to $x_{1,\mathrm{ref}}$, for two sets of closed-loop parameters $\omega_c = [0.33\ \ 0.45\ \ 0.67]$ and $\omega_c = [0.5\ \ 1.0\ \ 2.0]$, and the corresponding values of the estimation frequencies ω_e chosen according to the tuning guidelines depicted in the section above, that is, $\omega_e = [1.0\ \ 2.25]$ and $\omega_e = [2.0\ \ 5.25]$. Fig. 4(b) presents the corresponding control input and the drillstring twist φ. As expected, x_1 converges exponentially to $x_{1,\mathrm{ref}}$ and the larger the value of ω_{c1} and ω_{e1}, the faster the convergence. In this way, up to the point where undesirable high-frequency effects (e.g., noise, rippling, etc.), increments of ω_{c1} and ω_{e1} enhance the performance of the closed-loop system [20]. Notice that the applied torque reach the saturation limits for both cases of closed-loop parameters. This is not a serious drawback since these applied torques can be driven using a high-power electric motor. Fig. 5(a) shows that x_1 converges with damped oscillations to $x_{1,\mathrm{ref}}$, with controller parameters $\omega_c = [2.0\ 2.5\ 3.25]$ and $\omega_e = [2.75\ 4.0]$. It can be seen from Fig. 5(b) that the drilling twist φ resembles the oscillatory behavior

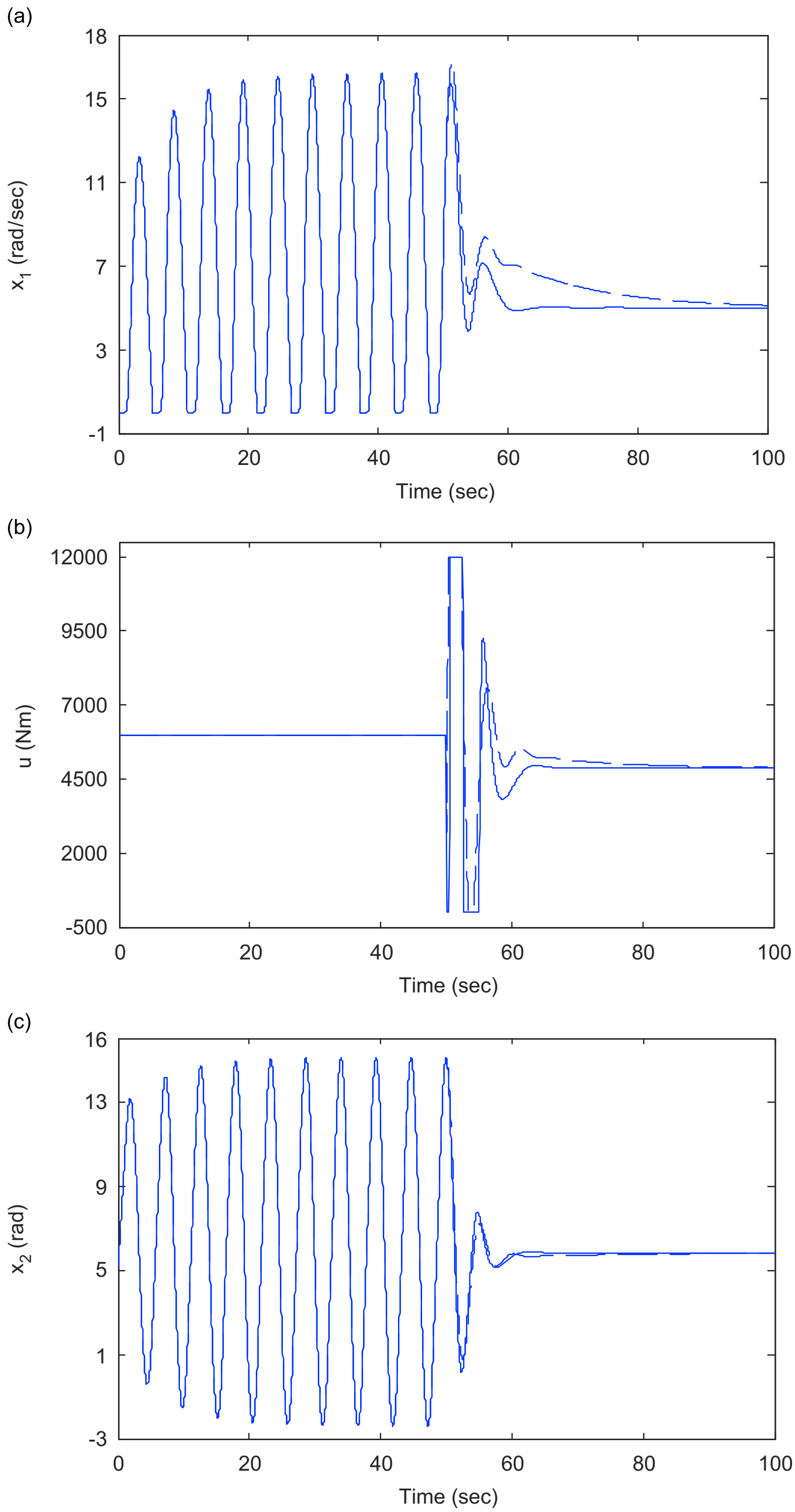

Fig. 4. Bit regulation for case study 2 and controller parameters $\omega_c = [0.33\ 0.45\ 0.67]$ and $\omega_e = [1.0\ 2.25]$ (solid line) and $\omega_c = [0.5\ 1.0\ 2.0]$ and $\omega_e = [2.0\ 5.25]$ (dashed line); (a) dynamics of bit velocity; (b) corresponding control input; (c) dynamics of the drilling twist φ.

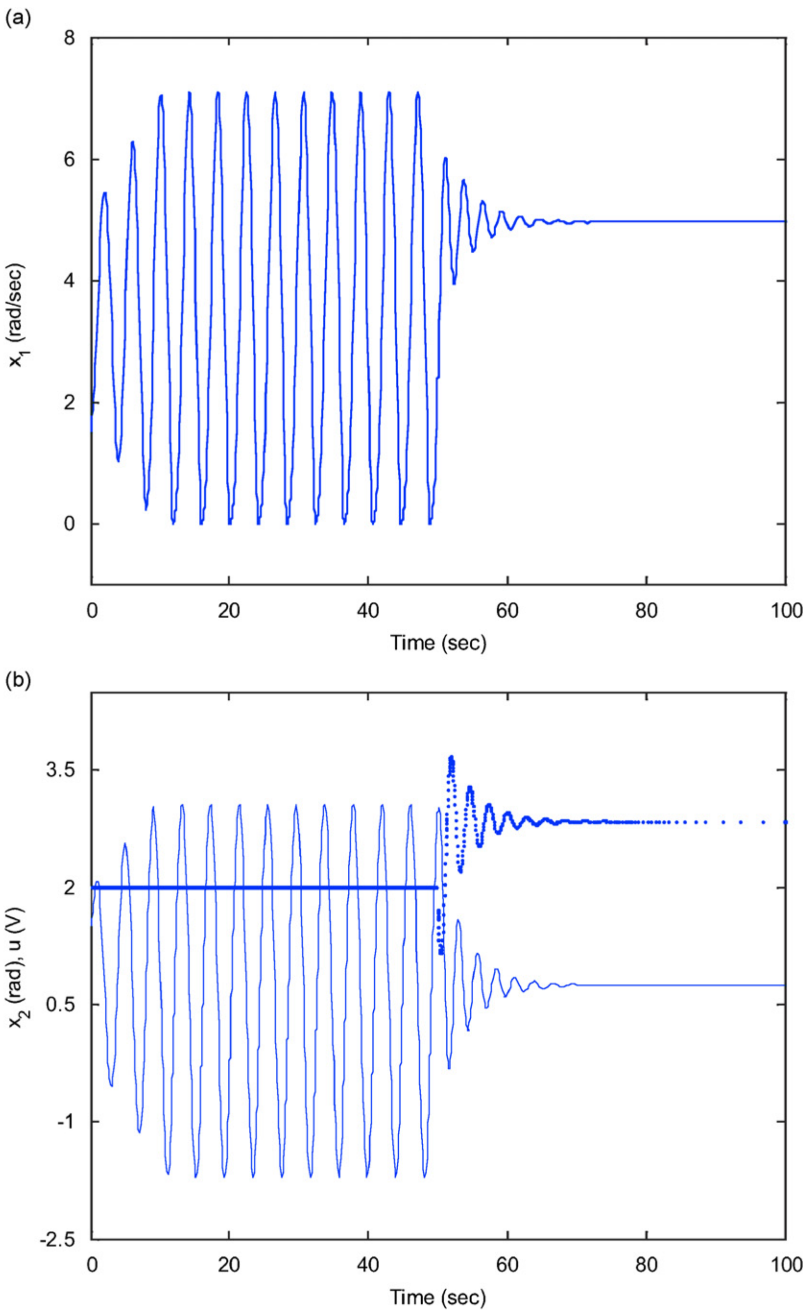

Fig. 5. Bit regulation for case study 3 and controller parameters $\omega_c = [2.0\ 2.5\ 3.25]$ and $\omega_e = [2.75\ 4.0]$: (a) dynamics of bit velocity; (b) dynamics of the corresponding control input (solid lines) and the drilling twist φ (marked points).

displayed by the control input u, which is a consequence of the cascade control structure of our control approach.

- *Set point change*. We consider both positive (+4 rad/s) and negative (−2 rad/s) set point changes, in the desired bit velocity, at $t = 100$ s, for case study 2 (Fig. 6) and case study 3 (Fig. 7), respectively. These set point changes can be required during the drilling process, for instance, to reach quickly a target depth level, and favorable or difficult rock drilling environment [7]. Figs. 6 and 7 shows that closed-loop system provides a stable transition operation with acceptable transient variations in the bit velocity. It can be seen from Fig. 6(a) that bit velocity oscillations are reduced as ω_{c1} and ω_{e1} takes larger values.

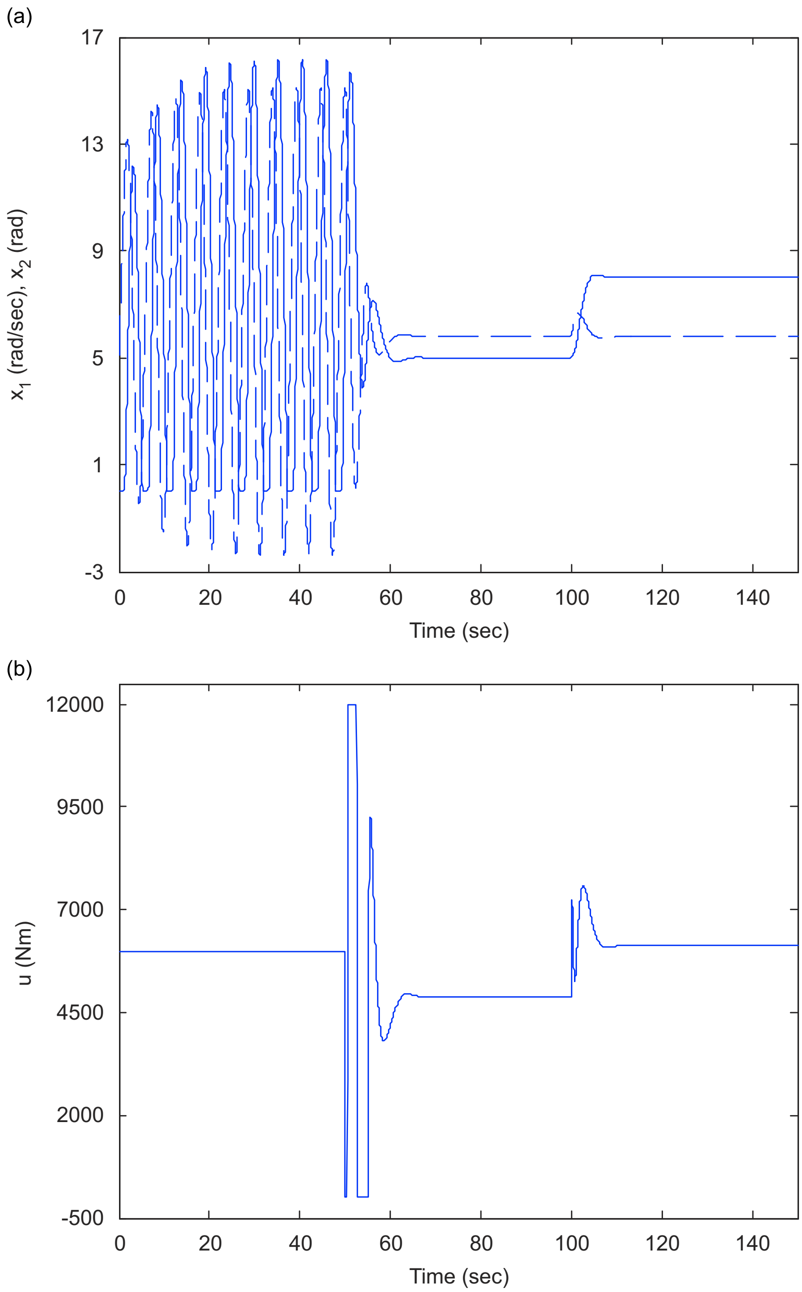

Fig. 6. Closed-loop performance for a set point change for case study 2 and controller parameters $\omega_c = [0.33\ 0.45\ 0.67]$ and $\omega_e = [1.0\ 2.25]$: (a) dynamics of bit velocity and the drilling twist φ (dashed line); (b) corresponding control input.

- *Effects of drilling and friction torque parameters.* In order to show the robustness properties of our control approach we consider the effect of changes in drilling and torque parameters in simulations below. For case study 2, at $t = 100$ s the drilling stiffness k changes to -20% of its nominal value. Fig. 8 shows that such a parameter mismatch introduces only small distortion in the bit velocity regulation. For case study 3 we consider an abrupt change in a friction torque parameter at $t = 100$ s. Namely, the viscous friction coefficient b_l, changes from 0.0042 to $0\,\mathrm{kg\,m^2/rad\,s}$. Fig. 9 shows that the control scheme is able to reject this sudden change in drilling conditions with acceptable control performance.

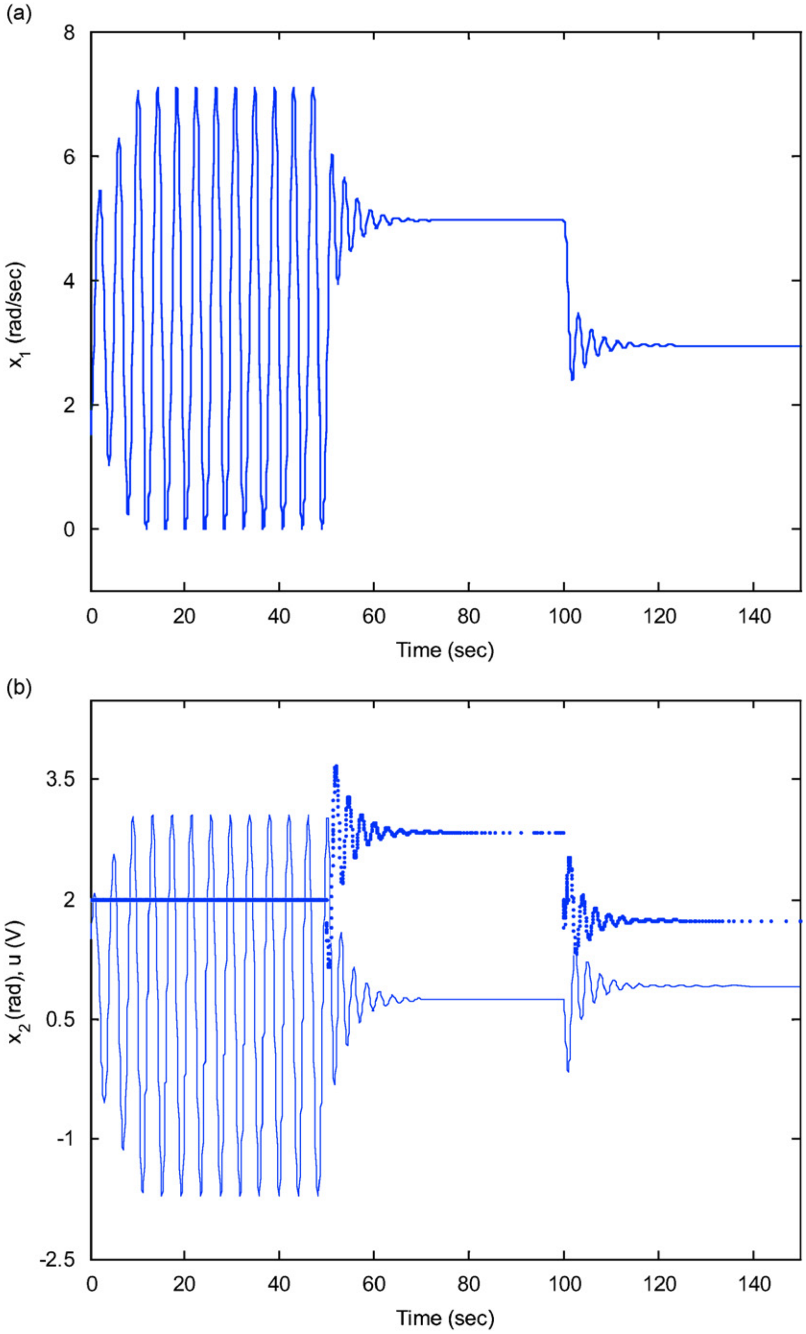

Fig. 7. Closed-loop performance for a set point change for case study 3 and controller parameter $\omega_c = [2.0\ 2.5\ 3.25]$ and $\omega_e = [2.75\ 4.0]$: (a) dynamics of bit velocity; (b) dynamics of the corresponding control input (solid line) and the drilling twist φ (marked points).

4.2. Controlling stick-slip oscillations via the applied torque and the weight-on-bit

For the case study 1, it is assumed that the applied torque u, and the weight-on-bit W_{ob}, are the variables available to regulate the drilling operation, as proposed in Refs. [9,13]. Therefore, it is possible to regulate two control targets: the rotary table and bit velocities.

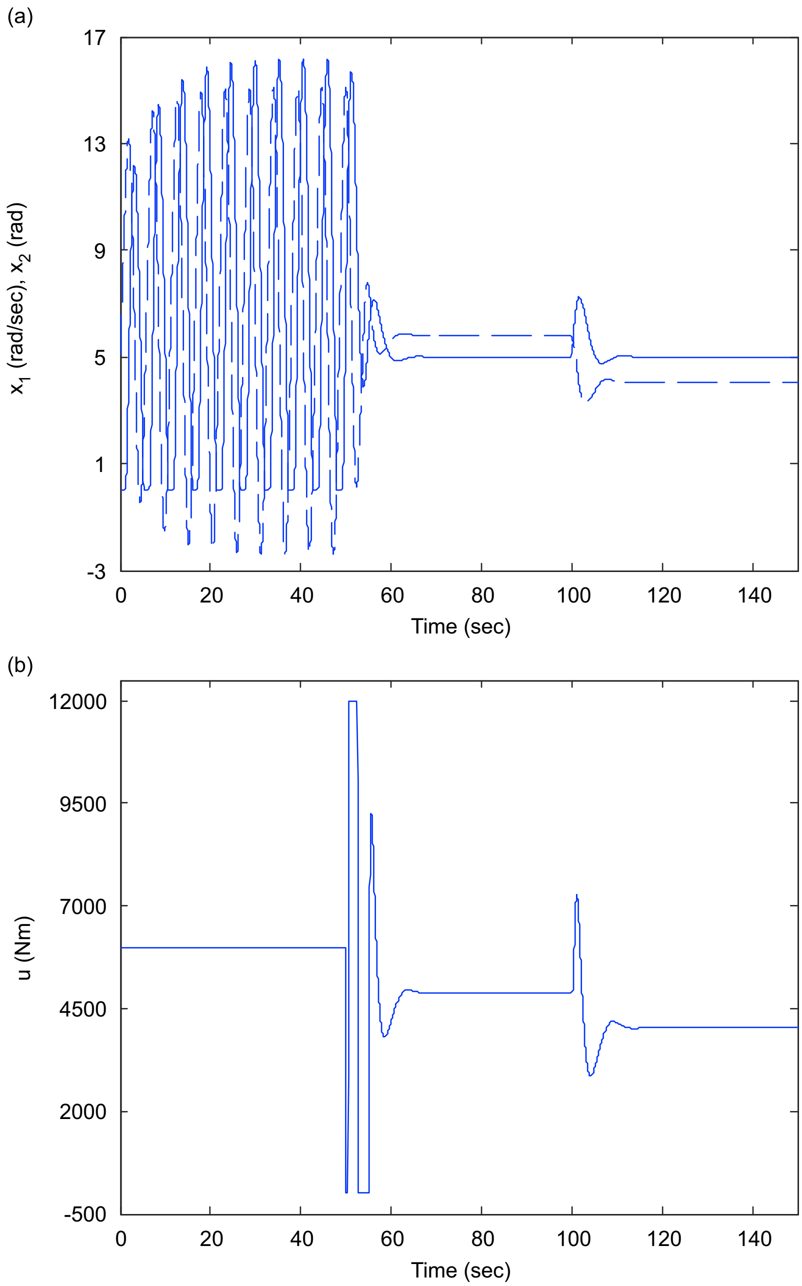

Fig. 8. Closed-loop performance for a −20% change in the drilling stiffness k and controller parameters as Fig. 6: (a) dynamics of bit velocity and the drilling twist φ (dashed line); (b) corresponding control input.

- *Regulation of rotary and bit velocities.* Fig. 10 shows the regulation of the rotary table and bit velocities with regulation set points $x_{1,\mathrm{ref}} = x_{3,\mathrm{ref}} = 10.0\,\mathrm{rad/s}$. The control law is turn on at $t = 100\,\mathrm{s}$ and the following controller parameters were used $\omega_{c1} = 0.5$, $\omega_{c2} = 0.9$, $\omega_{e1} = 1.5$ and $\omega_{e2} = 2.5$. It can be seen from Fig. 10(a) that the rotary table displays an oscillating behavior about a mean angular speed of 8.0 rad/s, while the bit is oscillating between a mean angular speed of 20.0 rad/s until the control actions are activated. This is in agreement with field measurements, which have shown that when significant torsional vibrations are present, the bit speed differs from the rotary table speed by as much as three times, as reported by

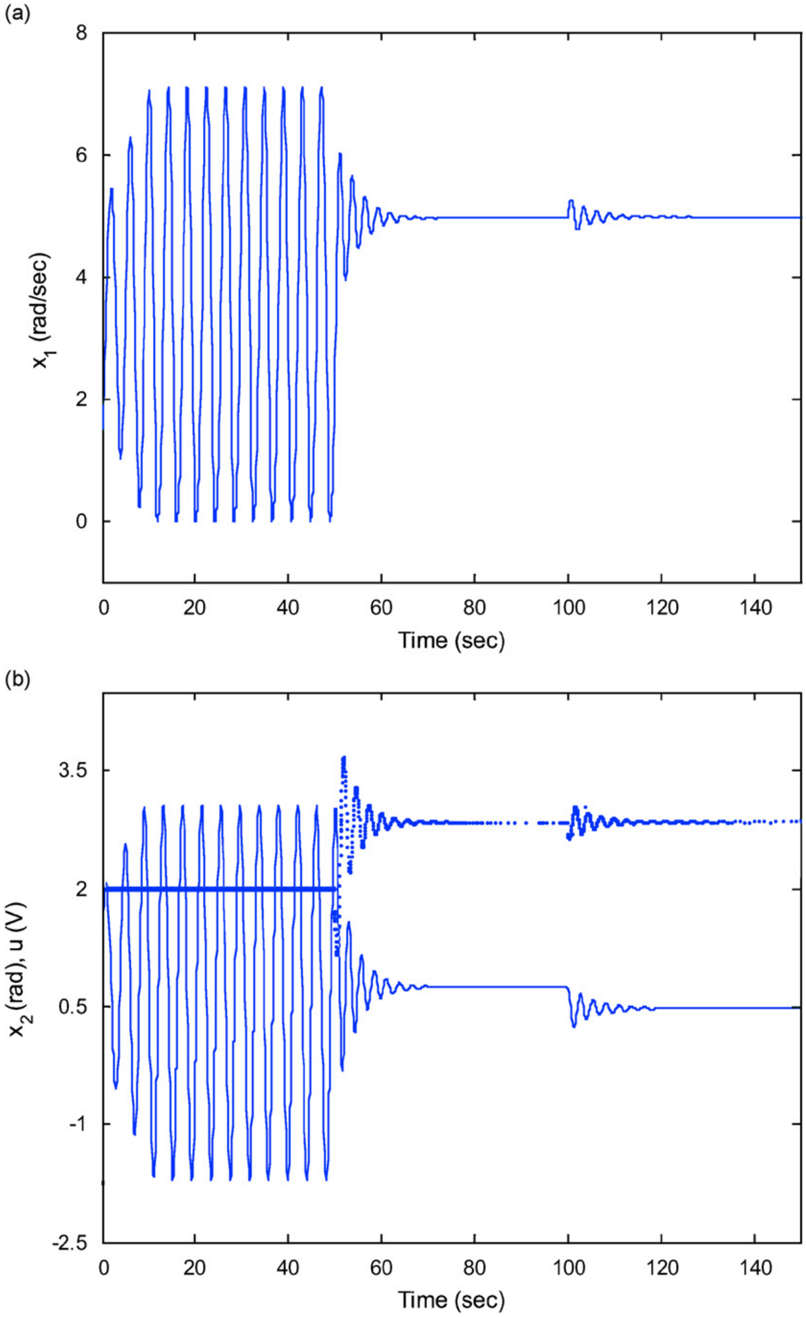

Fig. 9. Closed-loop performance for an abrupt change of b_l, from 0.0042 to $0\,\mathrm{kg\,m^2/rad\,s}$ and controller parameters as Fig. 7: (a) dynamics of bit velocity; (b) dynamics of the corresponding control input (solid line) and the drilling twist φ (marked points).

previous investigations [5]. On the other hand, Fig. 10(b) shows that during open-loop behavior there is a large fluctation in the drillstring twist around a mean value of 85.0 rad. The drilling twist may grow up to a very large values under the influence of the applied torques and the drillstring stiffness k. Since in this case the applied torques can take large values and the stiffness has a low value, the drillistring twsit takes these large values. Finally, it can be seen from Fig. 10 that after a slight peak in both control inputs, the decentralized control scheme based on modeling error compensation techniques is able to regulate both the

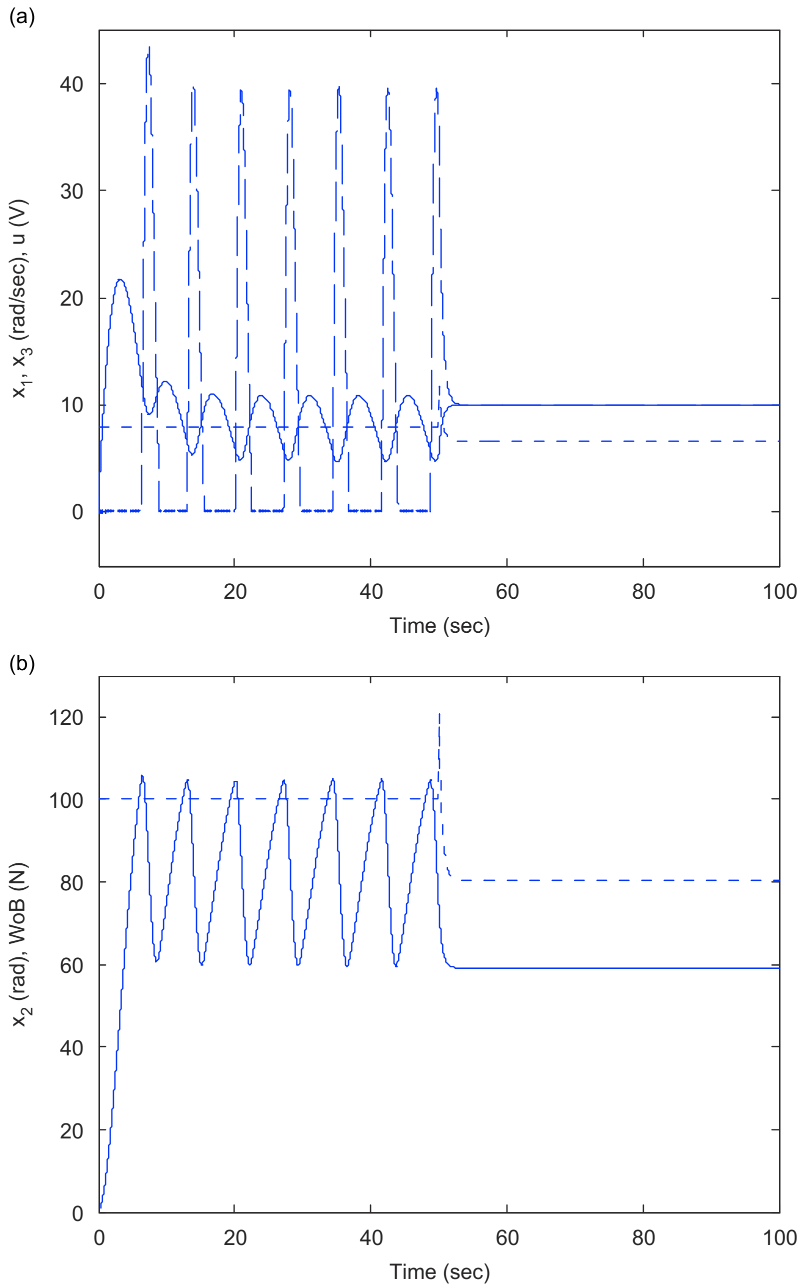

Fig. 10. Rotary table and bit regulation for case study 1 and controller parameters $\omega_{c1} = 0.5$, $\omega_{c2} = 0.9$, $\omega_{e1} = 1.5$ and $\omega_{e2} = 2.5$: (a) dynamics of rotary table (solid line) and bit velocities (dashed line), and control input u (dotted line); (b) dynamics of the drilling twist φ (solid line) and control input W_{ob} (dotted line).

rotary table and bit velocities at the desired reference. In this case, in order to eliminate the stick-slip oscillation at the bit velocity, the value of weight-on-bit need to be modified by a counterweight force [13].

- *Set point changes.* Fig. 11 shows the performance of the proposed control scheme under a change (+5 rad/s) in the desired velocities at $t = 100$ s. It can be seen from Fig. 11, that new set points are reached with acceptable dynamics of the rotary and bit velocities. In this case, both control inputs showed a slight peak before they are settled to constant values.

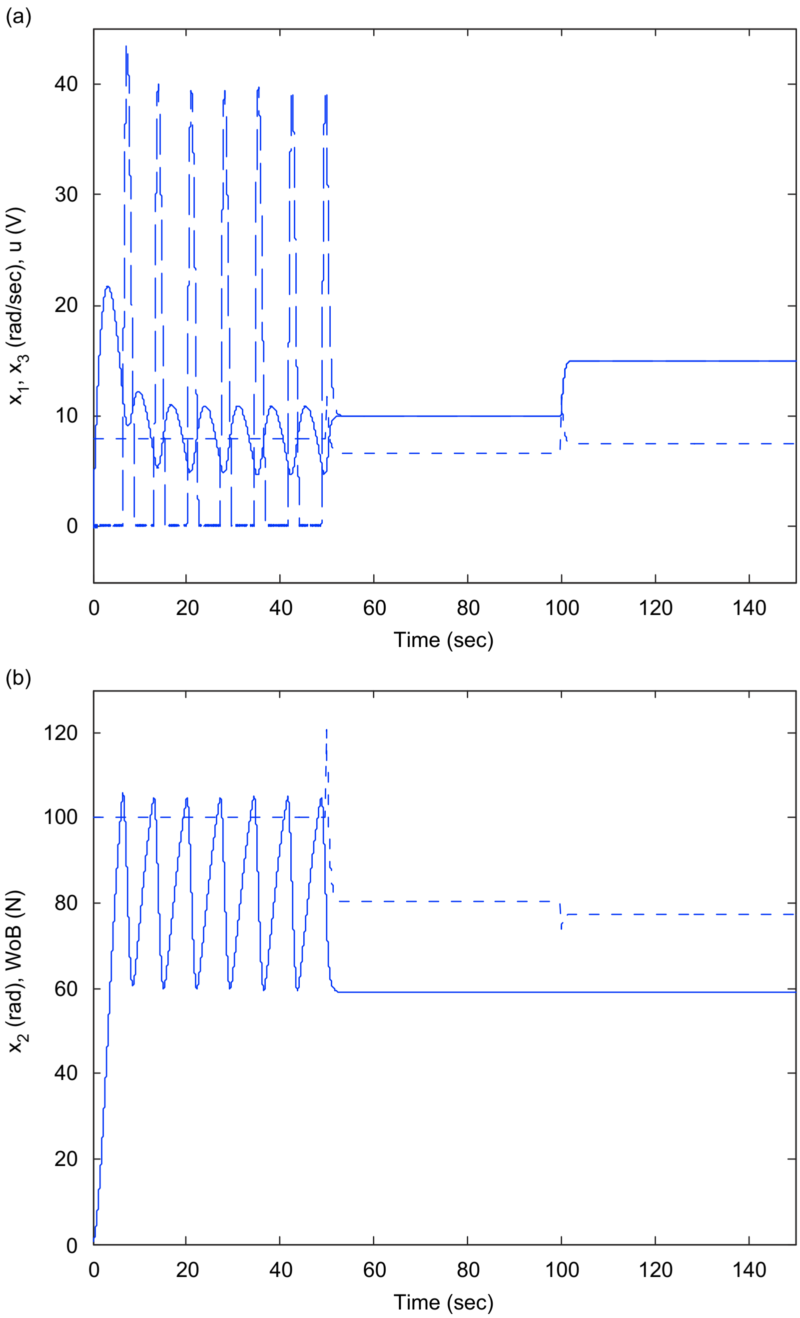

Fig. 11. Closed-loop performance for a set point change for case study 1 and controller parameter as Fig. 10: (a) dynamics of rotary table (solid line) and bit velocities (dashed line), and control input u (dotted line); (b) dynamics of the drilling twist φ (solid line) and control input W_{ob} (dotted line).

- *Effects of drilling parameters.* Finally, consider a change in the drilling stiffness $k(-20\%)$ plus a change in inertia J_1 $(+20\%)$, which can be due to changes in drillstring length. Fig. 12 presents the closed-loop performance for this parameter change. In this case, the controller is able to reject this perturbation without a significant change in the regulated velocities, with acceptable control efforts for both control inputs, u and W_{ob}.

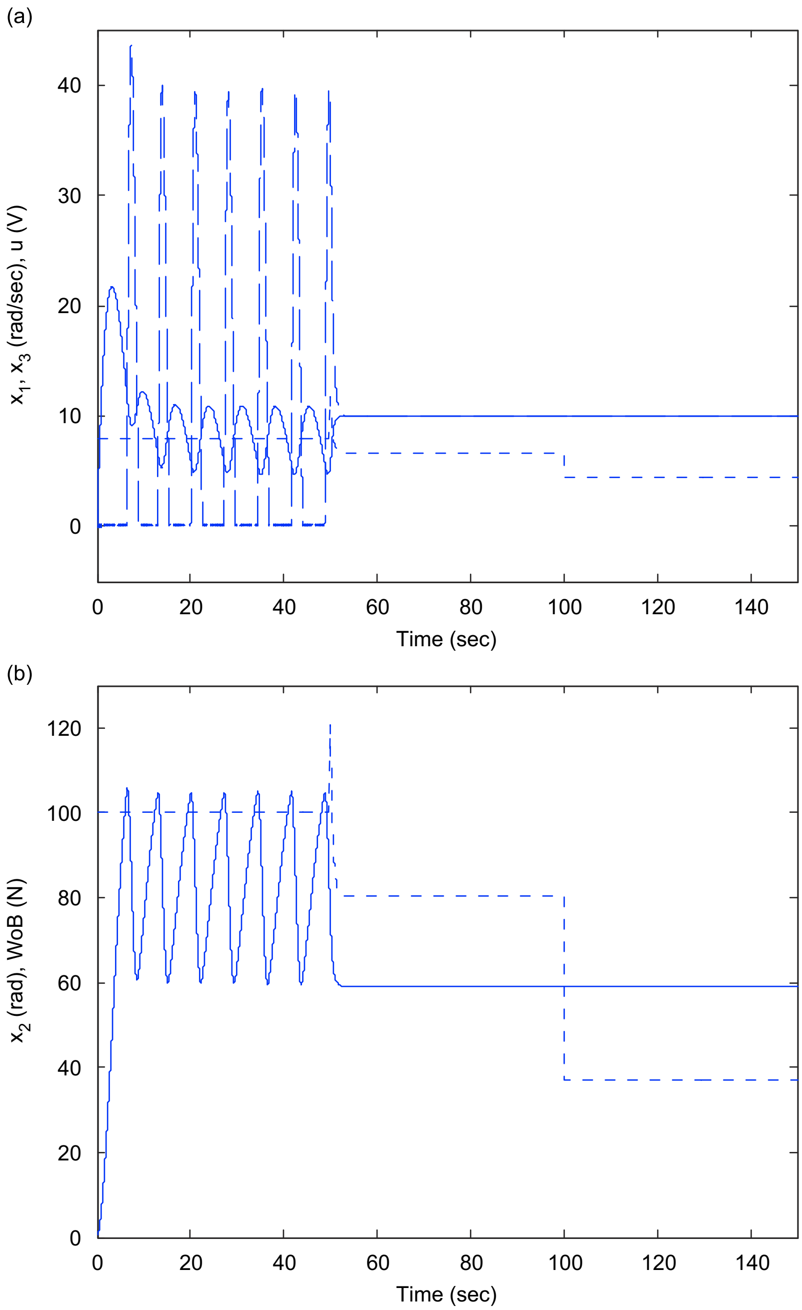

Fig. 12. Closed-loop performance for changes in the drilling stiffness $k(-20\%)$ plus inertia $J_1(+20\%)$, and controller parameters as Fig. 10: (a) dynamics of rotary table (solid line) and bit velocities (dashed line), and control input u (dotted line); (b) dynamics of the drilling twist φ (solid line) and control input W_{ob} (dotted line).

5. Conclusions

Stick-slip oscillations in oil drillstrings can lead to premature degradation of highly expensive mechanical and electronic components. Due to uncertainties and variations in environmental factors a mathematical model of the stick-slip dynamics in drillstrings present significant uncertainties. In this work, by means of modeling error compensation ideas and cascade and decentralized control schemes, we have derived a robust model-based feedback control approach that suppress stick-slip oscillations. The cascade control schemes exploits the generic model structure of oil drillstrings and maintains the bit velocity at a constant desired reference via a single control input. On the other hand, using the weight on the bit as an additional manipulable variable, a decentralized control scheme is able to suppress stick-slip oscillations in the rotary and bit velocities. The underlying idea behind the control approach is to lump terms with uncertain parameters and unknown terms in a single term, which is estimated and compensated via a suitable algorithm. The key feature of this control approach is that simple practical control design with good robustness and performance capabilities is obtained. We have shown via numerical simulations how stick-slip oscillations can be suppressed and stabilized to a desired references in presence of uncertainties in the control design and changes in model parameters. Although the control design is restricted to certain class of stick-slip models in drillstrings, the concepts presented in our work should find general applicability in the control of stick-slip oscillations in other mechanical systems.

Acknowledgments

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