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Sloshing damping in cylindrical liquid storage tanks with baffles

Abbas Maleki*, Mansour Ziyaeifar

International Institute of Earthquake Engineering and Seismology (IIEES), No. 26, Arghavan Street, Tehran 19395-3913, Iran

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Abstract

The potential of baffles in increasing the hydrodynamic damping of sloshing in circular-cylindrical storage tanks is investigated in this study. Based on the widespread use of baffles in moving liquid containers especially in space vehicles, the ability of baffles in reducing the sloshing effects in storage tanks that are especially broader than fuel containers was under question. Concerning horizontal ring and vertical blade baffles, an estimation of hydrodynamic damping ratio of liquid sloshing in baffled tanks undergoing horizontal excitation has been developed analytically using Laplace's differential equation solution. This method involves the assessment of dissipated fraction of total sloshing oscillation energy, which is caused by the flow separation around the baffles. A series of experiments employing a tank model on a shake-table has been carried out to validate the theoretically predicted damping ratio. A parametric study showed that the ring baffles are more effective in reducing the sloshing oscillations.

1. Introduction

Sloshing phenomenon is one of the major concerns in design of liquid storage tanks undergoing ground excitation. Reducing the sloshing and consequent environmental damages and economic losses can result in a safer and better design. To prevent such effects, obstacles like baffles can be placed in the oscillating liquid; fluid separation around baffles results in energy dissipation and reduction in sloshing amplitude and consequent hydrodynamic loads.

Sloshing in a tank without any anti-sloshing device is damped by viscous stresses. Although in principle the stresses can be determined theoretically, most of the available results have been obtained experimentally. Two of the more complete investigations which have been done by Mikishev and Dorozhkin [1] and Stephens et al. [2], show that in a cylindrical storage tank with rational dimensions which contains a liquid whose kinetic viscosity is even 100 times more than the viscosity of water, viscous damping is less than 0.5%.

Baffles are common devices in reducing sloshing effects in moving containers. Several investigations have been carried out in this regard, especially on their application in fuel containers of space vehicles whose stability is very sensitive to uncontrolled oscillations. The effects of a baffle on liquid oscillations can be divided into changes in frequency and damping ratio of sloshing mode. In circular-cylindrical tanks the resonant frequency can be up to 15% higher than the un-baffled tank value when a horizontal ring baffle

*Corresponding author. Tel.: +98 21 22831116.
E-mail address: meleki@iiees.ac.ir (A. Maleki).

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intersects the surface (partly by apparent reduction in the free surface diameter) and up to 10% lower than the un-baffled tank value when the free surface is about one baffle width above the baffle [3–6].

The changes in damping ratio of sloshing mode can be effective in reducing the sloshing effects such as hydrodynamic pressure and sloshing amplitude especially with an excitation frequency equal or close to the resonance frequency.

Miles [7] analyzed the damping caused by a ring baffle via analogy to the drag force that a flat plate exerts on an oscillatory flow. He obtained an analytical solution for damping ratio in terms of tank dimensions and sloshing height amplitude for circular-cylindrical tanks, when the liquid height is considerably greater than the tank radius. O'Neil [8] modified Miles' expression to be expressed in terms of lateral sloshing force, and these models have been tested by several experimental investigations, e.g. Refs. [2,3,9,10]. Isaacson and Premasiri [11] estimated the hydrodynamic damping of horizontal and vertical plate baffles in a rectangular tank by assessment of the total energy in the fluid associated with sloshing motions and the average rate of energy dissipation due to flow separation around the baffles.

Typical liquid storage tanks due to design considerations normally have height to radius ratios less than 2, therefore in the present paper a hydrodynamic model has been developed to estimate damping ratio of sloshing due to baffles in circular-cylindrical tanks with any given dimensions. In this model the damping ratio of sloshing at the presence of a baffle is estimated by the ratio of the energy dissipation rate to the total sloshing oscillation energy. The energy dissipation generally can be estimated with the work done by the drag force. The drag force is exerted on liquid flow by baffles. This methodology has been implemented for two kinds of baffles; horizontal ring and vertical blade baffles.

A series of experiments has been carried out using a tank model on a shake-table at the laboratory of International Institute of Earthquake Engineering and Seismology (IIEES) in Tehran for different liquid heights and baffles and the results of experiments and theoretical estimations have been compared. Also a parametric study has been carried out for analytically developed damping ratios to assess their effectiveness in tanks with different aspect ratios and with different baffle dimensions and locations.

2. Hydrodynamic damping

For a linear system, damping ratio can be computed as the fractional part of the mechanical energy E, dissipated into heat over one cycle of the oscillation, e.g. Refs. [7,11,12]:

$$\gamma = \frac{1}{2\omega}\frac{\overline{\mathrm{d}E/\mathrm{d}t}}{E}, \tag{1}$$

where $D = \overline{\mathrm{d}E/\mathrm{d}t}$ indicates the average energy dissipation rate per one cycle of oscillation, where the overbar denotes an average over an oscillation period, t is time, ω is the oscillation frequency, E is the total energy of sloshing wave and γ is the damping ratio. The parameters used in this equation can be estimated from the closed-form solution of the horizontal excitation of a rigid circular-cylindrical tank which is described later.

2.1. Basic theory

An incompressible, irrotational and inviscid fluid flow must satisfy Laplace's equation. With the assumptions that the tank is rigid and all liquid motions remain within the linearly elastic range of response, the solution of Laplace's differential equation is expressed as sum of 'impulsive' and 'convective' components. The impulsive component satisfies the actual boundary condition along the liquid and tank interfaces and the convective component corrects the surface shape for the sloshing effect. Assuming a $rz\theta$–cylindrical coordinate system fixed at the base of the tank and $\ddot{x}(t) = \ddot{x}_0 \mathrm{e}^{\mathrm{i}\omega t}$, in which $\ddot{x}_0$ is the horizontal acceleration amplitude of the input motion, Laplace's equation and its solution for convective component can be expressed as [13]

$$\nabla^2 \phi = 0, \tag{2}$$

$$\phi_c = \left(\frac{g\eta}{\mathrm{i}\omega}\right)\frac{\cosh(\lambda_1 z/R)}{\cosh(\lambda_1 H/R)}, \tag{3}$$

$$\eta(r,\theta,t) = R\left[\sum_{j=1}^{\infty} \frac{1}{1-(\omega/\omega_j)^2} \frac{2}{\lambda_j^2-1} \frac{J_1(\lambda_j r/R)}{J_1(\lambda_j)}\right] \frac{\ddot{x}_o}{g} \mathrm{e}^{\mathrm{i}\omega t} \cos\theta, \tag{4}$$

in which R is the cylinder radius, H is the total liquid height, ϕ is a velocity potential function which is a function of the both position and time coordinates, ϕ_c is the convective velocity potential function, η is the amplitude of the sloshing height, g is the gravitational acceleration, J_1 is the Bessel function of the first kind, λ_j is the jth zero of J'_1 and ω_j is the jth circular natural frequency of the sloshing:

$$\omega_j^2 = \frac{\lambda_j g}{R} \tanh\left(\lambda_j \frac{H}{R}\right). \tag{5}$$

In the case of earthquake motion, $\ddot{x}_o \mathrm{e}^{\mathrm{i}\omega t}/(\omega/\omega_j)^2$ can be replaced by pseudo-acceleration function $A_j(t)$ and maximum sloshing height can be obtained using earthquake response spectra, $S_a(\omega_j)$.

A component of liquid velocity in the direction of n-coordinate is given by differentiating the velocity potential function with respect to n-coordinate, therefore liquid velocities in cylindrical coordinates can be written as

$$u_z = \frac{\partial \phi_c}{\partial z} = \frac{g\eta(r,0,t)}{\omega} \frac{\lambda_1}{R} \frac{\sinh(\lambda_1 z/R)}{\cosh(\lambda_1 H/R)} \cos(\theta), \tag{6}$$

$$u_\theta = \frac{1}{r}\frac{\partial \phi_c}{\partial \theta} = \frac{1}{r}\frac{g\eta(r,0,t)}{\omega} \frac{\cosh(\lambda_1 z/R)}{\cosh(\lambda_1 H/R)} \sin(\theta). \tag{7}$$

The total energy of sloshing wave is found from classical theory as [10]

$$E = \frac{1}{4}\rho g \eta^2 \left(1 - \frac{1}{\lambda_1^2}\right) \pi R^2. \tag{8}$$

At the presence of the baffles, the energy dissipation is a result of flow separation and can be estimated by work done by the drag force. The drag force for dA, a small area of the baffle can be expressed as

$$\mathrm{d}F = \frac{1}{2}\rho C_d u_n |u_n| \,\mathrm{d}A, \tag{9}$$

in which $u_n = u_n(r,\theta,z,t)$ is the velocity of the flow normal to the baffle, ρ is the mass density of the liquid and C_d is the drag coefficient. Consequently the average energy dissipation rate can be expressed as

$$D = \overline{\int u_n \,\mathrm{d}F}. \tag{10}$$

Averaging D over an oscillation period gives

$$D = \int \frac{2}{3\pi}\rho C_d U_n^3 \,\mathrm{d}A \tag{11}$$

in which U_n is the amplitude of u_n.

2.2. Horizontal ring baffle

In addition to the dimensions shown in Fig. 1, following assumptions have been made on the derivation of damping ratio:

1. The width of the ring is small compared to the tank radius.
2. The baffle submergence depth is more than the sloshing wave amplitude; local flow in neighborhood of the baffle is not affected by the free surface and the baffle remains submerged during oscillation period.

The drag coefficient C_d, which is needed to estimate the energy dissipation rate, has been correlated by Keulegan and Carpenter [14]:

$$C_d = 15(UT/w)^{-0.5}, \quad 2 \leqslant UT/w \leqslant 20, \tag{12}$$

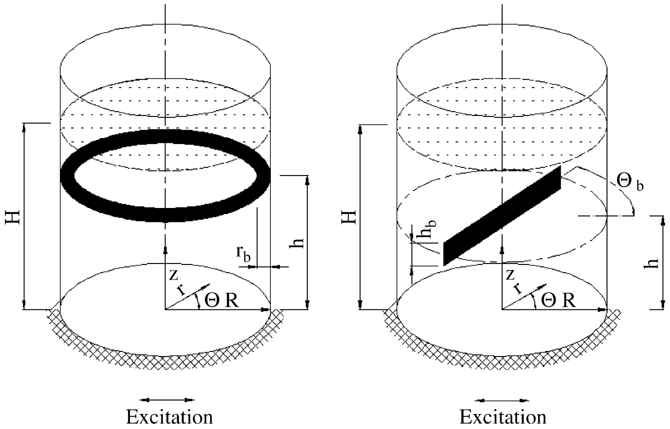

Fig. 1. Circular-cylindrical tanks with horizontal ring and vertical blade baffles.

where T is the flow oscillation period, w is the plate width and U is the flow velocity normal to the plate. The correlation is based on the experiments in which U did not vary along the plate [14]. Sarpkaya and O'Keefe [15] have carried out a series of experiments on free plates in addition to wall-bounded ones. Their studies on free plates have shown a good agreement with Keulegan and Carpenter's experiments. Moreover, it has been shown that drag forces caused by wall-bounded plates is higher than that caused by free plates. Although the ring baffles are wall bounded plates and blade baffles are free plates, due to the lack of correlation of drag coefficient for wall-bounded plates, Eq. (12) has been employed for both baffles.

Also despite the variation of velocity along a ring baffle in a tank, the circumferential variation is accounted for by basing C_d on the reasonable assumption that the effective velocity is the half of the maximum velocity, U_{hm} [12].

Substituting Eq. (12) into Eq. (11) and integrating it along the circumference with incorporating Eq. (6) for flow velocity along the ring baffle gives

$$D = 1.60\rho U_{hm}^{2.5}\sqrt{\omega}r_b^{1.5}\left(R - \frac{r_b}{2}\right). \tag{13}$$

Substituting Eqs. (8) and (13) into Eq. (1) leads to

$$\gamma_r = 4C_r\sqrt{\frac{\eta_m}{R}}\left(\frac{\sinh(1.84h/R)}{\sinh(1.84H/R)}\right)^{2.5}\tanh\left(1.84\frac{H}{R}\right), \tag{14}$$

where γ_r is the damping ratio of sloshing with a horizontal ring baffle and C_r is a function of relative baffle width:

$$C_r = \left(\frac{r_b}{R}\right)^{1.5}\left(2 - \frac{r_b}{R}\right). \tag{15}$$

Eq. (14) has been obtained for horizontal ring baffles in circular-cylindrical tanks without any limitations on tank dimensions, but while using this equation it should be considered that it has been developed using linear analysis, thus the result is valid if the sloshing amplitude remains linear.

Miles developed an equation for circular-cylindrical containers in which the liquid height is considerably greater than the tank radius. His approach has been verified by several experiments. Therefore his equation can be used as a benchmark to be compared with Eq. (14). His equation is given as [7]

$$\gamma_r = 2.83\mathrm{e}^{-4.6((H-h)/R)}C_1^{1.5}\sqrt{\frac{\eta_m}{R}}, \tag{16}$$

in which C_1 is the ratio of the baffle area to the cross-sectional area of the tank and may be expressed as

$$C_1 = \frac{r_b}{R}\left(2 - \frac{r_b}{R}\right) \approx 2\frac{r_b}{R}. \tag{17}$$

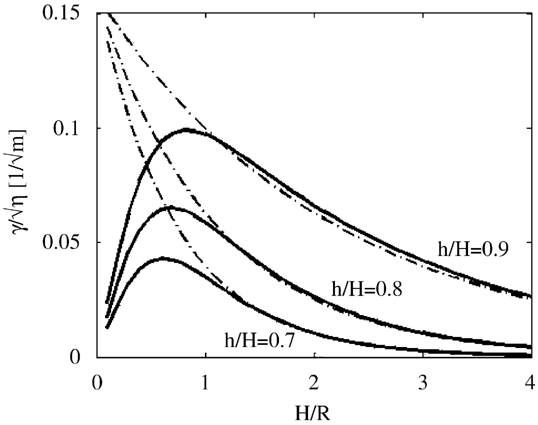

Fig. 2. Comparison between two predicted damping ratios: ——, this study (Eq. (14)); –·–·, Miles' equation (Eq. (16)).

Fig. 2 shows comparison between Eq. (14) and Miles' equation and apparently there is a conformity between these equations for $H/R>2$, but for $H/R<2$ Eq. (14) shows quantities less than Miles' equation. This difference between two equations increases with decrease of H/R and becomes significant for H/R. The source of the difference between two equations can be found in the simplifications that the derivation of Miles' equation was subject to.

Miles' equation can be obtained by relating the vertical velocity of liquid at the baffle location to horizontal oscillation of an equivalent mechanical model of sloshing [12]; it can be expressed in a more general form as below:

$$\gamma_r = \frac{40 C_1 A f_d^{2.5} \sqrt{\eta_m/R}}{3\pi^{2.5}(m_s/\rho)\Gamma^2}, \tag{18}$$

in which, m_s is the mass of the first sloshing mode which for slender tanks ($H/R>2$) can be considered as $0.456\pi R^3\rho$, Γ relates the peak oscillatory amplitude of an oscillatory mass (identical to m_s) to sloshing wave amplitude, which can be taken as 0.649 for slender tanks, and f_d is a parameter which relates liquid displacement amplitude at the baffle location to sloshing height amplitude, which for this kind of tanks ($H/R>2$) can be simply expressed as $\exp(1.84(H-h)/R)$.

In a more general form, these parameters can be re-written for circular-cylindrical tanks with any given aspect ratios, employing the solution of Laplace's equation:

$$m_s = \frac{2\pi}{\lambda_1^2 - 1}\frac{R^3}{\lambda_1}\tanh\left(\lambda_1 \frac{H}{R}\right)\rho, \tag{19}$$

$$\Gamma = \frac{\lambda_1^2 - 1}{2\lambda_1 \tanh\left(\lambda_1 (H/R)\right)}, \tag{20}$$

$$f_d = \left(\frac{\sinh(1.84h/R)}{\sinh(1.84H/R)}\right)^{2.5}. \tag{21}$$

Substituting these equations in Eq. (18) and rearranging it results in Eq. (14).

2.3. Vertical blade baffle

A vertical blade baffle can be treated the same as a ring baffle in order to estimate energy dissipation rate. To calculate energy dissipation rate, D, for a blade baffle with the dimensions shown in Fig. 1, the flow velocity normal to the baffle, u_θ, is required which was introduced before as Eq. (7). Therefore substituting Eq. (7) into Eq. (11) results in the following equation:

$$D = \frac{20}{\sqrt{2}\pi^{1.5}}\rho h_b^{1.5}\omega^3\eta_m^{2.5}\left(\frac{\sin(\theta_b)}{J_1(1.84)}\frac{\cosh(1.84h/R)}{\cosh(1.84H/R)}\right)^{2.5}\int_0^R\left(\frac{J_1(1.84r/R)}{r}\right)^{2.5}\mathrm{d}r, \tag{22}$$

where h_b is the baffle width in the vertical direction, θ_b is the angle between blade plane and excitation direction and h is the height of the centerline of the baffle. Integrating Eq. (22) and substituting the result and Eq. (8) into Eq. (1) leads to

$$\gamma_b = 2.12\left(\frac{h_b}{R}\right)^{1.5}\sqrt{\frac{\eta_m}{R}}\left(\frac{\cosh(1.84h/R)}{\sinh(1.84H/R)}\right)^{2.5}\tanh(1.84H/R)\sin^{2.5}(\theta_b), \tag{23}$$

in which γ_b is the damping coefficient for a vertical blade baffle.

3. Experiments

To assess the validity of the theoretical models, a series of experiments was carried out. According to Fig. 3, a steel tank with a diameter of 100 cm and a height of 120 cm was mounted on a 120 cm × 140 cm shake-table. Two pressure gauges and corresponding transducers were installed close to the bottom of the tank on the excitation axis. The shake-table operated in displacement control mode and the driving force was recorded by a load cell on the shaft of the table. Also two out of three used accelerometers were installed on the tank body and the other one on the table. With four installed strain gauges on the body of the tank, totally 10 channels of data were recorded.

Two ring baffles whose widths were 5 and 7.5 cm, equal to 10% and 15% of the radius of the tank and a vertical blade baffle with a width of 5 cm perpendicular to excitation axis were used. The ring baffles were supported by four bolts, which were hanged from four angles installed at the top edge of the tank. Also the blade baffle was fixed to the wall of the tank at any elevation by two bolts on each end of the baffle; by tightening these bolts, the friction between the end of the bolts and the wall of the tank provided the baffle with the required strength to resist the drag force on baffle and prevent it from moving. The baffles and their supporting setup are illustrated in Fig. 4.

To measure the sloshing damping, wave amplitude decay method was employed. In this method, the tank is oscillated at the resonant frequency until steady state is reached and then the oscillation is stopped quickly. The rate of decay of the free surface displacement is monitored and the logarithmic decay of successive wave

Fig. 3. The tank model on the shake-table.

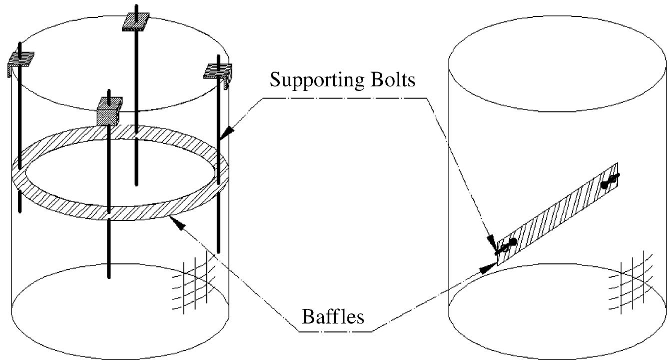

Fig. 4. Baffles and their supporting bolts.

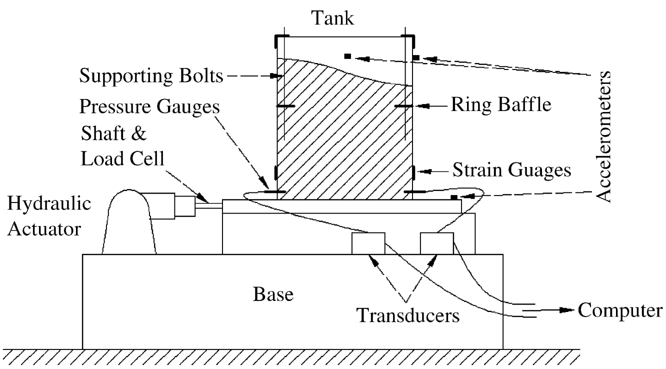

Fig. 5. Experimental apparatus.

heights is calculated from the following equation:

$$\gamma = \frac{1}{2\pi j} \ln \frac{\eta_i}{\eta_{i+j}}, \tag{24}$$

where η_i and η_{i+j} are the sloshing amplitudes in i and $i+j$ cycles of oscillation.

Experiments have been carried out for each ring baffle with water height equal to 20, 30, 50, 75 and 100 cm which are equivalent to height to radius ratios of 0.4, 0.6, 1.0, 1.5 and 2.0. In each case the baffle was located at two heights equal to 0.8 and 0.9 of the water height and the tank was excited at corresponding sloshing frequency with two different excitation displacement amplitudes of 0.4 and 1.5 mm.

Sloshing height during free-vibration phase can be obtained by the measured pressure at any point within the liquid. Having the location of pressure gauges in these experiments, sloshing height can be expressed in terms of measured pressures as

$$\eta_m = \frac{p}{\rho} \cosh(\lambda_1 H/R), \tag{25}$$

in which p is the pressure at the connection of the wall and base shells on the excitation axis. The experimental apparatus is illustrated in Fig. 5.

4. Results and discussion

As explained before, wave amplitude decay method was employed for determination of the sloshing damping. Sloshing height during free vibration of one of the experiment cases is illustrated in Fig. 6. The

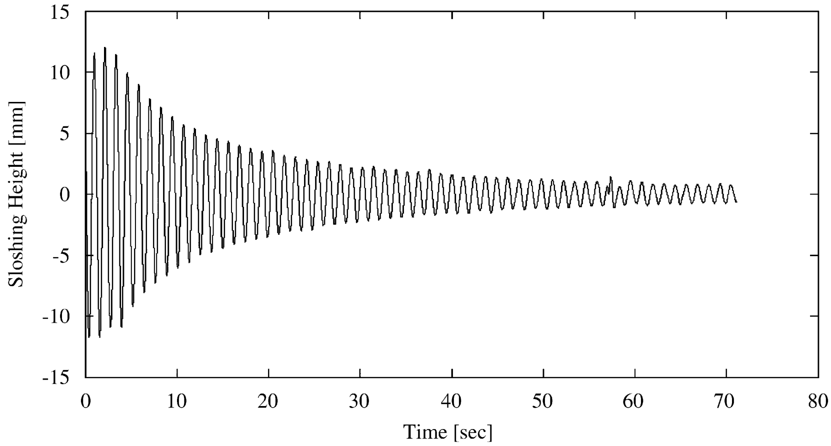

Fig. 6. Time variation of sloshing height in free vibration phase: $H = 30$ cm, $R = 50$ cm, $r_b = 7.5$ cm, $h = 24$ cm and excitation amplitude = 1.5 mm.

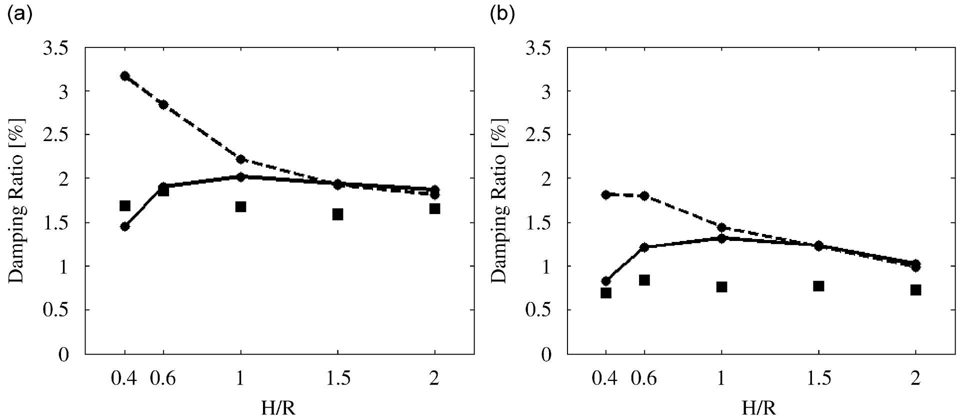

Fig. 7. Comparison of experimental and analytical damping ratios for a 7.5 cm width ring baffle located at $0.8H$ and for excitation amplitude equal to (a) 1.5 mm and (b) 0.4 mm. ——, this study (Eq. (14)); - - -, Miles' equation (Eq. (16)); ■, experiments.

amplitude of the first and seventh cycles of sloshing wave in free vibration phase in this case were measured equal to 11.4 and 5.7 mm. Experimental and analytical damping ratios calculated by Eqs. (24) and (14) were equal to 1.84% and 1.90%, respectively.

Figs. 7–10 illustrate the comparison between sloshing damping ratios measured from experiments employing the wave amplitude decay method and the damping ratios estimated by Miles' equation and Eq. (14). As it is deducible from the figures, there is an agreement between the equation developed in this paper and the experiments. Also as expected Miles' equation shows such an agreement for almost $H/R > 1$.

Fig. 11 shows the experimental results for sloshing damping due to a vertical blade baffle and estimated damping by Eq. (23) for the same sloshing heights. The agreement between two approaches can be seen.

The variation of Eqs. (14) and (23) against different parameters are plotted in Figs. 12–17. Fig. 12 shows the variation of damping ratio of sloshing mode with ring baffle against different relative liquid height (H/R) and relative baffle height (h/H) quantities and with sloshing height amplitude equal to $0.05R$ and r_b equal to $0.2R$. It can be concluded from the figure that the maximum damping ratio is obtained in height to radius ratios between 0.5 and 1 for $0 < h/H < 0.95$. The damping ratio for $h/H = 1$ has been plotted in dash-dot form because Eq. (14) is not valid in this case due to assumptions in its derivation (submergence depth is less than

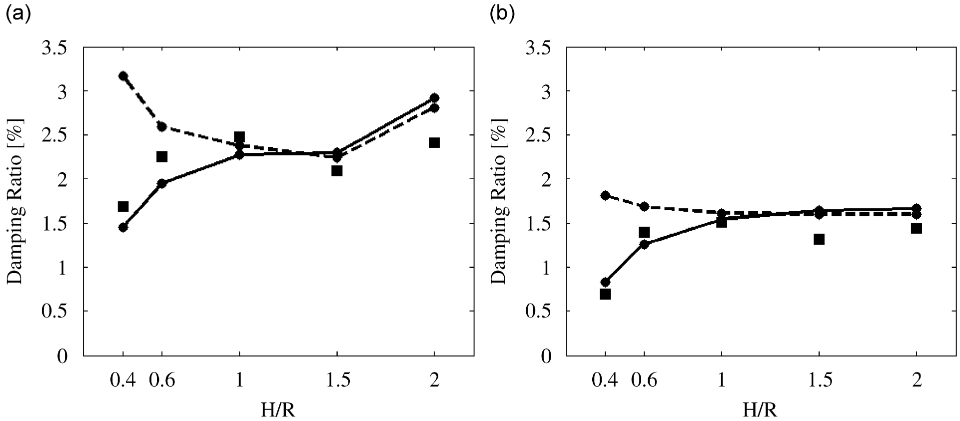

Fig. 8. Comparison of experimental and analytical damping ratios for a 7.5 cm width ring baffle located at $0.9H$ and for excitation amplitude equal to (a) 1.5 mm and (b) 0.4 mm. —◆—, this study (Eq. (14)); --◆--, Miles' equation (Eq. (16)); ■, experiments.

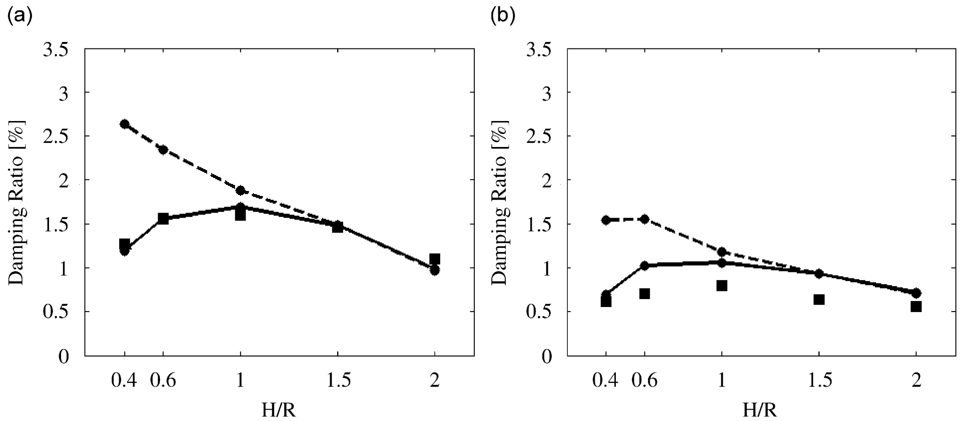

Fig. 9. Comparison of experimental and analytical damping ratios for a 5 cm width ring baffle located at $0.8H$ and for excitation amplitude equal to (a) 1.5 mm and (b) 0.4 mm. —◆—, this study (Eq. (14)); --◆--, Miles' equation (Eq. (16)); ■, experiments.

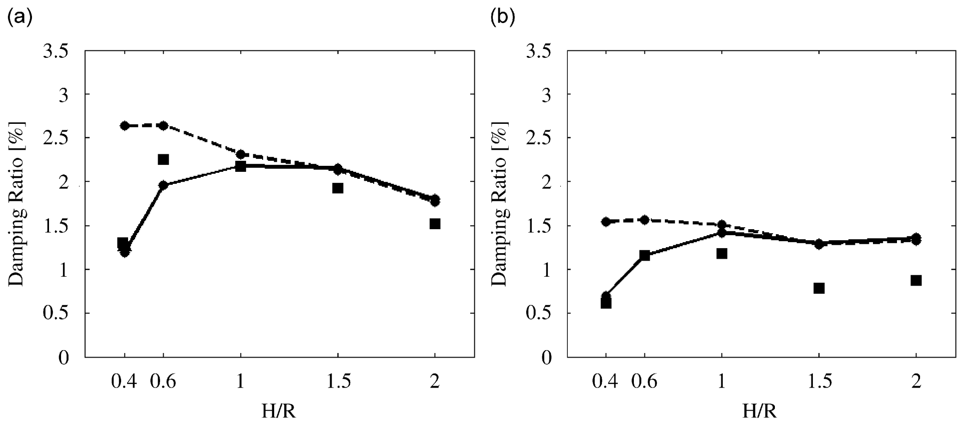

Fig. 10. Comparison of experimental and analytical damping ratios for a 5 cm width ring baffle located at $0.9H$ and for excitation amplitude equal to (a) 1.5 mm and (b) 0.4 mm. —◆—, this study (Eq. (14)); --◆--, Miles' equation (Eq. (16)); ■, experiments.

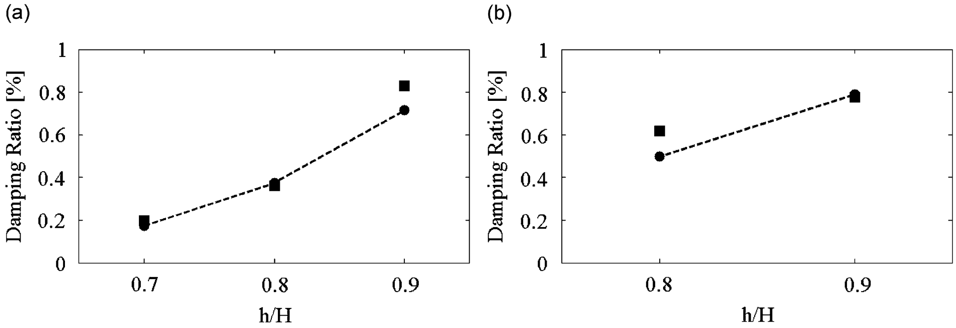

Fig. 11. Comparison of experimental and analytical damping ratios for a 5 cm width blade baffle in a tank with (a) $H/R=2$ and (b) $H/R=1.5$ and for excitation amplitude equal to 0.4 mm. —•—, this study (Eq. (23)); ■, experiments.

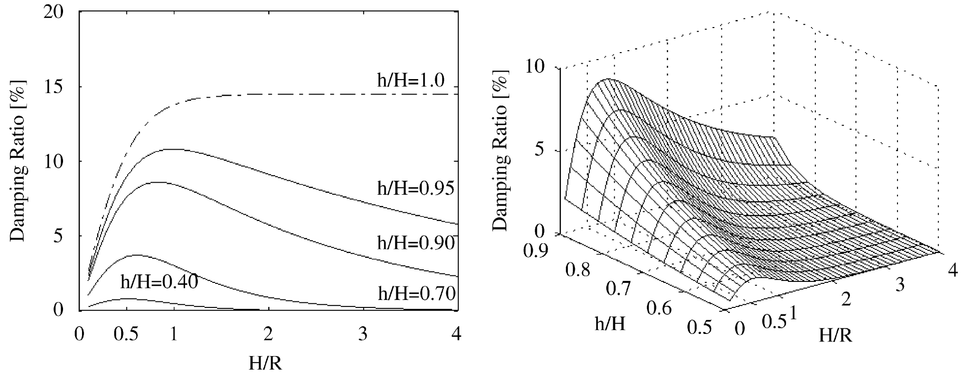

Fig. 12. Variation of the analytical damping ratio for ring baffles for $\eta/R=0.05$ and $r_b/R=0.2$.

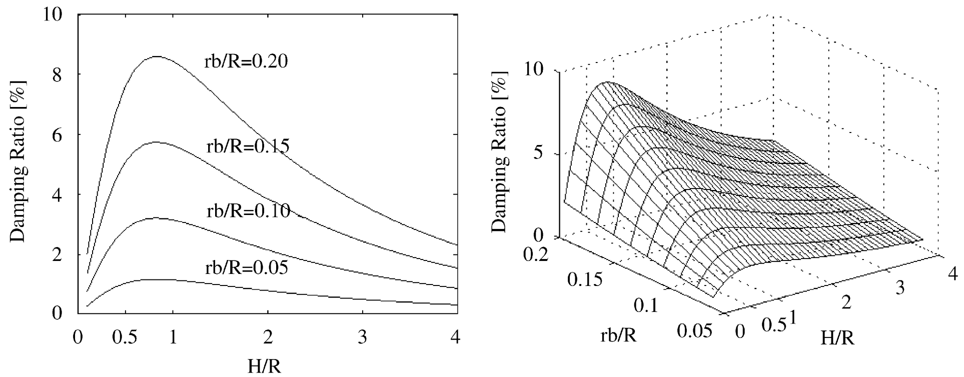

Fig. 13. Variation of the analytical damping ratio for ring baffles for $\eta/R=0.05$ and $h/H=0.9$.

sloshing amplitude). Also the figure shows that in slender tanks, a ring baffle provides damping only if it is located close to the liquid surface (not closer than the sloshing height), and this effect can be explained by the influence depth of the sloshing mode. For tanks with $H/R>2$, the sloshing mode affects just to a depth approximately equal to $2R$, thus for a given h/H and with increase of H/R, the baffle influences the sloshing mode less and less. In other words to achieve the same damping levels in different tanks and with H/R greater than 2, the baffle should be placed in a constant submergence depth (smaller than $2R$), not a constant height from the bottom of the tank.

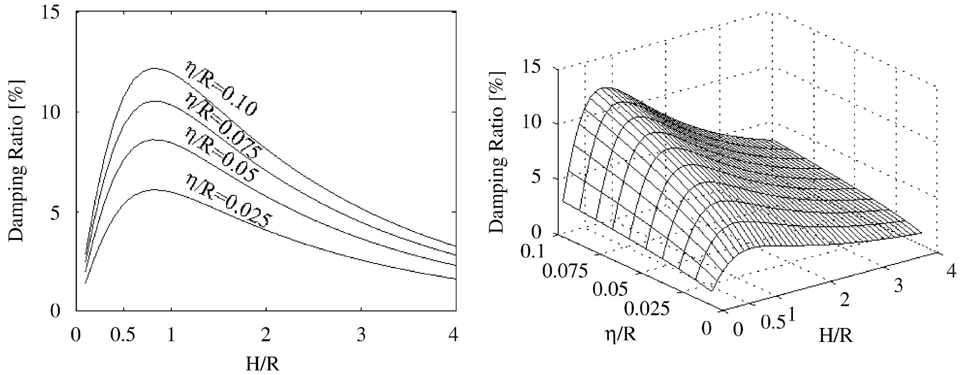

Fig. 14. Variation of the analytical damping ratio for ring baffles for $h/H = 0.9$ and $r_b/R = 0.2$.

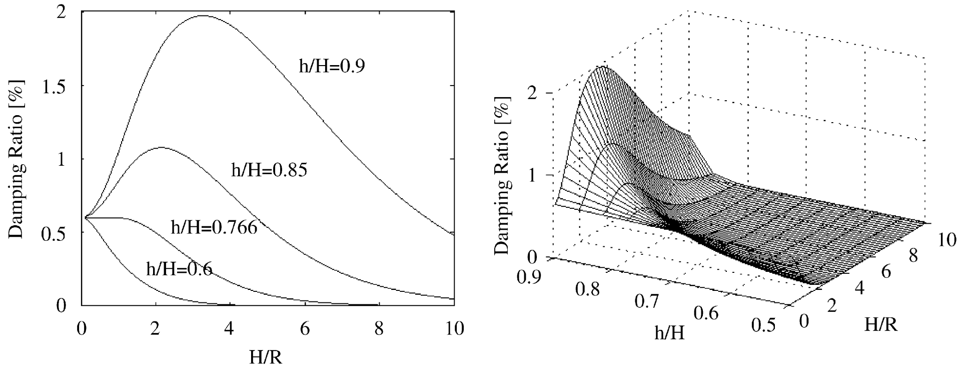

Fig. 15. Variation of the analytical damping ratio for blade baffles for $\eta/R = 0.05$ and $h_b/H = 0.1$.

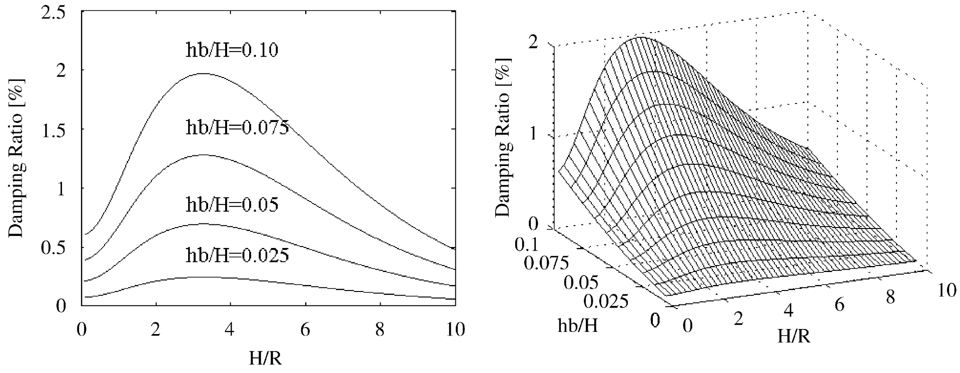

Fig. 16. Variation of the analytical damping ratio for blade baffles for $\eta/R = 0.05$ and $h/H = 0.9$.

A critical point which should be taken into consideration is that the relative sloshing height (η/R) in different tanks under the effect of an earthquake can be different due to different frequency characteristics of sloshing in different tanks, and this may affect the range of relative liquid height (H/R) which showed the highest damping ratios. Therefore for a given earthquake instead of a given relative sloshing height, the maximum damping ratio may occur out of the range of $0.5 < H/R < 1.0$.

Fig. 13 illustrates the variation of ring baffle damping ratio against relative liquid height and relative baffle width (r_b/R) for sloshing height amplitude equal to $0.05R$ and h equal to $0.9H$. As discussed about the previous figure, the

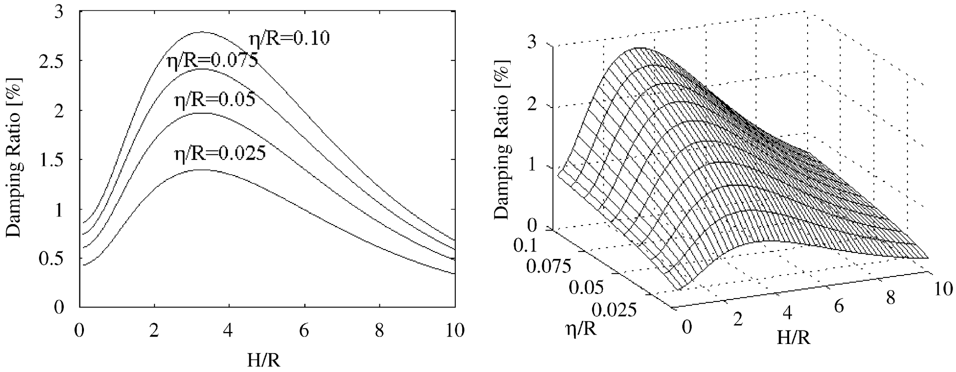

Fig. 17. Variation of the analytical damping ratio for blade baffles for $h/H = 0.9$ and $h_b/H = 0.1$.

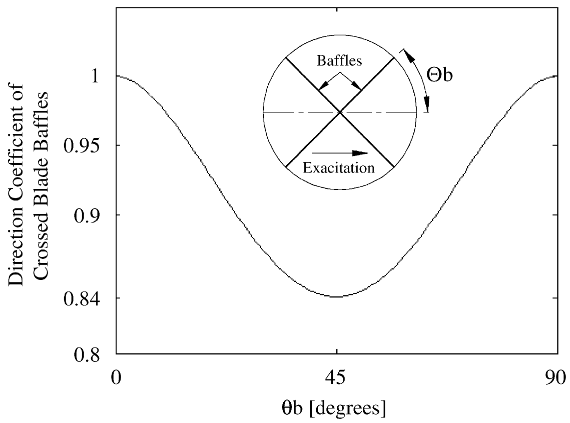

Fig. 18. Variation of the analytical damping ratio for crossed blade baffles against direction of excitation.

maximum damping ratio happens at a relative liquid height between 0.5 and 1, for $0 < h/H < 0.95$, consequently in this figure there is a peak at $H/R = 0.83$ for any given r_b/R because h/H has been chosen equal to 0.9.

The variation of sloshing damping ratio against relative liquid height and relative sloshing height (η/R) for a ring baffle width equal to $0.2R$ located at $0.9H$ is shown in Fig. 14. Like the previous figure there is a peak at $H/R = 0.83$ for any given η/R.

Figs. 15–17 are similar to the last three figures but for a blade baffle perpendicular to the horizontal excitation. Fig. 15 illustrates the damping ratio against relative liquid height (H/R) and relative baffle height (h/H) and with sloshing height amplitude equal to $0.05R$ and h_b equal to $0.1R$; quick decay in damping ratio with decrease of h/H is noticeable and there is a peak between $H/R = 0$ and $H/R = \infty$ for $0.766 < h/H < 1$, but for h/H less than 0.766, the peak is located at $H/R = 0$. In other words if H/R is considered less than 2 (ordinary storage tanks), then if the baffle is located below $0.766H$, the broader the tank is the higher the damping is, but for h/H greater than 0.766 the damping is almost constant or increasing versus H/R.

Fig. 16 demonstrates the variation of the blade baffle damping ratio against H/R and h_b/H for sloshing height amplitude equal to $0.05R$ and h equal to $0.9H$. In this case due to supposed h, the damping ratio for any sloshing height, show a maximum at $H/R = 3.2$.

The sloshing damping ratio against relative liquid height and relative sloshing height for the blade baffle width equal to $0.1H$ located at $0.9H$ has been shown in Fig. 17 and similar to the last figure there is a peak at $H/R = 3.2$ for any given η/R.

As it can be concluded from Eq. (23), the potential of a blade baffle in reducing the oscillation depends on the direction of excitation, while the damping of a ring baffle is independent of this parameter. To solve this problem, two crossed blade baffles can be placed in a tank instead of a single one. In this case if the total effect of crossed baffles is simply supposed as the superposition of the effect of each one, then the effect of excitation direction on total damping ratio will be according to Fig. 18 It can be concluded from the figure, the smallest damping is obtained when the angle between excitation and baffles is equal to 45°, and it is 84% of the largest value, which is achieved when one of the baffles is perpendicular to the excitation direction. Because when a baffle is parallel to the excitation, its influence on the sloshing is nearly zero (due to zero liquid velocity relative to the baffle), so the damping resulted from two crossed baffles when one of them is perpendicular to and the other one is parallel to excitation, is identical to the effect of a single perpendicular baffle. In other words using two crossed baffles can almost omit the direction effect.

5. Conclusions

To investigate the damping effect of horizontal ring and vertical blade baffles in seismic design of circular-cylindrical liquid storage tanks, a theoretical damping model has been developed based on Laplace's differential equation. A series of experiments employing a tank model on a shake-table has been carried out to study the validation of the theoretical models. The theoretically estimated and measured quantities of damping have shown reasonable conformity. The method can be extended to any kind of baffle, which is in conformity with the assumptions.

The developed models show that the damping ratio of sloshing mode at the presence of these two kinds of baffles depends on tank and baffle dimensions in addition to the location of baffle and sloshing height amplitude. A parametric study on the models confirmed that employing ring baffles in tanks with height to radius ratios (relative liquid height) between 0.5 and 1, can cause the highest damping ratios, provided the other parameters including relative sloshing amplitude remain constant. In the case of employing blade baffles, for baffles located at lower than 0.766 of liquid height, the damping ratio increases with relative liquid height, but for baffles located higher than that, there is a peak with variation of relative liquid height.

Also the damping ratio increases with increase of relative sloshing height and baffle height (from the bottom of tank) for both kinds of baffles, provided the baffle is not uncovered during the sloshing, because the theoretical models were subject to this assumption.

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