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The influence of cross-order terms in interface mobilities for structure-borne sound source characterization: Plate-like structures

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Abstract

With the introduction of the concept of interface mobilities, substantial simplifications can be obtained for vibrational source characterization and the description of the associated transmission process involving multi-point or continuous interfaces. The applicability of interface mobilities, however, depends on the admissibility of neglecting the so-called cross-order terms. Under the assumption of a uniform force-order distribution, the cross-order terms reduce to the cross-order interface mobilities. It is demonstrated that the cross-order interface mobilities represent the dependence of point and transfer mobilities on the location relative to boundaries and discontinuities. From theoretical and experimental analyses of circular interfaces located on plate-like structures, it is found that the influence of the cross-order terms features three distinct frequency regions. Provided that the structure can vibrate freely along the interface, the cross-order terms are significant only at intermediate Helmholtz numbers. For engineering practice, however, the omission of the cross-order interface mobilities appears to result in an acceptable estimate throughout the entire frequency range.

1. Introduction

The lack of a widely accepted approach for structure-borne sound source characterization in contrast to air-borne and liquid-borne sound sources causes substantial problems in engineering practice. Planning and low-noise design, especially at early design stages, is severely hampered, thereby effectively preventing lead-time reductions. Thus, a pressing research task is the development of an approach for the characterization of vibrational sources, which would provide the engineer with absolute source data and physical insight.

The majority of structure-borne sound sources and vibration transmission processes incorporate multi-point or continuous interfaces. Due to the inherent complexity of the underlying physics, simplifications are both desirable and necessary. The concept of interface mobilities [1] offers a scheme, which allows a subdivision of the physical source into a series of theoretical sources. The transmission problem is transposed into an equivalent single-point and single-component case, facilitating straightforward interpretation of the

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Nomenclature

Symbols

C	interface circumference
$C_{f,q}$	coupling function order
F	force
$\hat{F}_q$	force order
k_B	bending wave number
k_p, k_q	interface numbers
M	highest interface mobility order
N	number of contact points
p, q	order numbers
Q	complex power
r	interface radius
s	interface coordinate
S_q	source descriptor order
v	velocity
$\hat{v}_p$	velocity order
W	active power
Y	mobility
$\hat{Y}_{pq}$	interface mobility
$\hat{Y}_{q-q}$	equal-order interface mobility
Y^{∞}_{vF}	infinite plate point mobility

Subscripts

0	excitation
FS	free source
R	receiver
S	source

data. The applicability of this approach, however, relies upon the admissibility of neglecting the so-called cross-order terms. As recognized in Ref. [1], such a 'relaxed' formulation requires underpinning to clarify its validity.

Herein, the concept of interface mobilities is reviewed, providing physical insight into the various interface mobility terms. The application for source characterization is derived, manifesting the need for a physically underpinned relaxed definition of the concept of interface mobilities. Experimental and theoretical results are exemplified for the influence of the cross-order terms for plate-like structures.

2. Fundamental definitions

For multi-point connections or continuous interfaces larger than the governing wavelength, see Fig. 1, the field variables can be series expanded by means of a spatial Fourier decomposition [2],

$$v(s) = \sum_{p=-\infty}^{\infty} \hat{v}_p(k_p)\mathrm{e}^{\mathrm{j}k_p s}, \quad \hat{v}_p(k_p) = \frac{1}{C}\int_0^C v(s)\mathrm{e}^{-\mathrm{j}k_p s}\,\mathrm{d}s, \quad k_p = \frac{2p\pi}{C}. \tag{1}$$

Analogously, the decomposed force is given by

$$F(s_0) = \sum_{q=-\infty}^{\infty} \hat{F}_q(k_q)\mathrm{e}^{\mathrm{j}k_q s_0}, \quad \hat{F}_q(k_q) = \frac{1}{C}\int_0^C F(s_0)\mathrm{e}^{-\mathrm{j}k_q s_0}\,\mathrm{d}s_0, \quad k_q = \frac{2q\pi}{C} \tag{2}$$

with p, $q \in \mathbb{Z}$. Herein, $\hat{v}_p$ and $\hat{F}_q$ are termed velocity and force orders, respectively. A list of symbols is given in the nomenclature above.

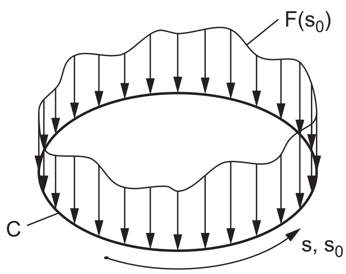

Fig. 1. Schematic illustration of a source–receiver interface.

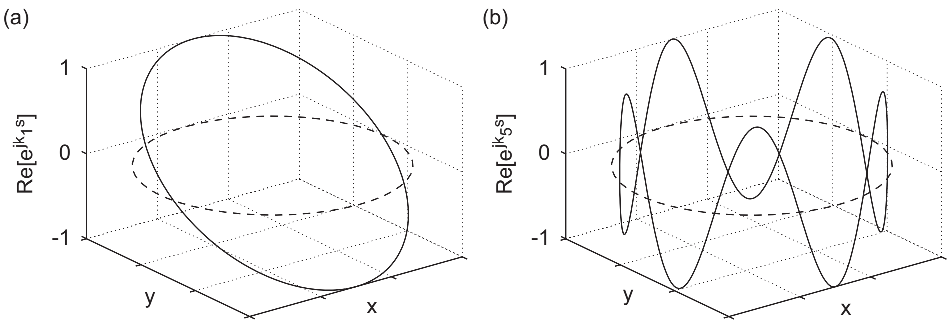

Fig. 2. Schematic illustration of two velocity or force order distributions over a circular line interface: (a) first order; (b) fifth order. —, Order distribution; - - -, interface.

Single velocity or force orders can be interpreted as waves travelling along the interface. The exponential terms e^{jk_ps} or $e^{jk_qs_0}$ describe these waves, while $\hat{v}_p$ or $\hat{F}_q$ represent the corresponding complex amplitudes. The instantaneous amplitudes of such orders, i.e. their real parts, are illustrated in Fig. 2. The summation over all orders with the associated complex amplitudes will reestablish the original distribution, as depicted in Fig. 1 for the case of forces.

By similarly expanding the mobilities, the interface mobilities can be formed. They represent the complex amplitudes in the double Fourier series in the following equation:

$$Y(s|s_0) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{Y}_{pq}(k_p, k_q) e^{jk_ps} e^{jk_qs_0}. \tag{3}$$

Hence, the interface mobilities are defined as the Fourier decomposition of the point and transfer mobilities,

$$\hat{Y}_{pq}(k_p, k_q) = \frac{1}{C^2} \int_0^C \int_0^C Y(s|s_0) e^{-jk_ps} e^{-jk_qs_0} \, ds \, ds_0. \tag{4}$$

3. Equal- and cross-order terms

In order to gain a better understanding of the link between interface mobilities and force and velocity orders, the relationship $Y = v/F$ in the interface order domain will be derived.

For a continuous force distribution $F(s_0)$, the velocity at any point along the contour C is given by

$$v(s) = \int_0^C Y(s|s_0) F(s_0) \, ds_0. \tag{5}$$

By substituting Eqs. (1) and (3) and applying Eq. (2), the sought relationship can be written as

$$\hat{v}_p(k_p) = C \sum_{q=-\infty}^{\infty} \hat{Y}_{p-q}(k_p, k_{-q}) \hat{F}_q(k_q). \tag{6}$$

The above equation states, that the coupling between a force of order q and the pth velocity order is described by the interface mobility $\hat{Y}_{p-q}$. The interaction between equal-order force and velocity is characterized by the term $\hat{Y}_{q-q}$, which thus is termed equal-order interface mobility or equal-order term. The coupling between different orders of force and velocity is described by the cross-order terms or cross-order interface mobilities.

The reciprocity relation of transfer mobilities in the spatial domain can be written as $Y(s|s_0) = Y(s_0|s)$. By means of substitution of Eq. (3), the reciprocity relation for interface mobilities is readily found to be given by $\hat{Y}_{pq} = \hat{Y}_{qp}$. With Eq. (6), one may infer that the coupling between the pth order velocity and the qth force order equals the coupling between the $-q$th velocity order and the $-p$th order force.

To enable a physical interpretation of the different interface mobility terms, their spatial distribution over the interface domains s and s_0 can be analyzed. The product of the two exponential functions in Eq. (3) forms

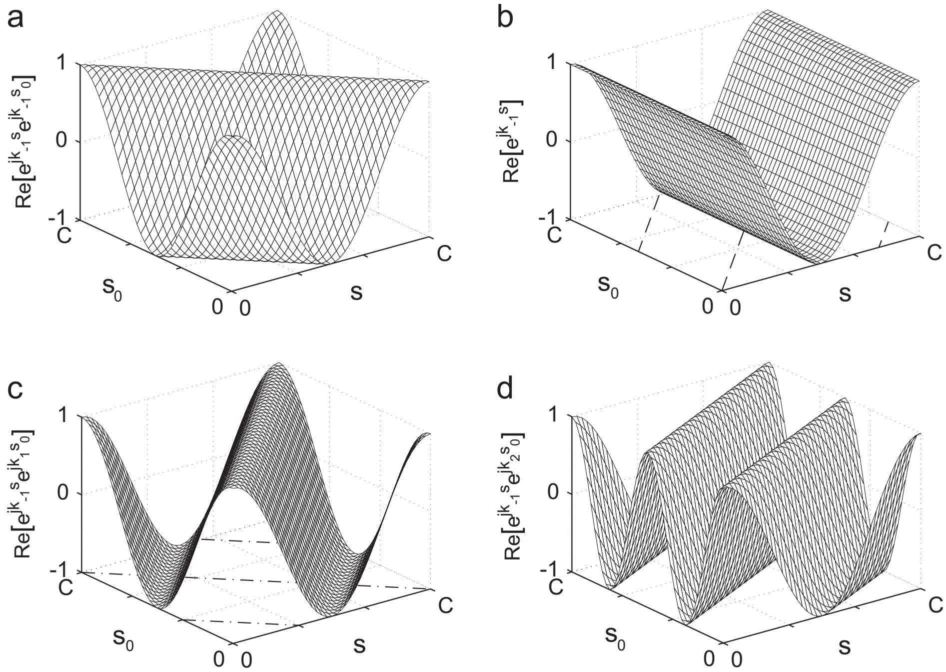

Fig. 3. Interface mobility shape functions with $p=-1$: (a) $\hat{Y}_{-1-1}$; (b) $\hat{Y}_{-10}$; (c) $\hat{Y}_{-11}$; (d) $\hat{Y}_{-12}$. - - -, Main diagonal and parallel lines with $s-s_0=$ const.; $-\cdot-$, codiagonal and parallel lines.

this spatial distribution, while $\hat{Y}_{pq}$ represents the corresponding complex amplitude. The interface mobility shape functions presented in Fig. 3 can be interpreted as components of the shapes of ordinary mobility matrices for an arbitrary line interface on any kind of structure. Considering the real part of such distributions facilitates the physical interpretation. For a unity excitation force at s_0, the mobility shape functions represent the instantaneous vibration amplitude of the structure along the interface.

The equal-order interface mobilities are seen to be symmetric and constant along the main diagonal and parallel lines. The symmetry of the matrices represents the physical attribute reciprocity. The variation of the mobility shape functions along the codiagonal and lines parallel thereto, see Fig. 3(c), can be interpreted as the transfer mobility dependence on the distance between excitation and response positions. The equal-order interface mobility of order zero is constant for all combinations of s and s_0 and thus describes the in-phase motion of the structure along the interface.

The cross-order terms are seen to be asymmetric, except for the special case of $\hat{Y}_{qq}$. The interface mobility matrices of type $\hat{Y}_{qq}$ are constant along the codiagonal and lines parallel thereto. The physical characteristic which the cross-order interface mobilities represent is manifested by the variation along the main diagonal and parallel lines, see Fig. 3(b). Along these lines $s-s_0=$ const. and hence equal to zero for the case of point mobilities.

Moving along the main diagonal of the mobility matrix or along any other line where $s-s_0=$ const., is equivalent to moving the respective mobility along the interface. The dynamic characteristics of point and transfer mobilities are sensitive to their location relative to nearby boundaries or structural discontinuities. Thus, the cross-order terms describe the dependence of any mobility with $s-s_0=$ const. on its position on the structure.

For non-circular interface geometries, the actual distance between the excitation and response positions will vary when moving along the interface with $s-s_0=$ const. A variation of this distance will inevitably alter the dynamic characteristics of the respective transfer mobility. Thus, for non-circular interfaces, the cross-order terms additionally describe the transfer mobility dependence on the distance between the excitation and response positions with respect to the complexity of the interface geometry.

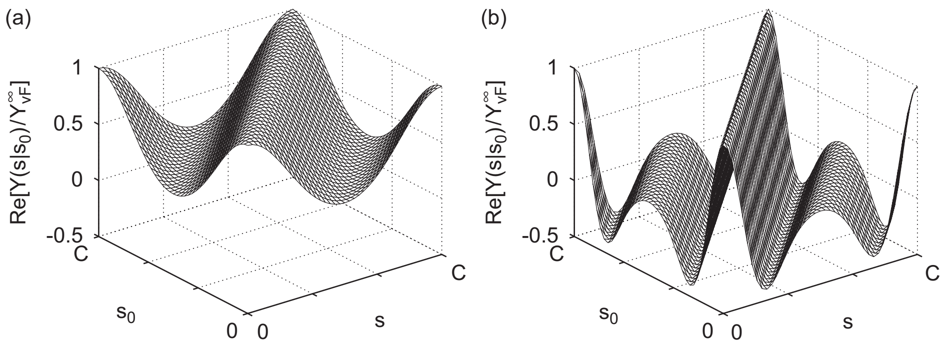

Fig. 4. Normalized mobility shape functions along a circular interface on a homogeneous, infinite, thin plate: (a) $k_B r = 1$; (b) $k_B r = 3$.

For point mobilities, the distance between the excitation and response positions is zero and does not vary when moving along the interface. Hence, the cross-order interface mobilities only describe the dependence of the point mobilities on their location along the interface. In order to prove the latter, the point mobility can be averaged over the interface:

$$\overline{Y}(s|s) = \frac{1}{C}\int_0^C Y(s|s)\,\mathrm{d}s \tag{7}$$

After substituting Eq. (3) and solving the integral, the cross-order terms are seen to cancel,

$$\overline{Y}(s|s) = \sum_{q=-\infty}^{\infty} \hat{Y}_{q-q}. \tag{8}$$

When averaging the point mobility over all possible locations along the interface, the dependence on such locations, which is described by the cross-order terms, drops out. Hence, this spatial dependence of the point mobility is proven to be the physical characteristic represented by the cross-order interface mobilities.

For the trivial case of a circular interface on a homogeneous, infinite structure, the mobility matrices are constant along the main diagonal and off-diagonal lines with $s - s_0 = \text{const.}$, see Fig. 4. With the circular interface geometry, the distance between the excitation and response positions are constant when moving along the interface with $s - s_0 = \text{const}$. Furthermore, there are no boundaries or discontinuities to which the positions of excitation and response could be related. Hence, the dynamic characteristics of the mobilities are independent of the location on the structure. As a result, all cross-order interface mobilities are equal to zero. Thus, the equal-order terms will fully manage to reestablish the mobility shape functions presented in Fig. 4.

From the above examination, two hypotheses can be posed for the relation between the cross- and equal-order interface mobilities when re-establishing the ordinary mobility shape functions. The cross-order interface mobilities are expected to be of smaller magnitude than the equal-order terms at low frequencies, where the interface is smaller than a fraction of the wavelength. In this Helmholtz number range, the structure along the interface moves primarily in-phase, suggesting a definite predominance of the equal-order interface mobility of order zero [2]. Furthermore, for homogeneous structures, the cross-order terms are expected to vanish asymptotically at high frequencies. This is due to the fact that the structural characteristics tend towards those of the corresponding infinite one, where it is seen from the trivial case that the cross-order interface mobilities are equal to zero.

4. Complex power

As recognized in Ref. [3], a proper source characterization should relate to the complex power. Therefore, in this section, the complex power will be discussed in conjunction with the concept of interface mobilities.

For a continuous line interface, the complex power can be written as

$$Q = \frac{1}{2}\int_0^C v(s)F^*(s)\,\mathrm{d}s. \tag{9}$$

Upon substituting the series expansions of the velocity and the force in Eqs. (1) and (2), respectively, one obtains

$$Q = \frac{1}{2}\int_0^C \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \hat{v}_p \hat{F}_q^* \mathrm{e}^{\mathrm{j}(k_p-k_q)s}\,\mathrm{d}s. \tag{10}$$

In Eq. (10), two types of power terms have to be distinguished. The product of equal-order force and velocity is seen to be independent of s and therefore constant along the contour C. For the case of different orders of force and velocity, the corresponding product will alternate around zero along the interface as illustrated in Figs. 2(a and b). As the integral over complete cycles of periodically alternating functions equals zero, the cross-order power terms in Eq. (10) cancel. Hence, the complex power for the concept of interface mobilities is given as

$$Q = \frac{C}{2}\sum_{q=-\infty}^{\infty} \hat{v}_q \hat{F}_q^*. \tag{11}$$

Although the complex power does not include cross-order terms, any further processing by means of Eq. (6) will reintroduce them. Moreover, the additional sum from Eq. (6) prevents the development of quantities relevant for source characterization. It is therefore interesting from a practical point of view to relax the concept of interface mobilities by neglecting the cross-order terms.

For the relaxed definition of the concept of interface mobilities, the relationship between velocity and force orders reduces to

$$\hat{Y}_{q-q} = \frac{1}{C}\frac{\hat{v}_q}{\hat{F}_q}. \tag{12}$$

By treating the contribution of each order separately, the transmission problem is formally brought back to the single-point and single-component case. The active power is readily obtained by taking the real part of the complex power,

$$W = \frac{C^2}{2}\sum_{q=-\infty}^{\infty} \mathrm{Re}[\hat{Y}_{q-q}]|\hat{F}_q|^2. \tag{13}$$

5. Source descriptor and coupling function

Under the aforementioned assumption of neglecting the cross-order terms, the source descriptor and the coupling function, cf. [4], can be derived in terms of interface mobilities.

Consider a source–receiver assembly with an interface covering more than a fraction of the governing wavelength, see Fig. 5. It is assumed that only the vertical translatory force component is present. By applying the concept of interface mobilities, the dynamic characteristics along the interface can be treated in terms of their Fourier orders. As a direct consequence of neglecting the cross-order terms, each order can be treated separately. This is because of the absence of an interdependence between different orders in Eq. (12).

The qth order source velocity is comprised of the velocity order due to the internal vibration, i.e. the free source velocity [5], and that arising from the force order acting on the source at the interface.

$$\hat{v}_{q,S} = \hat{v}_{q,FS} + C\hat{Y}_{q-q,S}\hat{F}_{q,S}. \tag{14}$$

In the connected state, the boundary conditions at the interface are given as $\hat{v}_{q,S} = \hat{v}_{q,R}$ and $\hat{F}_{q,S} = -\hat{F}_{q,R}$. By means of substitution, Eq. (14) can be solved for the receiver force order:

$$\hat{F}_{q,R} = \frac{1}{C}\frac{\hat{v}_{q,FS}}{\hat{Y}_{q-q,S}+\hat{Y}_{q-q,R}}. \tag{15}$$

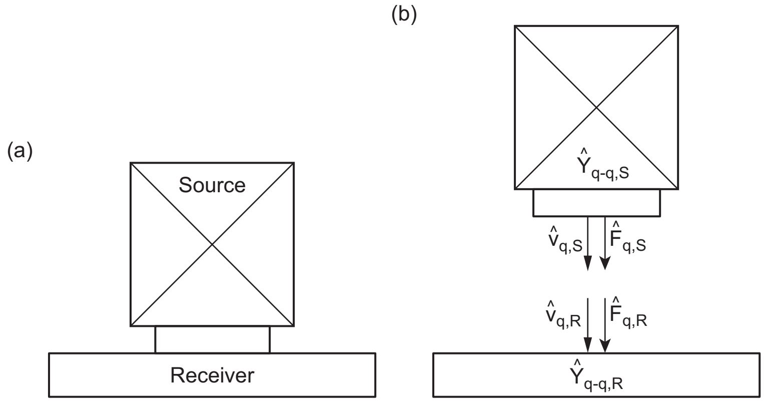

Fig. 5. Idealized continuous single-component interface between a source and a receiver with indication of the field variables: (a) configuration; (b) notation.

With Eqs. (11) and (12), the complex power is readily obtained as

$$Q = \frac{1}{2}\sum_{q=-\infty}^{\infty} \frac{|\hat{v}_{q,FS}|^2 \hat{Y}_{q-q,R}}{|\hat{Y}_{q-q,S} + \hat{Y}_{q-q,R}|^2} = \sum_{q=-\infty}^{\infty} \frac{1}{2}\frac{|\hat{v}_{q,FS}|^2}{\hat{Y}^*_{q-q,S}} \frac{\hat{Y}^*_{q-q,S}\hat{Y}_{q-q,R}}{|\hat{Y}_{q-q,S} + \hat{Y}_{q-q,R}|^2}. \tag{16}$$

Upon rewriting the complex power in order to separate a term consisting of source characteristics only, the source descriptor and the coupling function orders for the relaxed definition of the concept of interface mobilities are formed:

$$S_q = \frac{1}{2}\frac{|\hat{v}_{q,FS}|^2}{\hat{Y}^*_{q-q,S}}, \tag{17}$$

$$C_{f,q} = \frac{\hat{Y}^*_{q-q,S}\hat{Y}_{q-q,R}}{|\hat{Y}_{q-q,S} + \hat{Y}_{q-q,R}|^2}. \tag{18}$$

6. Influence of cross-order terms for plate-like structures

From the two preceding sections it is clear that the applicability of interface mobilities critically depends on the admissibility of neglecting the cross-order terms. The characteristic equation in which the cross-order terms are to be neglected is given by

$$\hat{v}_p = C \sum_{q=-\infty}^{\infty} \hat{Y}_{p-q}\hat{F}_q. \tag{19}$$

As indicated in the above relationship, the products of force order and cross-order interface mobility represent the cross-order terms. In this work, however, only the case is considered where all force orders are equal. Henceforth, therefore, the cross-order terms only refer to the cross-order interface mobilities. When dealing with the complex power, the influence of the cross-order terms can be investigated by comparing the superposition of all interface mobility terms with the sum of all cross-order interface mobilities.

In Section 3 it was hypothesized that the cross-order terms are smaller than the equal-order terms at small and large Helmholtz numbers. In order to support this, and to gain insight to the intermediate Helmholtz number range, the influence of the cross-order interface mobilities will be analyzed in the present section for plate-like structures.

The analytical investigation comprises simply supported, semi-infinite and finite, homogeneous, thin plates. For the semi-infinite plates the point and transfer mobilities are calculated by the mirror image approach [6]. A modal summation is applied for the case of finite plates [7].

Furthermore, built-up, box-like structures are included in the theoretical analysis with homogeneous, thin, plate-like elements. The recipient plate on which the box is placed is of either infinite, semi-infinite or finite extent. Here, the mobility method is applied. The top plate is modelled as an infinite one [8] and the side-plate mobilities are obtained from the higher-order extension in Ref. [9]. The studied interface is located on the top plate of the box.

For all analyzed structural configurations, interfaces of circular geometry are chosen. For circular interface geometries, the actual distance between excitation and response positions is constant when moving along the interface with $s - s_0 = \text{const}$. Hence, the cross-order terms only describe the dependence of point and transfer mobilities on the location of excitation and response positions relative to boundaries and discontinuities. The dimension and material of the plates as well as interface size and location on the plates are varied.

Analytical solutions for the interface mobilities could not be found for the majority of the structures, which is why the interfaces had to be discretized. As outlined in Ref. [10], two conditions have to be fulfilled for a proper solution of the highest interface mobility order sought. The spatial counterpart of the Nyquist criterion prescribes at least twice as many sampling points N along the interface as the highest order M, i.e. $N \geqslant 2M$. Furthermore, the distance between the sampling points has to be smaller than half the governing wavelength. For a circular interface of radius r, the latter condition yields, $k_B r \leqslant N/2$. For the discretized case, the interface mobilities can be obtained by means of a double FFT of the mobility matrix.

The overall characteristics and interrelations of the interface mobilities were found to be identical for the various structural configurations under consideration. In the following, these findings are illustrated through the case of a circular interface located on a simply supported, finite plate as shown in Fig. 6. The results from the numerical analysis are supported by an experimental investigation. For both the experimental and the theoretical analysis, the interface is sampled at 24 equidistant points. Thus, the results are valid up to $k_B r = 12$ with the highest interface mobility of order 12.

For the experimental setup, an 8 mm thick wooden chipboard plate was used. Simple supports were realized by thin and flexible plate strips attached to the edges of the chipboard plate and supported by a wooden beam along the other side, see Ref. [10]. Hence, the edges of the chipboard were supported rotationally flexible but translationally stiff. The plate was excited with an impulse hammer on one side, while the accelerometers were connected to the other side.

For a first assessment of the influence of the cross-order terms, the ordinary mobility shape functions can be compared with those of the interface mobilities. From Fig. 7, it is seen that for small Helmholtz numbers the mobility matrix is approximately constant, indicating that the structure along the interface moves in-phase. Thus, a predominance of the equal-order interface mobility of order zero can be expected at low frequencies and consequently a negligible influence of the cross-order terms. Substantial variations of the mobility matrices along the main diagonals and parallel lines are observed at intermediate Helmholtz numbers. This indicates a more pronounced involvement of the cross-order interface mobilities.

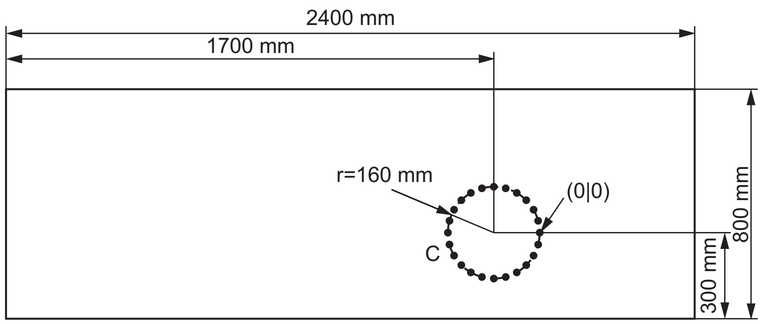

Fig. 6. Simply supported, finite plate with circular interface. •, Discretization point.

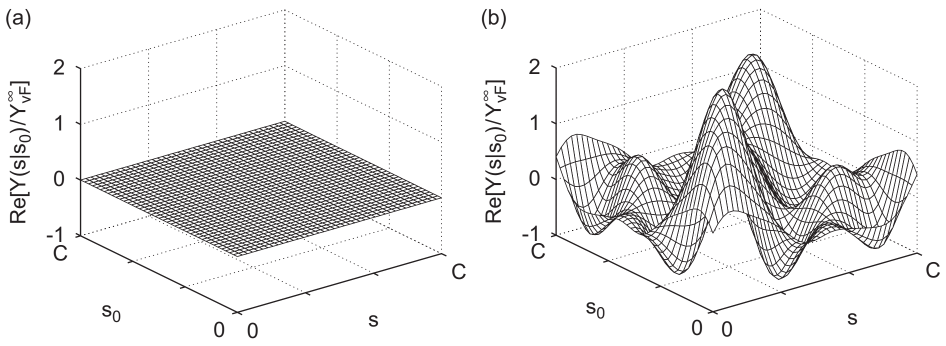

Fig. 7. Normalized, theoretical mobility shape functions along a circular interface on a simply-supported, finite plate: (a) $k_Br = 0.2$; (b) $k_Br = 2$.

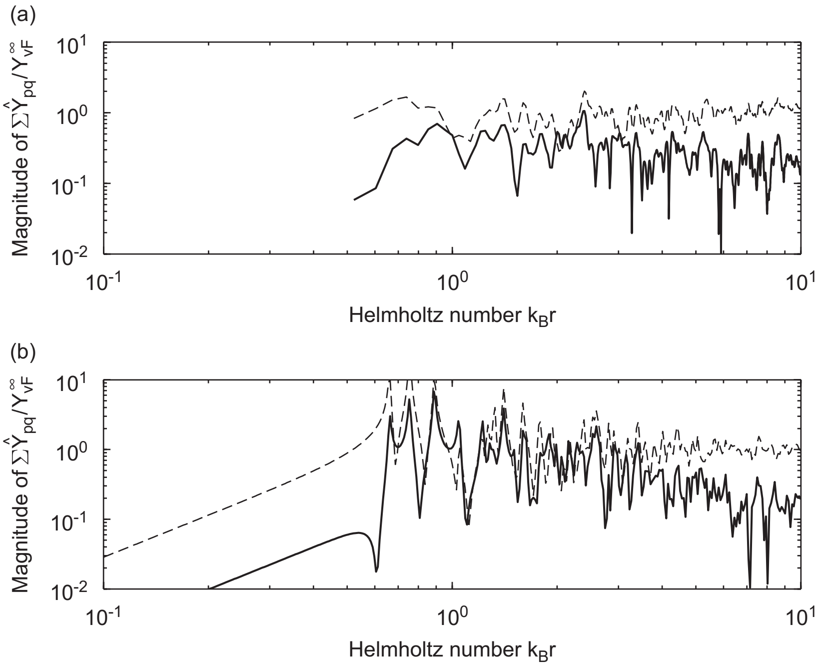

Fig. 8. Superposition of cross-order interface mobilities for a circular interface on a simply-supported, finite plate: (a) experimental analysis; (b) theoretical analysis. —, Cross orders; - - -, all orders.

A more thorough investigation can be made from the comparison in Fig. 8. Herein, the normalized sum over the cross-order terms is compared with the normalized superposition of all interface mobilities. As shown in Eq. (3), the point mobility $Y(0|0)$ equals the summation of all interface mobilities. Henceforth, the interface mobility superposition will be normalized with respect to the driving point mobility of the corresponding infinite plate, Y_{vF}^{∞}.

The mobility shape function in Fig. 7(b) shows large variations along the main diagonal and off-diagonal lines with $s - s_0 = \text{const}$. This indicates that the point and transfer mobilities are highly sensitive to their location relative to the boundaries at intermediate Helmholtz numbers. Consequently, as shown in Fig. 8, the superposition of all cross-order terms is seen to be of the same order of magnitude as the sum of all orders in this Helmholtz number range. With increasing frequency, the influence of the cross-order interface mobilities asymptotically vanishes. This is due to the fact that the boundaries virtually get out of range. The dynamic characteristics of the structure therefore more and more approach those of the corresponding infinite one.

It must be noted here, that the Helmholtz number used throughout this work refers to the interface radius. The frequency characteristics outlined above, however, depend on the distance to the nearest structural discontinuity or boundary.

For small Helmholtz numbers, the influence of the cross-order terms is comparatively low by reason of the dominating in-phase motion of the structure along the interface, see Fig. 7(a). However, if this in-phase motion is obstructed, e.g. by means of a constraint at some point along the interface, the influence of the cross-order terms is amplified. In such a case, the vibration of the structure along the interface caused by a zero-order excitation will be comprised of both the zero and the first order. As furthermore shown in Fig. 8, the influence of the cross-order terms is approximately constant at low frequencies. This indicates that the effect of the simple supports is independent of the wavelength in this range. An exception constitutes the case of an interface on the top plate of a built-up, box-like structure with infinite recipient. Here, the influence of the cross-order interface mobilities asymptotically vanishes with decreasing frequency. In absence of a support, the built-up structure moves entirely in-phase with constant amplitude at small Helmholtz numbers, which allows the equal-order term $\hat{Y}_{00}$ to fully describe the vibration.

From the theoretical and experimental analysis performed within the scope of this work, the equal-order interface mobilities were found to have positive real parts. The real parts of the cross-order terms, however, exhibit alternating signs. Specifically for the active power, cancellation between the cross-order interface mobilities is possible. However, the interference process is observed to be frequency dependent resulting in destructive as well as constructive interference of different cross-order terms.

With the knowledge of the cross-order term influence, it is now interesting to investigate how accurate the equal-order interface mobilities approximate the superposition of all terms. As shown in Fig. 9, the equal-order terms represent a good overall approximation with occasional local under- and overestimations. As described above, the explicitly good agreement at low frequencies is not granted for cases where the in-phase motion of the structure along the interface is obstructed. Owing to frequency shifts of peaks and troughs particularly at intermediate Helmholtz numbers, the deviations between the two curves can locally range up to a factor ten. Furthermore, even larger discrepancies can be observed in this frequency region when peaks of

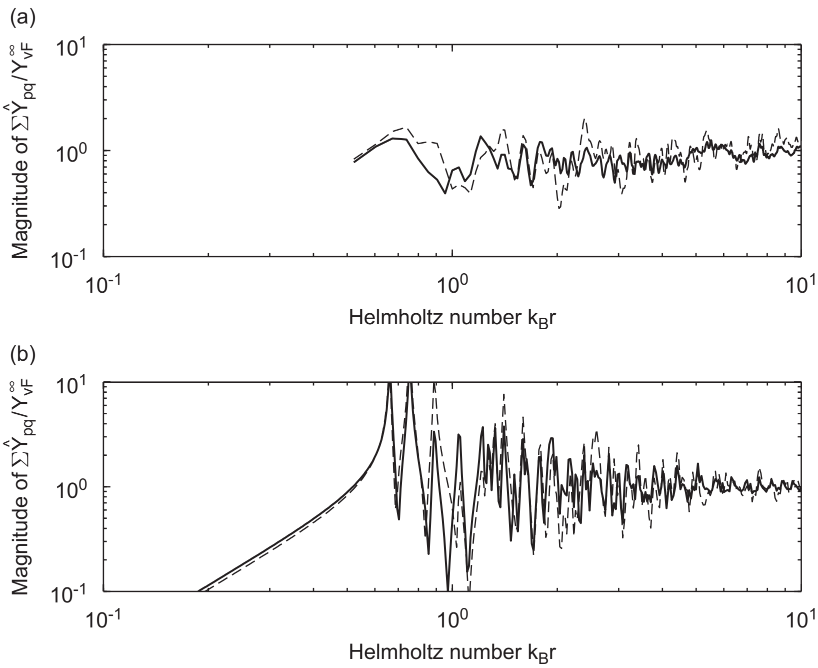

Fig. 9. Superposition of equal-order interface mobilities for a circular interface on a simply supported, finite plate: (a) experimental analysis; (b) theoretical analysis. —, Equal orders; - - -, all orders.

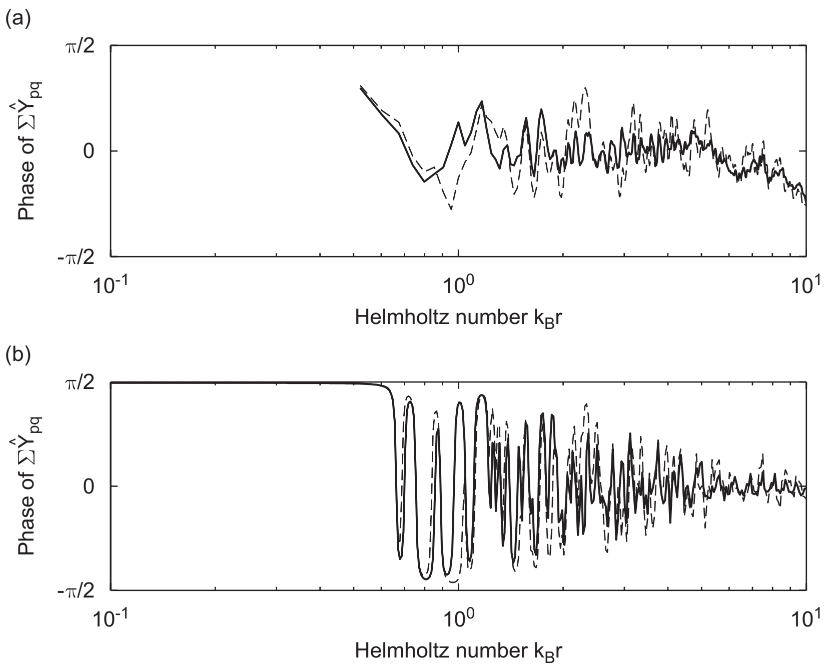

Fig. 10. Phase of equal-order interface mobility superposition for a circular interface on a simply supported, finite plate: (a) experimental analysis; (b) theoretical analysis. —, Equal orders; - - -, all orders.

the superposition of all interface mobilities coincide with troughs of the equal-order terms. For a lightly damped structure, the deviations in this case can be of a factor 10^2 or more.

As the phase relations of source and receiver are crucial for the coupling process, Fig. 10 displays the phase of the equal-order terms and that of the sum of all interface mobilities. Again, the equal-order term approximation offers a useful estimate of the complete representation. Explicit discrepancies are found at intermediate Helmholtz numbers, where distinct frequency shifts are observed for the magnitude of the superpositions, see Fig. 9.

At large Helmoltz numbers, the phase of point mobilities on plate-like structures tends towards zero. However, as seen in Fig. 10(a), the phase of the sum of all orders, corresponding to the mobility $Y(0|0)$, drops at high frequencies. Therefore, the experimental results are valid up to approximately $k_B r = 6$. The validity of the theoretical results ranges beyond $k_B r = 10$. Yet, the influence of the cross-order terms shows the same trends for the experimental and the theoretical analysis also above $k_B r = 6$, see Figs. 8–10. Hence, it can be presumed that measurement errors do not promote cross-order interface mobilities.

7. Concluding remarks

The application of the concept of interface mobilities for source characterization and the description of the transmission process critically depends on the significance of the cross-order terms. It is demonstrated that the cross-order interface mobilities represent the dependence of point and transfer mobilities on the location of excitation and response positions relative to boundaries and discontinuities. For non-circular interfaces, additionally, the cross-order terms take into account the dependence of the transfer mobilities on the distance between excitation and response positions with respect to the complexity of the interface geometry.

For circular interfaces located on plate-like structures, three distinct frequency regions can be distinguished regarding the influence of cross-order interface mobilities on the complex power. For small Helmholtz numbers, the structure moves predominantly in-phase at all points along the interface. Thus, the point and transfer mobilities show a low sensitivity to their location on the structure. Hence, the equal-order interface mobility of order zero singly captures the structural behaviour. When the motion of the structure at some point along the interface is constrained, however, the influence of the cross-order terms in this range is amplified. At high frequencies, the dynamic characteristics of the structure tend towards those of the corresponding infinite one and the cross-order interface mobilities vanish asymptotically.

In the intermediate frequency range, where the boundaries or discontinuities are at distances of the order of the governing wavelength, point and transfer mobilities are highly sensitive to the location along the interface. In this region, the cross-order terms are of the same order of magnitude as the equal-order terms. Yet, for engineering practice, the omission of the cross-order terms results in an acceptable estimate also at intermediate Helmholtz numbers. If precise information of resonances and anti-resonances is required, however, the relaxed definition of the concept of interface mobilities is insufficient.

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