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Short Communication

Analysis of misequalization in a narrowband active noise equalizer system

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Abstract

This paper presents the misequalization phenomenon in a narrowband active noise equalizer (ANE) system. The origin and characteristics of such phenomenon are investigated. Analysis shows that the estimation error of secondary path transfer function introduces this undesired phenomenon. Relationship between the estimation error and misequalization is also examined. It can be concluded that, for different gain factor values, importance of phase and amplitude estimation accuracy is different. Simulations based on both theoretical calculation and experiments are conducted to support the analysis.

1. Introduction

Active noise control (ANC) [1] systems based on the principle of superposition have evolved and developed in many applications to cancel undesirable noise. However, in some applications, it is desired to independently control some frequency components of the noise signal, either to keep some portions of the noise signal or to reshape the sound field. This demand leads to an extension of the ANC concept to include noise equalization.

In order to adjust the level of noise cancellation, a single-frequency active noise equalizer (ANE) was proposed by Kuo et al. [2,3], which used the filtered-X least mean square (FXLMS) [4] algorithm to adapt the coefficients of a two-weight filter minimizing a pseudo-error signal, instead of the residual noise. The idea behind this equalizer lies in independently controlling some given frequencies of a primary signal. However, in the practical ANC environment, a phenomenon called asymmetric out-of-band overshoot was discovered, and researchers [5] have shown its relationship with imperfect secondary path estimation. This imperfect estimation is expected in many practical applications.

This paper is organized as follows. Section 2 briefly reviews the overshoot problem in the ANC system. Section 3 introduces the narrowband ANE system, followed by the misequalization problem. Analysis and effects of the estimation errors are presented. Simulation results are included to support the analysis. Conclusions are drawn in Section 4.

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2. Overshoot problem in ANC

Under an ideal condition when the secondary path $S(z)$ is perfectly estimated (i.e. $\hat{S}(z) = S(z)$), there is no overshoot as observed in practical environment [5]. This implies that the overshoot is caused by the imperfect estimation of secondary path, which is also a factor leading to the instability of both ANC and ANE systems.

Kong et al. [5] analyzed this phenomenon in narrowband ANC systems by modeling the secondary path as a pure delay. Estimation error for this delay is included in the analysis. Their results showed that the phase estimation error of the secondary path alters the poles of the system transfer function.

3. Misequalization in a narrowband active noise equalization system

The narrowband ANE system was first proposed by Kuo et al. [2,3], and an extended version using FXLMS algorithm was developed for the ANE system. Transfer functions of both narrowband ANE systems were presented under the ideal secondary path estimation condition. Steady-state system behaviors were derived and validated with computer simulations.

Rees et al. [6] proposed four adaptive algorithms with similar function as the ANE and classified them as active sound-profiling algorithms. The four algorithms are the command-FXLMS, the internal model FXLMS, the phase scheduled command-FXLMS (PSC-FXLMS) and the automatic phase command FXLMS (APC-FXLMS) algorithms, and they are all variants of the FXLMS algorithm. The algorithms were analyzed for their stability properties and control effort load, and compared with the conventional ANE system. It was found that, both in simulations and experiments, the PSC-FXLMS and APC-FXLMS algorithms possess low control effort and good stability in the case of imperfect secondary path estimation.

Due to its ability to control over the signal and simplicity to implement, ANE systems are still preferable in many practical cases and continue to receive high interest from researchers [7–9]. However, its sensitivity to estimation errors [6] and lack of analysis of the estimation error effects have limited its further development. This was the motivation behind the work presented in this paper. The aim here is to investigate the estimation errors in both phase and amplitude, and the consequence of such estimation errors, which may aid system designers to focus on the more dominant estimation error in the ANE system.

3.1. Narrowband ANE system

In Ref. [2], the narrowband ANE system (Fig. 1) with two adaptive weights LMS algorithm was proposed. Functions of the narrowband ANE system are classified into four working modes depending on the gain factor β:

(i) *Cancellation mode* ($\beta = 0$): In this mode, ANE system functions as the conventional narrowband ANC.
(ii) *Attenuation mode* ($0<\beta<1$): The amount of attenuation is determined by factor β. So it is possible to retain some portion of the noise signal at the frequency of the reference signal.
(iii) *Neutral mode* ($\beta = 1$): The ANE system has no effect as there is no attenuation.
(iv) *Enhancement mode* ($\beta>1$): The ANE system functions as an amplifier that intends to enhance the noise at the frequency of the reference signal.

3.2. Equivalent transfer function of narrowband ANC/ANE system

Most of the researchers analyze the system stability in frequency domain by finding the z-transfer function of the system. The theoretical basis that enables this analysis was proposed by Glover [10]. The behavior of the adaptive filter is exactly described by a linear time-invariant filter between the primary signal and the error signal. As a result, the system stability property can be obtained according to the poles placement of the system transfer function.

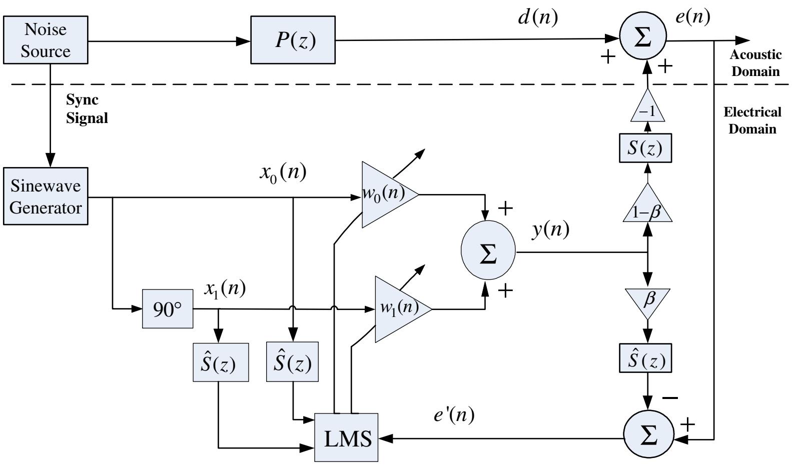

Fig. 1. Block diagram of single-frequency ANE system.

3.3. Origin of the misequalization in a narrowband ANE system

Under an ideal condition when the secondary path is perfectly estimated (i.e. $\hat{S}(z) = S(z)$), there is no misequalization. However, in a practical environment, the narrowband ANE system does not converge to the desired system gain factor value β. This implies that the misequalization is caused by the imperfect estimation of secondary path.

As mentioned in Section 2, Kong et al. [5] analyzed this phenomenon in narrowband ANC systems and concluded that the phase estimation error of the secondary path altered the poles of the system transfer function, which in turn leads the system to overshoot or become unstable.

On the other hand, Rees et al. [6] emphasized on the amplitude estimation error of the secondary path. In Ref. [6], analysis was carried out under the condition when there is no phase estimation error. The secondary path estimation error is modeled as amplitude difference. Under such condition, the estimation error causes the ANE system to become unstable, especially under a high system gain (when β is large).

3.4. Equivalent transfer function of the narrowband ANE system with imperfect secondary path estimation

In Refs. [2,3], Kuo proposed the narrowband ANE system with two adaptive weights LMS algorithm. The steady-state transfer function of the narrowband ANE was obtained and analyzed using the method derived by Glover et al. [10]:

$$H(z) = \frac{z^2 - 2z(1-\beta K)\cos\omega_0 + 1 - 2\beta K}{z^2 - 2z(1-K)\cos\omega_0 + 1 - 2K}, \tag{1}$$

where $K = \mu A^2/2$. (μ is the step size, and A is the amplitude of the reference signal.) This equation assumes perfect secondary path estimation. However, perfect secondary path is not achieved in most practical cases. To derive the equivalent transfer function of a narrowband ANE system with imperfect secondary path estimation, the equivalent flow diagram is shown in Fig. 2.

As presented in Ref. [10], the behavior of a synchronously sampled single-frequency adaptive system can be fully described by an equivalent transfer function as $G(z)$ in Fig. 1. Following this method, $G(z)$ can be found as

$$G(z) = \mu|\hat{S}|\frac{z\cos(\omega_0-\theta) - \cos\theta}{z^2 - 2z\cos\omega_0 + 1}, \tag{2}$$

where $|\hat{S}|$ and θ are the amplitude and phase shift of the estimated secondary path transfer function at the reference frequency ω_0, respectively.

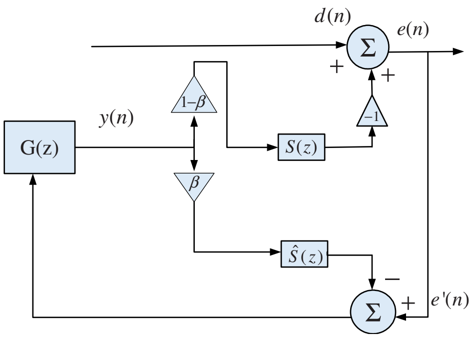

Fig. 2. Equivalent flow diagram of the active noise equalizer.

According to Fig. 2, the equivalent closed-loop transfer function of the narrowband ANE system from $d(n)$ to $e(n)$ is derived as

$$H(z) = \frac{1 + \beta G(z)\hat{S}(z)}{1 + G(z)[\beta\hat{S}(z) + (1-\beta)S(z)]}. \tag{3}$$

Note that, if perfect estimation $\hat{S}(z) = S(z) = 1$ is assumed, Eq. (3) is identical to the transfer function as listed in Eq. (1). Since the system is designed for control over a single frequency, the secondary path and its estimation can be represented as a delay and amplitude scaling. This can be shown as

$$S(z) = |S|z^{-D}, \tag{4}$$

$$\hat{S}(z) = |\hat{S}|z^{-\hat{D}}. \tag{5}$$

Relation between the estimated phase shift θ and the estimated delay $\hat{D}$ can be easily found as

$$\hat{D} = \frac{\theta}{\omega_0}. \tag{6}$$

Substituting Eqs. (2), (4) and (5) into Eq. (3), gives the closed-loop transfer function of the narrowband ANE system with imperfect secondary path estimation as

$$H(z) = \frac{z^2 - 2z\cos\omega_0 + 1 + \beta\mu|\widehat{S}|^2 z^{-\hat{D}}[z\cos(\omega_0-\theta) - \cos\theta]}{z^2 - 2z\cos\omega_0 + 1 + \mu|\widehat{S}|[\beta|\hat{S}|z^{-\hat{D}} + (1-\beta)|S|z^{-D})][z\cos(\omega_0-\theta) - \cos\theta]}. \tag{7}$$

Eq. (7) allows a full analysis of the narrowband ANE system for any given secondary path transfer function and its estimation. To examine the overall gain of the closed-loop system, let $z = e^{j\omega_0}$ and substitute into Eq. (7). The transfer function at the reference frequency is reduced to

$$H(e^{j\omega_0}) = \frac{\beta|\widehat{S}|e^{-j\omega_0\hat{D}}}{\beta|\hat{S}|e^{-j\omega_0\hat{D}} + (1-\beta)|S|e^{-j\omega_0 D})}. \tag{8}$$

Similar to Eq. (6), the phase error Φ between the actual secondary path transfer function and its estimation can be represented as

$$\Phi = (\hat{D} - D)\omega_0. \tag{9}$$

Substitute Eq. (9) into Eq. (8) and taking the norm of both sides, the overall gain factor of the narrowband ANE system is given as:

$$|H(e^{j\omega_0})| = \left|\frac{\beta|\widehat{S}|}{\beta|\hat{S}| + (1-\beta)|S|e^{j\Phi}}\right|. \tag{10}$$

3.5. *Effects of estimation error*

Assuming slow convergence, Eq. (10) describes the misequalization problem in a narrowband ANE system with imperfect secondary path estimation. This estimation error can be further categorized into three conditions.

3.5.1. *Amplitude estimation error*

Given perfect phase estimation, the overall gain of the narrowband ANE system can be derived as

$$|H(e^{j\omega_0})| = \left| \frac{\beta}{1 + (\beta - 1)(1 - (|S|/|\hat{S}|))} \right|. \tag{11}$$

For $\beta = 1$, according to Eq. (11) there is no misequalization. Experimental simulations were conducted to support this. For $\beta \neq 1$, if $|\widehat{S}| > |S|$, the second term in the denominator shifts the system gain away from the desired value of β. Under this condition and for $\beta < 1$, the system gain is shifted to a larger value but still smaller than 1. For the $\beta > 1$, the system gain is shifted to a smaller value but still larger than 1. In other words, with amplitude estimation $|\widehat{S}| > |S|$, the error compresses the system gain to a smaller range. Fig. 3 shows the actual system gain for different amplitude estimation errors when $|\widehat{S}| > |S|$. In general, the actual system gain is compressed to a smaller range. For the extreme case of $|\widehat{S}|/|S| = 10$, misequalization is obvious, which compresses the range to a constant close to 1. This suggests that, under this condition, the ANE system does not equalize the signal at all.

If $|\widehat{S}| < |S|$, the error expands the system gain to a larger range. For $\beta < 1$, the system gain is shifted to a value smaller than β. For $\beta > 1$, the system gain is shifted to a value larger than β. Furthermore, under the condition $|\widehat{S}| < |S|$, the system stability is affected. The second term in the denominator of Eq. (11) may reach -1, which causes the system to become unstable. Consequently, there is a constraint for β to prevent instability [6]:

$$\beta < \frac{|S|}{|S| - |\hat{S}|}. \tag{12}$$

The simulation results for amplitude estimation errors when $|\widehat{S}| < |S|$ are shown in Fig. 4. In contrast to Fig. 3 ($|\widehat{S}| > |S|$), the estimation error now expands the actual system gain from the desired β. This also causes the system to become unstable. For the extreme case of $|\widehat{S}|/|S| = 0.2$, the misequalization drives the system to become unstable even for a β slightly larger than 1.

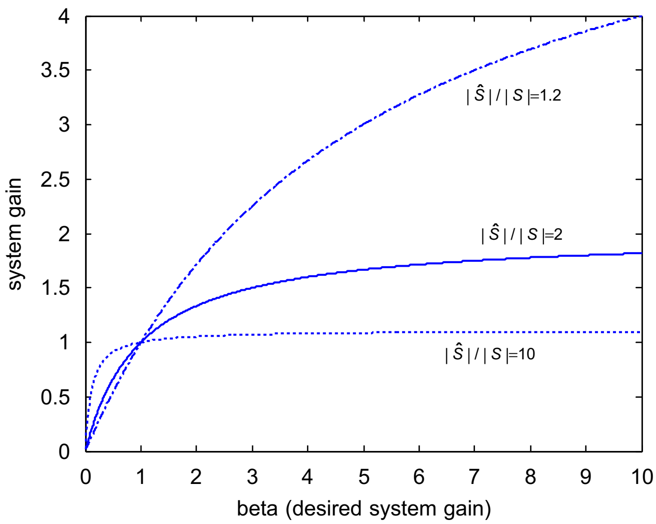

Fig. 3. Misequalization under different amplitude estimation errors ($|\hat{S}| > |S|$): - - -, $|\hat{S}|/|S| = 1.2$; —, $|\hat{S}|/|S| = 2$; …, $|\hat{S}|/|S| = 10$.

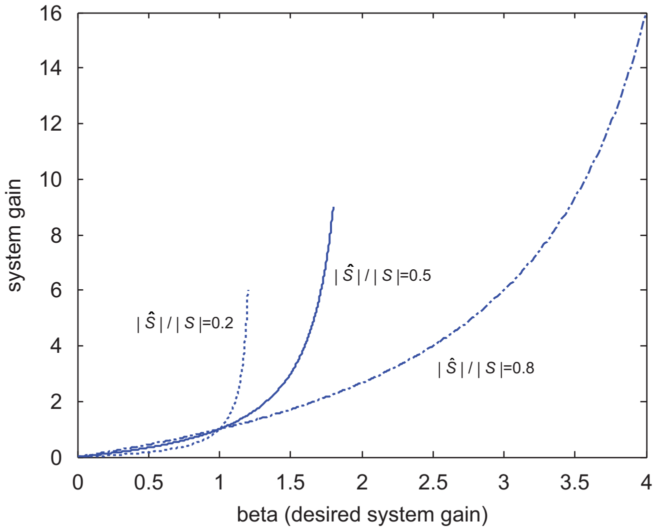

Fig. 4. Misequalization under different amplitude estimation errors ($|\hat{S}|<|S|$): —, $|\hat{S}|/|S| = 0.5$; - - -, $|\hat{S}|/|S| = 0.8$; …, $|\hat{S}|/|S| = 0.2$.

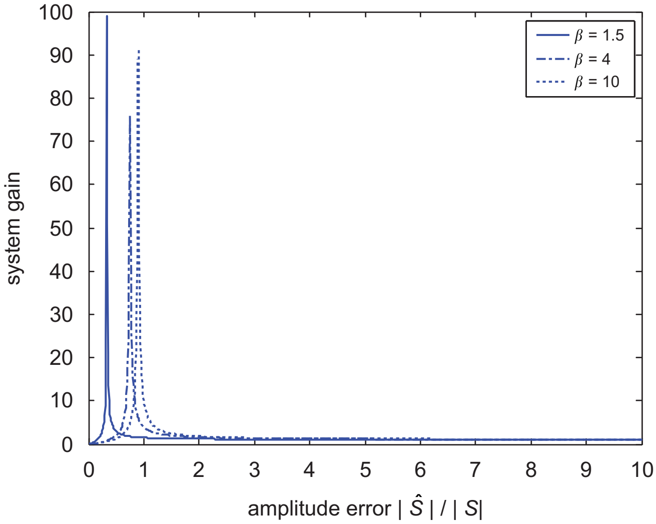

Fig. 5. Misequalization under different amplitude estimation errors ($\beta > 1$): —, $\beta = 5$; - - -, $\beta = 4$; …, $\beta = 10$.

Fig. 5 shows the narrowband ANE system actual gain against the amplitude error when $\beta > 1$. In this simulation, β is constant and we examined the system performance with $|\widehat{S}|/|S|$ as the measure of the amplitude estimation error. The peaks in this figure indicate that the system is unstable. This result coincides with Eq. (12).

In Fig. 6, we present the simulation results for system actual gain against the amplitude error when $\beta < 1$. From the results, it can be seen that, with estimation error $|\widehat{S}| < |S|$, the actual system gain tends to be smaller than the desired β, which leads the system towards ANC. With estimation error $|\widehat{S}| > |S|$, the system actual gain is shifted towards 1. This indicates that the ANE system is not equalizing the signal.

3.5.2. Phase estimation error

Given a perfect amplitude estimation, the overall gain of the narrowband ANE system can be derived as:

$$|H(e^{j\omega_0})| = \left|\frac{\beta}{1 + (\beta - 1)(1 - e^{j\Phi})}\right|. \tag{13}$$

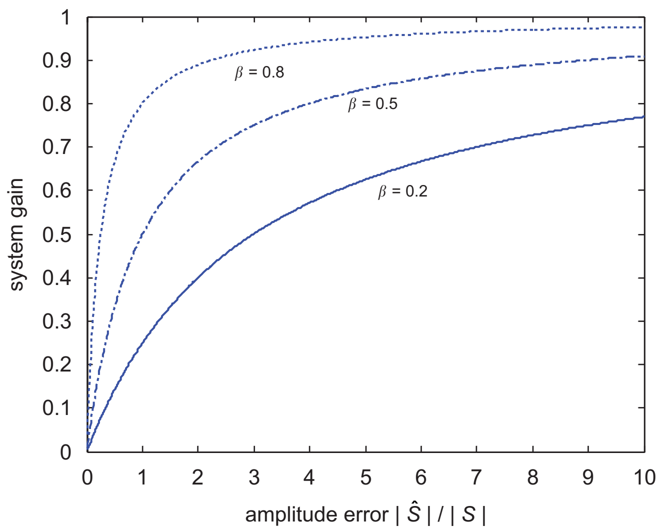

Fig. 6. Misequalization under different amplitude estimation errors ($\beta < 1$): —, $\beta = 0.2$; - - -, $\beta = 0.5$; …, $\beta = 0.8$.

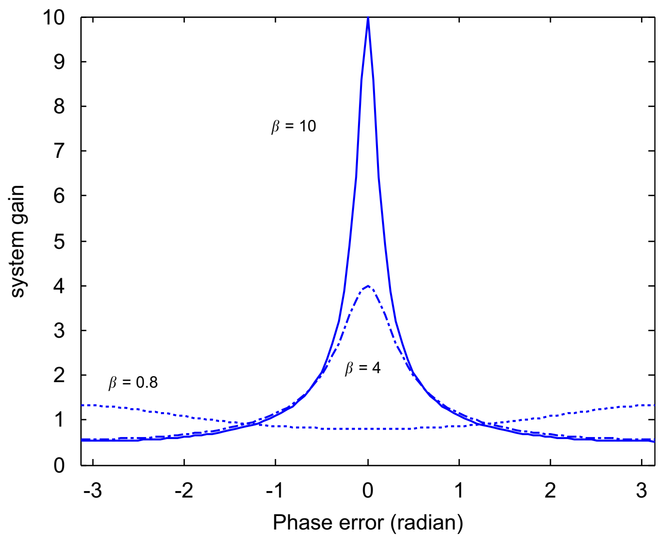

Fig. 7. Misequalization under different phase estimation errors ($\beta > 0.5$): —, $\beta = 10$; - - -, $\beta = 4$; …, $\beta = 0.8$.

The second term in the denominator is a vector of $(1 - e^{j\Phi})$ scaled by $(\beta - 1)$. For $\beta > 1$, the denominator of Eq. (13) is always larger than 1, which ensures the system stability. However, system gain is reduced to a smaller value than β. For $0.5 < \beta < 1$, the denominator of Eq. (13) is smaller than 1, but larger than 0. As a result, the system gain is misequalized to a larger value than β. For $\beta = 1$, there is no misequalization. Simulation results for $\beta > 0.5$ are shown in Fig. 7. It can be seen that for larger β, the system is more susceptible to phase estimation errors. For $\beta = 10$, a significant misequalization is observed for small-phase estimation error. However the system is stable for all phase estimation errors from $-\pi$ to π radians and for all $\beta > 0.5$.

For $\beta \leqslant 0.5$, the denominator tends to reach zero when the phase error is over $\pi/2$ radians. In this case, the system becomes unstable. As a result, Eq. (13) is not applicable here. This coincides with the classical constraint for the ANC system [4], which is a special case of $\beta = 0$.

3.5.3. Both amplitude and phase estimation error

In practical condition, both amplitude and phase errors are presented. These two error interact with each other and map the error effects and shift the system stability constraint and misequalization across the plane. Both amplitude and phase estimation errors are taken care in the following simulations.

As seen from Fig. 8, the system is robust against phase estimation error. The amplitude estimation error leads the system to be unstable according to Eq. (12). Both phase and amplitude estimation errors shift the system's overall gain from the desired value of 10. It is noticeable that, at the corners of the plot which indicate both severe phase and amplitude estimation error, the system is stable but misequalized from the desired value of 10. Compared with the previous case with only one estimation error, the introduction of the other estimation error stabilizes the system. However, the misequalization is still observed.

From the simulation shown in Fig. 9, we can observe that both phase and amplitude estimation errors shift the system gain from 0.8. However, the system is stable in most conditions. At extreme conditions, with both severe phase and amplitude estimation error, the system becomes unstable.

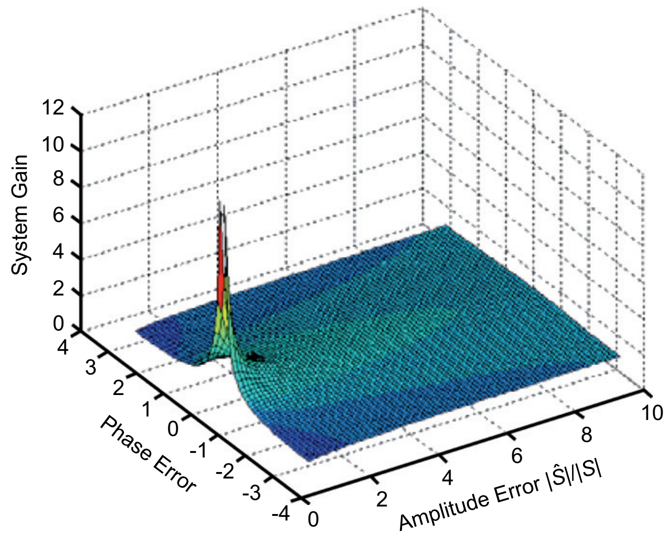

Fig. 8. Misequalization with both phase and amplitude estimation errors ($\beta = 10$).

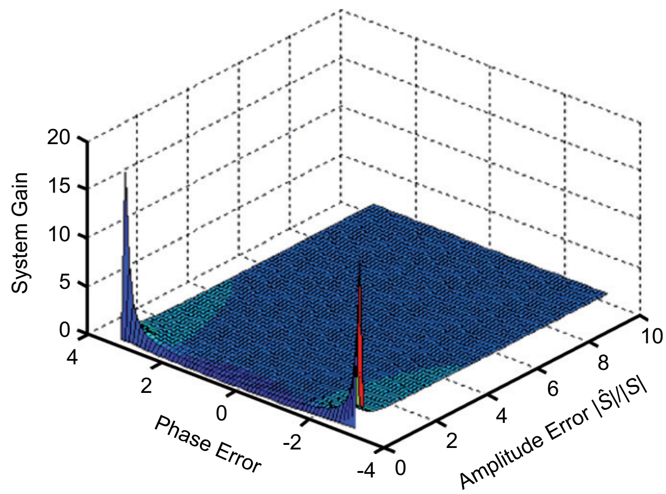

Fig. 9. Misequalization with both phase and amplitude estimation errors ($\beta = 0.8$).

Table 1
Dominant estimation error of secondary path for different gain factors

System gain factor β	Dominant estimation error of secondary path
$0 \leqslant \beta \leqslant 0.5$	Phase
$0.5 < \beta < 1$	Phase and amplitude
$\beta > 1$	Amplitude

4. Conclusions

The misequalization phenomenon in a narrowband ANE system is presented and investigated. Table 1 shows the range of system gain and its dominant estimation error of secondary path. Based on the analysis, we are able to conclude that for smaller gain factor, the phase estimation error is the dominant factor in system stability and misequalization, whereas for a gain factor larger than 1, the amplitude estimation error becomes the dominant one. For a gain factor in between 0.5 and 1, both estimation errors are important. This allows ANE system designers to focus more on the dominant estimation of secondary path.

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