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The influences of path characteristics on multichannel feedforword active noise control system

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Abstract

In this paper, a new analysis method for the performance of multiple channel feedforward active noise control (ANC) system in frequency domain is proposed. The reference paths from original sources to reference sensors and the secondary paths are considered when reference sensors cannot be installed close to the original sources. A closed expression of eigenvalues of spectral density matrix is given to determine the fixed step size. The convergence performance of the ANC algorithm is analyzed in detail, and the result shows that the reference paths and the secondary paths determined by the position of the reference sensors, loudspeakers and error sensors are the main factors to the final performance. Four impulse responses measured in a real laboratory are used in the simulations to verify the conclusions.

1. Introduction

Multichannel active noise control (ANC) system has many potential applications in aircraft, engine and mechanics, and many algorithms have made their success, like the multichannel filtered-x LMS (MFxLMS) algorithm, all summarized in Ref. [1]. The convergence performance of this algorithm, however, is limited by the reference signals and the structure of the secondary paths, which has been analyzed in Refs. [2–4]. Most ANC algorithms are performed in time domain, but their performance analysis is usually discussed in frequency domain, due to the complicated inherent structure and high dimension of the autocorrelation matrix of the input vector to the MFxLMS algorithm. Moreover, many papers [5–7] assume that the reference paths from the original sources to the reference sensors can be ignored, or just discuss the ANC algorithm's performance under tonal disturbance. This analysis is not accurate when the reference sensors cannot be put close to the original sources, or more frequency range needed to be controlled. Wang and Ren [7] proved a sufficient condition for the stability of the MFxLMS algorithm with the estimated secondary path, but the reference path was not considered. Elliott [8] proposed an optimal controller in frequency domain to

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accelerate the MFxLMS convergence speed by including the prior information of the reference signals and the secondary paths, however, which is difficult to obtain in time-varying environment.

In this paper, the reference paths are included into the multichannel ANC system, and the corresponding MFxLMS algorithm in frequency domain is given. A closed expression for spectral density matrix of the input vectors is established to analyze the effect of the reference paths and the secondary paths with fixed step size. The results show that the fixed step size of the ANC algorithm in time domain should be chosen in detail according to the transfer function matrix of the reference paths and the secondary paths. A simple criterion to evaluate MFxLMS algorithm's performance and some suggestions are given. Finally, two simple experiments are carried out to testify the conclusions.

2. MFxLMS algorithm in frequency domain

For a general multichannel ANC system shown in Fig. 1, K reference sensors and L error sensors receive data from I original noise sources via the reference path $\mathbf{B}_k$ and the primary path $\mathbf{H}_k$, which are time-invariant respectively. A system with control weight $\mathbf{W}_k$ is adapted to control M loudspeakers in order to minimize the sum of square error of L error sensors. Such a system is referred to as CASE[I, K, M, L] in this paper. The most conventional algorithm is the MFxLMS algorithm, but it is difficult to analyze, since the autocorrelation matrix of multichannel ANC system include the time and spatial correlation information of the filtered reference signals. In addition, high dimension of this matrix is also a problem, so it is easier to analyze its performance in frequency domain by discrete time Fourier transform (DTFT).

It is assumed that the step size is small and the system $\mathbf{W}$ has approximated an optimal solution. If the length of DTFT N is greater than the length of all filters, the linear convolution is approximately replaced by the circular convolution. The MFxLMS algorithm for CASE[I, K, M, L] in frequency domain is summarized as follows [3]:

$$\mathbf{e}(n,\omega) = \mathbf{d}(n,\omega) + \mathbf{U}(n,\omega)\mathbf{w}(n,\omega), \tag{1}$$

$$\mathbf{w}(n+1,\omega) = \mathbf{w}(n,\omega) - 2\mu\mathbf{U}^H(n,\omega)\mathbf{e}(n,\omega), \tag{2}$$

where the superscript H denotes Hermite transpose, μ is the fixed step size. $\mathbf{e}(n, \omega)$, $\mathbf{d}(n, \omega)$ and $\mathbf{w}(n, \omega)$ are the DTFT of the $L\times 1$ error vector $\mathbf{e}(n) = [e_1(n)\ldots e_L(n)]^T$, the $L\times 1$ desired vector $\mathbf{d}(n) = [d_1(n)\ldots d_L(n)]^T$ and the $LM\times 1$ weights vector $\mathbf{w}(n) = [\mathbf{w}_{11}^T(n)\ldots\mathbf{w}_{M1}^T(n)\ldots\mathbf{w}_{MK}^T(n)]^T$, respectively. $\mathbf{w}_{mk}(n)$ is a filter from the kth reference signal to the mth loudspeaker. $\mathbf{U}(n, \omega)$ is the $L\times MK$ filtered reference signals matrix in frequency domain, and the element of the lth row, the mkth column $u_{l,mk}(n, \omega)$ can be expressed as

$$u_{l,mk}(n,\omega) = c_{lm}(\omega)x_k(n,\omega), \tag{3}$$

where $c_{lm}(\omega)$ is the transfer function of the secondary path from the mth loudspeaker to the lth error sensor, which is also time-invariant, and the kth reference signal $x_k(n, \omega)$ in Fig. 1 is

$$x_k(n,\omega) = \sum_{i=1}^{I} b_{ki}(\omega)s_i(n,\omega).$$

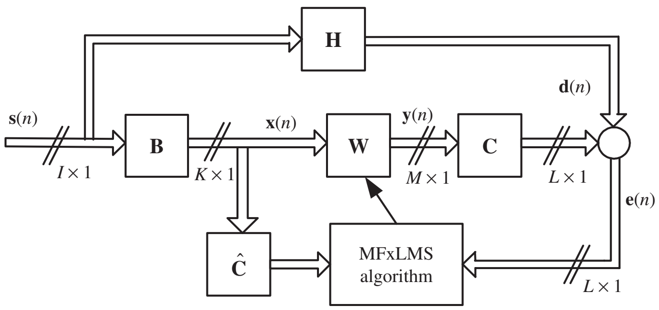

Fig. 1. The block diagram for multichannel ANC system, defined as CASE[I, K, M, L].

The filtered reference signals matrix $\mathbf{U}(n, \omega)$ can also be written in vector forms as

$$\mathbf{U}(n,\omega) = \mathbf{x}^T(n,\omega) \otimes \mathbf{C}(\omega), \tag{4}$$

$$\mathbf{x}(n,\omega) = \mathbf{B}(\omega)\mathbf{s}(n,\omega), \tag{5}$$

where $\otimes$ is the Kronecker product, and the corresponding matrix and vectors are defined as

$$\mathbf{C}(\omega) = \begin{bmatrix} c_{11}(\omega) & \ldots & c_{1M}(\omega) \\ \vdots & c_{lm}(\omega) & \vdots \\ c_{L1}(\omega) & \cdots & c_{LM}(\omega) \end{bmatrix}, \quad \mathbf{B}(\omega) = \begin{bmatrix} b_{11}(\omega) & \cdots & b_{1I}(\omega) \\ \vdots & b_{ki}(\omega) & \vdots \\ b_{K1}(\omega) & \cdots & b_{KI}(\omega) \end{bmatrix},$$

$$\mathbf{x}(n,\omega) = [x_1(n,\omega),\ldots,x_K(n,\omega)]^T, \quad \mathbf{s}(n,\omega) = [s_1(n,\omega),\ldots,s_I(n,\omega)]^T.$$

Eqs. (1) and (2) are similar to the standard LMS algorithm in Ref. [9], thus the stability and convergence performance of this algorithm are determined by the autocorrelation matrix of the filtered reference signals $\mathbf{U}(n, \omega)$, which is defined as

$$\mathbf{R}(\omega) = \mathbf{E}\{\mathbf{U}^H(n,\omega)\mathbf{U}(n,\omega)\}. \tag{6}$$

$LM \times LM$ matrix $\mathbf{R}(\omega)$, is called spectral density matrix as well. The upper limit of the step size μ at each frequency can be given by

$$0 < \mu < \frac{1}{\lambda_{\max}(\omega)}, \tag{7}$$

where $\lambda_{\max}(\omega)$ is the maximum eigenvalue of $\mathbf{R}(\omega)$. Over the whole frequency range of interest, μ in Eq. (7) will be reduced to

$$0 < \mu < \frac{1}{\max_\omega\{\lambda_{\max}(\omega)\}}. \tag{8}$$

Therefore, small step size should be chosen to keep this algorithm convergent over all frequency of interest. The convergence speed at each frequency, however, is decided by the mode related with the smallest eigenvalue, and is also proportional to the step size. Under the condition of stability, a suitable step size should be chosen to trade off between convergence speed and final mean square error (MSE) according to the eigenvalue spread, which is the ratio of the maximum eigenvalue to the minimum eigenvalue. Large eigenvalue spread leads to small step size and slow convergence speed, so a small eigenvalue spread over the frequency of interest is expected.

3. The structure of spectral density matrix R

The eigenvalue distribution of spectral density matrix $\mathbf{R}(\omega)$ will determine the final performance of the MFxLMS algorithm. It will be proved that $\mathbf{R}(\omega)$ can be expressed completely by the reference path $\mathbf{B}(\omega)$ and the secondary path $\mathbf{C}(\omega)$.

Without loss of generality, the original sources are assumed to be zero mean white noises with unit variance, uncorrelated with each other, so their spectral density matrix becomes

$$\mathbf{R}_s(\omega) = E\{\mathbf{s}(n,\omega)\mathbf{s}^H(n,\omega)\} = \mathbf{I}. \tag{9}$$

If $\mathbf{R}_s(\omega)$ is not diagonal, a causal and minimum-phase matrix $\mathbf{F}(\omega)$ can be obtained by using the spectrum decomposition theorem [9],

$$\mathbf{R}_s(\omega) = \mathbf{F}(\omega)\mathbf{F}^H(\omega).$$

A new reference path $\mathbf{B}'(\omega)$ is given by combining $\mathbf{F}(\omega)$ with the reference path $\mathbf{B}(\omega)$, $\mathbf{B}'(\omega) = \mathbf{B}(\omega)\mathbf{F}(\omega)$. The original noise sources are transformed into white noises, $\mathbf{s}'(n, \omega) = \mathbf{F}^{-1}(\omega)\mathbf{s}(n, \omega)$, whose spectral density matrix satisfies Eq. (9).

Combining Eqs. (4) and (5) into Eq. (6), and utilizing the Kronecker product properties

$$\begin{aligned}\mathbf{R}(\omega) &= E\{[\mathbf{x}^T(n,\omega)\otimes\mathbf{C}(\omega)]^H[\mathbf{x}^T(n,\omega)\otimes\mathbf{C}(\omega)]\}\\ &= [E\{\mathbf{x}(n,\omega)\mathbf{x}^H(n,\omega)\}]^*\otimes[\mathbf{C}^H(\omega)\mathbf{C}(\omega)]\\ &= [\mathbf{B}(\omega)E\{\mathbf{s}(n,\omega)\mathbf{s}^H(n,\omega)\}\mathbf{B}^H(\omega)]^*\otimes[\mathbf{C}^H(\omega)\mathbf{C}(\omega)]\\ &= [\mathbf{B}(\omega)\mathbf{B}^H(\omega)]^*\otimes[\mathbf{C}^H(\omega)\mathbf{C}(\omega)]. \end{aligned} \tag{10}$$

Under the assumption of Eq. (9), $LM\times LM$ spectral density matrix $\mathbf{R}(\omega)$ is dependent completely on the reference path $\mathbf{B}(\omega)$ and the secondary path $\mathbf{C}(\omega)$.

It is easy to prove that $\mathbf{R}(\omega)$ is Hermite matrix, i.e. $\mathbf{R}(\omega)=\mathbf{R}^H(\omega)$, so all eigenvalues of $\mathbf{R}(\omega)$ are nonnegative, that is to say, $\mathbf{R}(\omega)$ is a nonnegative define matrix. $\mathbf{B}(\omega)\mathbf{B}^H(\omega)$ and $\mathbf{C}^H(\omega)\mathbf{C}(\omega)$ are also Hermite. According to the properties of the Kronecker product, the determinant and trace of $\mathbf{R}(\omega)$ satisfy the following equations:

$$\det[\mathbf{R}(\omega)] = \det[\mathbf{C}^H(\omega)\mathbf{C}(\omega)]^K \det[\mathbf{B}(\omega)\mathbf{B}^H(\omega)]^M, \tag{11}$$

$$\mathrm{Trac}[\mathbf{R}(\omega)] = \mathrm{Trac}[\mathbf{C}^H(\omega)\mathbf{C}(\omega)]\mathrm{Trac}[\mathbf{B}(\omega)\mathbf{B}^H(\omega)]. \tag{12}$$

The relationship between eigenvalues of $\mathbf{R}(\omega)$ and those of $\mathbf{C}^H(\omega)\mathbf{C}(\omega)$ and $\mathbf{B}(\omega)\mathbf{B}^H(\omega)$ is determined by the following theorem.

Theorem. *If eigenvalues and eigenvectors of $M\times M$ matrix $\mathbf{A}$ are $\lambda_i^A,\mathbf{q}_i^A, i=1,2,\ldots,M$, and $N\times N$ matrix $\mathbf{B}$ has eigenvalues and eigenvectors $\lambda_j^B,\mathbf{q}_j^B, j=1,2,\ldots,N$, respectively. If $\lambda_i^A\neq 0$ and $\lambda_j^B\neq 0$, the eigenvalues and eigenvectors of $MN\times MN$ matrix $\mathbf{C}=\mathbf{A}\otimes\mathbf{B}$ satisfy*

$$\lambda_l^C=\lambda_i^A\lambda_j^B,\quad \mathbf{q}_l^C=\mathbf{q}_i^A\otimes\mathbf{q}_j^B,\quad \begin{aligned} l&=1,2,\ldots,MN,\\ i&=1,2,\ldots,M,\\ j&=1,2,\ldots,N.\end{aligned} \tag{13}$$

Proof. According to the definitions of eigenvalues, there are:

$$\begin{aligned}\mathbf{A}\mathbf{q}_i^A &= \lambda_i^A\mathbf{q}_i^A,\quad i=1,2,\ldots,M,\\ \mathbf{B}\mathbf{q}_j^B &= \lambda_j^B\mathbf{q}_j^B,\quad j=1,2,\ldots,N.\end{aligned}$$

Multiplying right $\mathbf{C}$ by $\mathbf{q}_i^A\otimes\mathbf{q}_j^B$ gives

$$\begin{aligned}\mathbf{C}(\mathbf{q}_i^A\otimes\mathbf{q}_j^B) &= (\mathbf{A}\otimes\mathbf{B})(\mathbf{q}_i^A\otimes\mathbf{q}_j^B)\\ &= (\mathbf{A}\,\mathbf{q}_i^A)\otimes(\mathbf{B}\,\mathbf{q}_j^B)\\ &= (\lambda_i^A\mathbf{q}_i^A)\otimes(\lambda_j^B\mathbf{q}_j^B)\\ &= (\lambda_i^A\lambda_j^B)(\mathbf{q}_i^A\otimes\mathbf{q}_j^B).\end{aligned}$$

Then Eq. (13) is established. □

Over a frequency range of interest $[\omega_1, \omega_2]$, the maximum eigenvalue of $\mathbf{R}(\omega)$ is the product of the maximum eigenvalue of $\mathbf{C}^H(\omega)\mathbf{C}(\omega)$ and $\mathbf{B}(\omega)\mathbf{B}^H(\omega)$, and the minimum eigenvalue and eigenvalue spread of $\mathbf{R}(\omega)$ have similar results. $\mathbf{B}(\omega)$ has the same effects on the performance analysis of $\mathbf{R}(\omega)$ as $\mathbf{C}(\omega)$, hence only $\mathbf{C}(\omega)$ will be discussed in the following paragraphs.

Consider a singular value decomposition of matrix $\mathbf{C}(\omega)$, $\mathbf{C}=\mathbf{R}\Sigma\mathbf{Q}^H$, $\mathbf{R}$ is the $L\times L$ eigenvector matrix of $\mathbf{C}\mathbf{C}^H$, $\mathbf{Q}$ is the $M\times M$ eigenvector matrix of $\mathbf{C}^H\mathbf{C}$, $L\times M$ matrix Σ is a diagonal matrix with M singular values (for $L\geqslant M$), which are the square roots of nonzero eigenvalues of $\mathbf{C}^H\mathbf{C}$. There are two factors to decide the singular values of $\mathbf{C}(\omega)$, one is the fluctuation of the transfer function of each channel, another is the correlation between channels, which is especially important for multichannel ANC system. All factors are determined by the positions of loudspeakers and error sensors, as well as the environment where the ANC system works [7].

If the estimated secondary path $\hat{\mathbf{C}}(\omega)$ is different from the true value, $\mathbf{C}^H\mathbf{C}$ is replaced by $\hat{\mathbf{C}}^H(\omega)\mathbf{C}(\omega)$. $\mathbf{R}(\omega)$ is not Hermite and has negative eigenvalue. The sufficient condition of stability is [7]

$$\mathrm{eig}[\hat{\mathbf{C}}^H(\omega)\mathbf{C}(\omega) + \mathbf{C}^H(\omega)\hat{\mathbf{C}}(\omega)] > 0, \quad (14)$$

where eig[] means the matrix eigenvalue. An effort weighting parameter β is used to improve the MFxLMS algorithm's performance for small random estimated error [4], and Eq. (2) becomes

$$\mathbf{w}(n+1,\omega) = (1-\mu\beta)\mathbf{w}(n,\omega) - 2\mu\mathbf{U}^H(n,\omega)\mathbf{e}(n,\omega). \quad (15)$$

It is noted that the algorithm in Eq. (15) is insensitive to small random estimation error of $\hat{\mathbf{C}}(\omega)$.

In practice, the step size is difficult to be chosen for the ANC system considering the reference path $\mathbf{B}(\omega)$ due to larger eigenvalue spread. Determination of the number and position of reference sensors is still an open problem, since the number of original sources is unknown and the reference path is not measurable. If a small step size is chosen to satisfy Eq. (8), error at different frequency will converge at different speed. After some iteration, error will result in a certain power spectrum, where there is more error power at some frequencies with large eigenvalue spread than those with small eigenvalue spread. In practical applications, it is very difficult to decide whether the current positions of loudspeakers and sensors can provide a "good" transfer function matrix, and the proposed analysis method in this paper provide a possible way. Some techniques, such as the blind deconvolution, can be applied to the reference signals to eliminate the effect of the reference path. Accurate secondary path modeling and optimal controller discussed in Ref. [8] is also a prospective way.

4. Numerical examples

Four impulse responses measured in a room with reverberation time 50 ms are used in the MFxLMS algorithm. The distance of two microphones is 20 cm, and two loudspeakers are placed in front of them with 70 cm and 65 cm, respectively. The detailed experiment is described in Ref. [10]. The measured impulse responses, with the sampling rate 2 kHz are shown in Fig. 2, where only 128 coefficients are used due to fast fading. The transfer functions with hamming window are shown in Fig. 3, where every transfer function has less fluctuation in [0.2, 2.5]. If each of the transfer functions is applied to the single channel ANC algorithm, error signals will converge very fast due to small eigenvalue spread at each frequency.

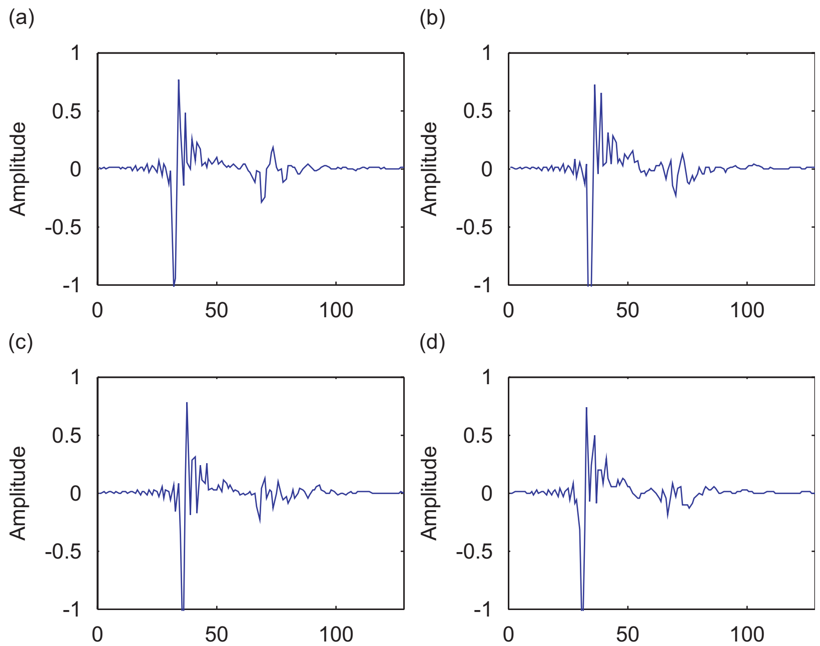

Fig. 2. Four impulse responses measured in a room with reverberation time 50 ms.

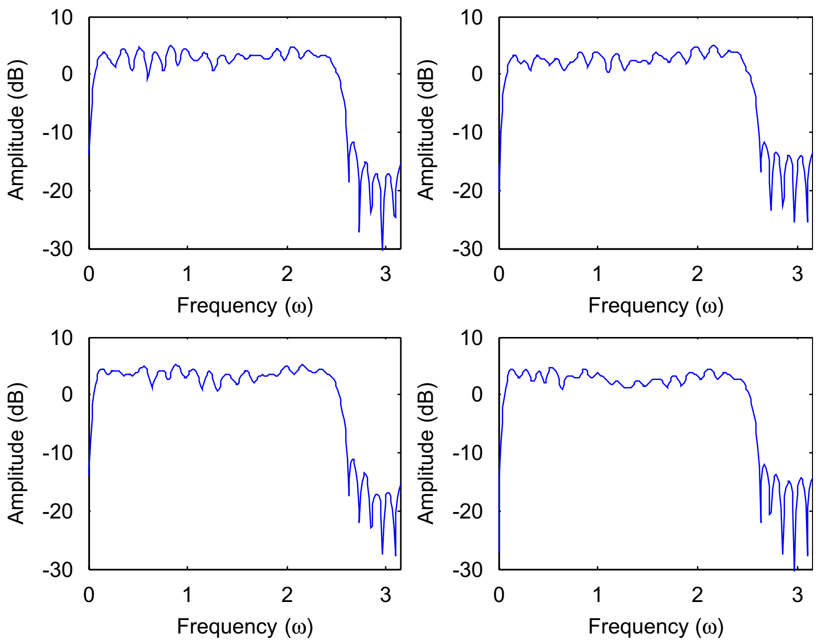

Fig. 3. The transfer function of four paths in Fig. 2, hamming widow with the length of 128 is applied.

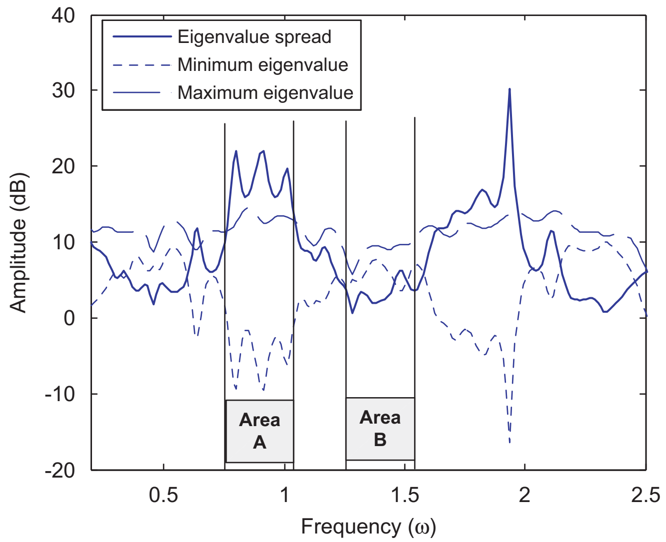

Fig. 4. The maximum eigenvalue (the dotted line), the minimum eigenvalue (the dashed line) and the eigenvalue spread (the solid line) of the 2×2 transfer matrix $\mathbf{C}^H(\omega)\mathbf{C}(\omega)$, which consist of four impulse responses in Fig. 2.

Combining four transfer functions into a 2×2 transfer function matrix, then the maximum eigenvalue, the minimum eigenvalue and the eigenvalue spread in the frequency of interest [0.2, 2.5], are plotted by the dotted line, the dashed line and the solid line in Fig. 4, respectively. The eigenvalue spread outside of [0.2, 2.5] will not affect the final performance since no signals appear in this region. From Fig. 4, it can be found that the maximum eigenvalues are approximately identical in [0.2, 2.5], and there are small eigenvalues at some particular frequency range. Choose two frequency area, labeled "*Area A*" with [0.75, 1.75] and "*Area B*" with [1.25,1.55], to test the MFxLMS algorithm's performance. There are large eigenvalue spread with "*Area A*" and small eigenvalue spread with "*Area B*", thus different performance will be obtained.

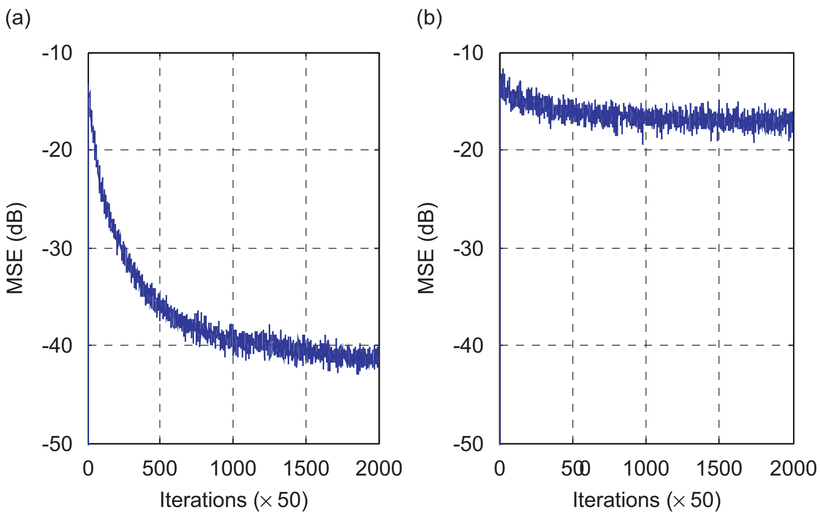

Fig. 5. The error learning curves with 50 times averages in two experiments, (a) the error learning curve with the bandpass Butterworth filter within "*Area B*", (b) the curve with the bandpass Butterworth filter within "*Area A*". Fixed step size 0.00001 and 100 000 samples are used.

Both the reference path $\mathbf{B}(\omega)$ and the secondary path $\mathbf{C}(\omega)$ are the four transfer functions shown in Fig. 3. A small step size is required to keep the ANC algorithm stable. In the experiments, two uniform distributed white noises with unit variance are filtered by a 10-order bandpass Butterworth filter, and the filter outputs are original noise sources, where the filter bandwidth is chosen as frequency range of "*Area A*" and "*Area B*", respectively. For convenience, an ideal primary path is chosen with the identify matrix at the 100th delay unit, and the estimated secondary path is assumed to be perfect, a 2×2 256-order FIR matrix is adopted in the ANC system. A step size $\mu = 0.00001$ and the same initial condition are used in two experiments. Two error learning curves with 50 times averages are shown in Fig. 5, where (a) is with "*Area B*" and (b) is with "*Area A*". Comparing Fig. 5(a) and (b), "*Area B*" has faster convergence speed and smaller MSE than "*Area A*" due to small eigenvalue spread.

5. Conclusions

In this paper, the stability and performance of the MFxLMS in frequency domain are analyzed in detail, where the spectral density matrix of MFxLMS algorithm depends completely on the secondary paths and the reference paths. A simple criterion has been proposed to estimate the algorithm's performance by analyzing the singular value of the transfer function matrix at different frequency. With the proposed method, the position of sensors and loudspeakers can be adjusted to improve the performance. It can also to be used to indicate the frequency range, which can be controlled more efficiently with current transfer paths in multichannel ANC system.

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