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Two simple methods to suppress the residual vibrations of a translating or rotating flexible cantilever beam

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Abstract

In this paper, two methods for controlling the residual vibrations of a translating or rotating Euler–Bernoulli cantilever beam are considered. Although a beam has an infinite number of vibration modes, when it simply changes its position by translation or rotation the first mode is the main contributor to the total response. Thus, the problem can be reduced to the base acceleration excitation of a single-degree-of-freedom system. Two simple methods are suggested for suppressing the residual vibration of such a system without considering any control algorithms. Both methods are based on the transient response of the system—namely, the shock response spectrum (SRS). The first method is simple and can be used for lightly damped systems, while the second method can be applied to more general situations. The result of the second method is similar to that of the input shaping method; however, in the method proposed here, both position and time to move from one position to another can be controlled simultaneously.

1. Introduction

When a flexible structure changes its position by suddenly translating or rotating following a command input, residual vibration is inevitable due to the inertial load imposed on the flexible structure. Suppressing this type of residual vibration has become very important in many engineering applications such as space structures, cranes and flexible robot manipulators. There are two main categories for controlling the residual vibrations; one is closed-loop control, for example PD, PID, and adaptive control [1–4], and the other is open-loop control, using methods such as pre-shaping the command input [5–9]. The latter method has been applied widely since being suggested by Singer and Seering [5]. It can be implemented easily once the dynamics of the structure, namely the natural frequencies and damping ratios, are known. Although there are many variants of the method to enhance the robustness, the basic principle is the same, which is to design the best *filter* to suppress the residual vibrations. For the input shaping method, the filter consists of a series of impulses (Fig. 1).

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Fig. 1. Input shaping method.

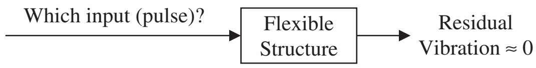

Fig. 2. The conceptual approach of the transient response method.

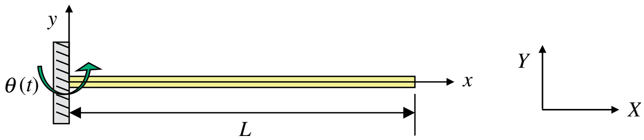

Fig. 3. Cantilever beam model.

The aim of this paper is to show that the same results can be achieved with only knowledge of the system dynamics, *without considering the filter, i.e., we do not rely on any control strategy*. Two methods are proposed, one is simple and fast and the other is somewhat related to the input shaping method. Both methods are purely based on the transient response of the structure—e.g., the shock response spectrum (SRS), and are much easier to implement because there is no need to consider a control algorithm. Moreover, the proposed methods control both the position and the time taken to change the position simultaneously. The basic idea follows from the SRS [11,12]. For an undamped single-degree-of-freedom system with base excitation, when the excitation force is a pulse-like input (e.g., rectangular, half-sine, etc.), the residual response is zero if the duration of the pulse is appropriately chosen. If this principle is applied to our case, then the problem becomes to determine an *input* (*pulse*) that *automatically* suppresses the residual vibration by considering the response of the structure excited by a pulse-like input. The problem can be depicted as in Fig. 2.

A typical application of the method may be a slewing flexible robot arm that follows a command input signal. Although there are many modes of vibration, for this particular problem, it is generally sufficient to consider the first mode only. This is demonstrated in Section 2. The details of the new approach follow in subsequent sections.

2. Response of a flexible beam under an inertial force

Consider a flexible robot arm crudely modelled as a uniform cantilever beam rigidly attached to a rotating hub as in Fig. 3, where the hub rotates to a desired angle following the command input. In this model, inertia of the hub is neglected. As in the references, for example [1,3,6,9,10], we neglect the centrifugal force, which may arise due to the rotation of beam. (Note that we do not consider the vibration while the beam is rotating, but the main concern is the 'residual' vibration after the beam stops.) We also assume that the beam neither spins around its axis nor translates along it.

In this case, the cantilever beam is effectively excited by a distributed inertial force, so the equation of motion can be written as [10]

$$\mathrm{EI}\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} = -mx\ddot{\theta}(t), \tag{1}$$

where EI is the bending stiffness, m the mass per unit length, and $-mx\ddot{\theta}(t)$ the inertial force per unit length, which is linearly proportional to x. Note that, as in Fig. 3, $y(x, t)$ is defined in the rotating coordinate system $(x-y)$, not in the fixed coordinate system $(X-Y)$. The solution to Eq. (1) is

$$y(x,t) = \sum_{r=1}^{n} \phi_r(x) q_r(t), \tag{2}$$

where $\phi_r(x)$ are the mode shapes which are given by

$$\phi_r(x) = \cosh \beta_r x - \cos \beta_r x - \frac{\cosh \beta_r L + \cos \beta_r L}{\sinh \beta_r L + \sin \beta_r L}(\sinh \beta_r x - \sin \beta_r x), \tag{3}$$

which are orthogonal and normalised such that [13]

$$\int_0^L \phi_r(x)\phi_s(x)\,\mathrm{d}x = \begin{cases} 0 & \text{for } r \neq s, \\ L & \text{for } r = s. \end{cases} \tag{4}$$

The natural frequencies are given by $\omega_r = \beta_r^2\sqrt{\mathrm{EI}/m}$, where β_r is the wavenumber, which can be determined from the characteristic equation $\cos\beta_r L \cosh\beta_r L = -1$. The values of $\beta_r L$ for the first three natural frequencies are [13]

$$\beta_1 L = 1.875104, \quad \beta_2 L = 4.694091, \quad \beta_3 L = 7.854757. \tag{5}$$

Using the natural frequencies and corresponding mode shapes, the equation of motion given in Eq. (1) can be written in terms of generalised coordinates q_r as [10] (in the reference a different normalisation factor is used)

$$\ddot{q}_r + \omega_r^2 q_r = -\ddot{\theta}\frac{1}{L}\int_0^L x\phi_r(x)\,\mathrm{d}x. \tag{6}$$

This equation describes a set of simple oscillators, one for each mode of vibration, each subject to base excited acceleration. Thus, the solution for q_r is the same as that of a single-degree-of-freedom system excited by $-\ddot{\theta}(t)$ and scaled by

$$\frac{1}{L}\int_0^L x\phi_r(x)\,\mathrm{d}x.$$

For a beam that is initially at rest, q_r is given by

$$q_r(t) = -\left[\frac{1}{L}\int_0^L x\phi_r(x)\,\mathrm{d}x\right]\frac{1}{\omega_r}\int_0^t \ddot{\theta}(\tau)\sin\omega_r(t-\tau)\,\mathrm{d}\tau. \tag{7}$$

To determine the contributions of each mode to the total response $y(x, t)$ at x, the amplitude of each mode is normalised with respect to the first mode so that

$$A_r(x) = \left|\frac{\phi_r(x)}{\phi_1(x)}\right| \frac{\int_0^L x\phi_r(x)\,\mathrm{d}x \Big/ \omega_r}{\int_0^L x\phi_1(x)\,\mathrm{d}x \Big/ \omega_1} = \left|\frac{\phi_r(x)}{\phi_1(x)}\right| \left(\frac{\beta_1}{\beta_r}\right)^2 \frac{\int_0^L x\phi_r(x)\,\mathrm{d}x}{\int_0^L x\phi_1(x)\,\mathrm{d}x}. \tag{8}$$

In most applications, the tip motion of the beam is generally of interest, and the scaling factor at the tip $A_r(L)$ is given by

$$A_1(L) = 1, \quad A_2(L) = 0.0225, \quad A_3(L) = 0.0032. \tag{9}$$

It can be seen that more than 97% of the displacement amplitude at the tip is from the contribution of the first mode. This is because the distributed force due to the inertia, which acts only in one direction, is linearly proportional to x and so predominantly excites the first mode. For translational motion, the contribution of the first mode is slightly less, but the first mode still contributes to more than 90% of the total response. Thus, this paper focuses on the control of this mode only. In the following section, two methods for suppressing residual vibrations of a single-degree-of-freedom system are discussed.

3. Control of residual vibration using the transient response method

As mentioned earlier, the methods described in this section are closely related to the SRS [11,12]. From the SRS of a single-degree-of-freedom system it is possible to determine the shock duration such that the residual amplitude of the SRS is zero for a given shock pulse shape. Because the first mode of a rotating beam is of interest, a single-degree-of-freedom system subject to base acceleration as shown in Fig 4 is considered.

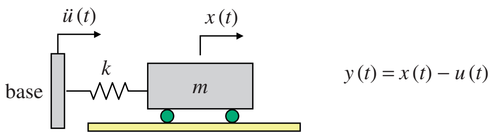

Fig. 4. Base acceleration of a simple oscillator.

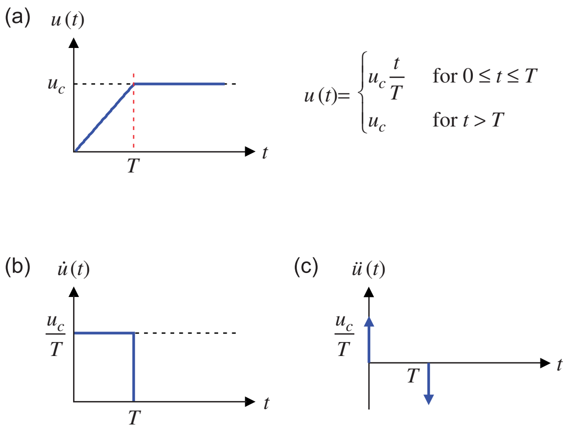

Fig. 5. (a) Displacement profile, (b) velocity profile, and (c) acceleration profile of the base motion of the system in Fig. 4.

The equation of motion is given by

$$\ddot{y}(t) + \omega_n^2 y(t) = -\ddot{u}(t). \tag{10}$$

Now the problem is to suppress the residual vibration of $y(t)$ after the base moves to a desired position and stops. For zero initial conditions, the solution to Eq. (10) is

$$y(t) = -\frac{1}{\omega_n}\int_0^t \ddot{u}(\tau)\sin\,\omega_n(t-\tau)\,\mathrm{d}\tau. \tag{11}$$

The problem is to determine a particular pulse that ensures that the residual amplitude of the relative response $y(t)$ is zero. Assuming that the base moves from rest, the base displacement $u(t)$ satisfies

$$u(0) = 0 \quad \text{and} \quad u(t \geqslant T) = u_c, \tag{12}$$

where u_c is the desired position for the tip, and T the time at which this is to be achieved. Since it is impossible to make $u(0) = u_c$, the *simplest* form of $u(t)$ as it moves from rest to the desired position could be linear motion as shown in Fig. 5a. The corresponding velocity profile is a constant velocity pulse as shown in Fig. 5b, and the acceleration profile consists of two spikes as shown in Fig. 5c.

Note that the two spikes of the acceleration profile have opposite signs, i.e.,

$$\ddot{u}(t) = \frac{u_c}{T}[\delta(t) - \delta(t-T)], \tag{13}$$

where $\delta(t)$ is the delta function. Also, note that the area under the acceleration profile must be *zero* over the time T since the corresponding velocity profile is pulse-like, i.e., $\dot{u}(0) = \dot{u}(T)$. This is due to the fact that, for the base to stop moving at u_c, the base must decelerate as much as it accelerates. Thus, the following must be satisfied:

$$\int_0^T \ddot{u}(t)\,\mathrm{d}t = 0. \tag{14}$$

Moreover, the area of the velocity pulse must be equal to the desired displacement; thus, $u(T) = u_c$, i.e.,

$$u(T) = \int_0^T \dot{u}(t)\,\mathrm{d}t = u_c. \tag{15}$$

Now consider the response $y(t)$ to the above acceleration profile. Substituting Eq. (13) into Eq. (11) and simplifying gives

$$y(t) = -\frac{u_c}{\omega_n T}\left(\sin\frac{2\pi}{T_n}t - \sin\left(\frac{2\pi}{T_n}t - \frac{2\pi T}{T_n}\right)\right), \quad t \geqslant T, \tag{16}$$

where $T_n = 2\pi/\omega_n$. Examining Eq. (16), it can be seen that for $t \geqslant T$ the response, $y(t)$, becomes zero when $T = kT_n$, where 'k' is a positive integer. Thus, for a constant velocity pulse, there is no residual vibration if the desired time is equal to an integer multiple of the natural period. This result may be considered as a generalisation of the well-known fact that can be found in a number of fundamental vibration texts; that is a rectangular force pulse results in zero residual vibration if the duration of the pulse is equal to the natural period of the undamped system.

The above results may also be interpreted in the frequency domain by considering the Fourier transform of the velocity profile, $FT[\dot{u}(t)]$. In this example, the velocity pulse is a rectangular pulse of duration T, and the 'zero crossing points' of the $FT[\dot{u}(t)]$ occur at frequencies equal to k/T. Thus, the system is not excited at the natural frequency if $T = kT_n$. Using this frequency domain interpretation, it can be seen that Fourier transforms of the input displacement or acceleration profile can also be used. However, the velocity profile may be the best to consider because it is always pulse-like, and thus it is much easier to find the zero crossing points.

Since the residual vibration is related to the velocity excitation pulse, any shape of velocity profile that satisfies the two conditions given in Eqs. (14) and (15) can be used. For example, a half-sine velocity excitation profile as shown in Fig. 6b can be used. The corresponding displacement profile and acceleration profiles are shown in Fig. 6a and c.

Note that the above pulses satisfy the two conditions. In this case, the residual vibration is zero if the desired time is

$$T = (k + 0.5)T_n \quad (k \text{ a positive integer}). \tag{17}$$

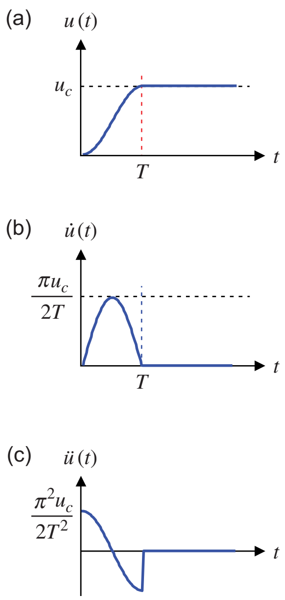

Fig. 6. (a) Displacement profile, (b) velocity profile, and (c) acceleration profile for a half-sine velocity pulse.

The procedure to ensure zero residual vibrations is summarised below:

1. Decide on the desired position u_c (e.g., a rotation angle).
2. Decide on any velocity pulse $\dot{u}(t)$ (rectangular, half-sine, cycloid, etc.).
3. Determine an appropriate pulse duration T for zero residual vibrations using the SRS or $FT[\dot{u}(t)]$.
4. Adjust the amplitude of $\dot{u}(t)$ such that

$$u(T) = \int_0^T \dot{u}(t)\,\mathrm{d}t = u_c.$$

Note that the condition $\int_0^T \ddot{u}(t)\,\mathrm{d}t = 0$ is satisfied automatically because $\dot{u}(0) = \dot{u}(T)$.

If the natural frequency does not match exactly with a zero in the SRS or $FT[\dot{u}(t)]$, some residual vibration will be excited. The largest amplitude of residual vibration is generated by a rectangular velocity pulse excitation. A half-sine excitation velocity pulse will result in a lower residual vibration amplitude, but this is at the expense of increasing the pulse duration (i.e., $T = 1.5T_n$ rather than $T = T_n$ for the rectangular pulse). In general, there is a trade-off between the *robustness* and the *desired time* (pulse duration) for control. For example, if the natural period is estimated 1.5 times the true natural period and if the rectangular pulse is used to minimise the desired time, then the maximum amplitude of the residual vibration is more than 6 times greater than the case if the half-sine pulse is used. This can be easily inspected by consulting SRS graphs of various pulses [12]. Note that, by examining the SRS, the residual vibration amplitude, the maximum amplitude of the response and the robustness can be determined at the same time.

So far, 'damping' of the system has not been considered. If a damped system is considered, the equation of motion is given by

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = -\ddot{u}(t) \tag{18}$$

and the solution for $y(t)$ is

$$y(t) = -\frac{1}{\omega_d}\int_0^t \ddot{u}(\tau)\mathrm{e}^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t-\tau)\,\mathrm{d}\tau, \tag{19}$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency. In this case, it is not possible to completely suppress the residual vibration using the method above. However, *minimal* residual vibration can be achieved by replacing T_n with the damped natural period, $T_d = 2\pi/\omega_d$. For example, using the rectangular velocity pulse, the desired time is $T = kT_d$.

In applications where the damping is very small, the method outlined above may be sufficient. However, when the damping is significant the method must be modified to take into account the decrease in amplitude due to the damping. In this case, zero residual vibration can be achieved by *splitting the pulse into two pulses with the same shape but with different amplitudes*, and adjusting the timing of the second pulse so that its response is exactly out of phase to the response due to the first pulse.

For this method, *any shape of pulse with any desired time T* that satisfies the two conditions given in Eqs. (14) and (15) is permitted, provided that the second pulse starts at a time $kT_d/2$ (k an odd positive integer). Consider two consecutive acceleration pulses $\ddot{u}_p$ scaled by A_1 and A_2 as shown in Fig. 7, where the second pulse starts at $T_d/2$ (i.e., $k = 1$ is used for simplicity).

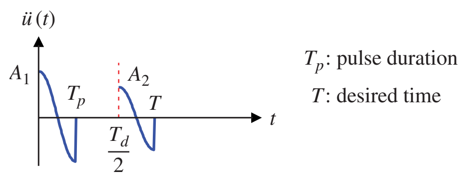

Fig. 7. Two consecutive acceleration pulses.

As it can be seen in Fig. 7, the acceleration profile is

$$\ddot{u}(t) = A_1\ddot{u}_p(t) + A_2\ddot{u}_p(t - T_d/2) \tag{20}$$

and so the response $y(t)$ for $t \geqslant T(= T_d/2 + T_p)$ is the sum of responses to two pulses, i.e.,

$$y(t) = y_1(t) + y_2(t), \tag{21}$$

where

$$\begin{aligned} y_1(t) &= -\frac{A_1}{\omega_d}e^{-\zeta\omega_n t}\int_0^{T_p}\ddot{u}_p(\tau)e^{\zeta\omega_n\tau}\sin\omega_d(t-\tau)\,d\tau, \\ y_2(t) &= -\frac{A_2}{\omega_d}e^{-\zeta\omega_n t}\int_{T_d/2}^{T_d/2+T_p}\ddot{u}_p(\tau - T_d/2)e^{\zeta\omega_n\tau}\sin\omega_d(t-\tau)\,d\tau. \end{aligned} \tag{22a, b}$$

Letting $\tau' = \tau - T_d/2$ for the second convolution integral, the response to the second pulse becomes

$$y_2(t) = \frac{A_2}{\omega_d}e^{-\zeta\omega_n t}e^{\zeta\omega_n T_d/2}\int_0^{T_p}\ddot{u}_p(\tau')e^{\zeta\omega_n\tau'}\sin\omega_d(t-\tau')\,d\tau'. \tag{23}$$

Note that the signs of $y_1(t)$ and $y_2(t)$ are opposite and the integral parts of $y_1(t)$ and $y_2(t)$ are similar. As a result, the total response $y(t)$ for $t \geqslant T$ can be set to zero if the amplitude of each pulse is scaled such that

$$\frac{A_1}{A_2} = e^{\zeta\omega_n T_d/2} = e^{\delta/2}, \tag{24}$$

where $\delta = \zeta\omega_n T_d$ is the logarithmic decrement. As an example of this result, consider a velocity input profile as in Fig. 8(a) to a damped system with $\zeta = 0.3$ and $\omega_n = 2\pi$. Then the displacement response $y(t)$ is as in Fig. 8(b), where it can be seen that $y(t) = 0$ for $t \geqslant T$.

If the second pulse starts at time $kT_d/2$ rather than at $T_d/2$, then the amplitude ratio between two pulses is $A_1/A_2 = e^{k\delta/2}$. This amplitude relationship is also valid for the velocity profiles, i.e.,

$$\dot{u}(t) = A_1\dot{u}_p(t) + A_2\dot{u}_p(t - kt_d/2), \tag{25}$$

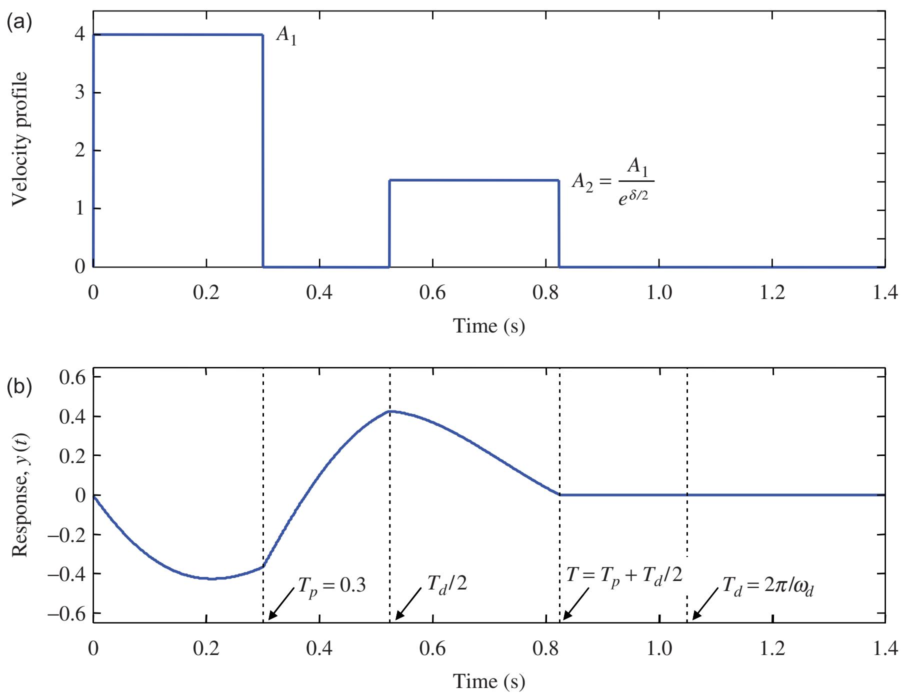

Fig. 8. (a) A sequence of input velocity pulses and (b) displacement response of a damped system.

where $A_1/A_2 = e^{k\delta/2}$. To calculate the amplitudes A_1 and A_2 for the velocity pulse, the condition given in Eq. (15) needs to be considered. In this case, it is

$$u(T) = u(T_P + kT_d/2) = \int_0^{T_P + kT_d/2} \dot{u}(t)\,\mathrm{d}t = u_c. \tag{26}$$

Because the duration of each pulse is the same, it follows that

$$\int_0^{T_p + kT_d/2} \dot{u}(t)\,\mathrm{d}t = (A_1 + A_2)\int_0^{T_p} \dot{u}_p\,\mathrm{d}t = u_c. \tag{27}$$

Noting that $A_1/A_2 = e^{k\delta/2}$, and if the integral term is set to unity for convenience, i.e., $\int_0^{T_P} \dot{u}_p\,\mathrm{d}t = 1$, the desired amplitudes are given by

$$A_1 = \frac{u_c e^{k\delta/2}}{1 + e^{k\delta/2}}, \quad A_2 = \frac{u_c}{1 + e^{k\delta/2}}. \tag{28}$$

For an *undamped* system $A_1 = A_2 = u_c/2$. The procedure to ensure zero residual vibrations for a heavily damped system is summarised below:

1. Decide on the desired position u_c.
2. Decide on the pulse duration T_p, and determine the desired time $T = T_p + kT_d/2$ (k an odd positive integer). (Note that the minimum desired time is $T = T_p + T_d/2$ for $k = 1$.)
3. Choose any velocity pulse (rectangular, half-sine, cycloid, etc.), and normalise the pulse such that $\int_0^{T_p} \dot{u}_p\,\mathrm{d}t = 1$.
4. Calculate the amplitudes of the velocity pulses using Eq. (28).
5. Obtain the velocity profile consisting of two pulses as in Eq. (25).

Note that this method can also be applied to an undamped system by replacing T_d with T_n. If $k = 1$ is used, the results are identical to the input-shaping method [5], which is described in Appendix A. However, an advantage of using this method is that one can control *the position and the desired time simultaneously provided that* $T > T_d/2$.

An example follows to illustrate the approach. Suppose we wish to rotate a flexible beam to the desired angle u_c as quickly as possible, before one period of the fundamental natural frequency (i.e., $T_d/2 < T < T_d$). For simplicity, $\zeta = 0$ and a rectangular velocity pulse is used as shown in Fig. 9.

For an undamped system, the amplitude of the two velocity pulses is the same, i.e., $A_1 = A_2 = u_c/2$. Thus, the velocity profile becomes

$$\dot{u}(t) = \frac{u_c}{2}[\dot{u}_p(t) + \dot{u}_p(t - T_d/2)]. \tag{29}$$

This velocity profile is depicted in Fig. 10(b), where the desired time T is $T_n/2 + T_p$; it will give the *zero* residual vibration for $t \geqslant T_n/2 + T_p$. From the figure, it is easily seen that the velocity profile satisfies the condition

$$\int_0^{T_n/2 + T_p} \dot{u}(t)\,\mathrm{d}t = u_c.$$

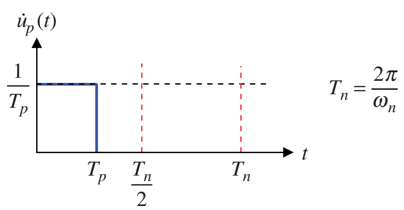

Fig. 9. Normalised rectangular velocity pulse.

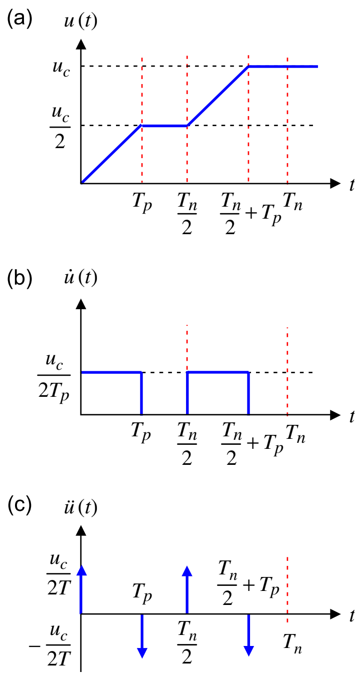

Fig. 10. (a) Displacement profile, (b) velocity profile, and (c) acceleration profile for a desired velocity profile.

The corresponding displacement and acceleration profiles are shown in Fig. 10(a) and (c), respectively. Note that the acceleration profile satisfies the condition $\int_0^{T_n/2+T_p} \ddot{u}(t)\,\mathrm{d}t = 0$ given in Eq. (15).

From the example given above, it can be seen that the proposed method to control residual vibration is potentially very effective and simple to use. In practice, a rectangular velocity pulse may not be appropriate due to the discontinuity in velocity that would be practically difficult for the hub to follow. Thus, a half-sine, versed-sine or trapezoidal shape of velocity pulse may be used although the use of these profiles would result in an increase of the time taken to move the beam to its desired position.

4. Conclusions

Two simple methods have been proposed to suppress the residual vibrations of a translating or rotating flexible cantilever beam excited as it is moved from one position to another. The methods are based on the transient response of the system, i.e., the SRS, and do not require any filtering processes or control algorithm. Once an appropriate velocity profile has been chosen by considering the SRS or the Fourier transform of the velocity pulse, the methods can be applied in a straightforward manner. The first method is the easiest to apply and is generally sufficient for most lightly damped structures, while the second method is more generally applicable. It is demonstrated that the second method produces an identical result to the input shaping method if the desired time is chosen as $T = T_p + T_d/2$. However, in the approach proposed here it gives more insight to the dynamical behaviour and is much simpler than the input shaping method. Moreover, the desired time and the position can be specified simultaneously, which may be beneficial in many practical problems. The proposed methods may be extended to a multi-degree-of-freedom system, although the analysis of transient response becomes very complicated. However, for most applications, where the structure is excited by an inertial force, the single-degree-of-freedom approximation may be sufficient.

Acknowledgements

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Appendix A. Relationship between the transient response method and the input shaping method

Consider the conceptual diagram of the input shaping method shown in Fig. 1 that depicts the relationship between the command input and the shaped input. For a single-degree-of-freedom system, the simplest form of input shaping filter can be realised with a two-impulse sequence as shown in Fig. A1. In this figure, A_1, A_2, and t_1 are given by [6]

$$A_1 = \frac{1}{1+E}, \quad A_2 = \frac{E}{1+E}, \quad t_1 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \tag{A.1}$$

where $E = \mathrm{e}^{-\zeta\pi/\sqrt{1-\zeta^2}}$.

Note that the values of A_1, A_2 in Eq. (A.1) are the same as those in Eq. (28) if $k = 1$ and $u_c = 1$. Also, t_1 becomes $T_d/2$ in this case. The shaped input is the convolution of the command input and the input shaping filter. To demonstrate the relationship between this method and the transient response method, consider the example shown in Section 3 (undamped system). Then A_1, A_2, and t_1 are given by

$$A_1 = \frac{1}{2}, \quad A_2 = \frac{1}{2}, \quad t_1 = \frac{T_n}{2}. \tag{A.2}$$

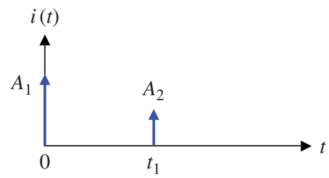

Fig. A1. Input shaping filter.

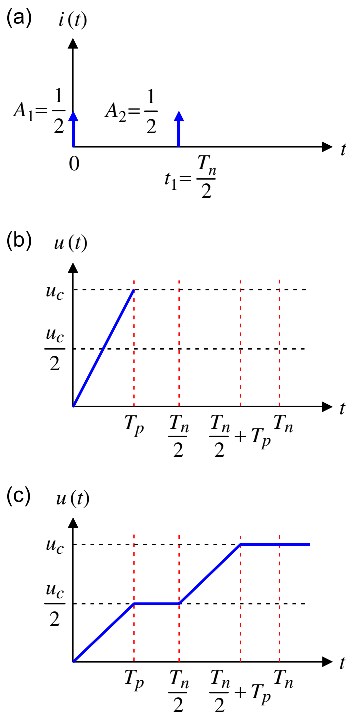

Fig. A2. (a) Input shaping filter of an undamped system, (b) command input, and (c) shaped input.

Thus, the input shaping filter can be depicted as in Fig. A2(a). If the command input is the straight line displacement profile, $u(t)$ as in Fig. A2(b), the shaped input, which is the convolution of Fig. A2(a) and (b), becomes as in Fig. A2(c). Note that Fig. A2(c) is exactly the same as Fig. 10(a).

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