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Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network

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Abstract

In this study, a combination of genetic algorithm (GA) and neural networks (NN) is proposed to find the optimal weight of structures subject to multiple natural frequency constraints. The optimization is carried out by an evolutionary algorithm using discrete design variables. The evolutionary algorithm employed in this investigation is virtual sub-population (VSP) method. To reduce the computational time of optimization process, the natural frequencies of structures are evaluated using properly trained radial basis function (RBF) and wavelet radial basis function (WRBF) neural networks. In the WRBF neural network, the activation function of hidden layer neurons is substituted with a type of wavelet functions. In this new network, the position and dilation of the wavelet are fixed and only the weights are optimized. The numerical results demonstrate the robustness and high performance of the suggested methods for structural optimization with frequency constraints. It is found that the best results are obtained by VSP method using WRBF network.

1. Introduction

It is well known that the natural frequencies are fundamental parameters affecting the dynamic behaviour of structures. Therefore, some limitations should be imposed on the natural frequency range to reduce the domain of vibration and also prevention of the resonance phenomenon in dynamic response of structures. Traditionally, the structure is iteratively analysed and designed to achieve this purpose. Therefore, a preliminary set of cross-sectional properties is assumed and then, the structural analysis is performed. If the demands of the design specifications are satisfied, the assumed sections are adopted. Otherwise, the cross-sectional properties are modified and the structure is reanalysed. The structure thus designed by trail and error is feasible but not necessarily optimal. Moreover, this trail-and-error procedure is very tedious. The process can be easily and reliably implemented using optimization techniques. In recent years, much progress has been made in optimal design of structures subject to stress, displacement and frequency constraints. They have mostly employed the conventional and traditional methods for constraints approximation and optimization [1–5]. These methods usually employ derivative calculations and may converge to a local optima.

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In this study, an efficient method is presented to find the optimal design of structures with multiple natural frequency constraints utilizing an evolutionary algorithm. In the optimization process, the weight of the structures is considered as the objective function. The design variables are cross-sectional areas of structural elements and the design constraints are taken as the multiple natural frequencies. The standard genetic algorithm (GA) is not efficient for optimization problems with large number of design variables. On the other hand, the virtual sub-population (VSP) method offers a robust tool for this purpose [6]. In this method, all the necessary mathematical models of the natural evolution are implemented on a small initial population to access the optimal solution in an iterative manner. The stochastic nature of the evolutionary algorithms makes the convergence of the process slow. Furthermore, evaluation of the natural frequencies of structures using finite element method during optimization process can be computationally intensive with slow convergence. Because of the above considerations, the natural frequencies of structures are evaluated in the present study using properly trained radial basis function (RBF) and wavelet radial basis function (WRBF) neural networks. Further, the activation function of hidden layer neurons of the WRBF is substituted with a specific kind of wavelet functions. By the proposed method, evaluating the eigenvalues is very fast as compared to the exact analytical methods; thus, employing the properly trained neural network in the process of optimization makes the process very efficient.

This is the first study based on application of WRBF for identifying the natural frequencies of structural systems. The other types of neural networks are employed by the researchers for the structural system identification [7–12]. The specific contribution of the present study is to develop the proposed WRBF neural network-based method for efficient evaluating the natural frequencies of the trial structural design. However, the combination of the VSP and the WRBF for efficient structural optimization is the other important goal of the study.

Two illustrative problems are solved to assess the robustness and efficiency of the suggested method for structural optimization subject to frequency constraints.

2. Formulation of optimization problem

In sizing optimization problems, the aim is usually to minimize the weight of the structure, under some constraints on stresses, displacements and frequencies. A discrete structural optimization problem can be formulated in the following form:

$$\begin{aligned} &\text{Minimize} && f(\mathbf{X}), \\ &\text{Subject to} && g_i(\mathbf{X}) \leqslant 0, \quad i = 1, 2, \ldots, m, \\ &&& X_j \in R^d, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $f(\mathbf{X})$ represents objective function, $g(\mathbf{X})$ is the behavioural constraint, m and n are the number of constraints and design variables, respectively. A given set of discrete values is expressed by R^d and design variables X_j can take values only from this set.

In this paper, objective function is taken as

$$f(\mathbf{X}) = \sum_{i=1}^{\mathrm{ne}} \rho_i X_i l_i \tag{2}$$

and constraints are chosen to be natural frequencies of trial structures:

$$g_i(\mathbf{X}) = \frac{\lambda_i}{\lambda_{\mathrm{all}}} - 1 \leqslant 0, \quad i = 1, 2, \ldots, m, \tag{3}$$

where ρ_i and l_i are weight of unit volume and length of ith element, respectively, ne is the number of the structural elements, λ_i and λ_{all} are the ith frequency and allowable frequency, respectively.

Further, constraints are handled by using the concept of penalty functions, i.e.,

$$f(\mathbf{X}) = \begin{cases} f(\mathbf{X}) & \text{if } \mathbf{X} \in \tilde{\Delta}, \\ f(\mathbf{X}) + f_p(\mathbf{X}) & \text{otherwise,} \end{cases} \tag{4}$$

where $f_p(\mathbf{X})$ is penalty function. Further, $\tilde{\Delta}$ denotes the feasible search space.

A simple form of penalty function is employed as

$$f_p(\mathbf{X}) = r_p \sum_{i=1}^{m} \left(\max(g_i(\mathbf{X}), 0)\right)^2, \tag{5}$$

where r_p is an adjusting coefficient.

As we employ an improved evolutionary algorithm for optimization, the main concepts of the algorithm are briefly explained in Section 3.

3. Virtual sub-population evolutionary algorithm

In structural optimization problems, the objective function and the constraints are highly nonlinear functions of the design variables. Hence, the computational effort for gradient calculations required by the mathematical programming algorithms is usually large. In recent years, it was found that probabilistic search algorithms are relatively computationally efficient even if greater number of optimization cycles is needed to reach the optimal point [13]. These cycles are computationally less expensive in comparison with mathematical programming algorithms, because they need no gradient evaluation. Furthermore, probabilistic methodologies were found to be more robust in finding the global optimum due to their random search, whereas mathematical programming algorithms may be trapped into local optima.

In the field of evolutionary algorithms, GA has been widely used in the recent decade. However, the stochastic nature of GA makes the convergence of the procedure slow. In particular, structural optimization with a great number of degrees of freedom is very time consuming. In general, the standard GA is not convenient to find the solution of problems with huge number of design variables. To overcome this shortcoming of GA and reduce the computational rigour of the method, an improved GA is utilized in this paper. In this modified GA, an initial population with a small number of individuals is selected. As a result the population is much smaller than that in standard GA. Using the reduced initial population, all the necessary operations of the standard GA are carried out and the optimal solution is achieved. As the size of the population is not adequate, the method converges to a pre-mature solution. In each generation, individual with a lower value of supplemental function satisfying the design constraints is saved. Then, the best solution is chosen and repeatedly copied to create a new population. In the new population, the majority of the individuals are the best-repeated solution of the previous results. The remaining members of the population are randomly selected. Thereafter, the optimization process is repeated using standard GA with a reduced population to achieve a new solution. The process of creating the reduced population with repeated individuals in each iterations is continued until the method converges. These reduced populations are called virtual sub-populations (VSPs) and the optimization process with VSPs is called VSP method. As demonstrated in Ref. [6], the computational effort by VSP is less in comparison to the standard GA.

In order to make the optimal design process more efficient, neural networks are employed to identify the necessary parameters of the trial structural design.

4. Wavelet RBF (WRBF) neural network

There are many types of neural networks,which are broadly utilized in civil and structural engineering applications [14–19]. One of the most popular neural networks is RBF [20] neural network. RBFs take an approach by viewing the design of neural networks as a curve-fitting problem by finding a best fit to the training data in a multidimensional space [21]. RBF neural network is a two layers feed forward network in which the hidden layer consists of RBF neurons with Gaussian activation functions [20]. RBF neural networks are widely used in the field of structural engineering due to their fast training, performance generality and simplicity [22,23].

A new mapping neural network called wavelet neural network (WNN) or wavenet proposed as an alternative to feed forward neural networks in order to approximate arbitrary nonlinear functions. A brief description of wavelet theory is presented as follows.

4.1. Fundamentals of wavelet theory

Wavelet theory is the outcome of multidisciplinary endeavours that brought together mathematicians, physicists, and engineers. This relationship creates a flow of ideas that goes well beyond the construction of new bases or transforms. The term wavelet means a little wave. A function $h \in L^2(R)$ (the set of all square integrable or a finite energy function) is called a wavelet if it has zero average on $(-\infty, +\infty)$ [24]:

$$\int_{-\infty}^{+\infty} h(t)\,\mathrm{d}t = 0. \tag{6}$$

This wavelet must have at least a minimum oscillation and a fast decay to zero in both the positive and negative directions of its amplitude. These three properties are the Grossmann–Morlet admissibility conditions of a function that is required for the wavelet transform. The wavelet transform is an operation, which transforms a function by integrating it with modified versions of some kernel functions. The kernel function is called the mother wavelet and the modified version is its daughter wavelet. A function $h \in L^2(R)$ is admissible if

$$c_h = \int_{-\infty}^{+\infty} \frac{|H(\omega)|^2}{|\omega|}\,\mathrm{d}\omega < \infty, \tag{7}$$

where $H(\omega)$ is the Fourier transform of $h(t)$. The constant c_h is the admissibility constant of the function $h(t)$. For a given $h(t)$, the condition $c_h < \infty$ holds only if $H(0) = 0$. The wavelet transform of a function $k \in L^2(R)$ with respect to a given admissible mother wavelet $h(t)$ is defined as

$$W_k(a,b) = \int_{-\infty}^{+\infty} k(t) h^*_{a,b}(t)\,\mathrm{d}t, \tag{8}$$

where * denotes the complex conjugate. However, most wavelets are real valued.

Sets of wavelets are employed for approximation of a signal and the goal is to find a set of daughter wavelets constructed by dilated and translated original wavelets or mother wavelets that best represent the signal. The daughter wavelets are generated from a single mother wavelet $h(t)$ by dilation and translation as follows:

$$h_{a,b}(t) = \frac{1}{\sqrt{a}} h\left(\frac{t-b}{a}\right), \tag{9}$$

where $a>0$ is the dilation factor and b the translation factor. The constant term of $1/\sqrt{a}$ is for energy normalization, which keeps the energy of the daughter wavelet equal to the energy of the original mother wavelet [24].

The combination of the wavelet transforms theory with the basic concept of RBF neural networks leads to a new network.

4.2. Designing of WRBF

The WNN use wavelets as activation functions of hidden layer neurons. Neural networks construction methods can be developed using theoretical features of the wavelet transform. These methods help to determine the neural networks parameters during the training process. In wavelet networks, both the position and dilation of the wavelets are optimized besides the weights.

There are different approaches to construct wavelet networks. In one approach, the position and dilation of the wavelets taken to be fixed and only the weights of network are optimized [21]. The key issues in design of wavelet networks are determination of the network structure and the learning algorithm that can be effectively

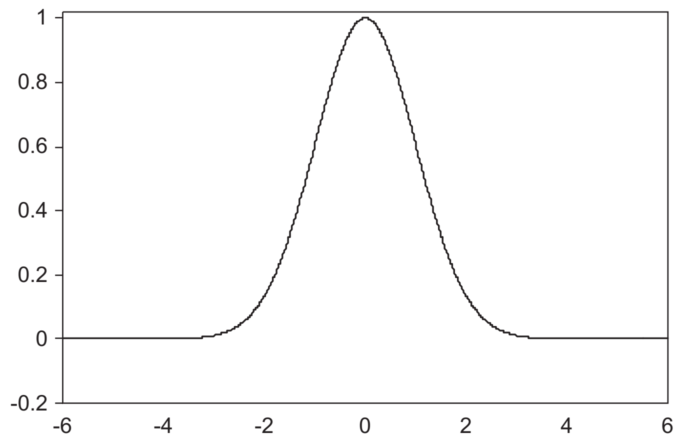

Fig. 1. Activation function of the RBF neurons.

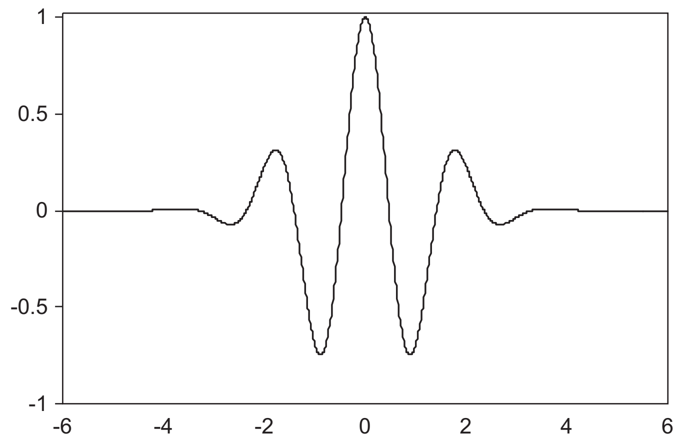

Fig. 2. Cosine-Gaussian Morlet wavelet with $\omega_0 = 4$.

used for training of the network. In this study, the above-mentioned approach is employed to design a WNN using RBF network topology and its training method.

In standard RBF neural networks, activation function of hidden layer neurons is a Gaussian kernel function shown in Fig. 1. In this study, in order to increase the performance generality of RBF neural network, the RBF neurons activation function is substituted with the Morlet's basic wavelet function.

The Morlet's basic wavelet function is a multiplication of the Fourier basis with a Gaussian window [25]:

$$h(t) = \exp(\mathrm{j}\omega_0 t)\exp(-0.5t^2), \tag{10}$$

$$h(t) = [\cos(\omega_0 t) + \mathrm{j}\ \sin(\omega_0 t)]\exp(-0.5t^2), \tag{11}$$

where the real part is a cosine-Gaussian and the imaginary part is a sine-Gaussian function. The cosine-Gaussian wavelet is a real even function. Fig. 2 provides the plot of the cosine-Gaussian Morlet wavelet, with $\omega_0 = 4$, which does not satisfy the wavelet admissibility condition, because

$$H(0) = \sqrt{2\pi}\ \exp(-0.5\omega_0^2) \neq 0, \tag{12}$$

which leads to $c_h = +\infty$. However, if ω_0 is sufficiently large, say $\omega_0 = 4$, $H(0)$ becomes very close to zero and practically is considered to be zero in numerical computations.

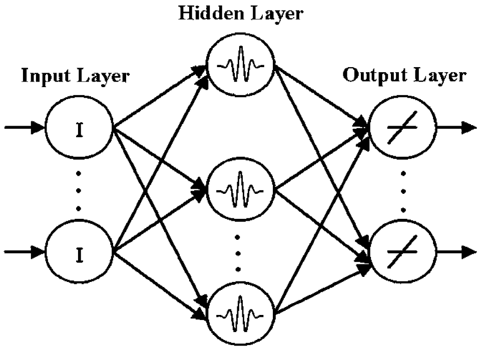

Fig. 3. Typical topology of WRBF networks.

In this study, the cosine-Gaussian Morlet wavelet with $\omega_0 = 4$ is employed as the activation function of RBF neurons. The resulted WRBF is trained by the same method employed for training the RBF network. With this training method, the location of each WRBF neuron is determined exactly in the input space. Therefore, the value of translation factor of hidden layer wavelet activation functions should be set to zero ($b = 0$). A simple procedure is used for determining the value of dilation factor of WRBF neurons. The WRBF network is trained using different values of dilation factors and the performance generality of the network is evaluated. The value that results in the highest performance generality is selected as optimal dilation factor of WRBF neurons. Because the training process of WRBF is implemented quickly, determination of optimal dilation factor is accomplished spending trivial effort. In this study, the best performance generality is found with $a = 4.5$ and consequently, WRBF neurons activation function is cosine-Gaussian Morlet daughter wavelet as follows:

$$\phi_i(\mathbf{Z}) = \frac{1}{\sqrt{4.5}} \cos(4(\mathbf{Z}/4.5)) \exp(-0.5(\mathbf{Z}/4.5)^2), \tag{13}$$

$$\phi_i(\mathbf{Z}) = 0.4714 \cos(0.889\mathbf{Z}) \exp(-0.0247\mathbf{Z}^2), \tag{14}$$

where $\phi_i(\cdot)$ and $\mathbf{Z}$ are WRBF neurons activation function and input vector, respectively.

Typical topology of WRBF networks is shown in Fig. 3.

4.3. Main steps for training and testing of WRBF

The important steps in training and testing of WRBF are summarized as follows:

(a) A data set is generated and divided into the training and testing data sets.
(b) The simple Gaussian activation function of RBF neurons is substituted with cosine-Gaussian Morlet wavelet function with $\omega_0 = 4$ and $b = 0$.
(c) A random value is assigned to dilation factor of WRBF neurons.
(d) The WRBF network is trained using training data set.
(e) Performance generality of WRBF network is checked using testing data set.
(f) If performance generality is satisfactory, the training process is terminated; otherwise step (g) is performed.
(g) Another value is assigned to dilation factor of WRBF neurons.
(h) Steps (d)–(h) are repeated until the proper solution is met.

Numerical results indicate that the proposed WNN with RBF networks structure and the new hidden layer activation function is much better in comparison to the standard RBF in terms of performance generality.

5. Main steps of structural optimization

The main steps for the structural optimization with multiple frequency constraints by VSP method using the RBF and WRBF networks are summarized as follows:

(a) Selecting some parent vectors from the design variables space.
(b) Evaluating the natural frequencies of the structure employing trained RBF and WRBF networks.
(c) Evaluating the objective function.
(d) Checking the constraints for feasibility of parent vectors.
(e) Generating offspring vectors using selection, crossover and mutation operators.
(f) Employing the trained RBF and WRBF networks for predicting the natural frequencies of the offspring population.
(g) Evaluating the objective function.
(h) Checking the constraints, if satisfied continue, else change the vector and go to step (f).
(i) Checking convergence criteria; if satisfied stop, else go to step (e).

Table 1
Specifications of GA method

Population size	50
Crossover method	One, two, and three points crossover
Crossover rate	0.9
Mutation rate	0.001
Maximum generation	300

Table 2
Specifications of VSP method

Population size	30
Crossover method	One, two, and three points crossover
Crossover rate	0.9
Mutation rate	0.001
Maximum generation in each run	30

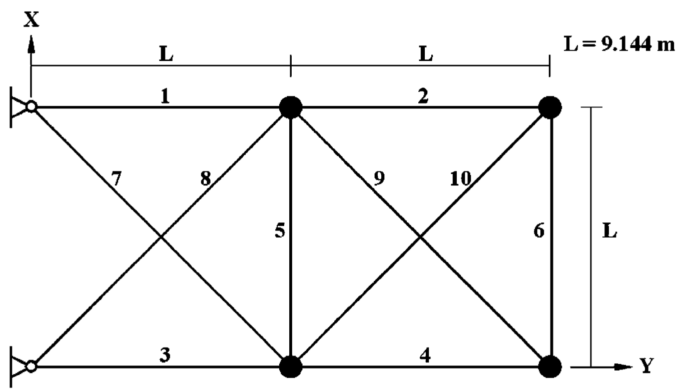

Fig. 4. 10-Bar aluminium truss.

Table 3
Available sections for Example 1

No.	Area (cm^2)
1	4.419
2	9.910
3	11.452
4	13.897
5	18.239
6	20.084
7	24.110
8	38.619

Table 4
Groups of the 10-bar steel truss elements

Group	Elements
1	1; 3
2	2
3	4
4	5; 6
5	7
6	8
7	9
8	10

Table 5
Information of training and testing of the networks for Example 1

Network	Training time (s)	Maximum errors (%)			Mean errors (%)		
		F_1	F_2	F_3	F_1	F_2	F_3
RBF	0.203	25.988	69.772	39.719	3.125	6.432	5.031
WRBF	0.205	13.815	22.351	23.696	2.091	3.286	2.980

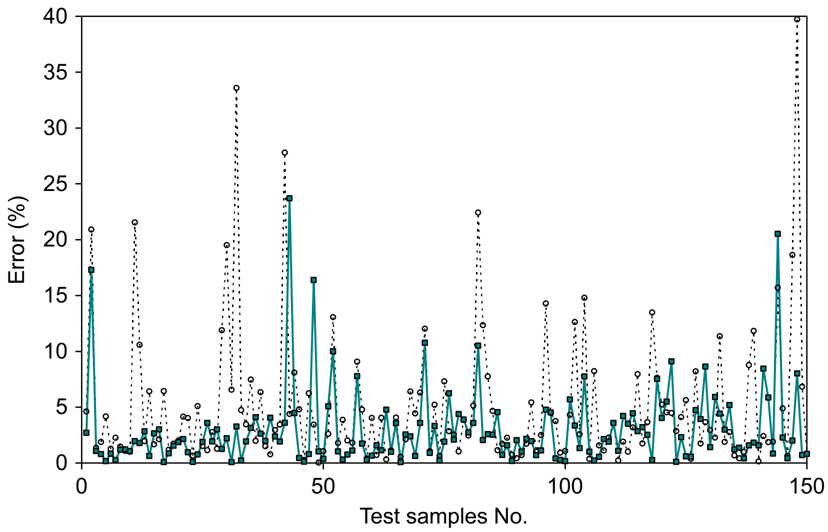

Fig. 5. Errors of the first approximate frequency. –■–, WRBF, ···○··, RBF.

(j) Selecting the majority parent vectors from the previous solution and some random design variables as a VSP.
(k) Repeating steps (c)–(k) until the proper solution is met.

As the size of populations in VSP method is small, the procedure converges rapidly. It can be observed that the modal analysis of the structures is not necessary during the optimization process.

6. Numerical results

In this study, two structures are selected as numerical examples for optimization. These structures are

(1) 10-bar aluminium truss and
(2) 200-bar steel double layer grid.

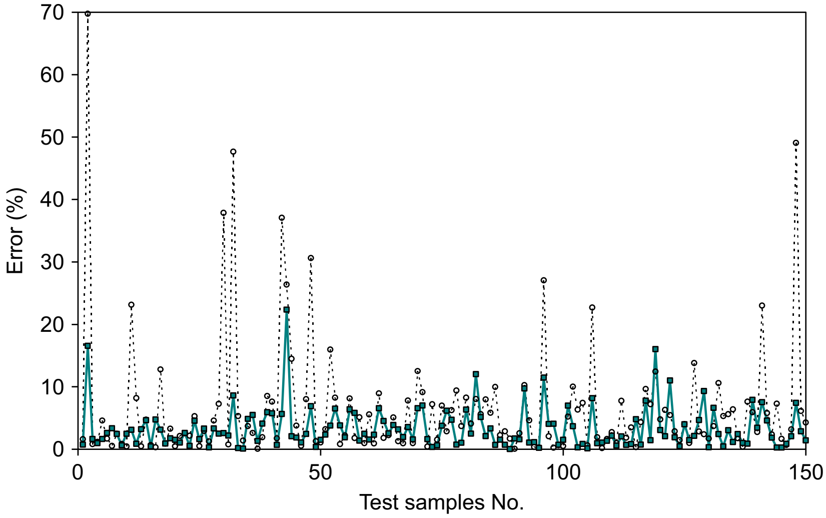

Fig. 6. Errors of the second approximate frequency. —■—, WRBF, ···○··, RBF.

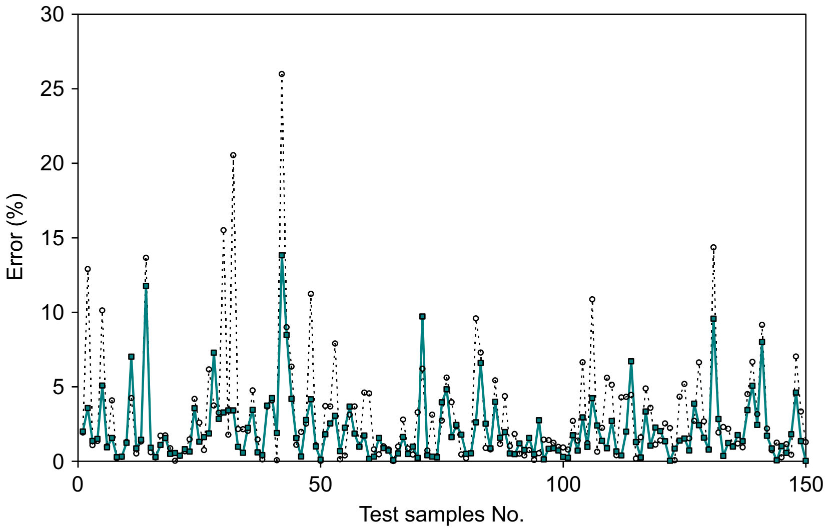

Fig. 7. Errors of the third approximate frequency. —■—, WRBF, ···○··, RBF.

Example 1 is taken from Ref. [1] for comparing the efficiency of the proposed and traditional methods. The other example is chosen arbitrarily. The optimization is carried out by the following methods:

(a) GA using exact analysis;
(b) GA using approximate analysis by RBF network;
(c) GA using approximate analysis by WRBF network;
(d) VSP using exact analysis;
(e) VSP using approximate analysis by RBF network; and
(f) VSP using approximate analysis by WRBF network.

The computational time is measured in terms CPU time required by a PC Pentium IV 3000 MHz. Also, the errors between exact and approximate frequencies are calculated using the following equation:

$$\text{error} = \frac{|\lambda_{\text{ap}} - \lambda_{\text{ex}}|}{\lambda_{\text{ex}}} \times 100, \tag{15}$$

where λ_{ap} and λ_{ex} represent the approximate and exact frequencies, respectively.

The specification of parameters for the GA and VSP methods are presented in Tables 1 and 2, respectively.

In this study, the input space consists of cross-sectional areas of the structural elements, while the corresponding natural frequencies of them are considered as the target space components. Therefore, in all the

Table 6
Optimal designs of the 10-bar truss obtained by the various methods

Variable no.	Optimal design (cm^2)					
	GA			VSP		
	Exact	RBF	WRBF	Exact	RBF	WRBF
1	38.619	38.619	38.619	38.619	38.619	38.619
2	11.452	24.110	18.239	18.239	18.239	13.897
3	24.110	13.897	13.897	9.910	9.910	20.084
4	4.419	4.419	4.419	4.419	4.419	4.419
5	20.084	18.239	20.084	24.110	20.084	20.084
6	24.110	24.110	20.084	20.084	24.110	20.084
7	13.897	11.452	9.910	11.452	18.239	11.452
8	11.452	13.897	20.084	13.897	9.910	13.897
Weight (kg)	557.03	556.62	550.58	538.27	548.29	538.61
Generations	98	86	93	72	60	56
Time (s)	15.1	14.2	15.0	11.6	7.2	6.5

Table 7
Comparison of approximate frequencies of optimal designs in terms of errors (%)

Frequency no.	GA		VSP	
	RBF	WRBF	RBF	WRBF
1	0.19	0.32	0.85	0.72
2	2.69	0.66	2.66	1.19
3	4.02	1.60	0.90	0.79

numerical examples, the number of the input vector components is equal to the number of the element groups, while the number of output vector components is equal to the number of the selected frequencies. MATLAB [26] is utilized for training and testing the neural networks while Lanczos [27] method is used for eigenvalue extraction.

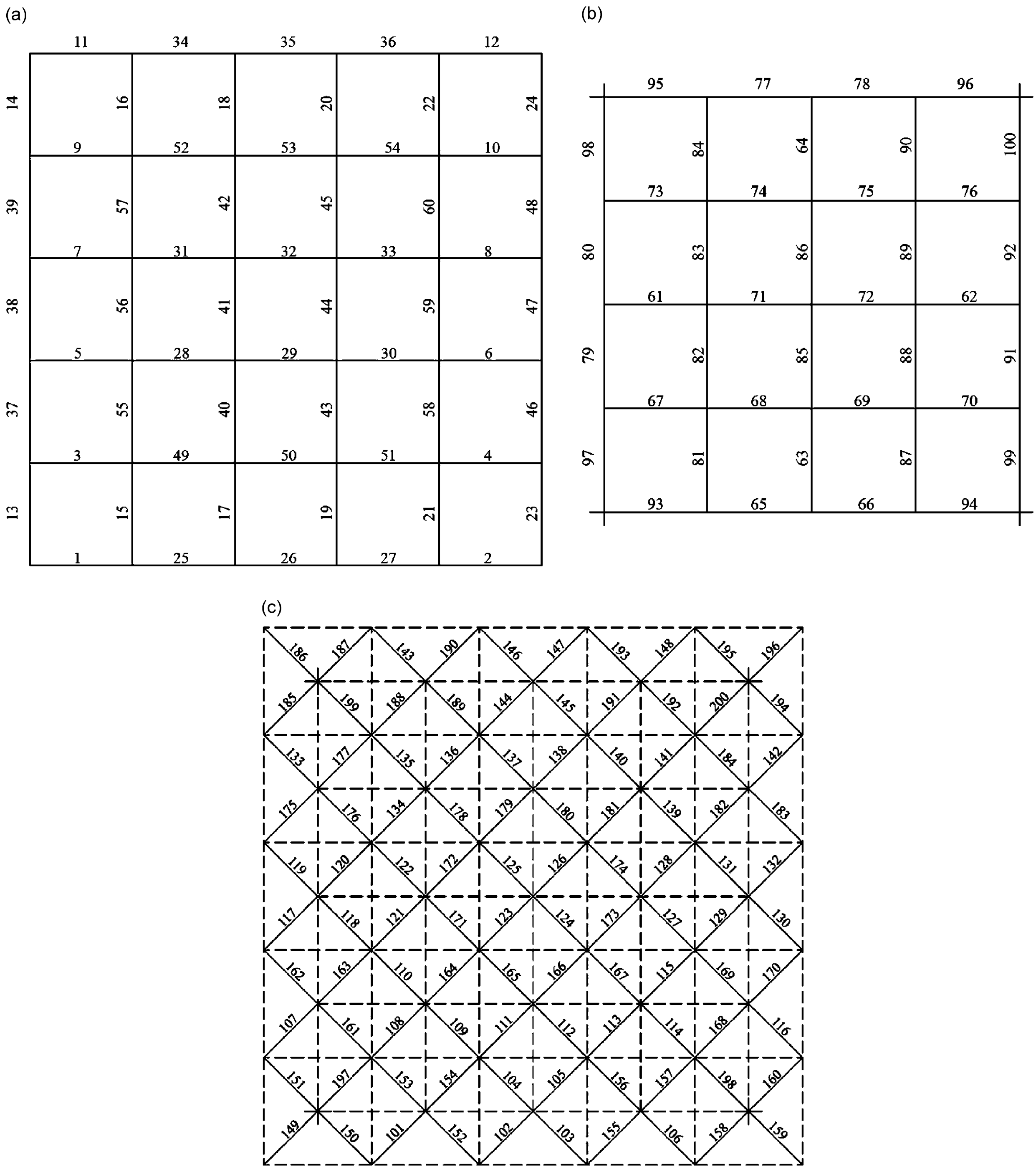

Fig. 8. Double layer grid: (a) top layer, (b) bottom layer, and (c) diagonal layer.

6.1. Example 1: 10-bar aluminium truss

The 10-bar aluminium truss, shown in Fig. 4, has been considered as the first illustrative example for optimization in this study. Cross-sectional areas of the elements, which are considered as the design variables, are selected from the available sections in Table 3. Modulus of elasticity and density are taken equal to $6.86 \times 10^{10}\,\mathrm{N/m^2}$ and $2770\,\mathrm{kg/m^3}$, respectively.

Mass of 454 kg is lumped at each free node. The multiple natural frequency constraints are considered as $\lambda_1 \geqslant 7\,\mathrm{Hz}$, $\lambda_2 \geqslant 15\,\mathrm{Hz}$, and $\lambda_3 \geqslant 20\,\mathrm{Hz}$.

For sake of simplicity and practicality, the truss elements are divided into 8 groups based on the cross-sections, as shown in Table 4.

6.1.1. Neural networks training and testing

The training and testing parameters of the neural networks used in this example are summarized in Table 5. The networks errors due to frequencies approximation in the testing mode are presented in Figs. 5–7. The number of samples in training and testing modes is 250 and 150, respectively.

Table 8
Available sections for Example 2

No.	Area ($\mathrm{cm^2}$)
1	1.213
2	2.540
3	3.733
4	4.534
5	5.229
6	6.669
7	10.670
8	11.810

Table 9
Groups of the double layer grid elements

Group	Elements
1	1–24; 61–64
2	65–92
3	93–100
4	25–48
5	49–60
6	101–148
7	149–196
8	197–200

Table 10
Information of training and testing of the networks for Example 2

Network	Training time (s)	Maximum errors (%)				Mean errors (%)			
		F_1	F_3	F_5	F_7	F_1	F_3	F_5	F_7
RBF	0.359	48.44	63.13	37.08	26.57	6.13	5.76	6.20	4.83
WRBF	0.365	7.61	8.29	12.76	7.99	1.59	1.57	2.31	1.89

The optimal solutions obtained by various methods employed are shown in Table 6. It can be noted from the table that the solutions found by the VSP method are more economical and the best solution is attained by the VSP method using WRBF network. The table also indicates that the number of generations, computational time and optimal weights are less for VSP method using WRBF network.

Table 7 presents the accuracy of approximate frequencies predicted by RBF and WRBF networks for optimum structures. As shown in the table, accuracy of the approximate frequencies computed by WRBF network is higher than RBF network. Based on the comparison with the results presented in Ref. [1], the combination of VSP method and neural networks provides a more reliable and powerful tool for structural optimization subject to multiple frequency constraints.

6.2. Example 2: 200-bar double layer grid

A 200-bar and 10 m × 10 m double layer grid with a height of 0.5 m is considered as the second example. The top, bottom and diagonal layers of the double layer grid are shown in Fig. 8. The structure is supported on the

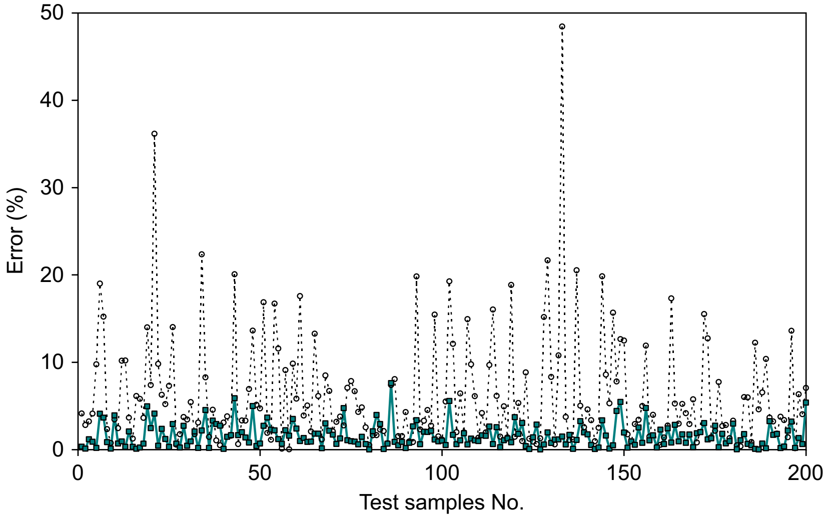

Fig. 9. Errors of the first approximate frequency. –■–, WRBF, ··○··, RBF.

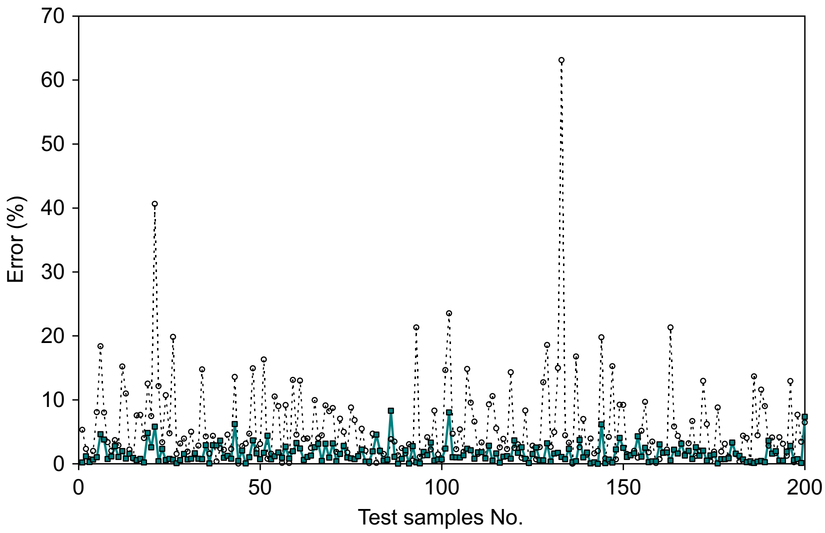

Fig. 10. Errors of the third approximate frequency. –■–, WRBF, ··○··, RBF.

corner nodes of the bottom layer. Cross-sectional areas of the elements are selected from the available sections in Table 8.

Owing to the practical considerations, the structural elements are grouped into 8 different types as shown in Table 9. Modulus of elasticity and density are taken equal to $2.1 \times 10^{11}\,\mathrm{N/m^2}$ and $7850\,\mathrm{kg/m^3}$, respectively. A mass of 19,620 kg is lumped at each free node of the top layer. To group the elements and also to select the frequency constraints, the double layer grid structure is optimized subject to gravity loads and with the natural frequencies considered as constraints in the example: $\lambda_1 \geqslant 3.5\,\mathrm{Hz}$, $\lambda_3 \geqslant 5\,\mathrm{Hz}$, $\lambda_5 \geqslant 7\,\mathrm{Hz}$, and $\lambda_7 \geqslant 9\,\mathrm{Hz}$.

6.2.1. Neural networks training and testing

The training and testing parameters of the neural networks used in the example are presented in Table 10. Also, the networks errors due to approximate frequencies in the testing mode are displayed in Figs. 9–12. The number of samples in training and testing modes is 400 and 200, respectively.

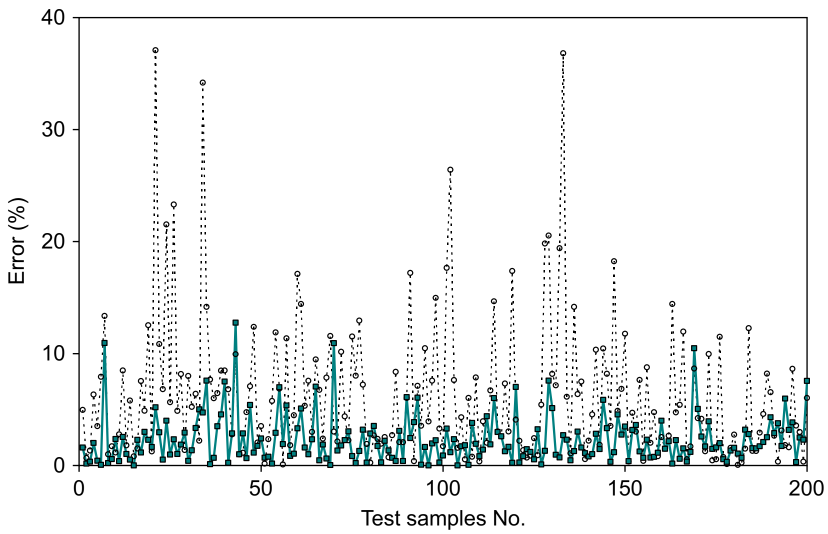

Fig. 11. Errors of the fifth approximate frequency. —■—, WRBF, ···○··, RBF.

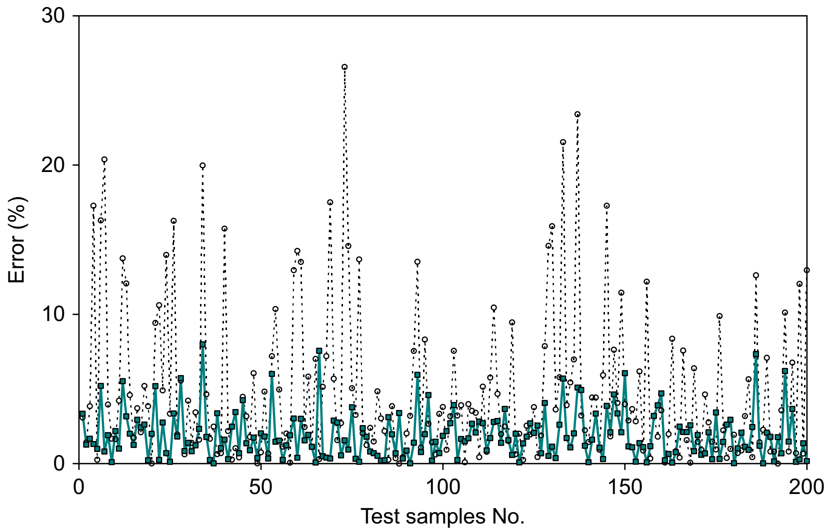

Fig. 12. Errors of the seventh approximate frequency. —■—, WRBF, ···○··, RBF.

Table 11
Optimal designs of the double layer grid obtained by the various methods

Variable no.	Optimal design (cm^2)					
	GA			VSP		
	Exact	RBF	WRBF	Exact	RBF	WRBF
1	5.229	3.733	3.733	3.733	3.733	4.534
2	6.669	6.669	6.669	5.229	6.669	6.669
3	10.670	10.670	6.669	10.670	6.669	5.229
4	6.669	6.669	10.670	10.670	6.669	6.669
5	6.669	5.229	5.229	6.669	5.229	6.669
6	2.540	4.534	3.733	2.540	4.534	3.733
7	6.669	5.229	4.534	4.534	5.229	5.229
8	6.669	10.670	4.534	4.534	10.670	6.669
Weight (kg)	1585.9	1543.1	1530.2	1476.9	1492.9	1483.2
Generations	156	187	190	196	140	112
Time (s)	582.0	22.5	23.0	315.0	8.2	6.5

Table 12
Comparison of approximate frequencies of optimal designs in terms of errors (%)

Frequency no.	GA		VSP	
	RBF	WRBF	RBF	WRBF
1	2.82	2.25	2.86	1.68
3	3.06	2.54	2.49	1.97
5	1.46	1.38	2.04	0.88
7	3.42	2.55	2.81	1.92

The optimal solutions obtained by the various methods are shown in Table 11. As observed from the table the solutions, which are found by VSP method are more economical and the best solution is achieved by VSP method using WRBF network.

Table 12 presents the accuracy of approximate frequencies predicted by RBF and WRBF networks for optimum structures. It can be observed that performance generality of WRBF network is higher than that of RBF network.

7. Conclusions

In this study, an efficient optimization procedure has been developed for the optimal design of structures with frequency constraints using discrete design variables. The proposed procedure utilizes a combination of the evolutionary algorithm, neural networks and wavelet theory. The employed evolutionary algorithm is VSP method. The VSP method alleviates the shortcomings of the standard GA such as convergence to a local optima and excessive computational effort for structures with a large number of degrees of freedom. The results demonstrate that VSP method results in a better solution and a greater efficiency in comparison the standard GA. To reduce the overall time of optimization process, the natural frequencies of structures are predicted using properly trained RBF neural networks. Further, in order to improve performance generality of RBF networks the activation function of hidden layer neurons is substituted with cosine-Gaussian Morlet daughter wavelet function. The resulted network is called WRBF network. Numerical results of testing the networks indicate that performance generality of WRBF network is higher in comparison to RBF network. Finally, the optimization is implemented by GA and VSP methods using RBF and WRBF networks and numerical results demonstrate that the combination of VSP and WRBF provides a more robust tool for optimization of structures with constrained frequencies.

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