

# Shape optimization of multi-chamber cross-flow mufflers by SA optimization

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## Abstract

It is essential when searching for an efficient acoustical mechanism to have an optimally shaped muffler designed specially for the constrained space found in today's plants. Because the research work of optimally shaped straight silencers in conjunction with multi-chamber cross-flow perforated ducts is rarely addressed, this paper will not only analyze the sound transmission loss (STL) of three kinds of cross-flow perforated mufflers but also will analyze the optimal design shape within a limited space.

In this paper, the four-pole system matrix used in evaluating acoustic performance is derived by using the decoupled numerical method. Moreover, a simulated annealing (SA) algorithm, a robust scheme in searching for the global optimum by imitating the softening process of metal, has been adopted during shape optimization. To reassure SA's correctness, the STL's maximization of three kinds of mufflers with respect to one-tone and dual-tone noise is exemplified. Furthermore, the optimization of mufflers with respect to an octave-band fan noise by the simulated algorithm has been introduced and fully discussed. Before the SA operation can be carried out, an accuracy check of the mathematical model with respect to cross-flow perforated mufflers has to be performed by Munjal's analytical data and experimental data.

The optimal result in eliminating broadband noise reveals that the cross-flow perforated muffler with more chambers is far superior at noise reduction than a muffler with fewer chambers. Consequently, the approach used for the optimal design of noise elimination proposed in this study is certainly easy and efficient.

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## 1. Introduction

The research of mufflers was started by Davis et al. [1]. To increase a muffler's acoustical performance, the assessment of a new acoustical element—cross-flow mechanism with double internal perforated tubes—was proposed and investigated by Munjal et al. [2]. On the basis of the coupled differential equations, a series of theories and numerical techniques in decoupling the acoustical problems have been widely proposed [3–7]. Considering the flowing effect, Munjal [8] and Peat [9] publicized the generalized decoupling and numerical decoupling methods, which supercede the drawbacks in previous studies. Because the constrained problem is

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Nomenclature	
$c_o$	sound speed ( $\text{m s}^{-1}$ )
$dh_i$	the diameter of perforated hole on $i$ th inner tube (m)
$D_i, D_{i+1}$	diameter of the inner perforated tubes inside the $i$ th chamber (m)
$D_o$	diameter of the outer tube (m)
$f$	cyclic frequency (Hz)
$H$	dynamic head (Pa)
$\text{iter}_{\max}$	maximum iteration
$j$	imaginary unit
$k$	wave number ( $= \omega/c_o$ )
$kk$	cooling rate in SA
$L_1, L_2$	lengths of inlet/outlet straight ducts (m)
$L_{Ai}, L_{Bi}$	length of the un-perforated segments within the $i$ th chamber (m)
$L_{Ci}$	length of the perforated segment within the $i$ th chamber (m)
$L_0$	total length of the muffler (m)
$L_{Zi}$	length of the $i$ th cross-flow chamber ( $= L_{Ai} + L_{Ci} + L_{Bi}$ ) (m)
$M$	mean flow Mach number
$\text{OBJ}_i$	objective function (dB)
$\bar{p}_i$	acoustic pressure at $i$ th node (Pa)
$\text{pb}(T)$	transition probability
$Q$	volume flow rate of venting gas ( $\text{m}^3 \text{s}^{-1}$ )
$S_i$	section area at $i$ th node ( $\text{m}^2$ )
STL	sound transmission loss (dB)
SWLO	unsilenced sound power level inside the muffler's inlet (dB)
$\text{SWL}_T$	overall sound power level inside the muffler's output (dB)
$t_i$	the thickness of the $i$ th inner perforated tube (m)
$\text{TS1}_{ij}, \text{TS2}_{ij}$	components of four-pole transfer matrices for straight ducts
$\text{TPCF}_{ij}$	components of a four-pole transfer matrix for a cross-flow perforated duct
$T_{ij}^*$	components of a four-pole transfer system matrix
$\bar{u}_i$	acoustic particle velocity at $i$ th node ( $\text{m s}^{-1}$ )
$V_i$	mean flow velocity at $i$ th node ( $\text{m s}^{-1}$ )
$x$	open area ratio
$\rho_o$	air density ( $\text{kg m}^{-3}$ )
$\eta_i$	the porosity of the $i$ th inner perforated tube
$\Delta p$	mean pressure drop (Pa)

mostly concerned with the necessity of operation and maintenance in practical engineering work, there is a growing need to optimize the acoustical performance under a fixed space. Yet, the need to investigate the optimal muffler design under space constraints is rarely tackled.

In previous papers, the shape optimizations of straight simple-expansion mufflers have been discussed [10–12]. In order to improve the performance of the noise control device, the cross-flow perforated mufflers with multi-chambers that were arrived at by using the novel scheme of simulated annealing (SA) is presented. In this paper, the numerical decoupling methods in conjunction with the SA to minimize the overall value of SWL by adjusting the shape, the perforated ratio, and the hole's diameter of the muffler under space constraints are used.

## 2. Theoretical background

In this paper, one-, two-, and three-chamber cross-flow perforated mufflers were adopted for the noise elimination in the fan room shown in Fig. 1. The outlines, acoustic pressure  $\bar{p}$  and acoustic particle velocity  $\bar{u}$ , of these mufflers are shown in Figs. 2–4.

### 2.1. A one-chamber cross-flow perforated muffler

As indicated in Fig. 2, individual transfer matrixes with respect to each case of straight ducts and cross-flow perforated tubes are described as follows [8–12]:

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = e^{-jM_1 k(L_1 + L_{A1}) / (1 - M_1^2)} \begin{bmatrix} \text{TS1}_{1,1} & \text{TS1}_{1,2} \\ \text{TS1}_{2,1} & \text{TS1}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix}, \quad (1a)$$

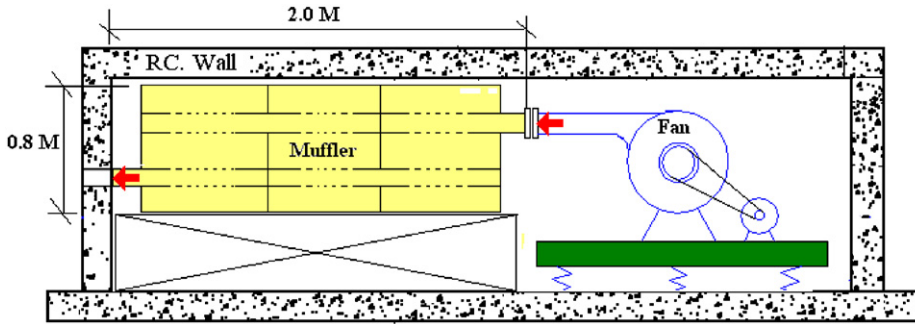


Fig. 1. Noise elimination of a fan noise inside a limited space.

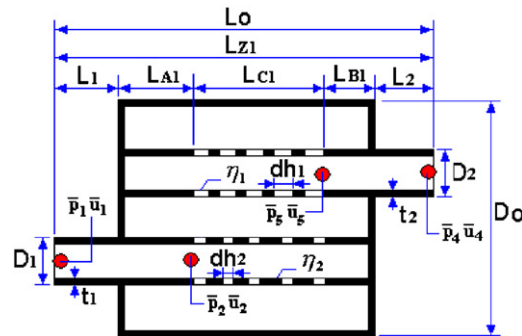


Fig. 2. The outline of a one-chamber cross-flow perforated muffler.

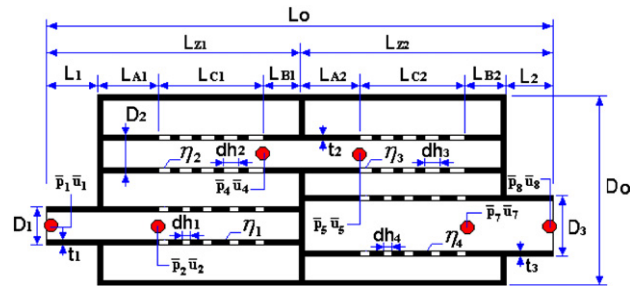


Fig. 3. The outline of a two-chamber cross-flow perforated muffler.

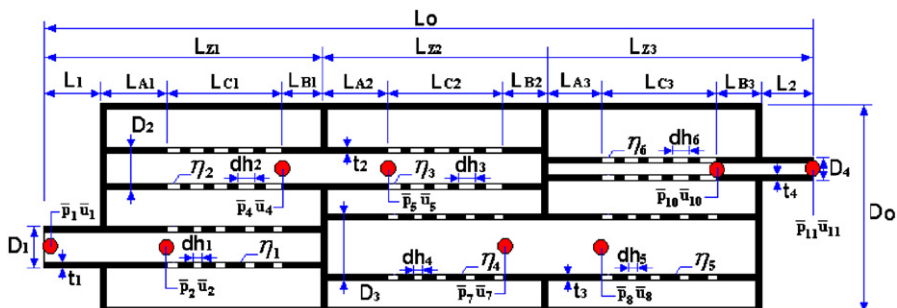


Fig. 4. The outline of a three-chamber cross-flow perforated muffler.

$$\begin{aligned} \text{TS1}_{1,1} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & \text{TS1}_{1,2} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], \\ \text{TS1}_{2,1} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & \text{TS1}_{2,2} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], \end{aligned} \quad (1b)$$

$$\begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix} = \begin{bmatrix} \text{TPCF1}_{1,1} & \text{TPCF1}_{1,2} \\ \text{TPCF1}_{2,1} & \text{TPCF1}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_4 \\ \rho_o c_o \bar{u}_4 \end{pmatrix}, \quad (2)$$

$$\begin{pmatrix} \bar{p}_4 \\ \rho_o c_o \bar{u}_4 \end{pmatrix} = e^{-jM_4 k(L_2 + L_{B1}) / (1 - M_4^2)} \begin{bmatrix} \text{TS2}_{1,1} & \text{TS2}_{1,2} \\ \text{TS2}_{2,1} & \text{TS2}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_5 \\ \rho_o c_o \bar{u}_5 \end{pmatrix}, \quad (3a)$$

$$\begin{aligned} \text{TS2}_{1,1} &= \cos \left[ \frac{k(L_2 + L_{B1})}{1 - M_4^2} \right], & \text{TS2}_{1,2} &= j \sin \left[ \frac{k(L_2 + L_{B1})}{1 - M_4^2} \right], \\ \text{TS2}_{2,1} &= j \sin \left[ \frac{k(L_2 + L_{B1})}{1 - M_4^2} \right], & \text{TS2}_{2,2} &= \cos \left[ \frac{k(L_2 + L_{B1})}{1 - M_4^2} \right]. \end{aligned} \quad (3b)$$

The total transfer matrix assembled by multiplication is

$$\begin{aligned} \begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} &= e^{-jk[(M_1(L_1 + L_{A1}) / (1 - M_1^2)) + (M_1(L_2 + L_{B1}) / (1 - M_1^2))]} \begin{bmatrix} \text{TS1}_{1,1} & \text{TS1}_{1,2} \\ \text{TS1}_{2,1} & \text{TS1}_{2,2} \end{bmatrix} \\ &\times \begin{bmatrix} \text{TPCF1}_{1,1} & \text{TPCF1}_{1,2} \\ \text{TPCF1}_{2,1} & \text{TPCF1}_{2,2} \end{bmatrix} \begin{bmatrix} \text{TS2}_{1,1} & \text{TS2}_{1,2} \\ \text{TS2}_{2,1} & \text{TS2}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_5 \\ \rho_o c_o \bar{u}_5 \end{pmatrix}. \end{aligned} \quad (4a)$$

A simplified form in the matrix is expressed as

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = \begin{bmatrix} T_{11}^* & T_{12}^* \\ T_{21}^* & T_{22}^* \end{bmatrix} \begin{pmatrix} \bar{p}_5 \\ \rho_o c_o \bar{u}_5 \end{pmatrix}. \quad (4b)$$

Under the assumption of the fixed thickness of the tubes ( $t_1 = t_2 = 0.001$ ) and the symmetric design ( $L_{A1} = L_{B1}$ ;  $L_1 = L_2$ ), the sound transmission loss (STL) of a muffler is defined as [8]

$$\text{STL}(Q, f, \text{Aff}_1, \text{Aff}_2, D_1, D_2, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2) = 20 \log \left( \frac{|T_{11}^* + T_{12}^* + T_{21}^* + T_{22}^*|}{2} \right) + 10 \log \left( \frac{S_1}{S_5} \right), \quad (5a)$$

where

$$\begin{aligned} \text{Aff}_1 &= L_{Z1} / L_0, & \text{Aff}_2 &= L_{C1} / L_0, & L_{Z1} &= L_{A1} + L_{B1} + L_{C1}, \\ L_{A1} &= L_{B1} = (L_{Z1} - L_{C1}) / 2, & L_1 &= L_2 = (L_0 - L_{Z1}) / 2. \end{aligned} \quad (5b)$$

The mean pressure drop ( $\Delta p$ ) of a one-chamber cross-flow muffler investigated by Munjal et al. [13] is

$$\Delta p = \text{Max}\{H_1(4.2e^{-0.06x_1} + 16.7e^{-2.03x_1}), H_2(4.2e^{-0.06x_2} + 16.7e^{-2.03x_2})\}, \quad (6a)$$

$$H_1 = \rho V_1^2 / 2, \quad H_2 = \rho V_2^2 / 2, \quad x_1 = 4L_{C1}\eta_1 / D_1, \quad x_2 = 4L_{C1}\eta_2 / D_2. \quad (6b)$$

## 2.2. A two-chamber cross-flow perforated muffler

As indicated in Fig. 3, individual transfer matrixes with respect to each case of straight ducts and cross-flow perforated ducts are described as follows [8–12]:

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = e^{-jM_1 k(L_1 + L_{A1}) / (1 - M_1^2)} \begin{bmatrix} \text{TS1}_{1,1} & \text{TS1}_{1,2} \\ \text{TS1}_{2,1} & \text{TS1}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix}, \quad (7a)$$

$$\begin{aligned} TS1_{1,1} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & TS1_{1,2} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], \\ TS1_{2,1} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & TS1_{2,2} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], \end{aligned} \tag{7b}$$

$$\begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix} = \begin{bmatrix} TPCF1_{1,1} & TPCF1_{1,2} \\ TPCF1_{2,1} & TPCF1_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_4 \\ \rho_o c_o \bar{u}_4 \end{pmatrix}, \tag{8}$$

$$\begin{pmatrix} \bar{p}_5 \\ \rho_o c_o \bar{u}_5 \end{pmatrix} = \begin{bmatrix} TPCF2_{1,1} & TPCF2_{1,2} \\ TPCF2_{2,1} & TPCF2_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_7 \\ \rho_o c_o \bar{u}_7 \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} \bar{p}_7 \\ \rho_o c_o \bar{u}_7 \end{pmatrix} = e^{-jM_7k(L_2+L_{B2})/(1-M_7^2)} \begin{bmatrix} TS2_{1,1} & TS2_{1,2} \\ TS2_{2,1} & TS2_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_8 \\ \rho_o c_o \bar{u}_8 \end{pmatrix}, \tag{10a}$$

$$\begin{aligned} TS2_{1,1} &= \cos \left[ \frac{k(L_2 + L_{B2})}{1 - M_7^2} \right], & TS2_{1,2} &= j \sin \left[ \frac{k(L_2 + L_{B2})}{1 - M_7^2} \right], \\ TS2_{2,1} &= j \sin \left[ \frac{k(L_2 + L_{B2})}{1 - M_7^2} \right], & TS2_{2,2} &= \cos \left[ \frac{k(L_2 + L_{B2})}{1 - M_7^2} \right]. \end{aligned} \tag{10b}$$

The total transfer matrix assembled by multiplication is

$$\begin{aligned} \begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} &= e^{-jk[(M_1(L_1+L_{A1})/(1-M_1^2)+(M_7(L_2+L_{B2})/(1-M_7^2))]} \begin{bmatrix} TS1_{1,1} & TS1_{1,2} \\ TS1_{2,1} & TS1_{2,2} \end{bmatrix} \begin{bmatrix} TPCF1_{1,1} & TPCF1_{1,2} \\ TPCF1_{2,1} & TPCF1_{2,2} \end{bmatrix} \\ &\times \begin{bmatrix} TPCF2_{1,1} & TPCF2_{1,2} \\ TPCF2_{2,1} & TPCF2_{2,2} \end{bmatrix} \begin{bmatrix} TS2_{1,1} & TS2_{1,2} \\ TS2_{2,1} & TS2_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_8 \\ \rho_o c_o \bar{u}_8 \end{pmatrix}. \end{aligned} \tag{11a}$$

A simplified form in the matrix is expressed as

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = \begin{bmatrix} T_{11}^* & T_{12}^* \\ T_{21}^* & T_{22}^* \end{bmatrix} \begin{pmatrix} \bar{p}_8 \\ \rho_o c_o \bar{u}_8 \end{pmatrix}. \tag{11b}$$

Under the assumption of the fixed thickness of the tubes ( $t_1 = t_2 = t_3 = 0.001$  m) and the symmetric design ( $L_{A1} = L_{B1}, L_{A2} = L_{B2}, L_1 = L_2$ ), the STL of a muffler is defined as [8]

$$\begin{aligned} STL(Q, f, Aff_1, Aff_2, Aff_3, Aff_4, D_1, D_2, D_3, dh_1, \eta_1, dh_2, \eta_2, dh_3, \eta_3, dh_4, \eta_4) \\ = 20 \log \left( \frac{|T_{11}^* + T_{12}^* + T_{21}^* + T_{22}^*|}{2} \right) + 10 \log \left( \frac{S_1}{S_7} \right), \end{aligned} \tag{12a}$$

where

$$\begin{aligned} Aff_1 &= L_{Z1}/L_0, & Aff_2 &= L_{Z2}/L_0, & Aff_3 &= L_{C1}/L_{Z1}, & Aff_4 &= L_{C2}/L_{Z2}, \\ L_{A1} &= L_{B1} = (L_{Z1} - L_{C1})/2, & L_{A2} &= L_{B2} = (L_{Z2} - L_{C2})/2, & L_1 &= L_2 = (L_0 - L_{Z1} - L_{Z2})/2. \end{aligned} \tag{12b}$$

Similarly, the mean pressure drop ( $\Delta p$ ) of a two-chamber muffler can be expressed as [13]

$$\begin{aligned} \Delta p &= \text{Max}\{H_1(4.2e^{-0.06x1} + 16.7e^{-2.03x1}), H_2(4.2e^{-0.06x2} + 16.7e^{-2.03x2})\} \\ &+ \text{Max}\{H_3(4.2e^{-0.06x3} + 16.7e^{-2.03x3}), H_4(4.2e^{-0.06x4} + 16.7e^{-2.03x4})\}, \end{aligned} \tag{13a}$$

$$\begin{aligned}
 H_1 &= \rho V_1^2/2, & H_2 &= \rho V_2^2/2, & H_3 &= \rho V_3^2/2, & H_4 &= \rho V_4^2/2, \\
 x_1 &= 4L_{C1}\eta_1/D_1, & x_2 &= 4L_{C1}\eta_2/D_2, & x_3 &= 4L_{C2}\eta_3/D_2, & x_4 &= 4L_{C2}\eta_4/D_3.
 \end{aligned}
 \tag{13b}$$

2.3. A three-chamber cross-flow perforated muffler

As indicated in Fig. 4, individual transfer matrixes with respect to each case of straight ducts and cross-flow perforated ducts are described as follows [8–12]:

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = e^{-jM_1 k(L_1+L_{A1})/(1-M_1^2)} \begin{bmatrix} \text{TS1}_{1,1} & \text{TS1}_{1,2} \\ \text{TS1}_{2,1} & \text{TS1}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix}, \tag{14a}$$

$$\begin{aligned}
 \text{TS1}_{1,1} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & \text{TS1}_{1,2} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], \\
 \text{TS1}_{2,1} &= j \sin \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right], & \text{TS1}_{2,2} &= \cos \left[ \frac{k(L_1 + L_{A1})}{1 - M_1^2} \right],
 \end{aligned}
 \tag{14b}$$

$$\begin{pmatrix} \bar{p}_2 \\ \rho_o c_o \bar{u}_2 \end{pmatrix} = \begin{bmatrix} \text{TPCF1}_{1,1} & \text{TPCF1}_{1,2} \\ \text{TPCF1}_{2,1} & \text{TPCF1}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_4 \\ \rho_o c_o \bar{u}_4 \end{pmatrix}, \tag{15}$$

$$\begin{pmatrix} \bar{p}_5 \\ \rho_o c_o \bar{u}_5 \end{pmatrix} = \begin{bmatrix} \text{TPCF2}_{1,1} & \text{TPCF2}_{1,2} \\ \text{TPCF2}_{2,1} & \text{TPCF2}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_7 \\ \rho_o c_o \bar{u}_7 \end{pmatrix}, \tag{16}$$

$$\begin{pmatrix} \bar{p}_8 \\ \rho_o c_o \bar{u}_8 \end{pmatrix} = \begin{bmatrix} \text{TPCF3}_{1,1} & \text{TPCF3}_{1,2} \\ \text{TPCF3}_{2,1} & \text{TPCF3}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_{10} \\ \rho_o c_o \bar{u}_{10} \end{pmatrix}, \tag{17}$$

$$\begin{pmatrix} \bar{p}_{10} \\ \rho_o c_o \bar{u}_{10} \end{pmatrix} = e^{-jM_{10} k(L_2+L_{B3})/(1-M_{10}^2)} \begin{bmatrix} \text{TS2}_{1,1} & \text{TS2}_{1,2} \\ \text{TS2}_{2,1} & \text{TS2}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_{11} \\ \rho_o c_o \bar{u}_{11} \end{pmatrix}, \tag{18a}$$

$$\begin{aligned}
 \text{TS2}_{1,1} &= \cos \left[ \frac{k(L_2 + L_{B3})}{1 - M_{10}^2} \right], & \text{TS2}_{1,2} &= j \sin \left[ \frac{k(L_2 + L_{B3})}{1 - M_{10}^2} \right], \\
 \text{TS2}_{2,1} &= j \sin \left[ \frac{k(L_2 + L_{B3})}{1 - M_{10}^2} \right], & \text{TS2}_{2,2} &= \cos \left[ \frac{k(L_2 + L_{B3})}{1 - M_{10}^2} \right].
 \end{aligned}
 \tag{18b}$$

The total transfer matrix assembled by multiplication is

$$\begin{aligned}
 \begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} &= e^{-jk[(M_1(L_1+L_{A1})/1-M_1^2)+(M_{10}(L_2+L_{B3})/1-M_{10}^2)]} \begin{bmatrix} \text{TS1}_{1,1} & \text{TS1}_{1,2} \\ \text{TS1}_{2,1} & \text{TS1}_{2,2} \end{bmatrix} \begin{bmatrix} \text{TPCF1}_{1,1} & \text{TPCF1}_{1,2} \\ \text{TPCF1}_{2,1} & \text{TPCF1}_{2,2} \end{bmatrix} \\
 &\times \begin{bmatrix} \text{TPCF2}_{1,1} & \text{TPCF2}_{1,2} \\ \text{TPCF2}_{2,1} & \text{TPCF2}_{2,2} \end{bmatrix} \begin{bmatrix} \text{TPCF3}_{1,1} & \text{TPCF3}_{1,2} \\ \text{TPCF3}_{2,1} & \text{TPCF3}_{2,2} \end{bmatrix} \begin{bmatrix} \text{TS2}_{1,1} & \text{TS2}_{1,2} \\ \text{TS2}_{2,1} & \text{TS2}_{2,2} \end{bmatrix} \begin{pmatrix} \bar{p}_{11} \\ \rho_o c_o \bar{u}_{11} \end{pmatrix}.
 \end{aligned}
 \tag{19a}$$

A simplified form in the matrix is expressed as

$$\begin{pmatrix} \bar{p}_1 \\ \rho_o c_o \bar{u}_1 \end{pmatrix} = \begin{bmatrix} T_{11}^* & T_{12}^* \\ T_{21}^* & T_{22}^* \end{bmatrix} \begin{pmatrix} \bar{p}_{11} \\ \rho_o c_o \bar{u}_{11} \end{pmatrix}. \tag{19b}$$

Under the assumption of the fixed thickness of the tubes ( $t_1 = t_2 = t_3 = t_4 = 0.001$  m) and the symmetric design ( $L_{A1} = L_{B1}, L_{A2} = L_{B2}, L_{A3} = L_{B3}, L_1 = L_2$ ), the STL of a muffler is defined as [8]

$$\begin{aligned} \text{STL} & \left( Q, f, \text{Aff}_1, \text{Aff}_2, \text{Aff}_3, \text{Aff}_4, \text{Aff}_5, \text{Aff}_6, D_1, D_2, D_3, D_4, \right. \\ & \left. \text{dh}_1, \eta_1, \text{dh}_2, \eta_2, \text{dh}_3, \eta_3, \text{dh}_4, \eta_4, \text{dh}_5, \eta_5, \text{dh}_6, \eta_6 \right) \\ & = 20 \log \left( \frac{|T_{11}^* + T_{12}^* + T_{21}^* + T_{22}^*|}{2} \right) + 10 \log \left( \frac{S_1}{S_{10}} \right), \end{aligned} \tag{20a}$$

where

$$\begin{aligned} \text{Aff}_1 & = L_{Z1}/L_0, \quad \text{Aff}_2 = L_{Z2}/L_0, \quad \text{Aff}_3 = L_{Z3}/L_0, \quad \text{Aff}_4 = L_{C1}/L_{Z1}, \quad \text{Aff}_5 = L_{C2}/L_{Z2}, \\ \text{Aff}_6 & = L_{C3}/L_{Z3}, \quad L_{A1} = L_{B1} = (L_{Z1} - L_{C1})/2, \quad L_{A2} = L_{B2} = (L_{Z2} - L_{C2})/2, \\ L_{A3} & = L_{B3} = (L_{Z3} - L_{C3})/2, \quad L_1 = L_2 = (L_0 - L_{Z1} - L_{Z2})/2. \end{aligned} \tag{20b}$$

Equally, the mean pressure drop ( $\Delta p$ ) of a two-chamber muffler can be expressed as [13]

$$\begin{aligned} \Delta p & = \text{Max}\{H_1(4.2e^{-0.06x_1} + 16.7e^{-2.03x_1}), H_2(4.2e^{-0.06x_2} + 16.7e^{-2.03x_2})\} \\ & + \text{Max}\{H_3(4.2e^{-0.06x_3} + 16.7e^{-2.03x_3}), H_4(4.2e^{-0.06x_4} + 16.7e^{-2.03x_4})\} \\ & + \text{Max}\{H_5(4.2e^{-0.06x_5} + 16.7e^{-2.03x_5}), H_6(4.2e^{-0.06x_6} + 16.7e^{-2.03x_6})\}, \end{aligned} \tag{21a}$$

$$\begin{aligned} H_1 & = \rho V_1^2/2, \quad H_2 = \rho V_2^2/2, \quad H_3 = \rho V_3^2/2, \quad H_4 = \rho V_4^2/2, \quad H_5 = \rho V_5^2/2, \\ H_6 & = \rho V_6^2/2, \quad x_1 = 4L_{C1}\eta_1/D_1, \quad x_2 = 4L_{C1}\eta_2/D_2, \quad x_3 = 4L_{C2}\eta_3/D_2, \\ x_4 & = 4L_{C2}\eta_4/D_3, \quad x_5 = 4L_{C3}\eta_5/D_3, \quad x_6 = 4L_{C3}\eta_6/D_4. \end{aligned} \tag{21b}$$

#### 2.4. Overall sound power level

The silenced octave sound power level emitted from a silencer’s outlet is

$$\text{SWL}_i = \text{SWLO}_i - \text{STL}_i, \tag{22}$$

where

- (1)  $\text{SWLO}_i$  is the original SWL at the inlet of a muffler (or pipe outlet), and  $i$  is the index of the octave band frequency;
- (2)  $\text{STL}_i$  is the muffler’s STL with respect to the relative octave band frequency;
- (3)  $\text{SWL}_i$  is the silenced SWL at the outlet of a muffler with respect to the relative octave band frequency.

Finally, the overall  $\text{SWL}_T$  silenced by a muffler at the outlet is

$$\begin{aligned} \text{SWL}_T & = 10 \log \left\{ \sum_{i=1}^7 10^{\text{SWL}_i/10} \right\} \\ & = 10 \log \left\{ \begin{aligned} & 10^{[\text{SWLO}(f=63) - \text{STL}(f=63)]/10} + 10^{[\text{SWLO}(f=125) - \text{STL}(f=125)]/10} + 10^{[\text{SWLO}(f=250) - \text{STL}(f=250)]/10} \\ & + 10^{[\text{SWLO}(f=500) - \text{STL}(f=500)]/10} + 10^{[\text{SWLO}(f=1000) - \text{STL}(f=1000)]/10} \\ & + 10^{[\text{SWLO}(f=2000) - \text{STL}(f=2000)]/10} + 10^{[\text{SWLO}(f=4000) - \text{STL}(f=4000)]/10} \end{aligned} \right\}. \end{aligned} \tag{23}$$

## 2.5. Objective function

By using the formula of Eqs. (5), (12), (20) and (23), the objective function used in SA optimization with respect to each type of muffler was established.

### 2.5.1. One-chamber cross-flow perforated muffler

(A) STL maximization for one-tone ( $f_1$ ) noise:

$$\text{OBJ}_{11} = \text{STL}(Q, f_1, \text{Aff}_1, \text{Aff}_2, D_1, D_2, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2). \quad (24)$$

(B) STL maximization for two-tone ( $f_2, f_3$ ) noise:

$$\text{STLA}_1 = \text{STL}(Q, f_2, \text{Aff}_1, \text{Aff}_2, D_1, D_2, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2), \quad (25a)$$

$$\text{STLB}_1 = \text{STL}(Q, f_3, \text{Aff}_1, \text{Aff}_2, D_1, D_2, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2), \quad (25b)$$

$$\text{OBJ}_{12} = \{\text{STLA}_1 + \text{STLB}_1\}/2. \quad (25c)$$

(C) SWL minimization for broadband noise:

To minimize the overall SWL, the objective function is

$$\text{OBJ}_{13} = \text{SWL}(Q, \text{Aff}_1, \text{Aff}_2, D_1, D_2, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2). \quad (26)$$

The related ranges of parameters are:

$$\begin{aligned} f_1 = 150 \text{ (Hz)}, \quad f_2 = 100 \text{ (Hz)}, \quad f_3 = 200 \text{ (Hz)}, \quad Q = 0.03 \text{ (m}^3 \text{ s}^{-1}\text{)}; \quad D_o = 0.8 \text{ (m)}, \quad L_0 = 2.0 \text{ (m)}; \\ \text{Aff}_1 : [0.2, 0.45]; \quad \text{Aff}_2 : [0.3, 0.8]; \quad D_1 : [0.1, 0.35]; \quad D_2 : [0.1, 0.35]; \quad \text{dh}_1 : [0.00175, 0.007]; \\ \eta_1 : [0.03, 0.1]; \quad \text{dh}_2 : [0.00175, 0.007]; \quad \eta_2 : [0.03, 0.1]. \end{aligned} \quad (27)$$

### 2.5.2. Two-chamber cross-flow perforated muffler

(A) STL maximization for one-tone ( $f_1$ ) noise:

$$\text{OBJ}_{21} = \text{STL}(Q, f_1, \text{Aff}_1, \text{Aff}_2, \text{Aff}_3, \text{Aff}_4, D_1, D_2, D_3, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2, \text{dh}_3, \eta_3, \text{dh}_4, \eta_4). \quad (28)$$

(B) STL maximization for two-tone ( $f_2, f_3$ ) noise:

$$\text{STLA}_2 = \text{STL}(Q, f_2, \text{Aff}_1, \text{Aff}_2, \text{Aff}_3, \text{Aff}_4, D_1, D_2, D_3, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2, \text{dh}_3, \eta_3, \text{dh}_4, \eta_4), \quad (29a)$$

$$\text{STLB}_2 = \text{STL}(Q, f_3, \text{Aff}_1, \text{Aff}_2, \text{Aff}_3, \text{Aff}_4, D_1, D_2, D_3, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2, \text{dh}_3, \eta_3, \text{dh}_4, \eta_4), \quad (29b)$$

$$\text{OBJ}_{22} = \{\text{STLA}_2 + \text{STLB}_2\}/2. \quad (29c)$$

(C) SWL minimization for broadband noise:

To minimize the overall SWL, the objective function is

$$\text{OBJ}_{23} = \text{SWL}_T(Q, \text{Aff}_1, \text{Aff}_2, \text{Aff}_3, \text{Aff}_4, D_1, D_2, D_3, \text{dh}_1, \eta_1, \text{dh}_2, \eta_2, \text{dh}_3, \eta_3, \text{dh}_4, \eta_4). \quad (30)$$



### 3. Model check

Before performing the SA optimal simulation on the mufflers, an accuracy check of the mathematical models on a one-chamber cross-flow perforated muffler is performed using both of the analytical data from Munjal [2] and the experimental data in our work. As indicated in Figs. 5 and 6, the frequency characteristic between the theoretical data and Munjal’s analytical data is different because of the shift in the fundamental resonance frequencies ( $\sin(kL/(1 - M^2))$ ) in which the flowing effect is considered. The performance curves are relatively accurate and in agreement. Therefore, the models of multi-chamber cross-flow and perforated mufflers in conjunction with the numerical searching method are applied to the shape optimization in the following section.

### 4. Case studies

In this paper, noise reduction with respect to a noisy induced fan (1200 rpm, 9 blades) installed inside a confined reinforced concrete (rc) room is exemplified and shown in Fig. 1. The sound power level (SWL) inside the fan’s outlet is shown in Table 1 where the overall SWL reaches 138.9 dB. To realize the acoustical performances with respect to three kinds of mufflers (one-, two-, and three-chamber) installed at the fan’s outlet, the numerical assessments linked to an optimizer will be performed. Before the minimization of a broadband noise is executed, the maximization of STL with respect to these mufflers at a targeted one tone (150 Hz) and two tones (100 and 200 Hz) will be carried out for the purpose of an accuracy verification on the SA method. As shown in Figs. 1–4, the available space for a muffler is 0.8 m in width, 0.8 m in height, and 2.0 m in length. The flow rate ( $Q$ ) and thickness of a perforated tube ( $t$ ) are preset as  $0.03 \text{ (m}^3 \text{ s}^{-1}\text{)}$  and 0.001 (m), respectively; the corresponding OBJ functions, space constraints, and the ranges of design parameters for each muffler are summarized in Eqs. (24)–(30). Moreover, to assure the steady venting rate of

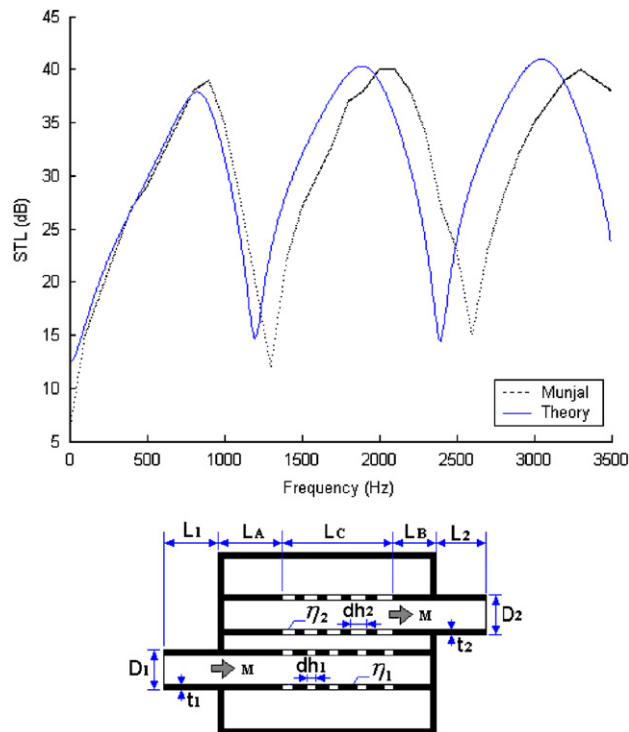


Fig. 5. Performance of a one-chamber cross-flow perforated muffler ( $D_1 = 0.0493 \text{ (m)}$ ,  $D_2 = 0.0493 \text{ (m)}$ ,  $D_o = 0.1481 \text{ (m)}$ ,  $L_A = L_B = 0.0064$ ,  $L_c = 0.1286 \text{ (m)}$ ,  $t_1 = t_2 = 0.0081 \text{ (m)}$ ,  $dh_1 = dh_2 = 0.0035 \text{ (m)}$ ,  $\eta_1 = \eta_2 = 0.039$ ,  $M_1 = 0.1$ ) (analytical data are from Munjal et al. [2]).

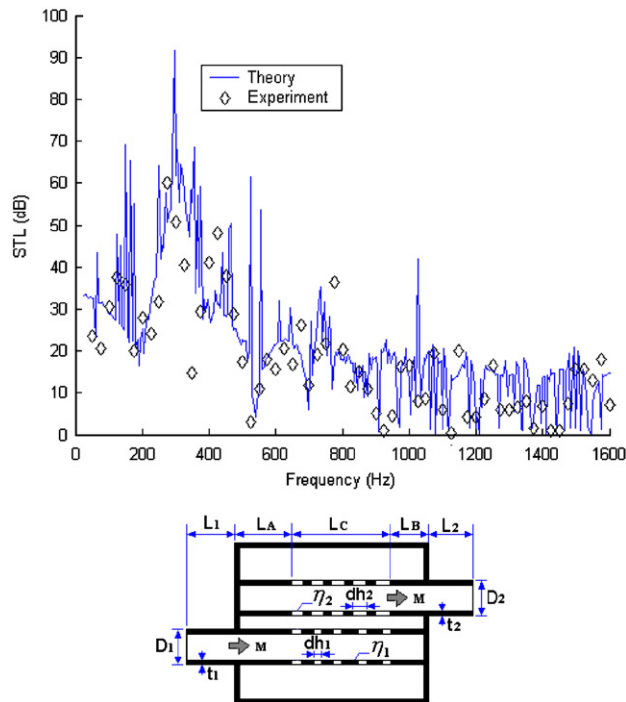


Fig. 6. Performance of a one-chamber cross-flow perforated muffler ( $D_1 = 0.0254$  (m),  $D_2 = 0.0254$  (m),  $D_0 = 0.254$  (m),  $L_A = L_B = 0.2$ ,  $L_c = 0.6$  (m),  $t_1 = t_2 = 0.0081$  (m),  $dh_1 = dh_2 = 0.003$  (m),  $\eta_1 = \eta_2 = 0.06$ ,  $M_1 = 0$ ).

Table 1  
Unsilenced SWLs of a fan inside a duct outlet

Frequency (Hz)	125	250	500	1000	2000	4000
SWLO (dB)	138	128	125	125	120	120

the fan, the assumed allowable pressure drop (or back pressure) of a proposed muffler not exceeding 100 (Pa) is obligatory.

### 5. Simulated annealing algorithm

The SA algorithm is one kind of local search process which imitates the softening process (annealing) of metal. The basic concept behind SA was first introduced by Metropolis et al. [14] and developed by Kirkpatrick et al. [15]. From the viewpoint of the physical system, annealing is the process of heating and keeping a metal at a stabilized temperature when cooling it slowly. The slow cooling will allow the particles to keep their state close to the minimal energy state; therefore, the particles have a more homogeneous crystalline structure; if not, a fast cooling rate will result in a higher distortion energy stored inside the imperfect lattice.

A variation of the hill-climbing algorithm shown in Fig. 7 can be an analog to the SA’s algorithm. The optimization process starts by generating a random initial solution. For a minimization process, all downhill movements for improvement are accepted for the decrement of the system’s energy. Simultaneously, SA also allows movement resulting in solutions that are worse (uphill moves) than the current solution in order to escape from the local optimum.

As indicated in Fig. 8, to imitate the evolution of the SA algorithm, a new random solution ( $X'$ ) from the neighborhood of the current solution ( $X$ ) is chosen and shown in Fig. 9. If the change in objective function (or energy) is negative (i.e.  $\Delta F \leq 0$ ), the new solution will be acknowledged as the new current solution with

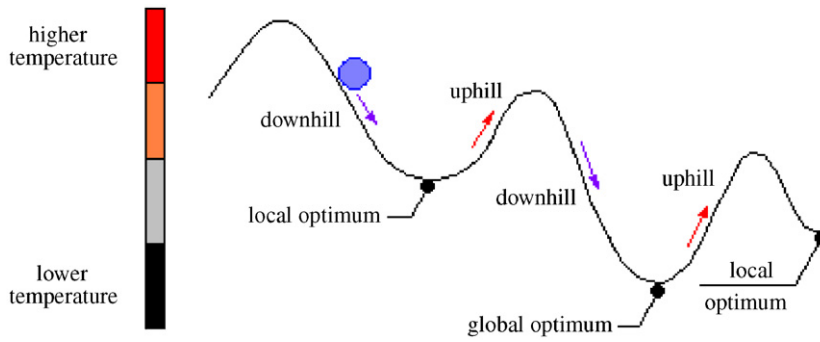


Fig. 7. SA algorithm from a physical viewpoint.

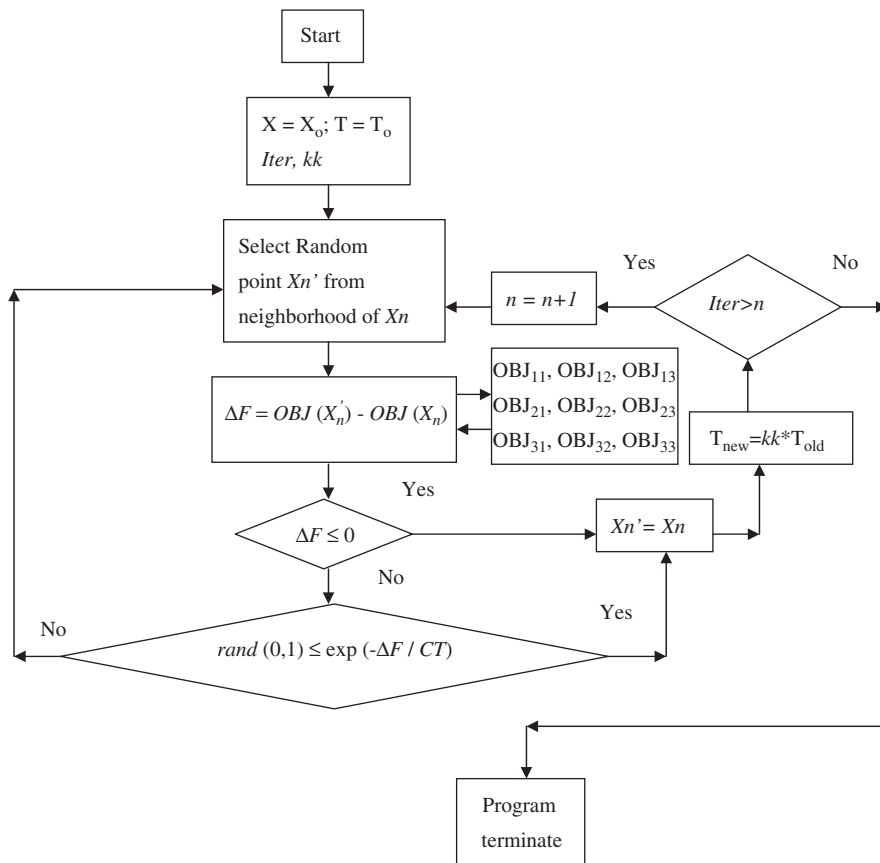


Fig. 8. Flow diagram of a SA optimization.

transition property ( $pb(X')$  of 1); if not (i.e.  $\Delta F > 0$ ), the new transition property ( $pb(X')$ ) varied from 0 to 1 will be first calculated by the Boltzmann's factor ( $pb(X') = \exp(\Delta F / CT)$ ) as shown in Eq. (31):

$$pb(X') = \begin{cases} 1, & \Delta F \leq 0, \\ \exp\left(\frac{-\Delta F}{CT}\right), & \Delta F > 0, \end{cases} \quad (31a)$$

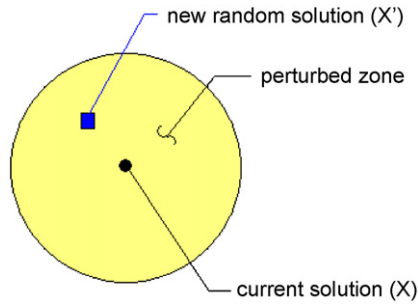


Fig. 9. New random solution in a perturbed zone.

Table 2  
Optimal STLs for a one-chamber cross-flow perforated muffler (pure tone)

Item	SA parameters		Results				
	kk	Iter					
1	0.90	50	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.6102	0.03653	0.1689	0.1689	38.7
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.003196	0.04929	0.003196	0.04929	14.69
2	0.93	50	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.5271	0.2406	0.1169	0.1169	54.9
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.002106	0.03474	0.002106	0.03474	15.62
3	0.96	50	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.5168	0.2252	0.1105	0.1105	56.9
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.001971	0.03294	0.001971	0.03294	18.23
4	0.99	50	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.5501	0.2752	0.1313	0.1313	51.5
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.002408	0.03878	0.002408	0.03878	11.33
5	<u>0.96</u>	100	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.5125	0.2188	0.1078	0.1078	57.9
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.001914	0.03219	0.001914	0.03219	19.49
6	<u>0.96</u>	200	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			0.5125	0.2188	0.1078	0.1078	57.9
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.001914	0.03219	0.001914	0.03219	19.49
7	<u>0.96</u>	<u>2000</u>	Aff <sub>1</sub>	Aff <sub>2</sub>	D <sub>1</sub>	D <sub>2</sub>	STL (dB)
			<u>0.5001</u>	<u>0.2001</u>	<u>0.1000</u>	<u>0.1000</u>	<u>61.0</u>
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	Δp (Pa)
			<u>0.001751</u>	<u>0.03001</u>	<u>0.001751</u>	<u>0.03001</u>	<u>23.86</u>

Underlined: selected parameter.

Underlined and bold: selected parameter and final results.

$$\Delta F = \text{OBJ}(X') - \text{OBJ}(X), \tag{31b}$$

wherein the  $C$  and  $T$  are the Boltzmann constant and the current temperature, respectively. Moreover, compared with the new random probability of rand (0, 1), if the transition property (pb( $X'$ )) is greater than a random number of rand (0, 1), the new worse solution which results in a higher energy (uphill moves)

Table 3  
Optimal STLs for a two-chamber cross-flow perforated muffler (pure tone)

Item	SA parameters		Results					
	kk	Iter	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	STL (dB)
1	0.90	50	0.3829	0.38290	0.66570	0.6657	0.2829	122.4
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.2829	0.2829	0.005590	0.08120	0.005590	11.25
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
			0.08120	0.005590	0.08120	0.005590	0.08120	
2	0.93	50	0.3855	0.38550	0.67100	0.6710	0.2855	129.0
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.2855	0.2855	0.005645	0.08193	0.005645	8.07
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
			0.08193	0.08193	0.005645	0.005645	0.08193	
3	0.96	50	0.3923	0.3923	0.6846	0.6846	0.2923	153.4
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.2923	0.2923	0.005788	0.08384	0.005788	10.50
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
			0.08384	0.005788	0.08384	0.005788	0.08384	
4	<u>0.99</u>	100	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	STL (dB)
			<b><u>0.3989</u></b>	<b><u>0.3989</u></b>	<b><u>0.6978</u></b>	<b><u>0.6978</u></b>	<b><u>0.2989</u></b>	<b><u>180.8</u></b>
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			<b><u>0.2989</u></b>	<b><u>0.2989</u></b>	<b><u>0.005926</u></b>	<b><u>0.08569</u></b>	<b><u>0.005926</u></b>	10.01
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
<b><u>0.08569</u></b>	<b><u>0.005926</u></b>	<b><u>0.08569</u></b>	<b><u>0.005926</u></b>	<b><u>0.08569</u></b>				

Underlined: selected parameter.  
Underlined and bold: selected parameter and final results.

condition will then be accepted. Otherwise, it will be abandoned. Nevertheless, the uphill at a higher temperature has a better chance of escaping from the local optimum. The algorithm will repeat the perturbation of the current solution and the measurement of the change in the objective function. As indicated in Fig. 8, each successful substitution of the new current solution will lead to the decay of the current temperature as

$$T_{\text{new}} = \text{kk } T_{\text{old}}, \tag{32}$$

where kk is the cooling rate. Moreover, to reach an initial transition probability pb(−ΔF/CT) of 0.5, which will allow uphill moves at a certain ΔF level, the related initial temperature (T<sub>0</sub>) is selected as 0.2 [16]. The process is repeated until the predetermined number (iter<sub>max</sub>) of the outer loop is reached.

## 6. Results and discussion

### 6.1. Results

As described in the above section, slow cooling is more efficient at maintaining a minimal energy state. Therefore, slow cooling (kk) with a range of 0.90–0.99, which was used in the previous work [17], is selected. To investigate the influences of the cooling rate and the number of iterations, the ranges of the SA parameters of the cooling rate and the iterations are:

$$\text{kk} = (0.90, 0.93, 0.96, 0.99), \quad \text{Iter}_{\text{max}} = (50-2000).$$

Table 4  
Optimal STLs for a three-chamber cross-flow perforated muffler (pure tone)

Item	SA parameters		Results					
	kk	Iter						
1	0.90	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			0.21540	0.2154	0.2154	0.2926	0.2926	100.3
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)
			0.2926	0.1386	0.1386	0.1386	0.1386	59.32
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)	
			0.002560	0.04080	0.002560	0.04080	0.002560	
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>	
0.04080	0.002560	0.04080	0.002560	0.04080				
dh <sub>6</sub> (m)	η <sub>6</sub>							
0.002560	0.04080							
2	0.93	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			0.2819	0.2819	0.2819	0.6913	0.6913	139.8
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)
			0.6913	0.3047	0.3047	0.3047	0.3047	8.04
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)	
			0.006049	0.08732	0.006049	0.08732	0.006049	
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>	
0.08732	0.006049	0.08732	0.006049	0.08732				
dh <sub>6</sub> (m)	η <sub>6</sub>							
0.006049	0.08732							
3	<u>0.96</u>	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			0.2962	0.2962	0.2962	0.7774	0.7774	199.1
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)
			0.7774	0.3406	0.3406	0.3406	0.3406	5.83
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)	
			0.006802	0.09736	0.006802	0.09736	0.006802	
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>	
0.09736	0.006802	0.09736	0.006802	0.09736				
dh <sub>6</sub> (m)	η <sub>6</sub>							
0.006802	0.09736							
4	0.99	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			0.2891	0.2891	0.2891	0.7348	0.7348	205.0
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)
			0.7348	0.3228	0.3228	0.3228	0.3228	6.82
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)	
			0.006429	0.09239	0.006429	0.09239	0.006429	
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>	
0.09239	0.006429	0.09239	0.006429	0.09239				
dh <sub>6</sub> (m)	η <sub>6</sub>							
0.006429	0.0923							
5	<u>0.96</u>	100	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			0.2908	0.2908	0.2908	0.7447	0.7447	217.5
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)
			0.7447	0.3270	0.3270	0.3270	0.3270	6.56
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)	
			0.006516	0.006516	0.09355	0.09355	0.006516	
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>	
0.09355	0.006516	0.09355	0.006516	0.09355				
dh <sub>6</sub> (m)	η <sub>6</sub>							
0.006516	0.09355							
6	<u>0.96</u>	<u>200</u>	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	STL (dB)
			<b><u>0.2905</u></b>	<b><u>0.2905</u></b>	<b><u>0.2905</u></b>	<b><u>0.7429</u></b>	<b><u>0.7429</u></b>	<b><u>218.8</u></b>

Table 4 (continued)

Item	SA parameters		Results					
	kk	Iter						
			<u>Aff<sub>6</sub></u>	<u>D<sub>1</sub></u>	<u>D<sub>2</sub> (m)</u>	<u>D<sub>3</sub> (m)</u>	<u>D<sub>4</sub> (m)</u>	$\Delta p$ (Pa)
			<b>0.7429</b>	<b>0.3262</b>	<b>0.3262</b>	<b>0.3262</b>	<b>0.3262</b>	6.61
			<u>dh<sub>1</sub> (m)</u>	<u><math>\eta_1</math></u>	<u>dh<sub>2</sub> (m)</u>	<u><math>\eta_2</math></u>	<u>dh<sub>3</sub> (m)</u>	
			<b>0.006501</b>	<b>0.09334</b>	<b>0.006501</b>	<b>0.09334</b>	<b>0.006501</b>	
			<u><math>\eta_3</math></u>	<u>dh<sub>4</sub> (m)</u>	<u><math>\eta_4</math></u>	<u>dh<sub>5</sub> (m)</u>	<u><math>\eta_5</math></u>	
			<b>0.09334</b>	<b>0.006501</b>	<b>0.09334</b>	<b>0.006501</b>	<b>0.09334</b>	
			<u>dh<sub>6</sub> (m)</u>	<u><math>\eta_6</math></u>				
			<b>0.006501</b>	<b>0.09334</b>				

Underlined: selected parameter.

Underlined and bold: selected parameter and final results.

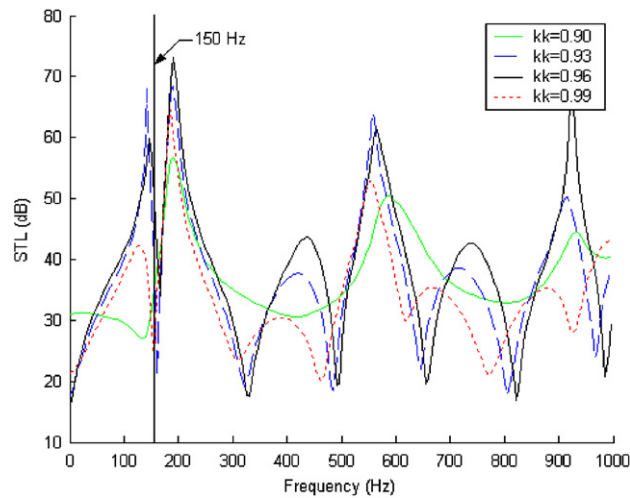


Fig. 10. STL curves with respect to frequencies at various cooling rates for a one-chamber muffler (iter<sub>max</sub> = 50, targeted tone = 150 Hz).

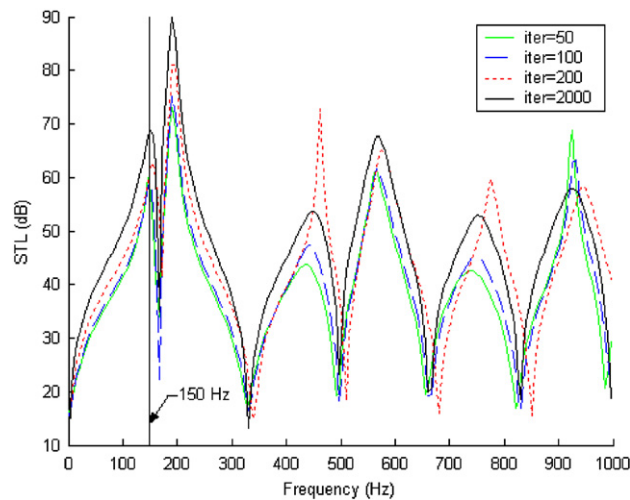


Fig. 11. STL curves with respect to frequencies at various iterations for a one-chamber muffler (kk = 0.96, targeted tone = 150 Hz).

The optimal results with respect to one-tone, two-tone, and broadband noise optimizations are described as follows.

### 6.1.1. One-tone noise optimization

The maximization of STL with respect to a one-, two-, and three-chamber cross-flow perforated muffler at 150 Hz was performed. As indicated in Tables 2–4, seven set, four set, and six set parameters with respect to three kinds of muffle are tried. Obviously, the optimal design data can be obtained at the last set of SA parameters at  $(kk, \text{Iter}) = (0.96, 2000)$ ,  $(0.99, 100)$ , and  $(0.96, 200)$ , respectively. Moreover, the pressure drops— $\Delta p$  (back pressure)—with respect to three kinds of mufflers are found to be 11.33–23.87 (Pa), 8.07–11.25 (Pa), and 5.83–59.32 (Pa). These drops will meet the specified maximal pressure drop of 100 (Pa).

For a one-chamber muffler, the related STLs with respect to various cooling rates ( $kk$ ) and iterations ( $\text{Iter}$ ) are plotted and illustrated in Figs. 10 and 11, respectively. Likewise, for a two-chamber muffler, the related STLs with respect to different parameters are plotted and illustrated in Fig. 12. Consequently, for a

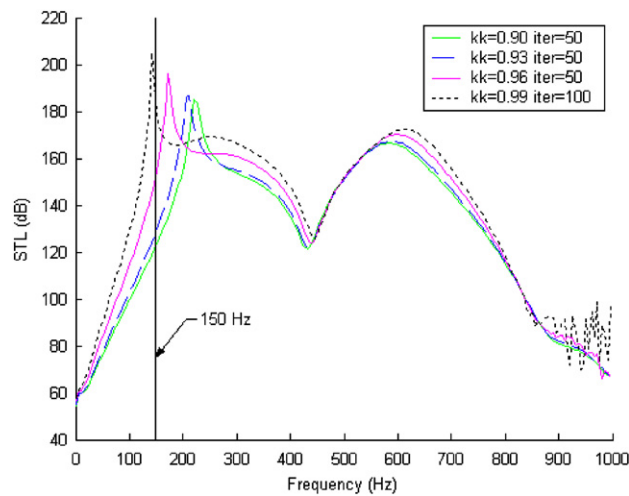


Fig. 12. STL curves with respect to frequencies at various cooling rates and iterations for a two-chamber muffler (targeted tone = 150 Hz).

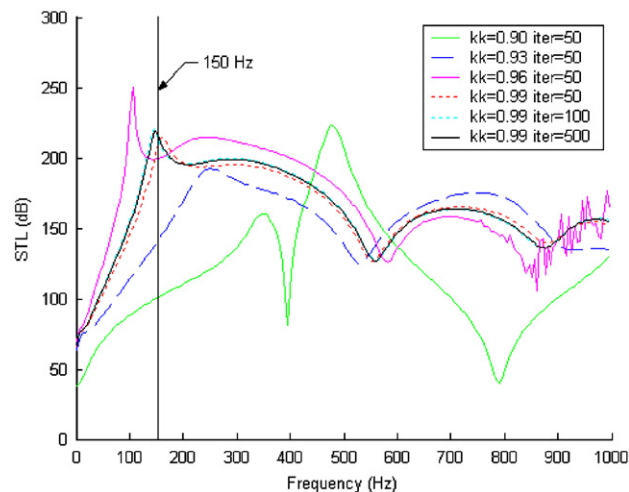


Fig. 13. STL curves with respect to frequencies at various cooling rates and iterations for a three-chamber muffler (targeted tone = 150 Hz).



three-chamber muffler, the related STLs with respect to different parameters are plotted and illustrated in Fig. 13. In addition, the spectrum of the STL curves with respect to these mufflers are plotted together in Fig. 14 simultaneously.

As indicated in Figs. 10–13, the better cooling rate ( $kk$ ) will occur within 0.96–0.99; moreover, the accuracy of the OBJ value will be significantly improved when the iteration increases. As illustrated in Fig. 14, the more chambers we have in a muffler the better the acoustical performance. Consequently, it is obvious that the maximal STLs with respect to three kinds of mufflers are precisely tuned at the targeted tone of 150 Hz.

6.1.2. Two-tone noise optimization

The maximization of averaged STLs with respect to a one-, two-, and three-chamber cross-flow perforated muffler at 100 and 200 Hz was performed. By using the optimal  $kk$  and iteration obtained in pure tone

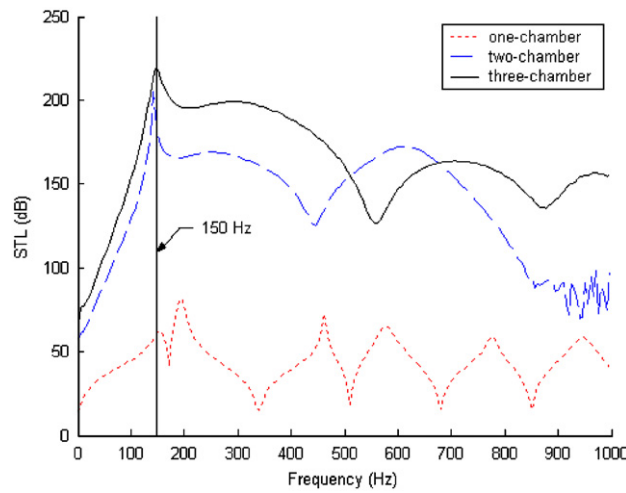


Fig. 14. STL curves with respect to frequencies for three kinds of mufflers (targeted pure tone of 150 Hz).

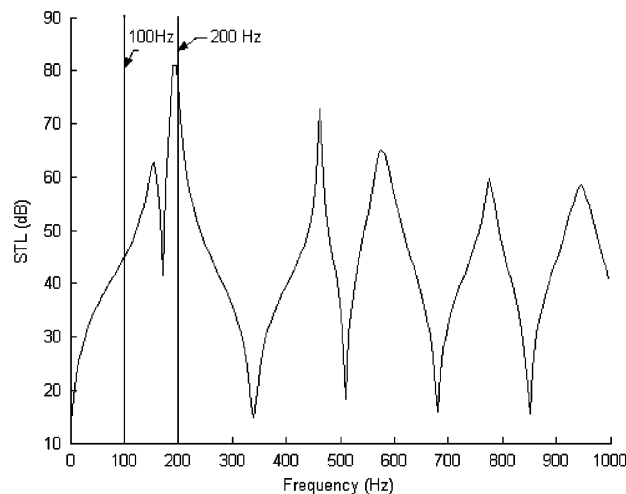


Fig. 15. STL curve with respect to frequencies for a one-chamber muffler (two targeted tones of 100 and 200 Hz). ( $kk = 0.96$ ; iter = 2000;  $Aff_1 = 0.5001E+00$ ;  $Aff_2 = 0.2001E+00$ ;  $D_1 = 0.1000E+00$ ;  $D_2 = 0.1000E+00$ ;  $dh_1 = 0.1751E-02$ ;  $\eta_1 = 0.3001E-01$ ;  $dh_2 = 0.1751E-02$ ;  $\eta_2 = 0.3001E-01$ ;  $(STL1 + STL2)/2 = 0.7114E+02$ ).

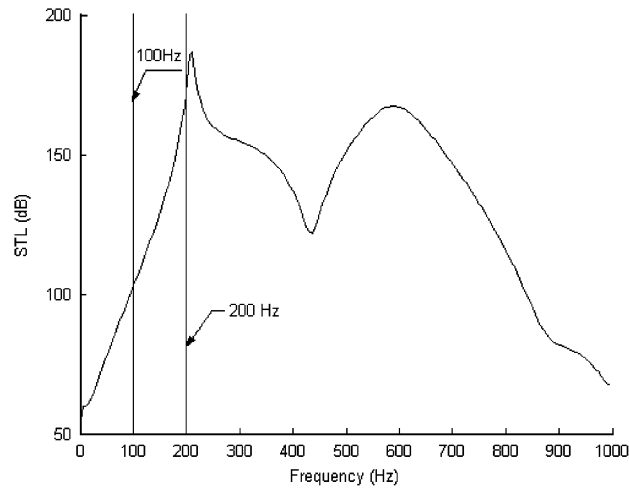


Fig. 16. STL curve with respect to frequencies for a two-chamber muffler (two targeted tones of 100 and 200 Hz) ( $kk = 0.99$ ;  $iter = 100$ ;  $Aff_1 =$ ;  $Aff_2 = 0.3855E+00$ ;  $Aff_3 = Aff_4 = 6710E+00$ ;  $D_1 = D_2 = D_3 = 0.2855E+00$ ;  $\eta_1 = 0.5645E-02$ ;  $dh_1 = 0.8193E-01$ ;  $\eta_2 = 0.5645E-02$ ;  $dh_2 = 0.8193E-01$ ;  $\eta_3 = 0.5645E-02$ ;  $dh_3 = 0.8193E-01$ ;  $\eta_4 = 0.5645E-02$ ;  $dh_4 = 0.8193E-01$ ;  $(STL1 + STL2)/2 = 0.1383E+03$ ).

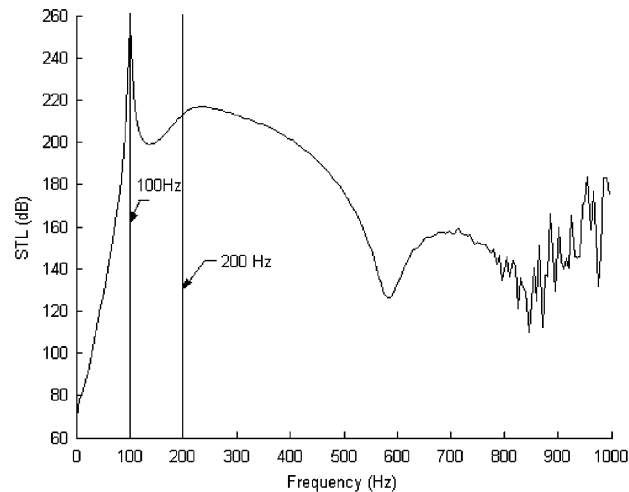


Fig. 17. STL curve with respect to frequencies for a three-chamber muffler (two targeted tones of 100 and 200 Hz) ( $kk = 0.99$ ,  $iter = 100$ ,  $Aff_1 = 0.2971E+00$ ,  $Aff_2 = 0.2971E+00$ ,  $Aff_3 = 0.2971E+00$ ,  $Aff_4 = 0.7824E+00$ ,  $Aff_5 = 0.7824E+00$ ,  $Aff_6 = 0.7824E+00$ ,  $D_1 = 0.3427E+00$ ,  $D_2 = 0.3427E+00$ ,  $D_3 = 0.3427E+00$ ,  $D_4 = 0.3427E+00$ ,  $dh_1 = 0.6846E-02$ ,  $\eta_1 = 0.9794E-01$ ,  $dh_2 = 0.6846E-02$ ,  $\eta_2 = 0.9794E-01$ ,  $dh_3 = 0.6846E-02$ ,  $\eta_3 = 0.9794E-01$ ,  $dh_4 = 0.6846E-02$ ,  $\eta_4 = 0.9794E-01$ ,  $dh_5 = 0.6846E-02$ ,  $\eta_5 = 0.9794E-01$ ,  $dh_6 = 0.6846E-02$ ,  $\eta_6 = 0.9794E-01$ ,  $(STL1 + STL2)/2 = 0.2407E+03$ ).

analysis, the resultant STL curves of three kinds of mufflers are obtained and shown in Figs. 15–17, respectively. In addition, the spectrum of the STL curves with respect to these mufflers is plotted together in Fig. 18 simultaneously.

### 6.1.3. Full-band noise optimization

Three kinds of optimal muffler design parameters and sizes in minimizing the fan's sound power level are achieved and summarized in Tables 5–7. As revealed in Tables 5–7, the optimal design data with respect to one-, two-, and three-chamber mufflers occurred in the sixth, fifth, and seventh set, respectively. The related silenced SWLs with respect to three silencers (i.e. one-, two-, and three-chamber muffler) are 96.5, 60.1 and

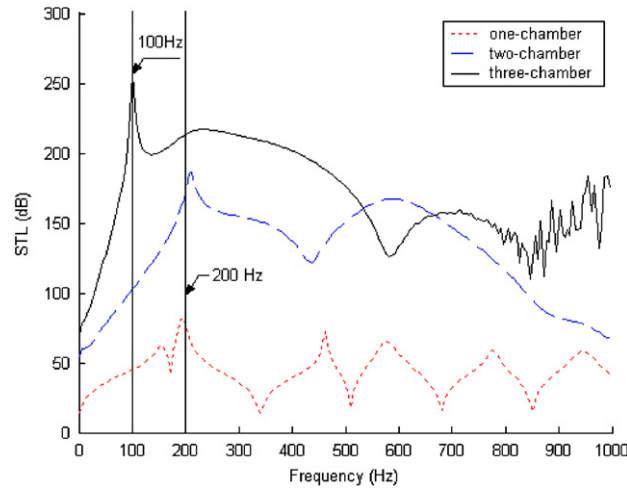


Fig. 18. STL curves with respect to frequencies for three kinds of mufflers (targeted two tones of 100 and 200 Hz).

Table 5  
Optimal STLs for a one-chamber cross-flow perforated muffler (broadband)

Item	SA parameters		Results				
	kk	Iter	$Aff_1$	$Aff_2$	$D_1$	$D_2$	SWL (dB)
1	0.90	50	0.7968 $dh_1$ (m) 0.005645	0.6452 $\eta_1$ 0.08193	0.2855 $dh_2$ (m) 0.005645	0.2855 $\eta_2$ 0.08193	110.2 $\Delta p$ (Pa) 1.31
2	0.93	50	0.8329 $dh_1$ (m) 0.006119	0.6993 $\eta_1$ 0.08825	0.3080 $dh_2$ (m) 0.006119	0.3080 $\eta_2$ 0.08825	104.9 $\Delta p$ (Pa) 1.08
3	0.96	50	0.6102 $dh_1$ (m) 0.003196	0.3653 $\eta_1$ 0.0929	0.1689 $dh_2$ (m) 0.003196	0.1689 $\eta_2$ 0.04929	103.8 $\Delta p$ (Pa) 5.60
4	<u>0.99</u>	50	0.5501 $dh_1$ (m) 0.002408	0.2752 $\eta_1$ 0.03878	0.1313 $dh_2$ (m) 0.002408	0.1313 $\eta_2$ 0.03878	99.6 $\Delta p$ (Pa) 11.33
5	<u>0.99</u>	100	0.5636 $dh_1$ (m) 0.002585	0.2954 $\eta_1$ 0.04113	0.1398 $dh_2$ (m) 0.002585	0.1398 $\eta_2$ 0.04113	96.6 $\Delta p$ (Pa) 9.53
6	<u><b>0.99</b></u>	<u><b>200</b></u>	<u><b>0.5467</b></u> $dh_1$ (m) <u><b>0.002363</b></u>	<u><b>0.2701</b></u> $\eta_1$ <u><b>0.03817</b></u>	<u><b>0.1292</b></u> $dh_2$ (m) <u><b>0.002363</b></u>	<u><b>0.1292</b></u> $\eta_2$ <u><b>0.03817</b></u>	<u><b>96.5</b></u> $\Delta p$ (Pa) 11.86

Underlined: selected parameter.

Underlined and bold: selected parameter and final results.

20.3 dB, individually. Moreover, their pressure drops— $\Delta p$  (back pressure)—with respect to three kinds of mufflers are found to be 1.08–11.86 (Pa), 12.83–59.79 (Pa), and 9.66–87.36 (Pa). They will meet the specified maximal pressure drop of 100 (Pa).

Table 6  
Optimal STLs for a two-chamber cross-flow perforated muffler (broadband)

Item	SA parameters		Results					
	kk	Iter						
1	0.90	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	SWL (dB)
			0.2256	0.2256	0.3512	0.3512	0.1256	73.8
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.1256	0.1256	0.002287	0.03716	0.002287	59.79
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
0.03716	0.002287	0.03716	0.002287	0.03716				
2	0.93	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	SWL (dB)
			0.3518	0.3518	0.6036	0.6036	0.2518	67.5
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.2518	0.2518	0.004938	0.07251	0.004938	14.36
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
0.07251	0.004938	0.07251	0.004938	0.07251				
3	0.96	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	SWL (dB)
			0.3405	0.3405	0.5809	0.5809	0.2405	75.3
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.2405	0.2405	0.004700	0.06933	0.004700	15.80
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
0.06933	0.004700	0.06933	0.004700	0.06933				
4	<u>0.99</u>	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	SWL (dB)
			0.2256	0.2256	0.3512	0.3512	0.1256	73.8
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			0.1256	0.1256	0.002287	0.03716	0.002287	59.79
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
0.03716	0.002287	0.03716	0.002287	0.03716				
5	<u><b>0.99</b></u>	<u><b>100</b></u>	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	D <sub>1</sub>	SWL (dB)
			<u><b>0.3657</b></u>	<u><b>0.3657</b></u>	<u><b>0.6315</b></u>	<u><b>0.6315</b></u>	<u><b>0.2657</b></u>	<u><b>60.1</b></u>
			D <sub>2</sub> (m)	D <sub>3</sub> (m)	η <sub>1</sub>	dh <sub>1</sub> (m)	η <sub>2</sub>	Δp (Pa)
			<u><b>0.2657</b></u>	<u><b>0.2657</b></u>	<u><b>0.005231</b></u>	<u><b>0.07641</b></u>	<u><b>0.005231</b></u>	12.83
			dh <sub>2</sub> (m)	η <sub>3</sub>	dh <sub>3</sub> (m)	η <sub>4</sub>	dh <sub>4</sub> (m)	
<u><b>0.07641</b></u>	<u><b>0.005231</b></u>	<u><b>0.07641</b></u>	<u><b>0.005231</b></u>	<u><b>0.07641</b></u>				

Underlined: selected parameter.

Underlined and bold: selected parameter and final results.

By using the above optimal design data sets obtained from Tables 5–7, the individual theoretical STL curves with three kinds of mufflers are plotted and compared to the SWLO—an un-silenced SWL—in Figs. 19–21. Based on plane wave theory, the proposed theoretical cutoff frequencies of  $f_{c1}$  ( $f_{c1} = (1.84c_o/\pi D)(1 - M^2)^{1/2}$ ) with respect to three kinds of mufflers (Figs. 19–21) are 652–1555 (Hz), 756–1600 (Hz), and 704–1718 (Hz). Actually, the frequency ranges in Figs. 19–21, which are valid, are below the mentioned cutoff frequencies.

Moreover, the spectrum of the STL curves with respect to the three kinds of mufflers is depicted together in Fig. 22.

### 6.2. Discussion

For the pure tone’s optimization discussed in Section 6.1 and shown in Fig. 14, the maximal STLs with respect to the three kinds of mufflers have been precisely tuned at the targeted pure tone of 150 Hz. Similarly, for a dual tone’s optimization depicted in Fig. 18, concerning the weighted factors for two tones that are equal to 0.5, the averaged STLs of the two tones are maximized uniformly. As a result of above observation, the SA method is reliably used in the muffler’s shape optimization.

Table 7  
Optimal STLs for a three-chamber cross-flow perforated muffler (broadband)

Item	SA parameters		Results						
	kk	Iter							
1	0.90	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2731	0.2731	0.2731	0.6388	0.6388	40.4	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.6388	0.2828	0.2828	0.2828	0.2828	9.92	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.005590	0.08120	0.005590	0.08120	0.005590		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.08120	0.005590	0.08120	0.005590	0.08120		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.005590	0.08120					
2	0.93	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2117	0.2117	0.2117	0.2701	0.2701	46.9	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.2701	0.1292	0.1292	0.1292	0.1292	69.67	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.002363	0.03817	0.002363	0.03817	0.002363		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.03817	0.002363	0.03817	0.002363	0.03817		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.002363	0.03817					
3	0.96	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2117	0.2117	0.2117	0.2701	0.2701	29.9	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.2701	0.1292	0.1292	0.1292	0.1292	69.67	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.002363	0.03817	0.002363	0.03817	0.002363		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.03817	0.002363	0.03817	0.002363	0.03817		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.002363	0.03817					
4	<u>0.99</u>	50	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2742	0.2742	0.2742	0.6452	0.6452	37.7	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.6452	0.2855	0.2855	0.2855	0.2855	9.66	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.005645	0.08193	0.005645	0.08193	0.005645		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.08193	0.005645	0.08193	0.005645	0.08193		

			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.005645	0.08193					
5	<u>0.99</u>	100	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2169	0.2169	0.2169	0.3012	0.3012	51.0	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.3012	0.1422	0.1422	0.1422	0.1422	55.90	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.002636	0.04181	0.002636	0.04181	0.002636		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.04181	0.002636	0.04181	0.002636	0.04181		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.002636	0.04181					
6	<u>0.99</u>	<u>200</u>	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			0.2075	0.2075	0.2075	0.2451	0.2451	21.7	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			0.2451	0.1188	0.1188	0.1188	0.1188	84.26	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			0.002144	0.03526	0.002144	0.03526	0.002144		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.03526	0.002144	0.03526	0.002144	0.03526		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.002144	0.03526					
7	<u>0.96</u>	<u>400</u>	Aff <sub>1</sub>	Aff <sub>2</sub>	Aff <sub>3</sub>	Aff <sub>4</sub>	Aff <sub>5</sub>	SWL (dB)	
			<u>0.2068</u>	<u>0.2068</u>	<u>0.2068</u>	<u>0.2406</u>	<u>0.2406</u>	<u>20.3</u>	
			Aff <sub>6</sub>	D <sub>1</sub>	D <sub>2</sub> (m)	D <sub>3</sub> (m)	D <sub>4</sub> (m)	Δp (Pa)	
			<u>0.2406</u>	<u>0.1169</u>	<u>0.1169</u>	<u>0.1169</u>	<u>0.1169</u>	87.36	
			dh <sub>1</sub> (m)	η <sub>1</sub>	dh <sub>2</sub> (m)	η <sub>2</sub>	dh <sub>3</sub> (m)		
			<u>0.002106</u>	<u>0.03474</u>	<u>0.002106</u>	<u>0.03474</u>	<u>0.002106</u>		
			η <sub>3</sub>	dh <sub>4</sub> (m)	η <sub>4</sub>	dh <sub>5</sub> (m)	η <sub>5</sub>		
			0.03474	0.002106	0.03474	0.002106	0.03474		
			dh <sub>6</sub> (m)	η <sub>6</sub>					
			0.002106	0.03474					

Underlined: selected parameter.

Underlined and bold: selected parameter and final results.

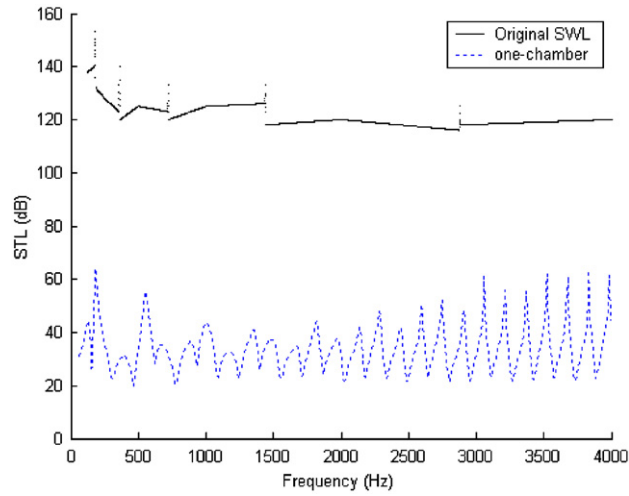


Fig. 19. STL curves and an original SWL with respect to frequencies for a one-chamber muffler (broadband).

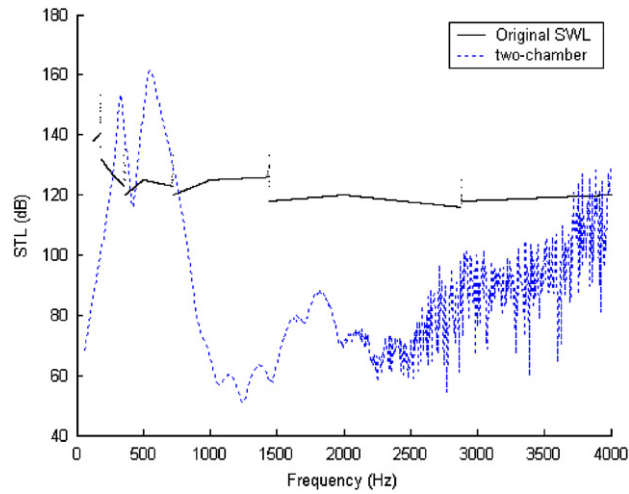


Fig. 20. STL curves and an original SWL with respect to frequencies for a two-chamber muffler (broadband).

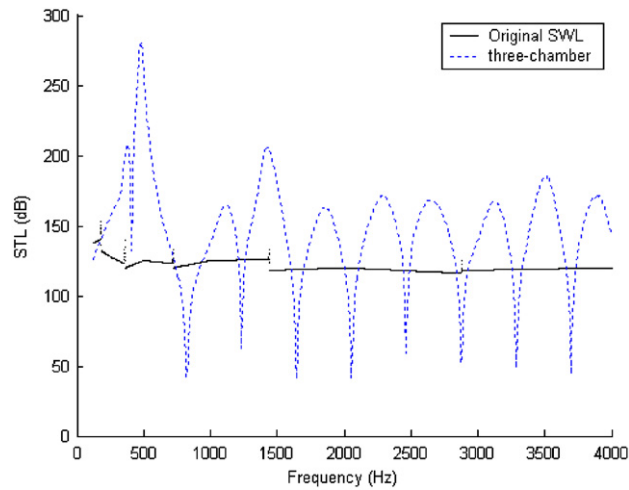


Fig. 21. STL curves and an original SWL with respect to frequencies for a three-chamber muffler (broadband).

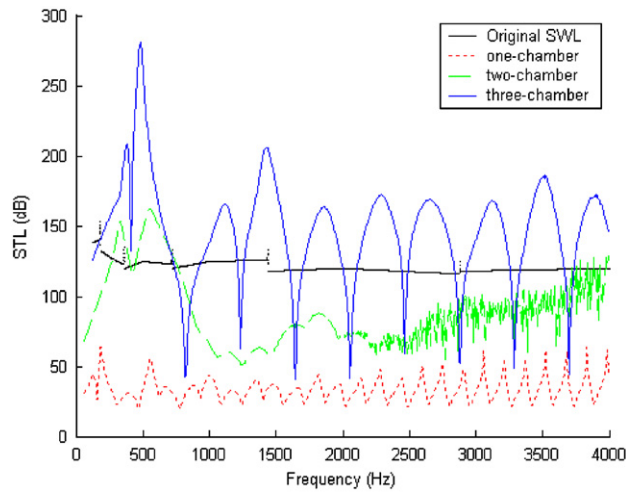


Fig. 22. STL curves and an original SWL with respect to frequencies for three kinds of mufflers (broadband noise).

Table 8  
Influence of STLs and  $\Delta p$  with respect to  $\eta_i$ ,  $M_i$ ,  $D_o/D_i$  and  $x_i$  for a one-chamber cross-flow perforated muffler (pure tone of 150 Hz)

Item	Parameters								Targeted OBJ	Back pressure
	$\eta_1$	$\eta_2$	$M_1$	$M_2$	$D_o/D_1$	$D_o/D_2$	$x_1$	$x_2$	STL (dB)	$\Delta p$ (Pa)
1	0.049	0.0493	0.0039	0.0039	11.84	11.84	0.0852	0.0852	38.7	14.69
2	0.0347	0.035	0.0081	0.0081	17.10	17.10	0.5720	0.5720	54.9	15.62
3	0.0329	0.033	0.0091	0.0091	18.09	18.09	0.5370	0.5370	56.9	18.23
4	0.0388	0.039	0.0064	0.0064	15.23	15.23	0.6502	0.6502	51.5	11.33
5	0.0322	0.032	0.0095	0.0095	18.55	18.55	0.5226	0.5226	57.9	19.49
6	0.003	0.003	0.0111	0.0111	20	20	0.4804	0.4804	61	23.87

In dealing with a broadband noise in which the spectrum is complicated and emitted from a noisy fan, the selection of the appropriate SA parameters set is indispensable in searching for a better shape design solution within the three kinds of mufflers. As illustrated in Tables 5–7, the optimal design data for three kinds of mufflers has been achieved at (kk, Iter) of (0.99, 200), (0.99, 100), and (0.96, 400), respectively. According to these tables, it is found that the overall noise reduction with respect to three mufflers (one, two, and three chambers) can reach 42.4, 78.8, and 118.6 dB, respectively.

To appreciate the influence of STL and  $\Delta p$  with respect to other parameters such as Mach number ( $M_i$ ), porosity ( $\eta_i$ ), expansion ratio ( $D_o/D_i$ ), and open area ( $x_i$ ) for a one-chamber cross-flow muffler, Table 2 is transformed into Table 8. As indicated in Table 8, it is obvious that the STL is proportional to the Mach number ( $M_i$ ) and expansion ratio ( $D_o/D_i$ ) and inversely proportional to the porosity ( $\eta_i$ ). As shown in Figs. 14, 18, and 22, the muffler with three chambers obviously has the best acoustical performance. Conversely, the single-chamber muffler has the worst. In addition, the pressure drop (i.e. back pressure) of the mufflers is proportional to the Mach number ( $M_i$ ) and inversely proportional to the porosity ( $\eta_i$ ). The above observation of one-chamber cross-flow mufflers is consistent with studies by our experimental data and Munjal et al. [13].

Consequently, the number of chambers, Mach number ( $M_i$ ), porosity ( $\eta_i$ ), and expansion ratio ( $D_o/D_i$ ) play essential roles in eliminating the noise level in mufflers.

## 7. Conclusion

It has been shown that two kinds of SA parameters—kk, Iter—play essential roles in seeking a better shape during the SA optimization. A higher iteration will lead to a set of enhanced data. Before broadband



noise optimization is performed, the pure-tone and dual-tone optimization of mufflers (one-, two-, and three-chamber mufflers) has been carried out. Results reveal that the maximal STL located around the desired tones is acceptable. To avoid the excessive backpressure occurring in a venting fan, which may lower the venting flow rate, the value of  $\Delta p$  has been calculated and rechecked. As stated in Section 6, several parameters—the number of chambers, Mach number ( $M_i$ ), porosity ( $\eta_i$ ), and expansion ratio ( $D_o/D_i$ )—dominate the acoustical performance. Without a doubt, the acoustical mechanism of a cross-flow muffler with three chambers in serial exhibits better noise reduction than that of the mufflers with less acoustical chambers.

Consequently, this study offers a quick and efficient methodology to comprehensively design well-shaped multi-chamber cross-flow mufflers within a confined space. It also satisfies the requirement of the allowable maximal pressure drop for the fan's venting system.

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