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Isom's thickness noise for axial and centrifugal subsonic fans

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Abstract

The thickness noise predicted by the Ffowcs Williams and Hawkings (FW&H) equation depends on the normal velocity v_n which is very sensitive to the meshing size. Isom showed that in a far field a monopolar source is equivalent to a dipolar source induced by a uniform distribution of the load on the entire moving surface. Consequently, the calculation of the thickness noise becomes completely independent of the normal velocity v_n. Its expression, as suggested by Farassat, is for any moving surface. The main objective of this work is to determine a specific expression of Isom's thickness noise in time and frequency domains for axial and centrifugal subsonic fans. The proposed form of the thickness noise enables to highlight the effect of each geometrical parameter of the fan on the overall thickness noise, on the one hand, and presents a fast computational mean and low memory storage capability since the acoustic pressure in the frequency domain is calculated for only one blade, on the other.

1. Introduction

In a free field, Isom showed in Ref. [1] that the acoustic pressure generated by a mopole is equivalent to these generated by a dipole induced by a uniform load equal to $\rho_0 c^2$ over a moving surface. Consequently, the calculation of the thickness noise becomes completely independent of the normal velocity v_n over the surface.

Farassat shows in Refs. [2,3] that it is more suitable to determine the thickness noise using Isom's formulation than methods based on the resolution of Ffowcs Williams and Hawkings (FW&H) equation [4] using fluctuating velocity. The latter presents an important variation on the surface which makes the numerical integration very sensitive to the meshing resolution. On the other hand, Isom's thickness noise formula can present some difficulties for rotating machinery because of its sensitivity to the geometry of the blade tips [5].

Isom's thickness noise formula was used by Ghorbaniasl and Hirsch [6] as a consistency benchmark to show that a discrepancy should not exist between FW&H's thickness noise and Isom's noise when Farassat 1A formulation presented by Farassat and Succi in Ref. [7] is numerically solved in the time domain.

The main objective of this paper is to define specific formulas of thickness noise in time and frequency domains for axial and centrifugal subsonic fans based on initial Isom's formulation. This approach has the

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Nomenclature

c	velocity of sound in a medium at rest
D	Doppler amplification factor due to the moving source
f	a function taking into account the geometry and the kinematics of the moving surface when $f(\mathbf{x}, t) = 0$
$\mathbf{F}$	force induced by the constant load $\rho_0 c^2$ on the moving surface
H	Heaviside function
m	rank of harmonic
M_r	Mach number of the sources in the direction of the listening point given by $M_r = (\mathbf{r}/r) \cdot \mathbf{M}$
$\mathbf{M}$	Mach number associated with the absolute velocity of the source
$\mathbf{n}$	a unit vector normal to the moving surface
p'	acoustic pressure
P'	overall acoustic pressure
$\mathbf{r}$	distance between the observer and the source given by $\mathbf{r} = \mathbf{x} - \mathbf{y}$
s	rank of harmonic
t	reception time
v_n	velocity normal to the moving surface
x	position of the observer
y	position of the source
α_n	angle between two successive fan blades
δ	Dirac function
θ	observer angular position
ρ_0	density of the propagation medium
τ	emission time
ϕ	observer angular position
Ω	rotational velocity in rad/s

double advantage of calculating the thickness noise in time and frequency domains without avoiding the calculation of the normal velocity v_n, on the one hand, and adding a new benchmark test of consistency for the thickness and the loading noise calculation in time and frequency domains using the free-field solution of FW&H's equation, on the other. The proposed formulas enable to highlight the effect of each parameter of the fan (shape of the blades, angular velocity, etc.) on the overall thickness noise and present a fast computational mean and low memory storage capability since the acoustic pressure in frequency domain is calculated for only one blade.

2. Isom's thickness noise

Let $f(\mathbf{x}, t) = 0$ be a function taking into account the geometry and the kinematics of a moving body surface, defined such as $f > 0$ outside the body; see Fig. 1. Let $H(f)$ be the Heaviside function.

According to Ref. [3], the wave equation corresponding to the trivial function $\rho_0 c^2[1 - H(f)]$ is given by

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\{\rho_0 c^2[1 - H(f)]\} = \frac{\partial}{\partial t}[\rho_0 v_n \delta(f)] + \frac{\partial}{\partial x_i}[\rho_0 c^2 n_i \delta(f)], \tag{1}$$

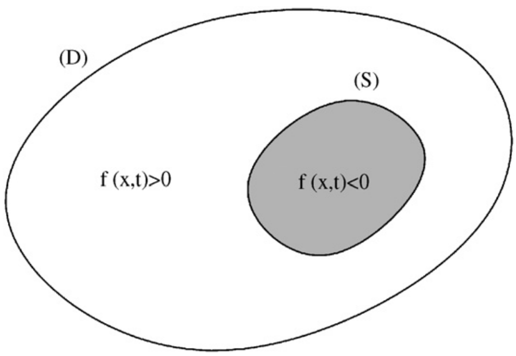

Fig. 1. Calculation domain.

where $n_i = \partial f/\partial x_i$ and $v_n = \mathbf{v}\cdot\mathbf{n}$ are, respectively, the local unit normal vector and the local normal velocity on the body surface.

2.1. Time domain solution

The far and free-field solution of Eq. (1) is given by

$$\rho_0 c^2[1 - H(f)] = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int_S \left[\frac{\rho_0 v_n}{rD}\right]_{\text{ret}} \mathrm{d}S + \frac{1}{4\pi}\frac{\partial}{\partial x_i}\int_S \left[\frac{\rho_0 c^2 n_i}{rD}\right]_{\text{ret}} \mathrm{d}S, \tag{2}$$

where $D = |1 - M_r|$ is the Doppler factor.

Outside the body $H(f) = 1$; then:

$$\rho_0 c^2[1 - H(f)] = 0. \tag{3}$$

Eq. (2) then gives

$$\frac{1}{4\pi}\frac{\partial}{\partial t}\int_S \left[\frac{\rho_0 v_n}{rD}\right]_{\text{ret}} \mathrm{d}S = \frac{1}{4\pi}\frac{\partial}{\partial x_i}\int_S \left[\frac{\rho_0 c^2 n_i}{rD}\right]_{\text{ret}} \mathrm{d}S. \tag{4}$$

This equality shows that the monopole source solution is not unique. It is, also, equal to a dipole source induced by a steady and uniform load $\rho_0 c^2$ over the moving body surface.

Isom's thickness noise is then given by

$$p'_{\text{Isom}}(\mathbf{x}, t) = \frac{1}{4\pi}\frac{\partial}{\partial x_i}\int_S \left[\frac{\rho_0 c^2 n_i}{rD}\right]_{\text{ret}} \mathrm{d}S. \tag{5}$$

In the far-field one can show that:

$$p'_{\text{Isom}}(\mathbf{x}, t) = -\frac{1}{4\pi}\int_S \left[\frac{\mathbf{r}}{cDr}\left\{\frac{1}{r}\frac{\partial}{\partial\tau}\left(\frac{\mathbf{F}}{D}\right) + \frac{\mathbf{F}}{r^3 D}c\ \mathbf{r}\cdot\mathbf{M}\right\}\right]_{\text{ret}} \mathrm{d}S, \tag{6}$$

where $t = \tau + (r/c_0)$ is the reception time and $\mathbf{F} = \rho_0 c^2\mathbf{n}$ is the force induced by the uniform load $\rho_0 c^2$.

In the far field, the term in $1/r^3$ is neglected and after expansion Eq. (6) becomes

$$p'_{\text{Isom}}(\mathbf{x}, t) = \frac{1}{4\pi}\int_S \left[\underbrace{\frac{\mathbf{r}\cdot\mathbf{F}}{cD^3 r^2}\frac{\partial D}{\partial\tau}}_{\text{I}} - \underbrace{\frac{\mathbf{r}\cdot\frac{\partial\mathbf{F}}{\partial\tau}}{cD^2 r^2}}_{\text{II}}\right]_{\text{ret}} \mathrm{d}S \tag{7}$$

D is given as a function of r by $D = |1 + [(1/c)(\partial r/\partial\tau)]|$.

Part (I) of Eq. (7) expresses the unsteadiness of the source motion while part (II) expresses the effect of the rotating load ($\rho_0 c^2$) source.

Eq. (7) can then be written as

$$p'_{\text{Isom}}(\mathbf{x}, t) = p'_{\text{I}}(\mathbf{x}, t) + p'_{\text{II}}(\mathbf{x}, t), \tag{8}$$

where

$$p'_{\text{I}}(\mathbf{x}, t) = \frac{1}{4\pi}\int_S \left[\frac{\mathbf{r}\cdot\mathbf{F}}{c^2 r^2\left|1 + \frac{1}{c}\frac{\partial r}{\partial\tau}\right|^3}\frac{\partial^2 r}{\partial\tau^2}\right]_{\text{ret}} \mathrm{d}S, \tag{9}$$

$$p'_{\text{II}}(\mathbf{x}, t) = -\frac{1}{4\pi}\int_S \left[\frac{\mathbf{r}\cdot\frac{\partial\mathbf{F}}{\partial\tau}}{cr^2\left|1 + \frac{1}{c}\frac{\partial r}{\partial\tau}\right|^2}\right]_{\text{ret}} \mathrm{d}S. \tag{10}$$

Eqs. (9) and (10) are nothing more than the loading noise parts of Formulation 1A of Farassat. As it is written, because of the existence of partial derivatives in the denominators, the formulas are ambiguous to interpretation. This ambiguity is a mathematical subtlety discussed in the NASA publication of Farassat [8]. In our case, this form of writing will be very useful for the continuation of the development and the ambiguity will be circumvented. An exact expression of r will be given below and partial derivatives will be fully defined.

2.2. Isom's thickness noise for fans

Let us consider a fan turning at a velocity Ω. The angular position of a point on the blade is related to the moment of noise emission τ by $\Psi = \Omega\tau + \Psi_0$ where Ψ_0 is the initial position at $\tau = 0$. Suppose that $\Psi_0 = 0$ at $\tau = 0$.

In Fig. 2, S is a noise source rotating around e_3 with an angular velocity Ω and a distance r_s, $\mathbf{F}$ a force applied by the fluid on the surface on S, defined by its radial, tangential and axial components (F_r, F_t, F_a), O is an observer defined by (r_0, φ, θ) and $\mathbf{r}$ the distance between S and O.

According to Fig. 2, the distance between the source and the observer is given by

$$\mathbf{r} = \begin{pmatrix} r_o \sin(\theta)\cos(\varphi) - r_s \cos(\Omega\tau) \\ r_o \sin(\theta)\sin(\varphi) - r_s \sin(\Omega\tau) \\ r_o \cos(\theta) \end{pmatrix}. \tag{11}$$

In the far-field r is approximated by

$$r \cong r_0 - r_s \sin(\theta)\cos(\Omega\tau - \varphi). \tag{12}$$

Over each element dS of the fan surface the applied force $\mathbf{F}$ is defined by

$$\mathbf{F} = \rho_0 c^2 \mathbf{n} = \begin{pmatrix} F_r \cos(\Omega\tau) + F_t \sin(\Omega\tau) \\ F_r \sin(\Omega\tau) - F_t \cos(\Omega\tau) \\ F_a \end{pmatrix}, \tag{13}$$

where F_r, F_t and F_a are constant in the moving reference frame.

If $\mathbf{n}$ is the local unit normal vector on the blade surface in the moving reference frame, then, $F_r = \rho_0 c^2(\mathbf{n} \cdot \mathbf{n}_r)$, $F_t = \rho_0 c^2(\mathbf{n} \cdot \mathbf{n}_t)$ and $F_a = \rho_0 c^2(\mathbf{n} \cdot \mathbf{n}_a)$, where $\mathbf{n}_r$, $\mathbf{n}_t$ and $\mathbf{n}_a$ are, respectively, the local unit vectors in radial, tangential and axial directions.

The direction of $\mathbf{F}$ components depends on the direction of the source rotation and position.

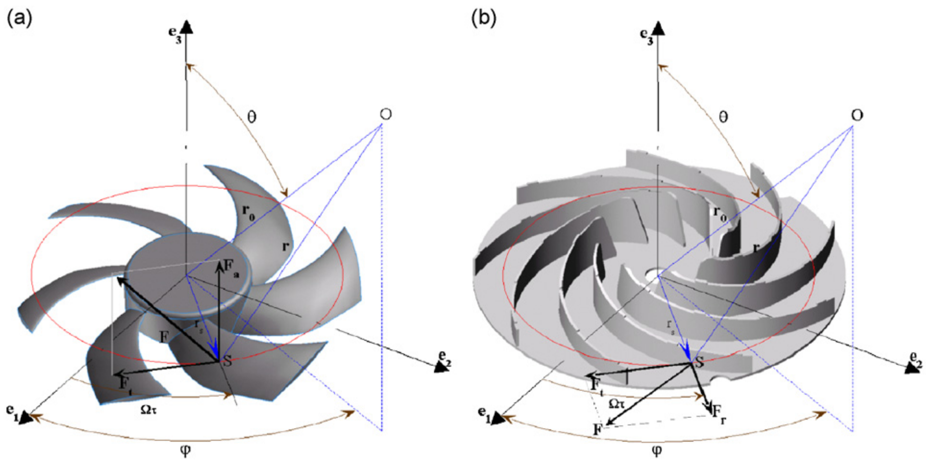

Fig. 2. Reference frame: (a) axial fan ($F_r = 0, F_t, F_a$), (b) centrifugal fan ($F_r, F_t, F_a = 0$).

According to the above development, the two parts of the thickness noise for both axial and centrifugal fans are given by

$$p'_{\mathrm{I}}(\mathbf{x},t)=\frac{\Omega^2\sin(\theta)}{4\pi r_0 c^2}\int_S \frac{r_s\cos(\Omega\tau-\varphi)\{\sin(\theta)\cos(\Omega\tau-\varphi)F_r+\sin(\theta)\sin(\Omega\tau-\varphi)F_t+\cos(\theta)F_a\}}{|1+A\sin(\Omega\tau-\varphi)|^3}\,\mathrm{d}S, \tag{14}$$

$$p'_{\mathrm{II}}(\mathbf{x},t)=\frac{\Omega\sin(\theta)}{4\pi r_0 c}\int_S \frac{\sin(\Omega\tau-\varphi)F_r+\cos(\Omega\tau+\varphi)F_t}{|1+A\sin(\Omega\tau-\varphi)|^2}\,\mathrm{d}S, \tag{15}$$

where $A=(r_s\Omega/c)\sin(\theta)$.

It is noticed that the axial component F_a of the load $\rho_0 c^2 n_i$ does not exist in Eq. (15). Hence, for centrifugal fans, part (II) of the thickness noise is influenced by both radial and tangential components of the load $\rho_0 c^2\mathbf{n}$, whereas, for axial fans, only the tangential component has an effect on it.

Let us consider a Z-blade fan; the overall acoustic pressure generated by the fan is given by

$$P'_{\mathrm{Isom}}(\mathbf{x},t)=\sum_{n=1}^{Z} p'_{\mathrm{Isom}}\left(\mathbf{x},t+\frac{\alpha_n}{\Omega}\right), \tag{16}$$

where p' is the acoustic pressure of one blade and α_n is the angle between two successive blades.

2.3. Frequency domain solution

2.3.1. General solution

The Fourier transform allows the passage from time to frequency domain of the acoustic pressure. It is given by

$$p'^{(s)}_{\mathrm{Isom}}(\mathbf{x})=\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} p'_{\mathrm{Isom}}(\mathbf{x},t)\mathrm{e}^{\mathrm{i}s\Omega t}\,\mathrm{d}t. \tag{17}$$

By replacing $p'_{\mathrm{Isom}}(\mathbf{x},t)$ by its literal expression in equation (17) and by considering $\xi=\Omega\tau-\varphi$, the two parts of the acoustic pressure in the frequency domain for both axial and centrifugal fans are given by

$$p'^{(s)}_{\mathrm{I}}(\mathbf{x})=\frac{\Omega^2\sin(\theta)}{8\pi^2c^2r_0}\mathrm{e}^{\mathrm{i}s(\Omega(r_0/c)+\varphi)} \times\int_S\int_{-\varphi}^{2\pi-\varphi}\frac{r_s\cos(\xi)\{\sin(\theta)\cos(\xi)F_r+\sin(\theta)\sin(\xi)F_t+\cos(\theta)F_a\}}{|1+A\sin(\xi)|^2}\mathrm{e}^{\mathrm{i}s(\xi-A\cos(\xi))}\,\mathrm{d}\xi\,\mathrm{d}S, \tag{18}$$

$$p'^{(s)}_{\mathrm{II}}(\mathbf{x})=\frac{\Omega\sin(\theta)}{8\pi^2cr_0}\mathrm{e}^{\mathrm{i}s(\Omega(r_0/c)+\varphi)}\int_S\int_{-\varphi}^{2\pi-\varphi}\frac{\sin(\xi)F_r+\cos(\xi+2\varphi)F_t}{|1+A\sin(\xi)|}\mathrm{e}^{\mathrm{i}s(\xi-A\cos(\xi))}\,\mathrm{d}\xi\,\mathrm{d}S, \tag{19}$$

where $A=(r_s\Omega/c)\sin(\theta)$.

2.3.2. Interferences between fan blades

The Fourier transform of Eq. (16) gives

$$P'^{(s)}_{\mathrm{Isom}}(\mathbf{x})=p'^{(s)}_{\mathrm{Isom}}(\mathbf{x})\sum_{n=1}^{Z}\mathrm{e}^{\mathrm{i}s\alpha_n} \tag{20}$$

knowing that $p'^{(s)}_{\mathrm{Isom}}(\mathbf{x})$ is the same for all blades.

If the blades are identical and equidistant, then $\alpha_n=2\pi n/Z$; in this case $\mathrm{e}^{\mathrm{i}s\alpha_n}$ is a geometric series with a common ratio $\mathrm{e}^{\mathrm{i}s2\pi/Z}$:

$$\sum_{n=1}^{Z}\mathrm{e}^{\mathrm{i}s\alpha_n}=\mathrm{e}^{\mathrm{i}s2\pi/Z}\frac{1-\mathrm{e}^{\mathrm{i}s2\pi}}{1-\mathrm{e}^{\mathrm{i}s2\pi/Z}}. \tag{21}$$

The equality in Eq. (21) is satisfied only and only if s is proportional to Z, i.e., $s = mZ$. In this case:

$$P'^{(m)}_{\text{Isom}}(\mathbf{x}) = Zp'^{(m)}_{\text{Isom}}(\mathbf{x}). \tag{22}$$

The two parts of the overall thickness noise given by the previous equation for Z blades are then given by

$$P'^{(m)}_{\text{I}}(\mathbf{x}) = \frac{Z\Omega^2 \sin(\theta)}{8\pi^2 c^2 r_0} \mathrm{e}^{\mathrm{i}mZ(\Omega(r_0/c)+\varphi)} \times \int_S \int_{-\varphi}^{2\pi-\varphi} \frac{r_s \cos(\xi)\{\sin(\theta)\cos(\xi)F_r + \sin(\theta)\sin(\xi)F_t + \cos(\theta)F_a\}}{(1 + A\sin(\xi))^2} \mathrm{e}^{\mathrm{i}mZ(\xi - A\cos(\xi))}\,\mathrm{d}\xi\,\mathrm{d}S, \tag{23}$$

$$P'^{(m)}_{\text{II}}(\mathbf{x}) = \frac{Z\Omega \sin(\theta)}{8\pi^2 c r_0} \mathrm{e}^{\mathrm{i}mZ(\Omega\frac{r_0}{c}+\varphi)} \int_S \int_{-\varphi}^{2\pi-\varphi} \frac{\sin(\xi)F_r + \cos(\xi + 2\varphi)F_t}{|1 + A\sin(\xi)|} \mathrm{e}^{\mathrm{i}mZ(\xi - A\cos(\xi))}\,\mathrm{d}\xi\,\mathrm{d}S, \tag{24}$$

where $A = (r_s\Omega/c)\sin(\theta)$.

Contrary to Eqs. (14) and (15), Eqs. (23) and (24) need a surface integral over only one blade, which constitutes a consequent time and numerical storage reduction.

Hawkings and Lowson showed in Ref. [9] that for high Mach numbers and high sound levels, the effects of nonlinear acoustic propagation must be taken into account. They stated that nonlinear propagation causes some noticeable changes in the observed acoustic field, especially in its spectral characteristics. Consequently, the formulas described above in the frequency domain will be more appropriate for low Mach numbers and linear acoustic propagation cases.

3. Conclusion

In this paper, specific formulas of thickness noise in time and frequency domains for axial and centrifugal subsonic fans based on initial Isom's formulation were defined. This approach enables to compute the thickness noise in time and frequency domains without performing any calculation of the normal velocity v_n. The proposed formulas highlight the effect of each geometrical parameter of the fan on the overall thickness noise and present a fast computational mean and low memory storage capability since the acoustic pressure in the frequency domain is calculated for only one blade. The overall acoustic pressure was decomposed into two parts: the first one highlighted the effect of the kinematics and the geometry of blades and the second one highlighted the effect of the moving load over blades expressed by $\rho_0 c^2$. Formulas (14), (15), (23) and (24) can also be used as a benchmark test of consistency for the thickness noise calculation and to test loading noise codes in both time and frequency domains when using the free-field solution of FW&H's equation. However, as the classic Isom's thickness noise formula proposed by Farassat presents some difficulties for rotating machinery because of its sensitivity to the geometry of the blade tips, it is expected that the proposed formulas present the same difficulties. It is then recommended to pay particular attention to the blade tips' modeling by choosing the adequate geometry and meshing.

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