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Love waves propagation in functionally graded piezoelectric materials with quadratic variation

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Abstract

The propagation behavior of Love waves in a semi-infinite functionally graded piezoelectric material (FGPM) with a quadratic variation is addressed. The coupled electromechanical field equations are solved, and the dispersion relations, displacement, electric potential, and stress fields are obtained analytically for both electrically open and short conditions. The effects of gradient coefficient on phase velocity, group velocity, and electromechanical coupling factor are plotted and discussed. It is shown that the phase velocity associated with the non-piezoelectric case coincides with that of the corresponding piezoelectric material under electrically open conditions. Because of gradual variation in electromechanical properties, the initial stress during manufacturing process is negligible. Therefore, this model serves as an excellent substitute for the typical layered piezoelectric structures used in surface acoustic wave (SAW) devices. This work provides with a theoretical foundation for the design and practical applications of SAW devices with high performance.

1. Introduction

Love waves propagation in a solid with semi-infinite extent has been one of the topics of great interests to many researchers and engineers in applied and engineering mechanics over the last century. Following Love [1], who examined the appearance of surface waves in a homogeneous half-space overlaid by a layer, many researchers have presented extensive fundamental results for the case of isotropic and anisotropic medium. For the treatments and explorations of such problems, the reader is referred to see the extensive list of references presented in Refs. [2–4] for isotropic material, and Refs. [5–8] for anisotropic one.

Recently, Love waves propagation in piezoelectric materials is of major concern because of their high performance in technological applications such as surface acoustic wave (SAW) devices, signal transmission, signal processing, and information storage [9]. Many researchers have theoretically considered the propagation of surface waves in the typical SAW devices consisting of a piezoelectric layer deposited on the elastic substrate or vice versa [10–17]. During the manufacturing processes of layered piezoelectric structures, due to the mismatch in electromechanical properties of adjacent layers, the presence of internal

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residual stresses within the layers is inevitable. Liu et al. [18], Qian et al. [19], Jin et al. [20], Su et al. [21], and Du et al. [22] studied the effects of homogeneous and inhomogeneous initial stress on the propagation behavior of Love waves in a layered piezoelectric medium.

With the development of the fabrication technology, functionally graded piezoelectric materials (FGPM) can be manufactured and used in SAW devices to improve their efficiency and other features. Li [23], Du et al. [24], and Qian et al. [25] studied the features of Love waves in an FGPM coating layer with an exponential variation in material properties bonded to a substrate, by applying direct and Wentzel–Kramers–Brillouin (WKB) methods.

In this work, Love waves propagation in an FGPM semi-infinite medium with a quadratic variation in material properties is considered. The analytical solution of dispersion relations is obtained under electrically open and short conditions, and the effects of gradient coefficient on phase velocity, group velocity, and electromechanical coupling factor are discussed. The effect of piezoelectricity on the phase velocity is examined by comparison of the results for the piezoelectric and the corresponding elastic materials. Due to the gradual spatial variation in the electromechanical properties of FGPMs, the amount of initial stress introduced in such materials during the manufacturing processes is negligible. Since the medium is piezoelectric, a source of excitation can be due to an applied electric potential. Thus, FGPM is desirable substitute for the typical layered piezoelectric structures used in SAW devices. A theoretical foundation relevant to the design and practical application of high-performance SAW devices is given in this paper.

2. Problem statement and governing equations

Let us consider an elastic transversely isotropic FGPM half-space as depicted in Fig. 1. The origin of the Cartesian coordinate system (x, y, z) is set on the surface with the z-axis pointing into the half-space, and it is assumed to be the axis of symmetry of the medium. The polling direction of the piezoelectric material is parallel to the z-axis, and the material properties change gradually with depth.

The governing equations of the piezoelectricity in the absence of body forces and free charges can be expressed as

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \qquad D_{i,i} = 0, \tag{1}$$

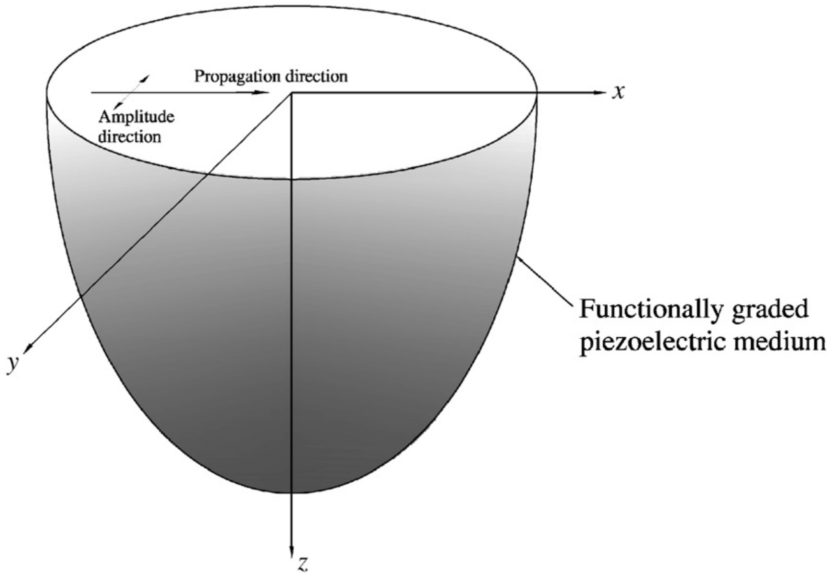

Fig. 1. A functionally graded piezoelectric medium with quadratic variation.

with $i,j = x, y, z$. The constitutive relations for transversely isotropic piezoelectric materials may be written as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{Bmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{zz} \\ \bar{\varepsilon}_{xz} \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} - \begin{pmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}, \tag{2}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{zz} \\ \bar{\varepsilon}_{xz} \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} - \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}, \tag{3}$$

where $c_{66} = (c_{11} - c_{12})/2$,

$$\bar{\varepsilon}_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}),$$
$$E_i = -\phi_{,i}. \tag{4}$$

In Eqs. (1)–(4), σ_{ij}, $\bar{\varepsilon}_{ij}$, u_i, E_i, and D_i are the components of stress, strain, displacement, electric field and electric displacement, respectively. ϕ is the electric potential, and ρ is the mass density, and c_{ij}, e_{ij}, and ε_{ij} are the elastic moduli, piezoelectric and dielectric constants, respectively.

On the assumption that Love waves propagate in the x-direction, it can be written that

$$u = w = 0,$$
$$v = v(x, z, t),$$
$$\phi = \phi(x, z, t). \tag{5}$$

where u, v, and w are the displacements in the x, y, and z-directions, respectively. Here, it is assumed that the transversely isotropic FGPM medium has a constant mass density ρ, whereas the other coefficients c_{ij}, e_{ij}, and ε_{ij} have a quadratic variation in the z-direction

$$c_{ij}(z) = c^0_{ij}(1 + bz)^2,$$
$$e_{ij}(z) = e^0_{ij}(1 + bz)^2,$$
$$\varepsilon_{ij}(z) = \varepsilon^0_{ij}(1 + bz)^2, \tag{6}$$

where $b > 0$ is the gradient factor. c^0_{ij}, e^0_{ij}, ε^0_{ij} is, respectively, the value of c_{ij}, e_{ij}, ε_{ij} on the surface. It is obvious that $b = 0$ corresponds to the homogeneous case, which has no significant interpretation in the context of Love waves propagation.

Employing Eqs. (1)–(6), the governing equations for the medium can be written as

$$\left(\nabla^2 + \frac{2b}{1 + bz}\frac{\partial}{\partial z}\right)\left(c^0_{44} v + e^0_{15}\phi\right) = \frac{\rho}{(1 + bz)^2}\frac{\partial^2 v}{\partial t^2},$$
$$\left(\nabla^2 + \frac{2b}{1 + bz}\frac{\partial}{\partial z}\right)(e^0_{15} v - \varepsilon^0_{11}\phi) = 0, \tag{7}$$

where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial z^2)$.

The air is regarded as vacuum since its dielectric constant is far less than that of the piezoelectric medium. Therefore, the governing equation associated with the electric potential in the air is

$$\nabla^2 \phi^a = 0. \tag{8}$$

The traction free condition at the surface implies that

$$\sigma_{yz}(x, 0, t) = 0. \tag{9}$$

The electrically open conditions at the free surface leads to

$$\phi(x, 0, t) = \phi^a(x, 0, t), \quad D_z(x, 0, t) = D^a(x, 0, t) \tag{10}$$

and the electrically short condition at the free surface yields

$$\phi(x, 0, t) = 0. \tag{11}$$

In addition to the above conditions, one must satisfy the following regularity conditions:

$$\begin{aligned} &\lim_{z \to -\infty} \phi^a(x, z, t) = 0, \\ &\lim_{z \to +\infty} v(x, z, t) = 0, \\ &\lim_{z \to +\infty} \phi(x, z, t) = 0. \end{aligned} \tag{12}$$

3. Analytical solution

In order to solve the coupled system of partial differential equations (7), it is assumed that

$$\psi = \phi - \frac{e_{15}}{\varepsilon_{11}} v. \tag{13}$$

Substituting Eq. (13) into Eq. (7) leads to

$$\begin{aligned} &\left(\nabla^2 + \frac{2b}{1+bz} \frac{\partial}{\partial z} \right) \psi = 0, \\ &\left(\nabla^2 + \frac{2b}{1+bz} \frac{\partial}{\partial z} - \frac{1}{c_{\mathrm{sh}}^2 (1+bz)^2} \frac{\partial^2}{\partial t^2} \right) v = 0, \end{aligned} \tag{14}$$

where c_{sh} is the shear wave velocity in the homogeneous piezoelectric medium, and it is given by

$$c_{\mathrm{sh}}^2 = \frac{1}{\rho} \left(c_{44}^0 + \frac{{e_{15}^0}^2}{\varepsilon_{11}^0} \right). \tag{15}$$

Taking the solutions of Eq. (14) to be in the following form:

$$\begin{aligned} \psi(x, z, t) &= \psi(z) \mathrm{e}^{\mathrm{i}k(x-ct)}, \\ v(x, z, t) &= v(z) \mathrm{e}^{\mathrm{i}k(x-ct)}, \end{aligned} \tag{16}$$

where k is the wavenumber, and c is the phase velocity of wave propagation, the following ordinary differential equations are obtained:

$$\begin{aligned} &\psi''(z) + \frac{2b}{1+bz} \psi'(z) - k^2 \psi(z) = 0, \\ &v''(z) + \frac{2b}{1+bz} v'(z) - k^2 \left(1 + \frac{c^2}{c_{\mathrm{sh}}^2 (1+bz)^2} \right) v(z) = 0. \end{aligned} \tag{17}$$

The exact solutions of these equations are

$$\psi(z) = \frac{1}{\sqrt{1+bz}}\left[A\mathscr{I}_{\frac{1}{2}}\left(k\frac{1+bz}{b}\right) + B\mathscr{K}_{\frac{1}{2}}\left(k\frac{1+bz}{b}\right)\right], \tag{18a}$$

$$v(z) = \frac{1}{\sqrt{1+bz}}\left[C\mathscr{I}_{s}\left(k\frac{1+bz}{b}\right) + D\mathscr{K}_{s}\left(k\frac{1+bz}{b}\right)\right], \tag{18b}$$

where

$$s = \sqrt{\frac{1}{4} - \left(\frac{k\,c}{b\,c_{\mathrm{sh}}}\right)^2}. \tag{19}$$

Here, A, B, C, and D are the unknown coefficients appropriate to the boundary conditions, and $\mathscr{I}_v(z)$ and $\mathscr{K}_v(z)$ are the vth-order modified Bessel functions of the first and second kind, respectively. As can be easily deduced, s can be either real for low-phase velocities ($c/c_{\mathrm{sh}} \leqslant b/2k$) or imaginary for high-phase velocities ($c/c_{\mathrm{sh}} > b/2k$).

Substituting Eqs. (18a) and (18b) into Eq. (13), the electric potential can be obtained as

$$\begin{aligned}\phi(x,z,t) = \frac{\mathrm{e}^{\mathrm{i}k(x-ct)}}{\sqrt{1+bz}}\Bigg\{&A\mathscr{I}_{\frac{1}{2}}\left(k\frac{1+bz}{b}\right) + B\mathscr{K}_{\frac{1}{2}}\left(k\frac{1+bz}{b}\right)\\ &+\frac{e_{15}^0}{\varepsilon_{11}^0}\left[C\mathscr{I}_{s}\left(k\frac{1+bz}{b}\right) + D\mathscr{K}_{s}\left(k\frac{1+bz}{b}\right)\right]\Bigg\}.\end{aligned} \tag{20}$$

The solution of Eq. (8) yields the electric potential in the air

$$\phi^a(x,z,t) = E\mathrm{e}^{\mathrm{i}k(x-\mathrm{i}z-ct)}, \tag{21}$$

where E is an unknown coefficient determined from the pertinent boundary conditions.

Using the asymptotic expansions of the modified Bessel functions for large arguments [26], one may express the asymptotic behavior of the displacement and the electrical potential in the limit of $z \to +\infty$ as

$$\begin{aligned}&v(z) \sim \sqrt{\frac{k}{2\pi b}}(C\mathrm{e}^{kz} - \pi D\mathrm{e}^{-kz}),\\ &\phi(z) \sim \sqrt{\frac{k}{2\pi b}}\left[\left(A + \frac{e_{15}^0}{\varepsilon_{11}^0}C\right)\mathrm{e}^{kz} - \pi\left(B + \frac{e_{15}^0}{\varepsilon_{11}^0}D\right)\mathrm{e}^{-kz}\right].\end{aligned} \tag{22}$$

From this analysis, it is evident that the regularity conditions (12) would require $A = C = 0$. As a result, the solutions for the displacement and the electrical potential in Eqs. (18b) and (20) can be reduced to

$$v(x,z,t) = \frac{D}{\sqrt{1+bz}}\mathscr{K}_s\left(k\frac{1+bz}{b}\right)\mathrm{e}^{\mathrm{i}k(x-ct)}, \tag{23a}$$

$$\phi(x,z,t) = \frac{\mathrm{e}^{\mathrm{i}k(x-ct)}}{\sqrt{1+bz}}\left\{B\mathscr{K}_{1/2}\left(k\frac{1+bz}{b}\right) + \frac{e_{15}^0}{\varepsilon_{11}^0}\left[D\mathscr{K}_s\left(k\frac{1+bz}{b}\right)\right]\right\}. \tag{23b}$$

Furthermore, from the integral representation of the modified Bessel function of the second kind [26]

$$\mathscr{K}_v(\xi) = \int_0^\infty \mathrm{e}^{-\xi\,\cosh(t)}\cosh(vt)\,\mathrm{d}t, \quad |\arg(\xi)| < \frac{\pi}{2}, \tag{24}$$

it follows that $\mathscr{K}_v(\xi)$ is a real-valued function for either real or imaginary order v, which in Eqs. (23a) and (23b) corresponds to the lower and higher phase velocity cases, respectively. This is in agreement with the assumption that there is no net wave propagation in the z-direction.

4. Phase velocity equations

In this section, the phase velocity equation is obtained for both electrically open and short conditions at the free surface. In addition, the dispersion relation for non-piezoelectric medium is also presented.

Substituting Eqs. (23a) and (23b) into Eq. (1), and using Eq. (6), the elastic and electrical fields of the medium are obtained in terms of the unknown coefficients B, D, and E. These unknown coefficients are determined analytically by employing the boundary conditions. The traction free boundary condition (9) yields the following equation:

$$2\alpha\mathscr{K}_{3/2}(\alpha)B + \rho c_{\rm sh}^2[2\alpha\mathscr{K}_{s+1}(\alpha) - (2s-1)\mathscr{K}_s(\alpha)]D = 0, \tag{25}$$

where $\alpha = k/b$. The details of the remaining conditions are discussed in Sections 4.1–4.3 below.

4.1. Electrically open conditions at the free surface

From the electrically open conditions at the free surface in Eq. (10), the following equations can be obtained:

$$\mathscr{K}_{1/2}(\alpha)B + \frac{e_{15}^0}{\varepsilon_{11}^0}\mathscr{K}_s(\alpha)D - E = 0, \tag{26}$$

$$\frac{\varepsilon_{11}^0}{\varepsilon_0}\mathscr{K}_{3/2}(\alpha)B - E = 0. \tag{27}$$

Using Eqs. (25)–(27) the unknown coefficients B, D, and E can be determined. As a result, the phase velocity equation is obtained to be

$$\rho\, c_{\rm sh}^2\left[\frac{\mathscr{K}_{1/2}(\alpha)}{\mathscr{K}_{3/2}(\alpha)} - \frac{\varepsilon_{11}^0}{\varepsilon_0}\right]\left[2\alpha\frac{\mathscr{K}_{s+1}(\alpha)}{\mathscr{K}_s(\alpha)} + (1-2s)\right] - 2\alpha\frac{{e_{15}^0}^2}{\varepsilon_{11}^0} = 0. \tag{28}$$

4.2. Electrically short conditions at the free surface

Substituting Eq. (23b) into the electrically short condition at the free surface Eq. (11) leads to

$$\mathscr{K}_{1/2}(\alpha)B + \frac{e_{15}^0}{\varepsilon_{11}^0}\mathscr{K}_s(\alpha)D = 0. \tag{29}$$

The unknown coefficients B and D can be determined from Eqs. (25) and (29). For non-trivial solutions, one must satisfy the following phase velocity equation:

$$\rho c_{\rm sh}^2\frac{\mathscr{K}_{1/2}(\alpha)}{\mathscr{K}_{3/2}(\alpha)}\left[2\alpha\frac{\mathscr{K}_{s+1}(\alpha)}{\mathscr{K}_s(\alpha)} + (1-2s)\right] - 2\alpha\frac{{e_{15}^0}^2}{\varepsilon_{11}^0} = 0. \tag{30}$$

4.3. Non-piezoelectric material

If the piezoelectric constant $e_{15} = 0$, that is the medium is elastic, then the dispersion relation reads

$$2\alpha\frac{\mathscr{K}_{s+1}(\alpha)}{\mathscr{K}_s(\alpha)} + (1-2s) = 0. \tag{31}$$

5. Electromechanical fields

After determining the unknown coefficients, the elastic and electrical fields of the medium can be expressed as

$$
\begin{aligned}
v(x,z,t) &= \frac{\beta C_1}{\sqrt{1+bz}}\mathscr{K}_s\left(k\frac{1+bz}{b}\right)\mathrm{e}^{\mathrm{i}k(x-ct)},\\
\phi(x,z,t) &= \frac{C_1}{\sqrt{1+bz}}\left[\mathscr{K}_{1/2}\left(k\frac{1+bz}{b}\right)+\beta\frac{e_{15}^0}{\varepsilon_{11}^0}\mathscr{K}_s\left(k\frac{1+bz}{b}\right)\right]\mathrm{e}^{\mathrm{i}k(x-ct)},\\
\sigma_{xy}(x,z,t) &= \mathrm{i}kC_1(1+bz)^{3/2}\left[\beta\rho c_{\mathrm{sh}}^2\mathscr{K}_s\left(k\frac{1+bz}{b}\right)+e_{15}^0\mathscr{K}_{1/2}\left(k\frac{1+bz}{b}\right)\right]\mathrm{e}^{\mathrm{i}k(x-ct)},\\
\sigma_{yz}(x,z,t) &= C_1\sqrt{1+bz}\left\{\frac{\beta\rho c_{\mathrm{sh}}^2 b}{2}\left[(2s-1)\mathscr{K}_s\left(k\frac{1+bz}{b}\right)-2\alpha(1+bz)\mathscr{K}_{s+1}\left(k\frac{1+bz}{b}\right)\right]\right.\\
&\quad\left.-ke_{15}^0(1+bz)\mathscr{K}_{3/2}\left(k\frac{1+bz}{b}\right)\right\}\mathrm{e}^{\mathrm{i}k(x-ct)},\\
\phi^a(x,z,t) &= C_1\frac{\varepsilon_{11}^0}{\varepsilon_0}\mathscr{K}_{3/2}(\alpha)\mathrm{e}^{\mathrm{i}k(x-\mathrm{i}z-ct)},
\end{aligned} \tag{32}
$$

where

$$
\beta = \begin{cases} \dfrac{\varepsilon_{11}^0}{\varepsilon_0}\mathscr{K}_{3/2}(\alpha)-\mathscr{K}_{1/2}(\alpha) & \text{for open conditions,}\\[2ex] \dfrac{\varepsilon_{11}^0\mathscr{K}_{1/2}(\alpha)}{e_{15}^0\ \mathscr{K}_s(\alpha)} & \text{for short conditions} \end{cases} \tag{33}
$$

and C_1 is a constant corresponding to the amplitude of excitation. It is evident that the electrical potential in the air ϕ^a is zero for electrically short condition.

6. Numerical results and discussion

In Section 4, the dispersion relations for the FGPM medium with quadratic variation under different boundary conditions were obtained analytically. In order to demonstrate the influences of the gradient coefficient on the phase velocity, group velocity, and coupled electromechanical factor, a numerical example is proposed. Consider an FGPM medium made of PZT-5H with the following properties: $c_{44}^0 = 23\times10^9\,\mathrm{N\,m^{-2}}$; $e_{15}^0 = 17\,\mathrm{C\,m^{-2}}$; $\varepsilon_{11}^0 = 15\times10^{-9}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$; $\rho = 7.5\times10^3\,\mathrm{kg\,m^{-3}}$. Assume that all the material properties, except for the mass density ρ which is constant, vary quadratically as specified in Eq. (6). In addition, $\varepsilon_0 = 8.85\times10^{12}\,\mathrm{F\,m^{-1}}$ is used as the dielectric constant of vacuum.

The phase velocity c can be calculated by numerical solution of the dispersion Eqs. (28), (30) and (31) for the electrically open and short conditions as well as for the non-piezoelectric medium, respectively. Furthermore, the group velocity c_g which expresses the rate of energy transmission, is introduced to explain the dispersion relation. The group velocity is defined as

$$c_g = c + k\frac{\mathrm{d}c}{\mathrm{d}k}. \tag{34}$$

The other significant parameter represents the coupled electromechanical factor K^2 which plays an important role in the design of SAW devices. It is defined for surface waves as [9]

$$K^2 = 2\frac{c_{\mathrm{open}}-c_{\mathrm{short}}}{c_{\mathrm{open}}}, \tag{35}$$

where c_{open} and c_{short} are the phase velocities for the electrically open and short cases, respectively.

Figs. 2 and 3 show the phase and group velocities of the first and second modes for the electrically open and short cases, respectively. The first and second modes for each case correspond to the two smallest roots of the pertinent dispersion relation. The results indicate that, for a given wavenumber the phase velocity of Love waves increases with an increase of the gradient factor. For both electrically short and open conditions, the energy propagates in dispersion behaviors, i.e. the rate of the energy propagation c_g, does not exceed that of the wave propagation c. It can be found that when k/b tends to infinity, phase velocity approaches to c_{sh}. For large wavenumbers the group velocity become less than c_{sh} for electrically short condition while it tends to c_{sh} for electrically open case.

A comparison between the first mode of the phase velocity for electrically short and open cases and non-piezoelectric medium is illustrated in Fig. 4. It is evident that the phase velocity for electrically short case is less than that for electrically open case. As a result, the phase velocity for the non-piezoelectric half-space is the

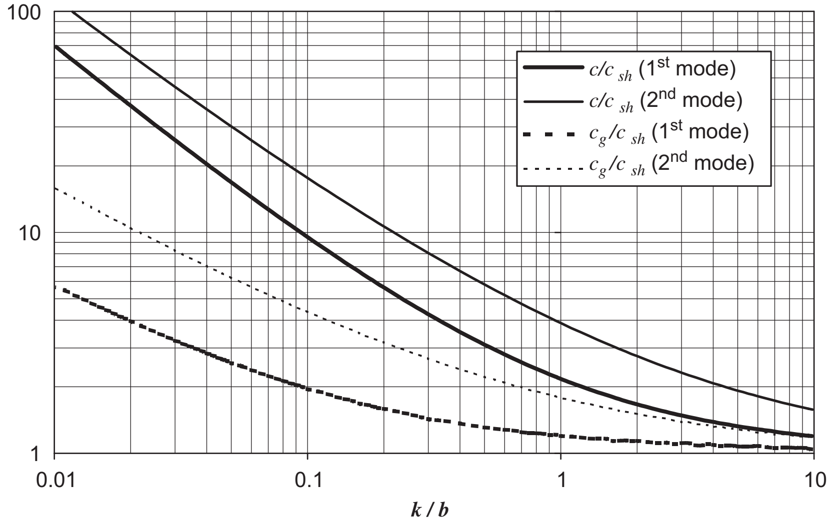

Fig. 2. Phase and group velocity of the first and second modes for the electrically open case.

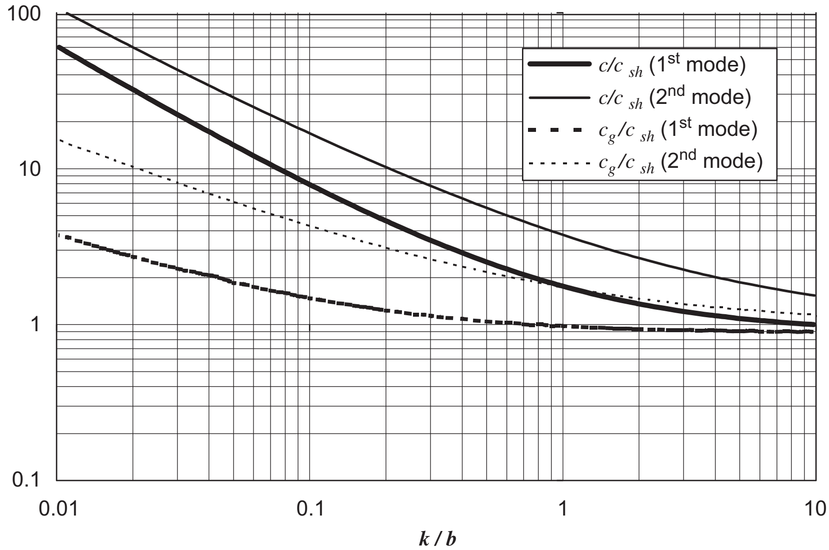

Fig. 3. Phase and group velocity of the first and second modes for the electrically short case.

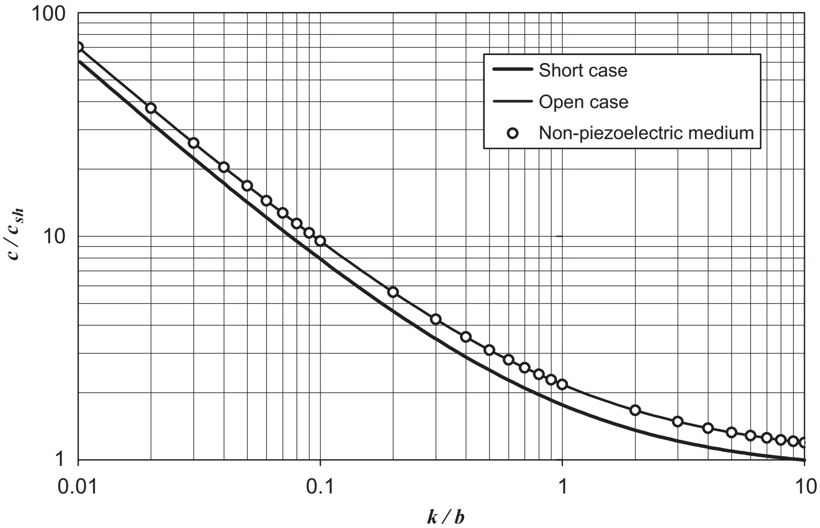

Fig. 4. Computed results for the phase velocity of the first mode for three different cases of non-piezoelectric medium, and piezoelectric medium under short and open conditions.

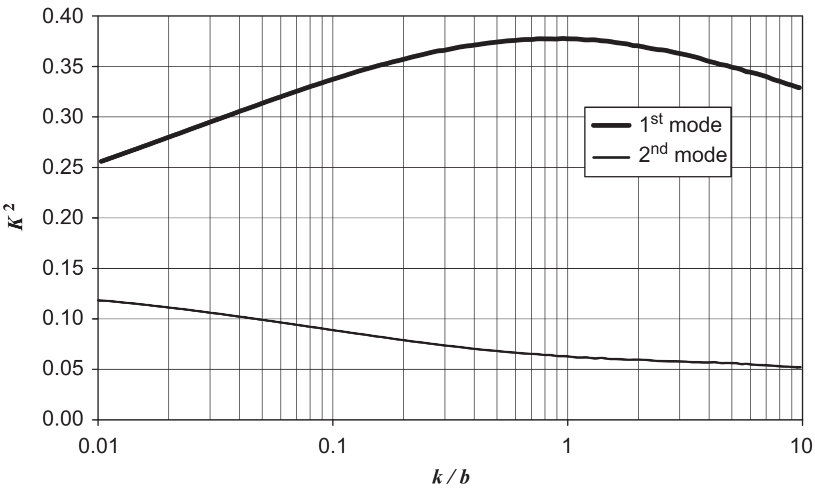

Fig. 5. Electromechanical coupling factor of the first and second modes.

same as that for electrically open case. This fact is consistent with the dispersion equations given by Eqs. (28) and (31).

Fig. 5 depicts the relation between the non-dimensional wavenumber and the coupled electromechanical factor. The coupled electromechanical factor increases with increase of gradient factor in the second mode. In the first mode, the coupled electromechanical factor reaches to a maximum value of about 0.38 for $0.6<k/b<1.3$ and attains the minimum value of 0.25 at very low wavenumbers, i.e. $k/b\approx 0.01$. In comparison with the typical layered piezoelectric systems and systems consisting of an FGPM layer bonded to a homogeneous substrate considered by previous investigators, the above-mentioned peak of 0.38 is a significant value for the coupled electromechanical factor that can improve the capability of SAW devices.

7. Conclusion

The propagation behavior of Love waves in an FGPM semi-infinite medium with a quadratic variation in electromechanical properties is studied. The coupled electromechanical field equations are solved exactly for the dispersion relations, displacement, electric potential, and stress fields under both electrically open and short conditions. In addition the case of a non-piezoelectric medium is considered. The effects of gradient coefficient on phase velocity, group velocity, and electromechanical coupling factor are plotted and discussed. It is found that the phase velocity for the non-piezoelectric half-space is the same as that for the electrically open case. An interesting phenomenon observed in this work is that FGPM medium exhibits higher peak for the coupled electromechanical factor as compared to the case of a typical layered piezoelectric medium and the case of a homogeneous medium coated by an FGPM layer considered in literature. This important feature of FGPM medium makes it a preferred candidate for use in SAW devices.

Acknowledgments

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