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Frequency equation for the in-plane vibration of a clamped circular plate

Chan Il Park*

Department of Precision Mechanical Engineering, Kangnung National University, Jibyeon 123 Gangneung, Gangwon-do 210-702, Republic of Korea

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Abstract

The in-plane vibration of the circular plate structure is important in the transmission of high-frequency vibration, but none have produced an exact frequency equation of the in-plane vibration of a clamped circular plate. Therefore, this paper focuses on deriving the frequency equation for the in-plane vibration of the clamped circular plate of uniform thickness with an isotropic material in the elastic range. To derive the frequency equation for the clamped circular plate in the cylindrical coordinate, kinetic and potential energy for in-plane behavior were first obtained by using the stress–strain–displacement expressions and applying Hamilton's principle, which led to two sets of highly coupled differential equations for the equations of motion. Substitution of Helmholtz decomposition for the coupled differential equations produced uncoupled equations of motion. The assumption of a harmonic solution for the uncoupled equations led to wave equations. Using the separation of the variable, the general solutions for the wave equations were obtained. The solutions generated the in-plane displacements in the r and θ directions. Finally, the application of boundary conditions yielded the frequency equation for the clamped circular plate. The derived frequency equation was validated by finite element analysis and by comparison of previously reported results.

1. Introduction

In platelike structures, transverse vibrations are of great practical importance, as transverse motion is easy to excite from external sources. Such motion can also interact strongly with the surrounding acoustic environment. However, rotating disks are important in in-plane characteristics, and platelike structures in most applications have direct in-plane forces or in-plane force components due to imperfections in the manufacturing, assembly or alignment of the supporting mounts. In many mechanical systems, the bearing radial force is transmitted to the housing by in-plane excitation, which can be modeled by the plate panels. The in-plane mode of a plate occurs at relatively high frequencies. They may nonetheless be important from a design point of view. In-plane vibration can play a major role in the transmission of high frequency vibration through a built-up structure. Therefore, an accurate analysis must consider the presence of these vibrations.

*Tel.: +82 33 640 2392; fax: +82 33 640 2244.
E-mail address: pci@kangnung.ac.kr

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Nomenclature

a	radius of plate
$\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$	unit vector in cylindrical coordinates
E	Young's modulus
G	shear modulus
h	plate thickness
i	complex number
J_m	Bessel function of the first kind
r, θ, z	cylindrical coordinates
t	time
u, v, w	displacements
Y_n	Bessel function of the second kind
γ	shear strain
ε	normal strain
ρ	density
σ	normal stress
τ	shear stress
ν	Poisson's ratio
ω	frequency (rad s^{-1})
$\mathbf{\Phi}$, $\mathbf{H}$	scalar and vector potentials
∇^2	Laplacian in polar coordinate

Superscripts

$\wedge$	frequency domain quantity
$\cdot$	dot, time derivative
$'$	derivative with respect to argument

Many studies have been devoted to transverse vibration, and the natural frequencies and mode shapes for transverse vibration have been well documented [1]. Wah [2] studied the transverse vibration of circular plates with a large initial tension for the case of simply supported and clamped edges. Reismann [3] calculated the response of a clamped, circular plate to the action of an arbitrarily placed, harmonically oscillating, transverse, concentrated force. For the in-plane response of finite rectangular plates and a general media, Farag and Pan [4,5] calculated the forced response of finite rectangular plates to in-plane point force excitations using the orthogonal properties of the assumed mode shapes and more-comprehensive modal characteristics of the in-plane forced vibration of rectangular plates with several boundary conditions. Bardell et al. [6] studied the in-plane vibration of isotropic rectangular plates with various boundary conditions using the Rayleigh–Ritz method. Rizzi and Doyle [7] developed a spectral method as a means of solving two-dimensional wave propagation problems in semi-infinite and finite media.

For research related to the in-plane vibration of circular plates, Love [8] solved the extensional vibration of an infinitely isotropic thin circular plate with free edge as an initial approach to the in-plane vibration of plates. Kane and Mindlin [9] investigated the coupling between extensional vibration and the first mode of thickness vibration on the circular disk. Holand [10] investigated the frequency parameters of the free in-plane vibration in circular plates with free edges on the variation of Poisson's ratio. Ambati et al. [11] analyzed the in-plane vibration of circular plates and annular rings with free boundaries. Chen and Liu [12] studied the free in-plane vibration of thin plates of various shapes with a free edge, including a circular plate, and compared the nodal pattern of plates with different shapes. Irie et al. [13] calculated the natural frequencies for the in-plane vibration of annual plates with four combinations of free and clamped boundary conditions at the inner and outer edges using a transfer matrix. Chen and Jhu [14] investigated the effects of the clamping ratio on the critical speeds and the natural frequencies of the in-plane vibration of a spinning annular disk. More recently, Farag and Pan [15] proposed natural frequencies and assumed mode shapes by trigonometric functions in the circumferential direction and by a series summation of Bessel functions in the radial direction according to three circumferential wavenumber categories of 0, 1, and numbers of more than 1, and investigated the modal characteristics of in-plane vibration of circular plates with clamped edge. However, no studies dealing with the exact frequency equation for the in-plane vibration of a clamped circular plate have been published.

Therefore, the purpose of this paper is to derive equations of motion for the in-plane vibration of a clamped circular plate and to obtain the frequency equation by solving the equations. Validation of the derived equation uses finite element analysis and a comparison of previously reported results.

2. Analysis

This study analyzes a clamped circular plate that lies in an r–θ plane, as shown in Fig. 1. The mid-plane of the plate is taken at $z = 0$. The circular plate of uniform thickness with an isotropic material is treated at the

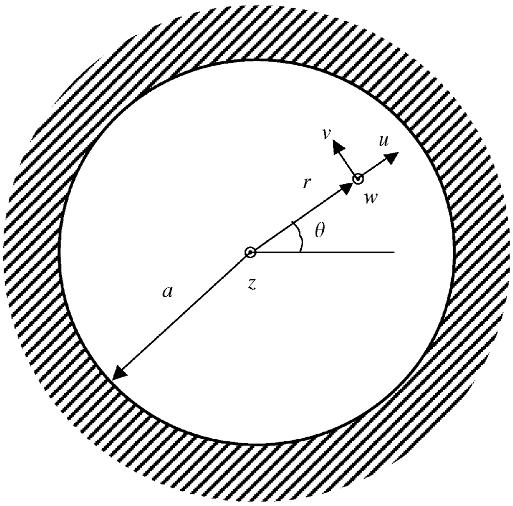

Fig. 1. A clamped circular plate.

elastic range for brevity. Using total energy and work and applying calculus of variation, Hamilton's principle formulates equations of motion for the mechanical system. Therefore, equations of motion for the in-plane vibration are derived using Hamilton's principle. Since the displacements are coupled, they are uncoupled by the Helmholtz decomposition. Then, the frequency equation for a clamped circular plate is derived.

2.1. Equations of motion for in-plane vibration

The strain–displacement relations in the cylindrical coordinate are given by [16,17]

$$\varepsilon_r = \frac{\partial u}{\partial r}, \tag{1}$$

$$\varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}, \tag{2}$$

$$\gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r},$$
$$\text{where } \mathbf{u} = u\mathbf{e}_r + v\mathbf{e}_\theta + w\mathbf{e}_z. \tag{3}$$

Substituting for the strains in Hooke's law then leads to [16,18]

$$\sigma_r = (\varepsilon_r + v\varepsilon_\theta)\frac{E}{1-v^2} = \left(\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{vu}{r}\right)\frac{E}{1-v^2}, \tag{4}$$

$$\sigma_\theta = (\varepsilon_\theta + v\varepsilon_r)\frac{E}{1-v^2} = \left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} + v\frac{\partial u}{\partial r}\right)\frac{E}{1-v^2}, \tag{5}$$

$$\tau_{r\theta} = G\gamma_{r\theta} = G\left(\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}\right). \tag{6}$$

Hamilton's principle, which is employed to derive equations of motion, uses kinetic energy and potential energy. Kinetic energy for the in-plane behavior is given by [16,18]

$$T = \frac{1}{2}\int_0^{2\pi}\int_0^a \rho(\dot{u}^2 + \dot{v}^2)hr\,\mathrm{d}r\,\mathrm{d}\theta. \tag{7}$$

In the definition of potential energy for plane stress, potential energy that substitutes strain and stress relations from Eqs. (1) to (5) is given by [16,18]

$$
\begin{aligned}
U &= \frac{1}{2}\int_0^{2\pi}\int_0^{a}(\sigma_r\varepsilon_r + \sigma_\theta\varepsilon_\theta + \tau_{r\theta}\gamma_{r\theta})hr\,\mathrm{d}r\,\mathrm{d}\theta \\
&= \frac{1}{2}\int_0^{2\pi}\int_0^{a}\left\{\left(\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + v\frac{u}{r}\right)\frac{E}{1-v^2}\frac{\partial u}{\partial r} + \left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r} + v\frac{u}{\partial r}\right)\frac{E}{1-v^2}\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right)\right. \\
&\quad \left. + G\left(\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}\right)^2\right\}hr\,\mathrm{d}r\,\mathrm{d}\theta.
\end{aligned} \tag{8}
$$

Hamilton's principle, which applies the calculus of variation to the total energy of system T-U, leads to two equations of motion:

$$
\rho\frac{\partial^2 u}{\partial t^2} = \frac{E}{1-v^2}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{v}{r}\frac{\partial^2 v}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial v}{\partial \theta} - \frac{u}{r^2}\right) + G\left(\frac{1}{r}\frac{\partial^2 v}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{1}{r^2}\frac{\partial v}{\partial \theta}\right), \tag{9}
$$

$$
\rho\frac{\partial^2 v}{\partial t^2} = \frac{E}{1-v^2}\left(\frac{v}{r}\frac{\partial^2 u}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial u}{\partial \theta}\right) + G\left(\frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial u}{\partial \theta} - \frac{v}{r^2}\right). \tag{10}
$$

2.2. *Frequency equation for in-plane vibration*

The previously derived equations of motion are highly complex and coupled. However, a simpler set of equations can be obtained by introducing scalar and vector potentials Φ and $\mathbf{H}$, known as the Helmholtz decomposition, such that [18,19]

$$
\mathbf{u} = \nabla\Phi + \nabla\times\mathbf{H}, \quad \nabla\cdot\mathbf{H} = 0. \tag{11}
$$

Therefore, the scalar components of the displacement vector $\mathbf{u}$ in the cylindrical coordinates can be expressed by [17,18]

$$
u = \frac{\partial\Phi}{\partial r} + \frac{1}{r}\frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z}, \tag{12}
$$

$$
v = \frac{1}{r}\frac{\partial\Phi}{\partial \theta} + \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}, \tag{13}
$$

$$
w = \frac{\partial\Phi}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r}\frac{\partial H_r}{\partial \theta}. \tag{14}
$$

Because the in-plane motion in the r and the θ direction is considered, the in-plane displacements with $(\partial/\partial z) = 0$ are given by

$$
u = \frac{\partial\Phi}{\partial r} + \frac{1}{r}\frac{\partial H_z}{\partial \theta}, \tag{15}
$$

$$
v = \frac{1}{r}\frac{\partial\Phi}{\partial \theta} - \frac{\partial H_z}{\partial r}. \tag{16}
$$

Substituting Eqs. (15) and (16) into Eq. (9) leads to two sets of differential equations:

$$
\rho\frac{\partial\ddot{\Phi}}{\partial r}\mathbf{e}_r = E^*\left(\frac{\partial^3\Phi}{\partial r^3} + \frac{1}{r}\frac{\partial^2\Phi}{\partial r^2} - \frac{2}{r^3}\frac{\partial^2\Phi}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial^3\Phi}{\partial r\partial\theta^2} - \frac{1}{r^2}\frac{\partial\Phi}{\partial r}\right)\mathbf{e}_r, \tag{17}
$$

$$
\rho\frac{\partial\ddot{H}_z}{r\partial\theta}\mathbf{e}_r = G\left(\frac{1}{r}\frac{\partial^3 H_z}{\partial r^2\partial\theta} + \frac{1}{r^3}\frac{\partial^3 H_z}{\partial \theta^3} + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial r\partial\theta}\right)\mathbf{e}_r. \tag{18}
$$

In the same manner, substituting Eqs. (15) and (16) into Eq. (10) leads to two other sets of differential equations:

$$\rho\frac{\partial\ddot{\Phi}}{r\partial\theta}\mathbf{e}_\theta = E^*\left(\frac{1}{r^3}\frac{\partial^3\Phi}{\partial\theta^3}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial r\partial\theta}+\frac{1}{r}\frac{\partial^3\Phi}{\partial r^2\partial\theta}\right)\mathbf{e}_\theta, \tag{19}$$

$$\rho\frac{\partial\ddot{H}_z}{\partial r}\mathbf{e}_\theta = G\left(\frac{\partial^3 H_z}{\partial r^3}+\frac{1}{r^2}\frac{\partial^3 H_z}{\partial r\partial\theta^2}+\frac{1}{r}\frac{\partial^2 H_z}{\partial r^2}-\frac{2}{r^3}\frac{\partial^2 H_z}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial H_z}{\partial r}\right)\mathbf{e}_\theta, \tag{20}$$

where

$$E^* = \frac{E}{1-\nu^2}.$$

Eqs. (17) and (19) are modified as follows:

$$\nabla^2\Phi = \frac{1}{c_1^2}\ddot{\Phi}. \tag{21}$$

Eqs. (18) and (20) are also converted as follows:

$$\nabla^2 H_z = \frac{1}{c_2^2}\ddot{H}_z, \tag{22}$$

where

$$c_1^2 = \frac{E^*}{\rho} = \frac{E}{\rho(1-\nu^2)}, \quad c_2^2 = \frac{G}{\rho}.$$

To solve Eq. (21), the solution assumes to be $\Phi = \hat{\Phi}(r,\theta)\mathrm{e}^{\mathrm{i}\omega t}$.

Then, Eq. (21) leads to

$$\nabla^2\hat{\Phi} + k_p^2\hat{\Phi} = 0, \tag{23}$$

where $k_p = \omega/c_1$.

Application of the separation of the variable as $\hat{\Phi} = X(r)\Theta(\theta)$ to Eq. (23) leads to

$$X''\Theta + \frac{1}{r}X'\Theta + \frac{1}{r^2}X\Theta'' + k_p^2 X\Theta = 0. \tag{24}$$

If the following condition is satisfied, separation occurs as follows:

$$r^2\frac{X''}{X} + r\frac{X'}{X} + k_p^2 r^2 = -\frac{\Theta''}{\Theta} = k^2. \tag{25}$$

Then, the solutions for Θ are given by

$$\Theta = \sin k\theta,\ \cos k\theta. \tag{26}$$

From the continuity conditions that require $\hat{\Phi}(r,\theta) = \hat{\Phi}(r,\theta+2\pi)$, $k = n$, where n is an integer.

Bessel's equation of order n from Eq. (25) is also separated by

$$X'' + \frac{1}{r}X' + \left(k_p^2 - \frac{n^2}{r^2}\right)X = 0. \tag{27}$$

The solution for X is obtained by

$$X(r) = CJ_n(k_p r), \tag{28}$$

where the second solution $Y_n(k_p r)$ has been discarded because of its singular behavior at the origin.

Assuming $H_z = \hat{H}_z\mathrm{e}^{\mathrm{i}\omega t}$, Eq. (22) also becomes

$$\nabla^2\hat{H}_z + k_s^2\hat{H}_z = 0, \tag{29}$$

where $k_s = \omega/c_2$.

The separation of the variable as $\hat{H}_z = Y(r)\Omega(\theta)$ yields the same solution for $\Omega(\theta)$ as Eq. (26). The solution for Y is also given by

$$Y(r) = DJ_n(k_s r). \tag{30}$$

Discarding either sine or cosine terms in the $\Theta(\theta)$ and $\Omega(\theta)$, the solutions for Eqs. (23) and (29) are assumed by

$$\hat{\Phi} = AJ_n(k_p r)\cos(n\theta), \tag{31}$$

$$\hat{H}_z = BJ_n(k_s r)\sin(n\theta). \tag{32}$$

Then, the displacements for $u = \hat{u}e^{i\omega t}$ and $v = \hat{v}e^{i\omega t}$ are given by

$$\hat{u} = A\frac{dJ_n(k_p r)}{dr}\cos(n\theta) + \frac{nB}{r}J_n(k_s r)\cos(n\theta), \tag{33}$$

$$\hat{v} = -\frac{nA}{r}J_n(k_p r)\sin(n\theta) - B\frac{dJ_n(k_s r)}{dr}\sin(n\theta). \tag{34}$$

The application of the clamped boundary condition of $v = 0$ at $r = a$ in Eq. (34) yields the following equations:

$$B = -\frac{nA}{a}J_n(k_p a)\bigg/\frac{dJ_n(k_s a)}{dr}. \tag{35}$$

Substituting Eq. (35) into Eqs. (33) and (34), Eqs. (33) and (34) reduce to

$$\hat{u} = A^* \cos(n\theta)\left[\frac{dJ_n(k_s a)}{dr}\frac{dJ_n(k_p r)}{dr} - \frac{n^2}{ar}J_n(k_p a)J_n(k_s r)\right], \tag{36}$$

$$\hat{v} = A^* \sin(n\theta)\left[-\frac{n}{r}\frac{dJ_n(k_s a)}{dr}J_n(k_p r) + \frac{n}{a}J_n(k_p a)\frac{dJ_n(k_s r)}{dr}\right], \tag{37}$$

where A^* is a modified constant.

The application of the clamped boundary condition of $u = 0$ at $r = a$ in Eq. (36) yields the frequency equation

$$\frac{dJ_n(k_p a)}{dr}\frac{dJ_n(k_s a)}{dr} - \frac{n^2}{a^2}J_n(k_p a)J_n(k_s a) = 0. \tag{38}$$

The numerical solution of Eq. (38) for a given n produces the natural frequencies.

3. Numerical results and validation

For the numerical example, a clamped circular plate of a radius of 0.5 m and a thickness of 5 mm is used. The material of the plate is aluminum, for which Young's modulus of 71 GPa, a density of 2700 kg m^{-3}, Poisson's ratio of 0.33, and a structural loss factor of 0.001 are assumed. With the derived frequency equation, natural frequencies are calculated in the MATLAB for a frequency range of 0–10 kHz. To validate the analytical results, the natural frequencies and mode shapes are also computed using the finite element method. The plate is modeled by 1200 and 3600 shell elements of the 4-node Q4 type using MSC NASTRAN for Windows. The natural frequencies and the mode shapes by the finite element analysis with elements of 3600 are shown in Fig. 2. Table 1 presents a comparison between the proposed equation and the finite element method of natural frequencies. Natural frequencies of the finite element analysis are the results of the finite elements of 3600, and the values in parentheses are those of the finite elements of 1200. The results of the finite elements of 3600 are close to those of the proposed equation compared to those of finite elements of 1200. The percentage difference is from 0.02% to 0.78% in the frequency range examined if the results of the finite element analysis with elements of 3600 are selected as true values. As the frequency increases, the difference between the finite element analysis and the proposed equation increases to 0.78%. However, if the results of

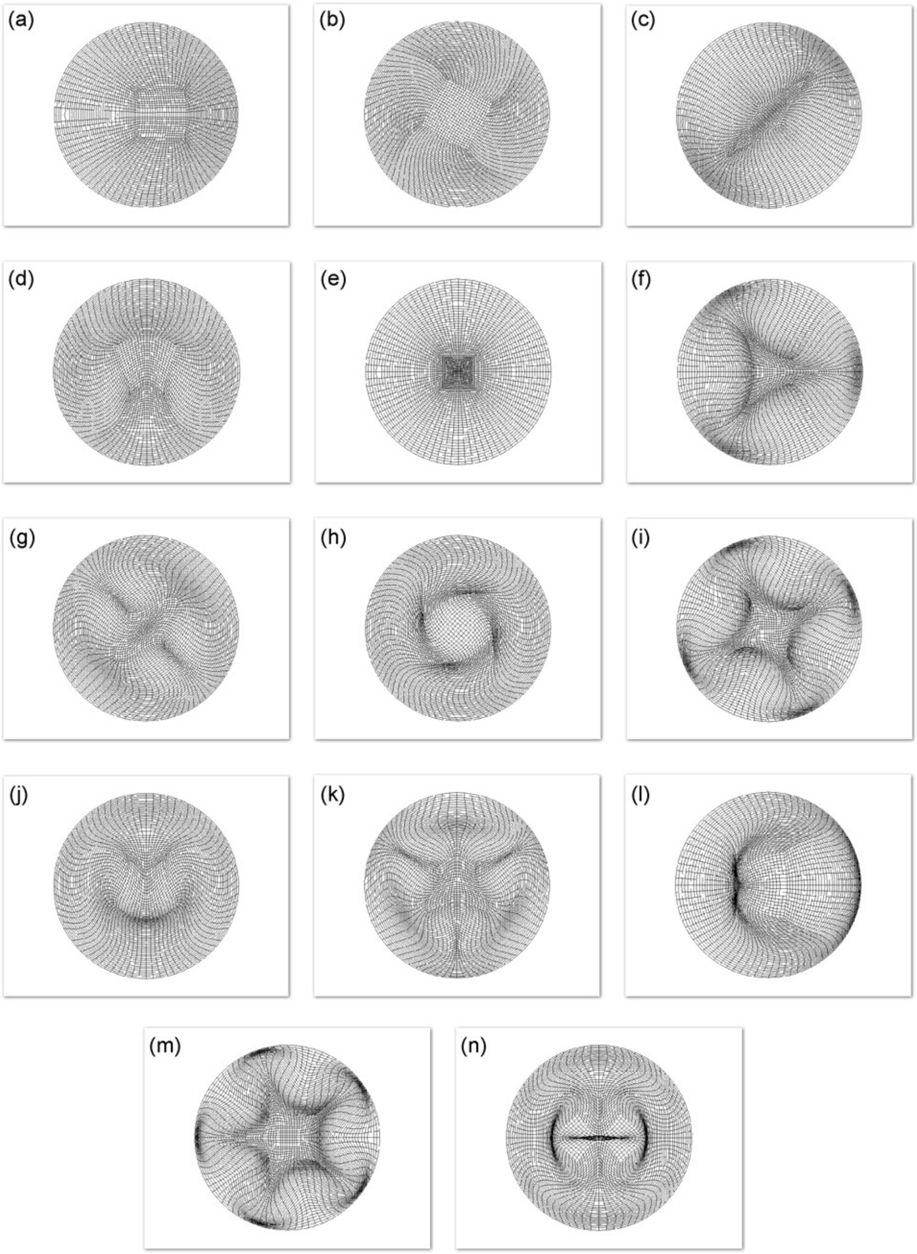

Fig. 2. Natural frequencies and mode shapes by the FEM. (a) 3363.6, (b) 3836.4, (c) 5217.5, (d) 5380.5, (e) 6624, (f) 6749.3, (g) 6929, (h) 7019.3, (i) 8093, (j) 8476.5, (k) 8530.6, (l) 9258, (m) 9328.1 and (n) 9887.7 Hz.

the finite element analysis with elements of 1200 are selected as true values, the percentage difference increase to 2.22%, which is caused by the poor behavior of the finite element method in the high-frequency region. Thus, finite element modeling with more elements can lead to a more accurate solution. The results of this work are also compared with those of Farag and Pan [15]. For a comparison of the frequency range of 0–20 kHz, natural frequencies for a clamped circular plate of a radius of 0.5 m with the steel material properties of Young's modulus of 200 GPa, a density of 7800 kg m^{-3}, and Poisson's ratio of 0.28 are calculated by the proposed equation. The results of 20 terms in the series summation by Farag and Pan except for circumferential wavenumber $n = 0$, are compared in Table 2. The results show good agreement with the

Table 1
Comparison of natural frequencies of the proposed equation and those of the FEM

$n=0$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)	$n=1$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)	$n=2$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)
1	3835	3836.4 (3838.9)	0.04	1	3362	3363.6 (3366.7)	0.05	1	5219	5217.5 (5213.7)	0.03
2	6626	6624 (6620.7)	0.03	2	5383	5380.5 (5375.5)	0.05	2	6939	6929 (6910.3)	0.14
3	7021	7019.3 (7013.4)	0.02	3	8489	8476.5 (8449.8)	0.15	3	9925	9887.7 (9817)	0.38
				4	9263	9258 (9247.3)	0.05				
$n=3$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)	$n=4$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)	$n=5$	Proposed eq. (Hz)	FEM (Hz)	Diff (%)
1	6764	6749.3 (6722.7)	0.22	1	8130	8093 (8022)	0.46	1	9401	9328.1 (9196.5)	0.78 2.22
2	8557	8530.6 (8481.7)	0.31								

Table 2
Comparison of natural frequencies of this work and those of Farag et al.

$n=0$	Farag et al. (Hz)	Present work (Hz)	$n=1$	Farag et al. (Hz)	Present work (Hz)	$n=2$	Farag et al. (Hz)	Present work (Hz)
1	3860	3860	1	3300	3300	1	5148	5147
2	6434	6433	2	5408	5409	2	6942	6942
3	7068	7067	3	8512	8510	3	9967	9966
4	10,249	10,248	4	9034	9033	4	11,301	11,300
5	13,423	13,422	5	11,792	11,792	5	13,289	13,289
6	16,593	16,592	6	14,315	14,314	6	16,356	16,355
7	17,082	17,081	7	14,995	14,993	7	16,848	16,848
			8	18,144	18,145			
			9	19,657	19,655			
$n=3$	Farag et al. (Hz)	Present work (Hz)	$n=7$	Farag et al. (Hz)	Present work (Hz)	$n=8$	Farag et al. (Hz)	Present work (Hz)
1	6709	6708	1	11,828	11,827	1	13,003	13,002
2	8520	8519	2	14,746	14,744	2	16,171	16,169
3	11,377	11,376	3	17,022	17,020	3	18,477	18,475
4	13,408	13,407	4	19,880	19,878			
5	14,786	14,785						
6	17,861	17,859						

frequency difference of only 0–2 Hz, even at a high frequency. Therefore, the proposed equation is well predicted.

4. Summary and conclusions

In this work, the equations of motion for in-plane vibration were derived using Hamilton's principle. The coupled equations were uncoupled by the Helmholtz decomposition. Then, the frequency equation for a clamped circular plate was derived. Through a numerical example, natural frequencies of a clamped circular plate were computed. The results of finite element analysis and the previous report were used to validate the proposed equation. Finite element analysis and comparison of the previously reported results showed that the proposed frequency equation was an accurate solution for the in-plane vibration of the clamped circular plate.

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