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Journal of Sound and Vibration 313 (2008) 363–366
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Rapid Communication

Estimation of the frequency response of a structure using its non-stationary vibration

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Received 26 August 2007; received in revised form 31 January 2008; accepted 2 February 2008
Handling Editor: L.G. Tham
Available online 25 March 2008

Abstract

The frequency distribution of a short-interval period, the SIP distribution, obtained from the vibration of a structure that is excited by the force of non-stationary vibration, reflects the dynamic property of the structure. This idea was validated by numerical experiments and it was shown that the SIP distribution was fairly stable and accurate in comparison with the FFT spectrum for random noise excitation. A theoretical study is shown to confirm the idea.

The frequency distribution of a short-interval period, the SIP distribution, obtained from the vibration of a structure that is excited by the force of non-stationary vibration, reflects the dynamic property of the structure. This idea was validated by numerical experiments and it was shown that the SIP distribution was fairly stable and accurate in comparison with the FFT spectrum for random noise excitation [1]. A theoretical study confirms the idea, which follows.

Let $V(t)$ be a bandpass filtered waveform of the vibration of a structure at time t, which is excited by Gaussian noise, and let the band-width of the filter, $\mathrm{d}f$, be narrow enough compared with its mid-frequency f_n. Hence we can express

$$V(t) = a(t)\sin(2\pi f_n t) + b(t)\cos(2\pi f_n t), \tag{1}$$

where $a(t)$ and $b(t)$ are time-varying quantities that may be considered constant for short intervals. The amplitude of $V(t)$ is given by

$$x_n = \sqrt{a^2(t) + b^2(t)}. \tag{2}$$

The probability density of the random variable $x_n(=x)$ is expressed by the Rayleigh distribution such that

$$p_n(x) = (x/c_n)\exp(-x^2/2c_n), \tag{3}$$

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2008.02.002

where

$$c_n = \frac{\overline{x^2}}{2}. \tag{4}$$

Since $V(t)$ is weighted by the frequency response of a structure, $G(f)$, the power c_n is given by

$$c_n = kG^2(f_n)\mathrm{d}f, \tag{5}$$

where k is a constant that is proportional to the magnitude of input noise. It should be noted that, in practice, the average of x_n^2 given by the finite number of samples does not give c_n [1].

Putting $n = 1, 2, \ldots, N$, we get random variables $x_1, x_2, \ldots, x_N$ for arbitrary frequencies $f_1, f_2, \ldots, f_N$, respectively, of which probability densities are expressed by Eq. (3).

Thus, the probability of x_j ($1 \leqslant j \leqslant N$), which takes the maximum among values $(x_1, x_2, \ldots, x_N)$ is given by

$$P_j = \int_0^\infty p_j(x) Q_j(x)\mathrm{d}x, \tag{6}$$

where

$$Q_j(x) = Q(x) \Big/ \int_0^x p_j(y)\mathrm{d}y, \tag{7}$$

where

$$Q(x) = \int_0^x p_1(y)\mathrm{d}y \int_0^x p_2(y)\mathrm{d}y \ldots \int_0^x p_N(y)\mathrm{d}y. \tag{8}$$

If $c_1 = c_2 = \cdots = c_N$,

$$Q_j(x) = \left(\int_0^x p_n(y)\mathrm{d}y\right)^{N-1}. \tag{9}$$

Since

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x p_n(y)\mathrm{d}y\right)^N = N p_n(x)\left(\int_0^x p_n(y)\mathrm{d}y\right)^{N-1}, \tag{10}$$

we have

$$P_j = \int_0^\infty p_n(x) Q_j(x)\mathrm{d}x = \frac{1}{N}\left(\int_0^\infty p_n(y)\mathrm{d}y\right)^N = \frac{1}{N}, \tag{11}$$

as is expected.

Let us put,

$$Q_{ij}(x) = Q(x) \Big/ \int_0^x p_i(y)\mathrm{d}y \int_0^x p_j(y)\mathrm{d}y, \tag{12}$$

where $1 \leqslant i, j \leqslant N$ ($i \neq j$). The difference of P_j and P_i is

$$P_j - P_i = \int_0^\infty Q_{ij}(x)\left\{p_j(x)\int_0^x p_i(y)\mathrm{d}y - p_i(x)\int_0^x p_j(y)\mathrm{d}y\right\}\mathrm{d}x. \tag{13}$$

Since $Q_{ij}(x)$ is an increasing function, then, applying the first mean value theorem to the integral of Eq. (13), we have

$$\begin{aligned} P_j - P_i &= Q_{ij}(r)\int_0^\infty \left\{p_j(x)\int_0^x p_i(y)\mathrm{d}y - p_i(x)\int_0^x p_j(y)\mathrm{d}y\right\}\mathrm{d}x \\ &= Q_{ij}(r)\,\frac{c_j - c_i}{c_j + c_i}, \end{aligned} \tag{14}$$

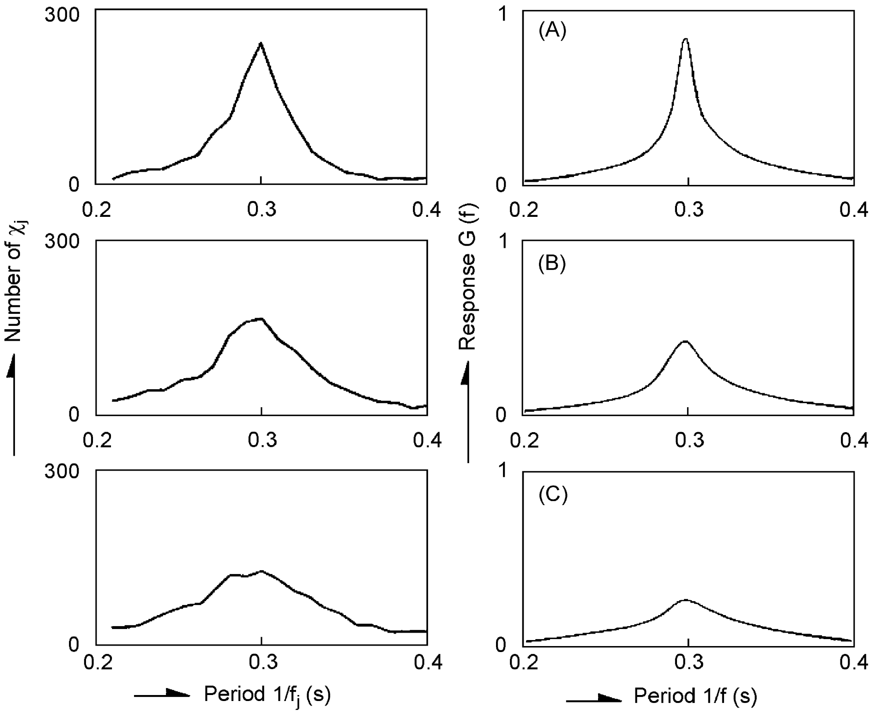

Fig. 1. SIP distribution (left) compared with frequency response (right), where (A) $h = 0.04$, (B) $h = 0.08$ and (C) $h = 0.12$.

where r is a proper value such that $0<r<\infty$ and $0<Q_{ij}(r)<1$. Thus, by Eq. (5), if $G(f_1)>G(f_2)>\cdots>G(f_N)$, we get $P_1>P_2>\cdots>P_N$. The SIP distribution is similar to a set of probabilities $\{P_n\}$, which is given by the finite number of samples where the value x_j of Eq. (6) is a dominant frequency component in a short-interval portion of vibration. Hence, if we assign proper frequencies to $\{f_n\}$ and increase the number of samples, the SIP distribution corresponds to the frequency response of a structure.

Examples of SIP distributions given by numerical experiments are shown in Fig. 1 where the frequency responses A, B and C are given by

$$G(f) = 1/\{(f/f_o - f_o/f)^2 + h^2\}^{1/2}, \tag{15}$$

where f_o and h denote the natural vibration frequency and damping ratio of a model structure, respectively. Each SIP distribution comprises 2400 samples, which were given by applying a short-interval frequency analysis to the vibration of the model structure, which was excited by random noise (see Ref. [1]). The prominent limitation of the conventional Fourier transform is that of frequency resolution, which occurs when processing short waveforms [2]. A non-harmonic Fourier analysis proposed by the author is available for the present purpose of frequency analysis [3]. The time length of short-interval waveforms is 0.8 s and the natural vibration period, which is the reciprocal of f_o, is 0.3 s. Although the set of frequencies $f_1, f_2, \ldots, f_N$ which gives the inequalities $G(f_1)>G(f_2)>\cdots>G(f_N)$ and $P_1>P_2>\cdots>P_N$ is not directly shown in Fig. 1, similarities of SIP distributions to the respective curves of A, B and C confirm the theoretical result discussed above.

Structural changes due to degradation or damage after an earthquake, for example, are attributed to the decrease of stiffness, which generally gives rise to the increase of the natural vibration period of a structure. In principle, we can check the changes using a huge shaker to obtain the frequency response of a structure. However, it is not practical to shake a building, for example, before and after an earthquake. Regardless of size and weight, all structures such as buildings, towers and bridges vibrate due to the natural force of winds, small ground motions or both. Hence, the non-stationary vibration of a structure is available and it is easy to obtain the SIP distribution, which may change in the lapse of time. Thus, a monitoring SIP distribution for investigating structural changes is both practical and reliable.

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