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Detection of two cracks in a rotor-bearing system using amplitude deviation curve

T. Ramesh Babu, A.S. Sekhar*

Machine Design Section, Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai 600 036, India

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Abstract

The dynamics and diagnostics of cracked rotors have been gaining importance in recent years. Relatively few authors have addressed the problem of multi-crack assessment for rotors. In many cases there exist more than one crack on the shaft. Then the solutions or the combinations of parameters characterizing the cracks are more and the problem becomes more complicated. In this study, a new technique called amplitude deviation curve (ADC) or slope deviation curve (SDC) has been developed, which is a modification of the operational deflection shape (ODS). The effectiveness of the SDC over ODS for small cracks detection has been demonstrated in the present paper.

1. Introduction

In large rotating machinery, such as turbine, generator and aero-engine, the rotor is one of the most important parts. Appearances and expansion of fatigue crack in large rotating machines may lead to catastrophic failures. Online detection of cracks is very important for engineers working in the areas of machine dynamics. The research in the past few decades on cracked structures and rotors is well documented in the review papers [1–3]. The thrust of most of the works in the past has been on a structure with a single transverse surface crack. In many cases there is more than one crack on the rotor. When more than one crack appears in a structure, the dynamic response becomes more complex depending upon the relative positions and depths of these cracks.

The work on the diagnosis of crack has been mainly based on vibration signature. The changes in the vibration response in the form of changes of frequency composition or a rising trend of higher harmonics of rotational frequency or an increased level of sub-harmonic resonances have been found to be some of the important crack indicators. Various investigators have recently addressed the effect of two surface cracks on the vibration response of structures. The transfer matrix method is used by Tsai and Wang [4] to obtain the frequency equation of the rotor system. A multi-section and multi-cracked shaft is simulated in this paper,

*Corresponding author. Tel./fax: +91442257 4709.
E-mail address: as_sekhar@iitm.ac.in (A.S. Sekhar).

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based on the Timoshenko beam theory. It has been concluded that the change of natural frequency due to orientations of cracks is very small except in higher modes.

In Ref. [5] the theory of model-based identification in a rotor system with two cracks has been carried out. In this work, the crack-induced change of the rotor system is taken into account by equivalent loads in the mathematical model. The case of two cracks with an angular distance from each other is considered in a Jeffcott or de Laval rotor in Ref. [6]. The cracks are considered to be both breathing or one open and the other breathing. A review based on the work related to multiple-cracked structures is presented in Ref. [7]. Recently a beam with multiple cracks under bending for identification of cracks has been analysed in Ref. [8]. For the first time the local compliance due to crack as a function of both the crack depth and the angle of rotation by integrating in every angle of rotation has been calculated. A method for crack identification in double-cracked beams based on wavelet analysis has been presented in Ref. [9]. The fundamental vibration mode of a double-cracked cantilever beam is analysed using a symlet wavelet, a continuous wavelet transform and both the location and depth of the cracks are estimated. The location of the cracks is determined by the sudden changes in the spatial variation of the transformed response.

Modal curvatures are highly sensitive to damage; by plotting the difference in modal curvature (MC) between the intact and the damaged case, a peak appears at the damaged element, indicating the presence of a fault. But it was observed that apart from a high peak at the fault position, there were some small peaks at different undamaged locations for the higher modes. This can cause confusion to the analyst in a practical application. The application of the technique to constructions in which more than one fault positions exist is further investigated in Ref. [10] on a continuous beam with simulated data using a curvature damage factor, (CDF).

Traditionally, an operational deflection shape (ODS) has been defined as the deflection of a structure at a particular frequency or speed. However, an ODS can be defined more generally as any forced motion of two or more points on a structure. Specifying the motion of two or more points defines a shape. Stated differently, a shape is the motion of one point relative to all others. ODSs are quite different from mode shapes. They depend on the forces or loads applied to a structure. They will change if the load changes. ODSs can have units, typically displacement, velocity, or acceleration, or perhaps displacement per unit of excitation force. They can be used to answer the question, "*How much is the structure really moving, at a particular time or frequency?*" Finally, ODSs can be defined for nonlinear and non-stationary structural motion, while mode shapes are defined only for linear, stationary motion [11]. The method for crack identification of bridge beam structures under a moving load based on wavelet analysis is presented in Ref. [12]. The locations of the cracks are determined from the sudden changes in the spatial variation of the transform responses.

In the present work the detection of two cracks with different configurations in a rotor-bearing system has been studied. The identification of small cracks using the concept of ODS is quite difficult, so a new curve called amplitude deviation curve (ADC) or slope deviation curve (SDC) has been found and the same has been used for the identification of small cracks in a rotor system. The information of the crack parameters like the number of cracks and position of cracks are also available from the SDC.

2. Cracked rotor modelling

The finite element model of a simple rotor-bearing system has been considered in Ref. [13] as shown in Fig. 1. The details of the cracked element used in the rotor shaft are given in Fig. 2. Here 'L' is the total length

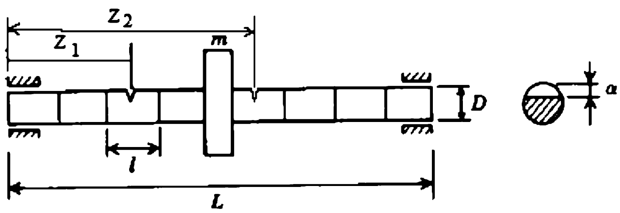

Fig. 1. Finite element model of the rotor.

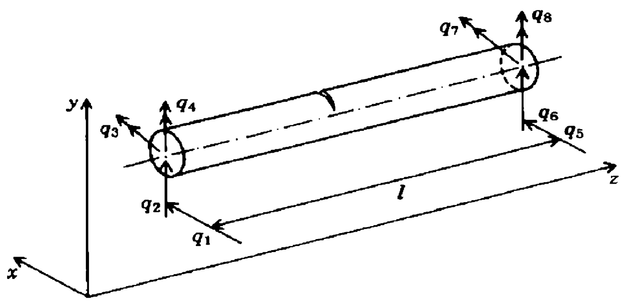

Fig. 2. Cracked element.

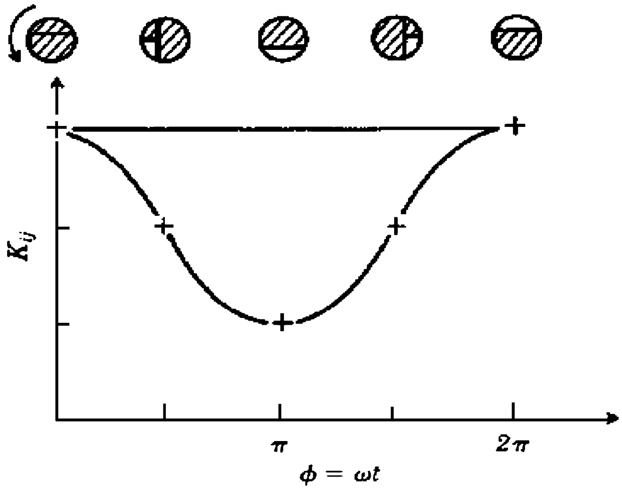

Fig. 3. Breathing crack model.

of the shaft, ‘D’ the diameter, ‘l’ the element length, ‘m’ the mass of the disc, ‘z’ the location of the crack and ‘α’ the crack depth.

The equation of motion of the complete rotor system in a fixed coordinate system can be written as

$$[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = \{F\} \tag{1}$$

The excitation matrix $\{F\}$ consists of the unbalance forces due to disc having mass m, eccentricity e and the weight of the disc. The breathing action of the crack [13], i.e. its opening and closing, is illustrated in Fig. 3. During the shaft’s rotation, the crack opens and closes, depending on the rotor deflection. For a large class of machines, the static deflection is much greater than the rotor vibration. With this assumption, the crack is closed when $\Phi = 0$ and it is fully open when $\Phi = 180$. The transverse surface crack on the shaft element introduces considerable local flexibility due to strain energy concentration in the vicinity of the crack tip under load. The local flexibility matrices for breathing cracks are utilized from Refs. [13,14]. These will be used for the stiffness calculations.

When the shaft is cracked, during rotation the stiffness varies with time, or with angle. The variation may be expressed by a truncated cosine series

$$[K] = [K_0] + [K_1]\cos\,\omega t + [K_2]\cos\,2\omega t + [K_3]\cos\,3\omega t + [K_4]\cos\,4\omega t \tag{2}$$

where $[K_\eta]$, $\eta = 0,\ 1,\ \ldots, 4$ are the fitting coefficient matrices, determined from the known behaviour of the stiffness matrix at certain angular locations as explained in Refs. [13,14].

3. Results and discussion

A steel shaft supported on two isotropic flexible bearings at both ends and having a disc at the centre with the following data is considered for the analysis: shaft diameter 20 mm, length 500 mm; disc mass 5.5 kg, polar moment of inertia = 0.01546 kg m^2, unbalance eccentricity = 0.01 mm; bearing stiffness = 10^5 N/m, damping = 100 Ns/m. The rotor shaft has been divided into 20 equal elements. Different crack depths ($\bar{\alpha} = 0.1$, 0.2, etc. or 10%, 20%, etc.) are considered for the analysis, where($\bar{\alpha} = \alpha/D$). The Houbolt time-marching technique ($\Delta t = 0.001$ s) has been used to model the system in time domain.

The transient analysis of a cracked rotor using continuous wavelet transform (CWT) has proved to detect small cracks both quantitatively and qualitatively. The presence of $\frac{1}{3}$ and $\frac{1}{2}$ critical peaks in the CWT plots are a clear indication of crack in the rotor system [5,15]. The CWT plots for single and two cracks are shown in Fig. 4, where it can be observed that the presence of multiple cracks in the system adds only quantitatively to the transient response.

The presence of $\frac{1}{2}$ critical peaks in the CWT (Fig. 4c, d) is a clear indication of crack in the system, but there is no qualitative difference between a single-crack (Fig. 4c) and a two-crack system (Fig. 4d). So the identification of multiple cracks becomes difficult using CWT of the transient response. There are many cases where two cracks are present in the rotor system, so the identification of the same is very important for better diagnosis of the system.

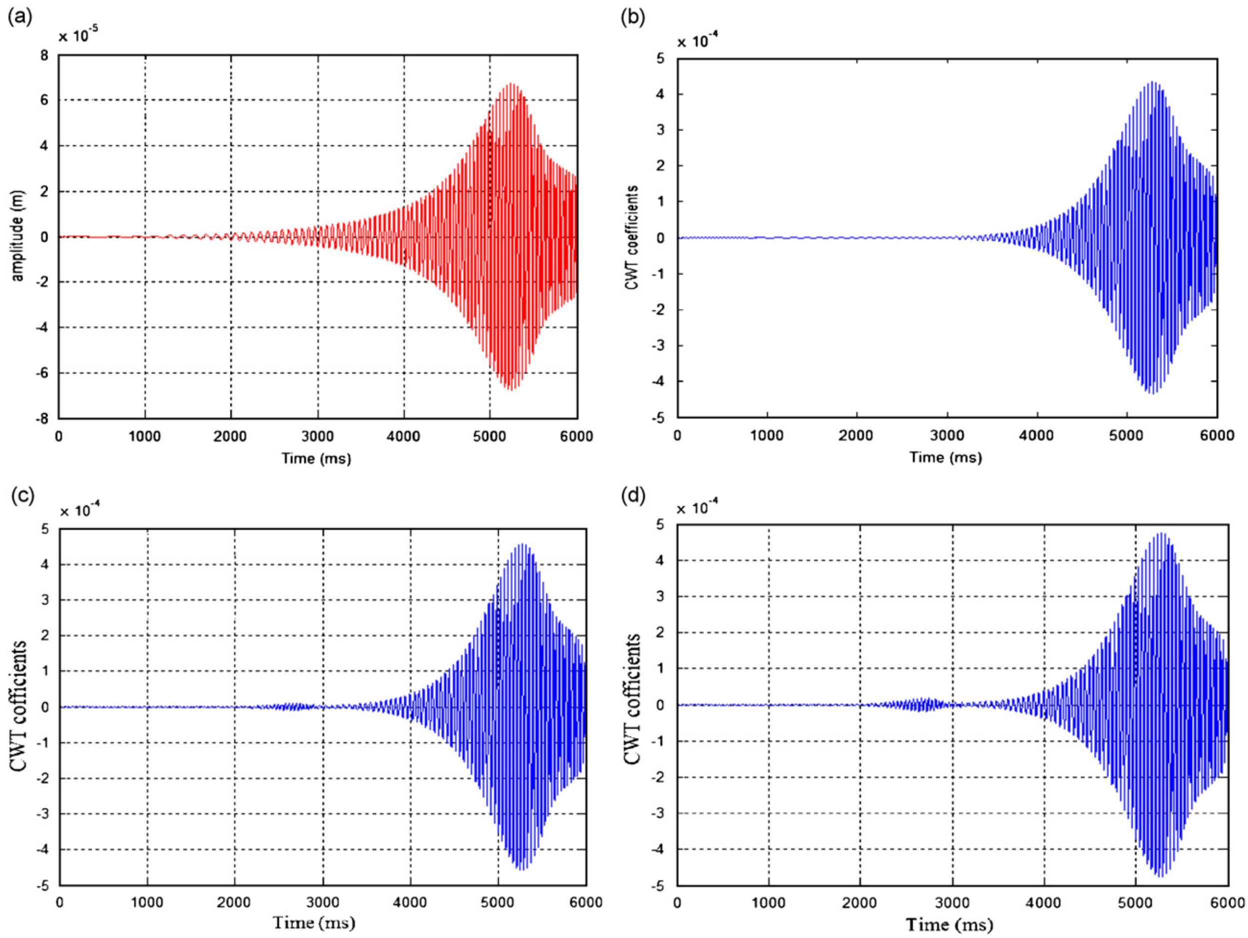

Fig. 4. Comparison of CWT plots. (a) Time plot for the no-crack system, (b) CWT plot for the no-crack system, (c) CWT plot for a single-crack ($\bar{\alpha} = 0.2$) system crack in the 10th element and (d) CWT plot for a two-crack ($\bar{\alpha} = 0.15$, 0.1) system crack in the 8th and 12th elements, respectively.

The above problem has been solved using the concept of ODS. The ODS indicates the displacement of the rotor along its length at a particular speed (generally operating speed). The response at a particular time along the length of the shaft is shown in Fig. 5(i-a). The cracks in the system introduce a local flexibility to the system, so the ODS of a cracked system slightly deviates from the uncracked system as shown by arrows in Fig. 5(i-b). This difference in ODSs of the cracked and uncracked system may not be sufficient to detect the cracks when the depths of the cracks are small.

Therefore a new curve called the SDC or the ADC is generated from the ODS by a simple transformation. Let A_1, A_2, $A_3 \ldots A_{21}$ be the response data along the length of the shaft at a particular time step, then $B_i = (A_{i+1} - A_i)$, the curve obtained by plotting B_i is called the SDC. Strictly speaking, the curve is called the ADC, but the amplitude deviations cause change in the slope so it can also be called as the SDC (deviation per equal element length). Fig. 5(i) shows the ODSs (at 3000 rev/min) of the undamaged and damaged system with two cracks (one in the 8th element and the other in the 12th element with a crack depth of 10% each). It is clearly understood from Fig. 5(i) that the identification of the damage through ODS is difficult. The SDC of the same is shown in Fig. 5(ii). The presence of cracks and their locations are clearly seen from the SDC of

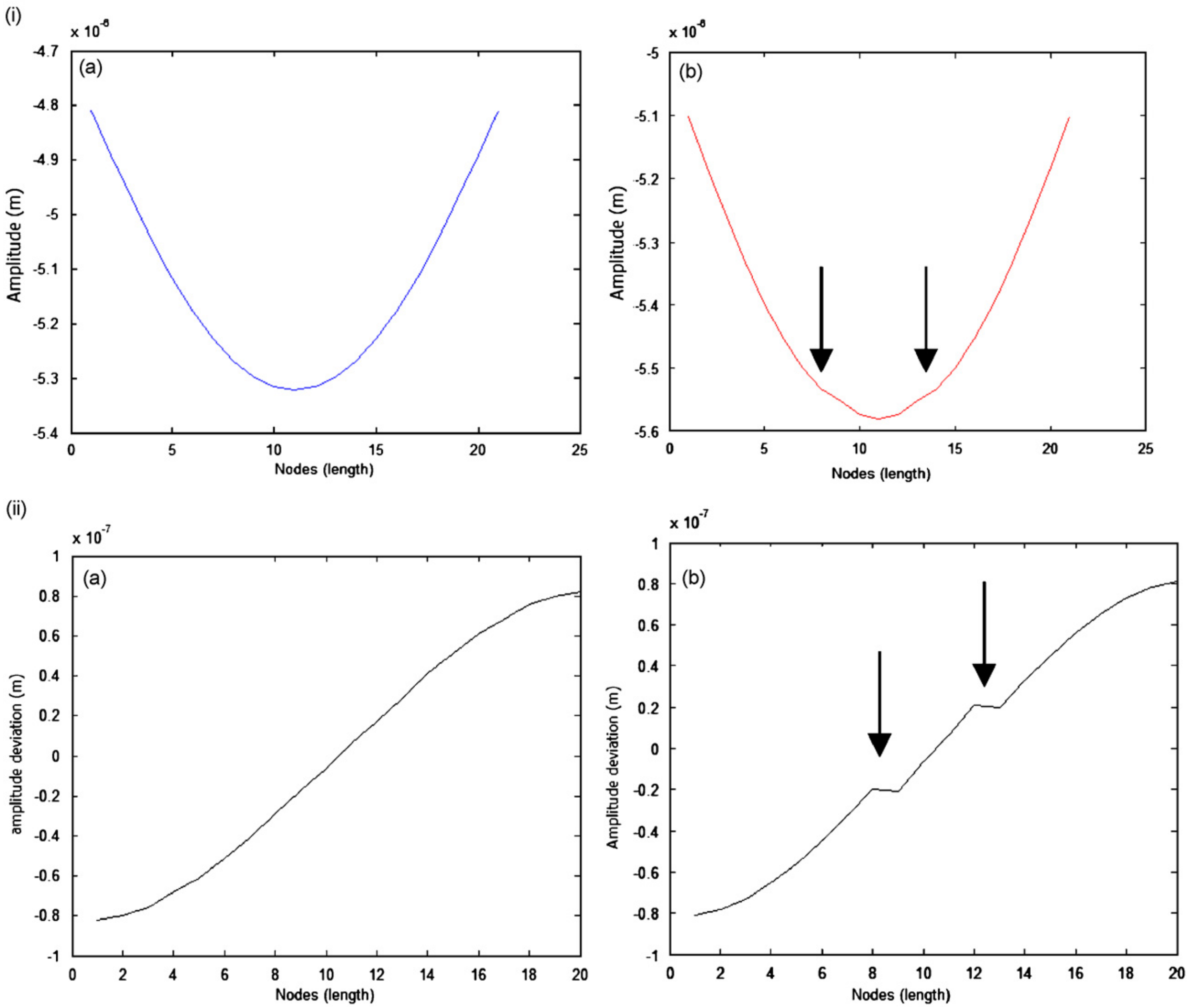

Fig. 5. Comparison of (i) ODS, (ii) SDC of a cracked rotor at an operating speed of 3000 rev/min: (a) no crack and (b) with cracks of ($\bar{\alpha} = 0.1$) each in the 8th and 12th elements.

Fig. 5(ii-b), which is pointed by arrows. The sudden rising peaks in the SDC represent the cracks; the number of peaks and position of peaks are clearly seen from the figure.

Consider the same shaft (whose data have been already mentioned) with a single crack in the 8th element and that the shaft is discretized into 20 elements. The SDCs of the same are shown in Fig. 6. Fig. 6(i) shows the 3D plot of the SDCs of the undamaged and damaged (20% crack) system at 3000 rev/min in one complete revolution. The time taken to complete one revolution at 3000 rev/min is 0.02 s. Hence, to complete one revolution it takes 20 sub-steps with 0.001 s as the numerical integration time step. The three axis of the 3D plot are length (along the shaft of the rotor), sub-steps (for one revolution) and amplitude of the SDC. Fig. 6(i-b) shows the 3D plot of the SDC for a damaged system, but only a few number of sub-steps are considered to draw the above plot. This clearly shows that even with few sub-steps the damage can be detected, but the considered sub-steps must have the crack information.

The 2D plots (with axes as length and amplitude) of the SDC shown in Fig. 6(ii) are the front view of the above 3D plots. It has a shape of an HOURGLASS (laid horizontally) when the system is undamaged; if the system has a crack then a peak is clearly seen at the damaged position as shown in Fig. 6(ii-b). The magnitude of maximum amplitude or height of the peak can be correlated to the depth of the crack. Since laser technology is developing at a rapid pace, laser vibrometers with single point and multi-point are available to capture the vibration data. Therefore, online detection of crack parameters by the present method can prove to be a good tool for crack detection.

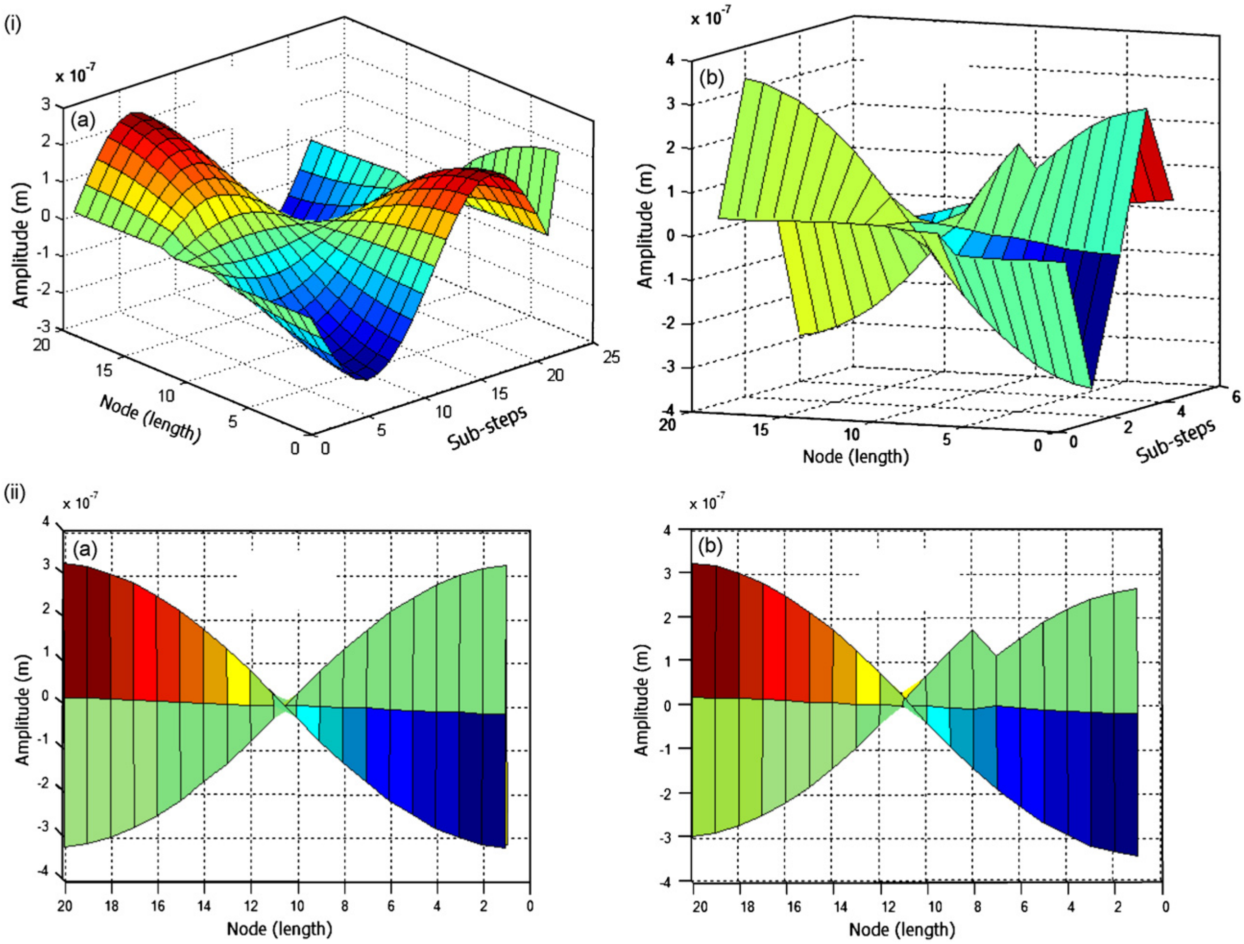

Fig. 6. Comparison of (i) 3D-SDC, (ii) 2D-SDC of a cracked rotor at an operating speed of 3000 rev/min: (a) no crack and (b) with crack of ($\bar{\alpha} = 0.2$) in the 8th element.

Fig. 7 shows some results of SDCs for different damage configurations at an operating speed of 3000 rev/min. The location of the crack can be clearly identified from the SDC based on the location of the peak. However, to quantify the amount of damage using the concept of SDC, the information of amplitudes or heights of peaks can be passed through techniques such as the genetic algorithm [16].

Fig. 8 shows the sensitivity curve of the cracked rotor system. The depth of the crack is inversely identified based on the amplitude or height of the peaks that appeared in the SDC curves. So, the variation in Fig. 8 indicates the sensitivity of the amplitudes of these peaks due to crack propagation. The vertical axis represents the values of amplitudes or heights of the peaks in the SDC and the horizontal axis represents the depth of crack in percentage with respect to the shaft diameter. It is clear from the curve that the crack is easily

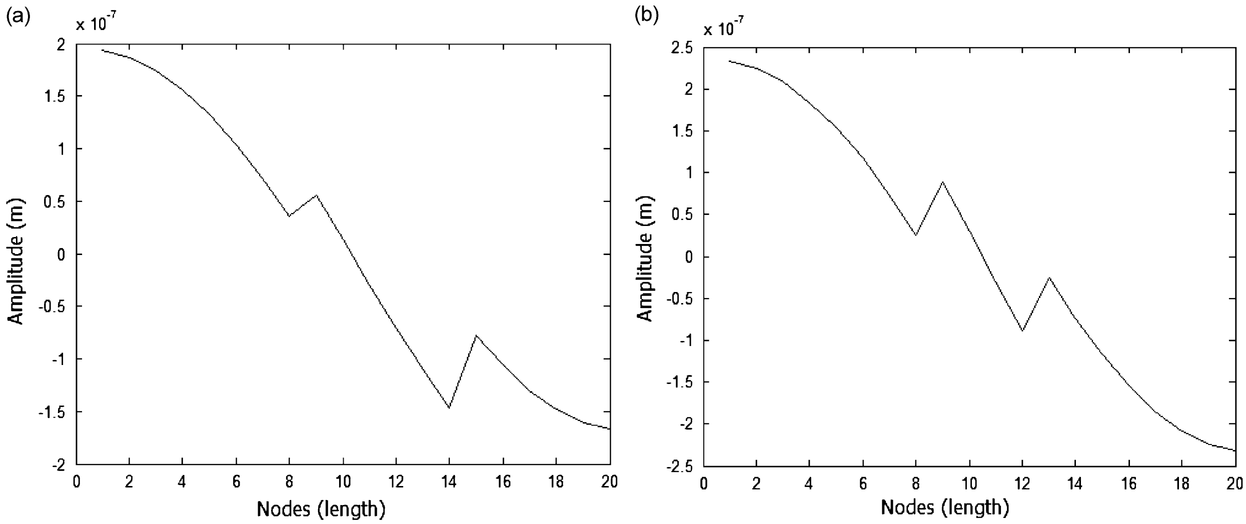

Fig. 7. The sample results of SDCs: (a) cracks at the 8th and 14th elements with 0.15 and 0.2 crack depths, respectively and (b) cracks in the 8th and 12th elements with 0.2 and 0.2 crack depths, respectively.

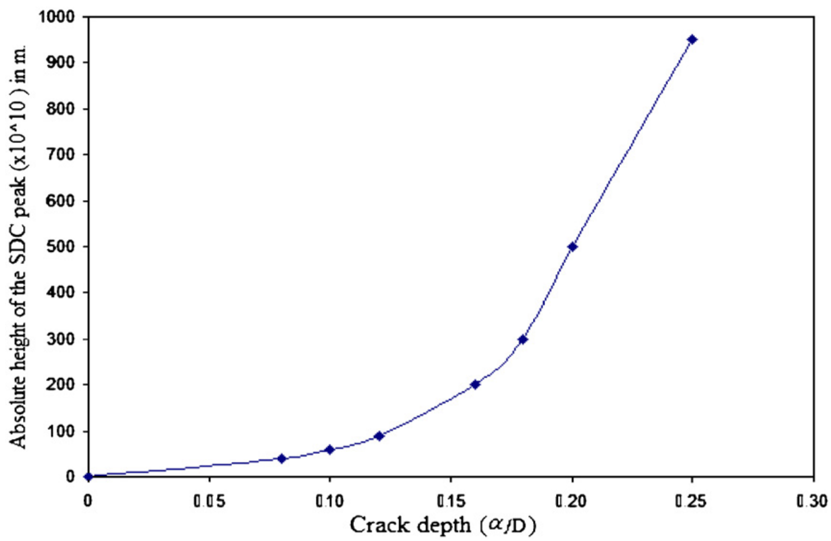

Fig. 8. Variation of SDC with crack depth.

detectable even slightly less than 10% depth of the crack. The curve becomes more sensitive above 15% of crack depth, so the propagation of the crack can also be easily monitored.

Recently, Loutridisa et al. [9] have analysed a beam with multiple cracks under bending for identification of cracks. The response data collected along the length of the damaged beam are passed to the symlet wavelet transform to obtain the depth and positions of cracks. But a large number of data points are needed to collect the response of the damaged system (200 points for the 1 m shaft) along the beam length to detect cracks in the system [9]. In the present study, as discussed in the results so far, a shaft of 0.5 m is considered. Accordingly, 100 points are required for the faults detection using the [9] symlet wavelet transform method. However, the present results show effective detection by using only 20 points. Hence there is a possibility that the number of points (sensors) can be reduced for vibration measurements and detection by using SDC.

4. Conclusions

The dynamic analysis of a rotor system with transverse breathing crack has been studied for flexural vibrations. The identification of small cracks using the concept of ODS is quite difficult, so a new curve called ADC or SDC has been found and the same is used for the identification of cracks in a rotor system. The effectiveness of the SDC over ODS for small cracks detection has been demonstrated. Therefore, online detection of crack parameters by the present method can prove to be a good tool in detecting even small cracks around 10% (depth of crack to shaft diameter).

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