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Adaptive tracking control of a class of uncertain chaotic systems in the presence of random perturbations

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Abstract

Considering the effect of random perturbations on the chaotic system, a new adaptive tracking control is presented for a large class of uncertain chaotic systems using the invariance principle of differential equations, where the bound of random perturbations is not necessarily known in advance and it is estimated through an adaptive control process. It is theoretically proved that this approach can make the perturbed chaotic system track any desired reference signal; in addition, we can see that this method can apply to almost all uncertain chaotic systems and it is simpler and easier to implement in practical application. In the end, we take the perturbed Lorenz system as an example to illustrate that the proposed scheme is effective.

1. Introduction

In the past decade or more, the investigation of chaos control and synchronization has undergone the process of vigorous development due to the pioneering work of Ott, Grebogi and Yorke (OGY) [1]. Now, it has been applied to various fields such as in physics, chemistry, biology, information science and secure communication [2–9]. There are many strategies for controlling chaos, such as feedback control, impulsive control, back-stepping design, adaptive control, and time-delay feedback control.

Among them, tracking is the most commonly discussed problem in the domain of chaos control. It can be explained that, for random reference signal, a controller is needed to be designed in order to cause the output of the chaotic system to follow the given reference signal asymptotically [10–17]. There are many references about tracking control in the literature. For some concrete chaotic systems, Refs. [10–16] studied the problem of tracking control. But it is a pity that all above chaotic systems' parameters are taken as known constants, and these methods can apply only to some special systems not a class of chaotic systems. In Ref. [17], adaptive tracking control strategy was proposed to approach the desired bounded trajectories for a class of uncertain chaotic systems with time-varying unknown parameters, but uncertain parameters in Ref. [17] are supposed to be bounded and random perturbations are not taken into consideration. In real life, random perturbation or

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noise exists inevitably in system, and even sometimes, we require certain disturbance to increase security in communication because it can induce complicated dynamics of the original system, so in recent years some people began to move the discussion to the chaotic system in the presence of random perturbations [18–20].

Motivated by all above works, in this paper, one more general method of tracking control, which is different from the previous works, is given for a class of uncertain chaotic systems with random perturbations, where the bound of random perturbations is not necessarily known in advance and it is estimated through an adaptive control process. Compared to the predecessor's method [10–17], our method can apply to almost all chaotic systems such as Chen system, Rössler system, Chua's system, Liu system and the adopted adaptive technique is convenient to implement in practical application. Numerical simulations verify the effectiveness of our method.

The organization of this paper is as follows: In Section 2, adaptive tracking control is theoretically introduced for a class of chaotic systems with random perturbations by means of the invariance principle of differential equations where random disturbances are taken into consideration in the design of controller. In Section 3, Lorenz chaotic system as a concrete example is used to illustrate the validity of the proposed method, and the results show that the perturbed Lorenz chaotic system can follow any desired signal. Section 4 draws some conclusions.

2. A general method for controller design

Consider an n-dimension chaotic system with bounded disturbances

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{d}, \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ denotes the state vector, and $\tilde{\boldsymbol{\theta}} \in \mathbf{R}^m$represents the uncertain parameter vector of the system, $\mathbf{f}(\mathbf{x})$ is an $n \times 1$ matrix and $\mathbf{F}(\mathbf{x})$ is an $n \times m$ matrix. $\mathbf{d}(t) \in \mathbf{R}^n$ may come from exoteric perturbation or is the random component added to the system for some purpose which assumed to satisfy bounded condition $||\mathbf{d}(t)|| \leqslant \tilde{k} < \infty$ for all t, where $\tilde{k} > 0$ is not necessarily known previously. Let $\boldsymbol{\Omega} \subset \mathbf{R}^n$ be a bounded closed set that contains the whole attractor of Eq. (1).

When a controller $\mathbf{u} \in \mathbf{R}^n$ is added to the original system (1), we get

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{d} + \mathbf{u}. \tag{2}$$

Let $\mathbf{h(t)}$ be an arbitrary given reference signal with first derivative. Our aim is that, according to the design of the controller, the output signal $\mathbf{x(t)}$ of system (2) follows the reference signal $\mathbf{h(t)}$ ultimately. That is

$$\lim_{t \to \infty} ||\mathbf{e}(t)|| = \lim_{t \to \infty} ||\mathbf{x(t)} - \mathbf{h(t)}|| = 0, \tag{3}$$

where $|| \cdot ||$ is the Euclidean norm.

Due to the randomicity of reference signal, it is easy to find that controlling of the equilibrium of chaotic system belongs to this class of problem if we take the equilibrium as a reference signal, and the synchronization phenomenon between chaotic systems also belongs to this class of problem if we consider the reference signal as being the output of one of the chaotic systems.

Now, Theorem 1 is given based on the above conditions in order to achieve the goal of Eq. (3).

Theorem 1. *If the controller is selected such that*

$$\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \hat{k}\mathbf{e}/||\mathbf{e}||, \tag{4}$$

where $l > 0$ is a constant and $\hat{\boldsymbol{\theta}} \in \mathbf{R}^m$ is updated according to the following law:

$$\dot{\hat{\boldsymbol{\theta}}} = [\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}, \tag{5}$$

and $\hat{k}$ obeys the following differential equation:

$$\dot{\hat{k}} = ||\mathbf{e}||, \tag{6}$$

then the signal $\mathbf{x(t)}$ of system (2) can track the reference signal $\mathbf{h(t)}$ ultimately.

Proof. Let $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}, k = \tilde{k} - \hat{k}$ and chose a Lyapunov function as

$$\mathbf{V} = \tfrac{1}{2}[\mathbf{e}^{\mathrm{T}}\mathbf{e} + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}) + (\tilde{k} - \hat{k})^{\mathrm{T}}(\tilde{k} - \hat{k})].$$

Thus, the time derivative of $\mathbf{V}$ is

$$\begin{aligned}
\dot{\mathbf{V}} &= \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{e} + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-\dot{\hat{\boldsymbol{\theta}}}) + (\tilde{k} - \hat{k})^{\mathrm{T}}(-\dot{\hat{k}}) \\
&= (\mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{d} + \mathbf{u} - \dot{\mathbf{h}})^{\mathrm{T}}\mathbf{e} + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}) + (\tilde{k} - \hat{k})^{\mathrm{T}}(-||\mathbf{e}||) \\
&\leqslant (\mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{u} - \dot{\mathbf{h}})^{\mathrm{T}}\mathbf{e} + ||\mathbf{d}||||\mathbf{e}|| + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}) + (\tilde{k} - \hat{k})^{\mathrm{T}}(-||\mathbf{e}||) \\
&\leqslant (\mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{u} - \dot{\mathbf{h}})^{\mathrm{T}}\mathbf{e} + \tilde{k}||\mathbf{e}|| + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}) + (\tilde{k} - \hat{k})^{\mathrm{T}}(-||\mathbf{e}||) \\
&= (\mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \mathbf{u} - \dot{\mathbf{h}})^{\mathrm{T}}\mathbf{e} + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}) + \hat{k}||\mathbf{e}|| \\
&= (\mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} + \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \hat{k}\mathbf{e}/||\mathbf{e}|| - \dot{\mathbf{h}})^{\mathrm{T}}\mathbf{e} + (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}(-[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e}) + \hat{k}||\mathbf{e}|| \\
&= (\mathbf{F}(\mathbf{x})\tilde{\boldsymbol{\theta}} - l\mathbf{e} - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \hat{k}\mathbf{e}/||\mathbf{e}||)^{\mathrm{T}}\mathbf{e} - (\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^{\mathrm{T}}[\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e} + \hat{k}||\mathbf{e}|| \\
&= -l\mathbf{e}^{\mathrm{T}}\mathbf{e} \leqslant 0.
\end{aligned}$$

It is found that the set $M = \{\mathbf{e} = 0, \tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}} = 0, \tilde{k} - \hat{k} = 0\}$ is the largest invariant set contained in the set $E = \{\mathbf{e} = 0 \in R^n\}$. Consequently, according to the well-known invariance principle of differential equations [21], the chaotic signal $\mathbf{x(t)}$ of system (2) will approach the reference signal $\mathbf{h(t)}$ asymptotically. □

3. Numerical simulations

In this section, with the aid of appropriate controller, the perturbed Lorenz system will track any given reference signal and corresponding numerical results are given to illustrate the validity of the proposed approach.

The Lorenz chaotic system is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_2 - x_1x_3 \\ x_1x_2 \end{bmatrix} + \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix} \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{bmatrix}, \tag{7}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} 0 \\ -x_2 - x_1x_3 \\ x_1x_2 \end{bmatrix}, \quad \mathbf{F}(\mathbf{x}) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix}.$$

when the system parameters are taken as $\tilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3)^{\mathrm{T}} = (10, 28, 8/3)^{\mathrm{T}}$, this system exhibits chaotic behavior, which can be seen in Fig. 1.

For the perturbed Lorenz system, it can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_2 - x_1x_3 \\ x_1x_2 \end{bmatrix} + \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix} \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \tag{8}$$

where d_1, d_2 and d_3 are all taken to be uniformly distributed random noise in the range $[-1.0,1.0]$ in numerical simulations, which means that the strength of perturbation is 1.0. Of course, d_1, d_2 and d_3 can be completely different, but the derived result is identical to the above ones.

The organization of this part is as follows: Firstly, according to the appropriate controller, the uncertain Lorenz system with disturbances will approach given desired signal $\mathbf{h} = (\sin 2t, \cos t, 0)^{\mathrm{T}}$. Secondly, the perturbed Lorenz chaotic system will track one of its own equilibriums $\mathbf{h} = (0,0,0)^{\mathrm{T}}$, that is to say, it will be

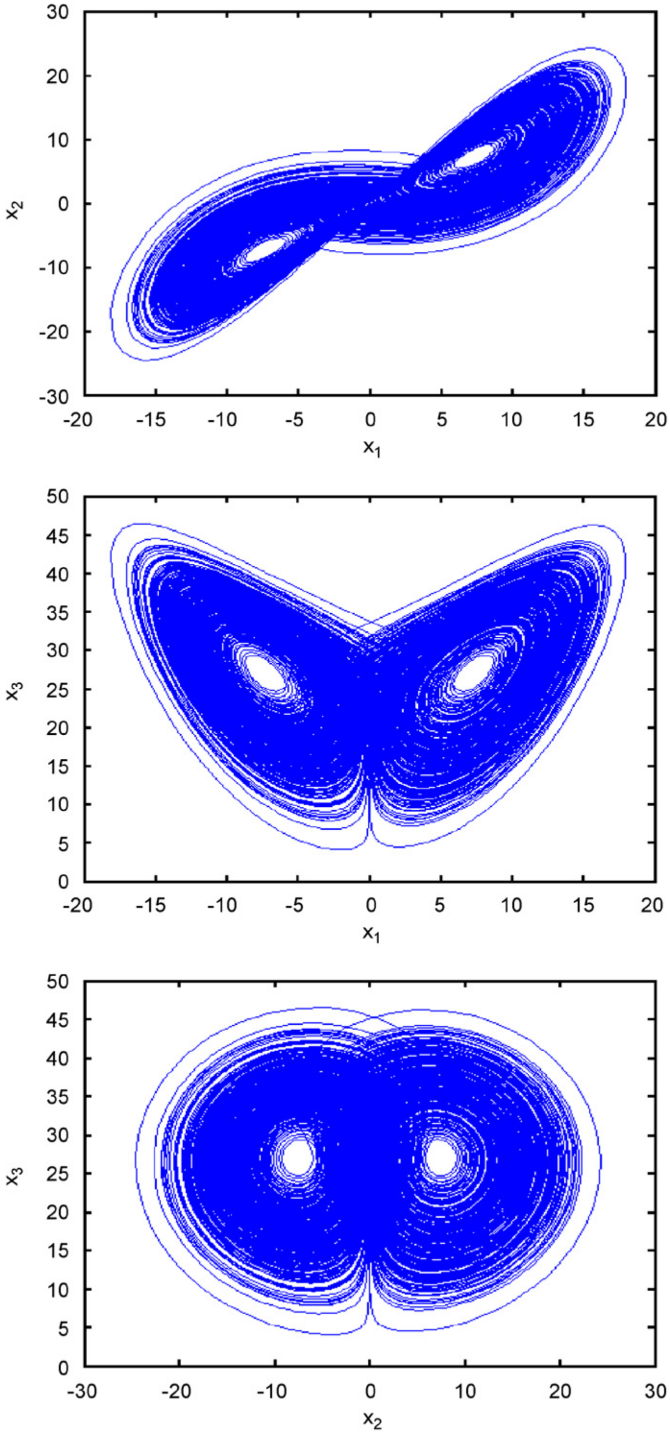

Fig. 1. The phase portrait of Lorenz system.

controlled to the equilibrium. Thirdly, based on Theorem 1, we try to make the perturbed Lorenz system close to the reference signal that is the output of identical Lorenz chaotic system. Lastly, synchronization of two different chaotic systems is discussed by means of our method. All these results show that our method is effective.

3.1. Tracking desired signal $\mathbf{h} = (\sin 2t, \cos t, 0)^T$

For the given reference signal $\mathbf{h} = (\sin 2t, \cos t, 0)^{\mathrm{T}}$, the controller is derived with the help of Theorem 1. So it can be expressed as

$$\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \hat{k}\mathbf{e}/||\mathbf{e}|| = \begin{pmatrix} (x_1 - x_2)\hat{\theta}_1 + 2\cos 2t - l(x_1 - \sin 2t) - \dfrac{\hat{k}}{||\mathbf{e}||}(x_1 - \sin 2t) \\ x_2 + x_1 x_3 - x_1\hat{\theta}_2 - \sin t - l(x_2 - \cos t) - \dfrac{\hat{k}}{||\mathbf{e}||}(x_2 - \cos t) \\ -x_1 x_2 + x_3\hat{\theta}_3 - l x_3 - \dfrac{\hat{k} x_3}{||\mathbf{e}||} \end{pmatrix}. \tag{9}$$

From Ref. [19], several approaches are introduced in order to eliminate the chattering phenomenon, which is as a result of the coming of $\hat{k}\mathbf{e}/||\mathbf{e}||$ in the controller $\mathbf{u}$, so in this simulation, we employ a continuous controller that is in the following:

$$\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \hat{k}\mathbf{e}/||\mathbf{e}|| + 0.0001 = \begin{pmatrix} (x_1 - x_2)\hat{\theta}_1 + 2\cos 2t - l(x_1 - \sin 2t) - \dfrac{\hat{k}}{||\mathbf{e}|| + 0.0001}(x_1 - \sin 2t) \\ x_2 + x_1 x_3 - x_1\hat{\theta}_2 - \sin t - l(x_2 - \cos t) - \dfrac{\hat{k}}{||\mathbf{e}|| + 0.0001}(x_2 - \cos t) \\ -x_1 x_2 + x_3\hat{\theta}_3 - l x_3 - \dfrac{\hat{k} x_3}{||\mathbf{e}|| + 0.0001} \end{pmatrix}, \tag{10}$$

and from Eq. (5), we derived the following:

$$\dot{\hat{\boldsymbol{\theta}}} = [\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e} = \begin{pmatrix} (x_2 - x_1)(x_1 - \sin 2t) \\ x_1(x_2 - \cos t) \\ -x_3^2 \end{pmatrix}. \tag{11}$$

The unknown parameter vector $\tilde{\boldsymbol{\theta}} = (10, 28, 8/3)^{\mathrm{T}}$ and the initial conditions $\mathbf{x}(\mathbf{0}) = (-1, 1, 2)^{\mathrm{T}}$, $\hat{\boldsymbol{\theta}}(\mathbf{0}) = (0, 0, 0)^{\mathrm{T}}$, $\hat{k}(0) = 1.0$ are chosen in this simulation; furthermore, constant l is fixed at 1.0 in this paper. The numerical results are shown in Fig. 2 that tells us time evolution of errors between the perturbed chaotic signal and reference signal. From Fig. 2(a)–(c), we can see that, with the passage of time, the errors are close to zero asymptotically; therefore, Eq. (3) is satisfied. This says that the perturbed Lorenz chaotic system follows the given reference signals successfully though its parameters are uncertain.

3.2. Tracking the equilibrium of its own system $\mathbf{h} = (0,0,0)^T$

In order to obtain the perturbed Lorenz chaotic system with disturbances close to the equilibrium $\mathbf{h} = (0,0,0)^{\mathrm{T}}$, from Eqs. (4) and (5), the controller is given in the following as a result of considering the

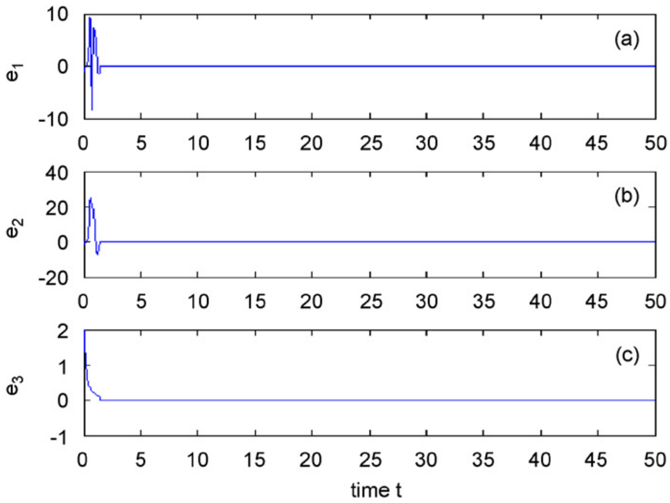

Fig. 2. Time evolution of error signals $\mathbf{e} = \mathbf{x}–\mathbf{h}$.

chattering phenomenon:

$$\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \frac{\hat{k}\mathbf{e}}{||\mathbf{e}|| + 0.0001}$$
$$= \begin{pmatrix} (x_1 - x_2)\hat{\theta}_1 - lx_1 - \dfrac{\hat{k}x_1}{||\mathbf{e}|| + 0.0001} \\ x_2 + x_1x_3 - x_1\hat{\theta}_2 - lx_2 - \dfrac{\hat{k}x_2}{||\mathbf{e}|| + 0.0001} \\ -x_1x_2 + x_3\hat{\theta}_3 - lx_3 - \dfrac{\hat{k}x_3}{||\mathbf{e}|| + 0.0001} \end{pmatrix}, \tag{12}$$

and

$$\dot{\hat{\boldsymbol{\theta}}} = [\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e} = \begin{pmatrix} x_1(x_2 - x_1) \\ x_1x_2 \\ -x_3^2 \end{pmatrix}. \tag{13}$$

We select the unknown parameter vector $\tilde{\boldsymbol{\theta}} = (10, 28, 8/3)^{\mathrm{T}}$ and the initial conditions $\mathbf{x(0)} = (-1, 1, 3)^{\mathrm{T}}$, $\hat{\boldsymbol{\theta}}\mathbf{(0)} = (0, 1, 0.5)^{\mathrm{T}}$, $\hat{k}(0) = 1.0$; Fig. 3 shows the numerical results and Fig. 3(a)–(c), give us time evolution of errors that the perturbed Lorenz system tracks its own equilibrium. We can see that the errors converge to zero ultimately as time goes on. Consequently, the conclusions are drawn that the perturbed Lorenz chaotic system is controlled to its equilibrium. In fact, Lorenz system can be close to other equilibriums of its own in a similar way.

3.3. Tracking Lorenz chaotic signal $\boldsymbol{h} = (y_1, y_2, y_3)^T$

Our aim is that the perturbed Lorenz chaotic system synchronizes with the signals that is the output of Lorenz chaotic system by means of the method of tracking control.

The reference signals $\mathbf{h} = (y_1, y_2, y_3)^{\mathrm{T}}$ satisfy

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1), \\ \dot{y}_2 &= by_1 - y_2 - y_1y_3, \\ \dot{y}_3 &= -cy_3 + y_1y_2. \end{aligned} \tag{14}$$

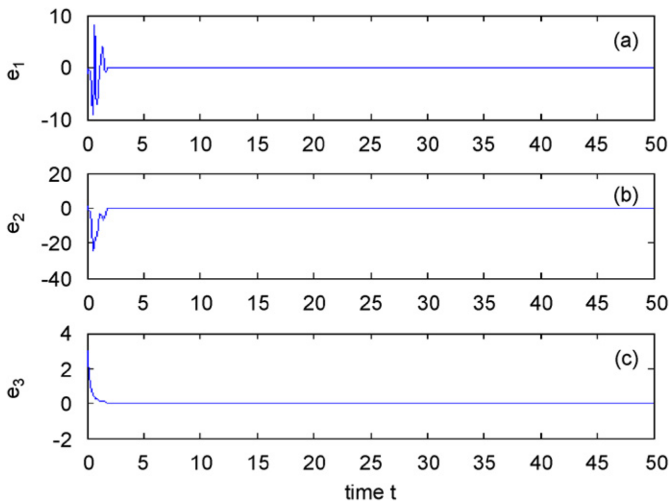

Fig. 3. Time evolution of error signals $\mathbf{e} = \mathbf{x}–\mathbf{h}$.

Similar to (12), the controller is as follows:

$$\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \frac{\hat{k}\mathbf{e}}{||\mathbf{e}|| + 0.0001}$$
$$= \begin{pmatrix} (x_1 - x_2)\hat{\theta}_1 + a(y_2 - y_1) - l(x_1 - y_1) - \dfrac{\hat{k}(x_1 - y_1)}{||\mathbf{e}|| + 0.0001} \\ x_2 + x_1x_3 - x_1\hat{\theta}_2 + by_1 - y_2 - y_1y_3 - l(x_2 - y_2) - \dfrac{\hat{k}(x_2 - y_2)}{||\mathbf{e}|| + 0.0001} \\ -x_1x_2 + x_3\hat{\theta}_3 - cy_3 + y_1y_2 - l(x_3 - y_3) - \dfrac{\hat{k}(x_3 - y_3)}{||\mathbf{e}|| + 0.0001} \end{pmatrix}, \tag{15}$$

and

$$\dot{\hat{\boldsymbol{\theta}}} = [\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e} = \begin{pmatrix} (x_2 - x_1)(x_1 - y_1) \\ x_1(x_2 - y_2) \\ -x_3(x_3 - y_3) \end{pmatrix}. \tag{16}$$

It is seen that the parameter vector of system (14) is $(a,b,c)^{\mathrm{T}} = (10,28,8/3)^{\mathrm{T}}$; thus, system (14) is chaotic. Simultaneously, we select initial conditions $\mathbf{y(0)} = (-0.5,1.0,-1.5)^{\mathrm{T}}$, $\mathbf{x(0)} = (-1,1,3)^{\mathrm{T}}$, $\hat{\boldsymbol{\theta}}\mathbf{(0)} = (0,0,1)^{\mathrm{T}}$, $\hat{k}(0) = 1.0$. The numerical results are shown in Fig. 4. From Fig. 4(a)–(c), we can see that the errors are indeed close to zero which means synchronization between chaotic systems can be achieved finally based on the method of tracking control in this paper.

3.4. Tracking another different chaotic signal $\boldsymbol{h} = (z_1,z_2,z_3)^T$

In this part, chaos synchronization of two different dynamical systems will be discussed with the aid of the tracking control. Now, we take the output of dynamical system in Ref. [22] as reference signals $\mathbf{h} = (z_1,z_2,z_3)^{\mathrm{T}}$, it can be described as

$$\begin{aligned} \dot{z}_1 &= \alpha z_1 - z_2 z_3, \\ \dot{z}_2 &= -\beta z_2 + z_1 z_3, \\ \dot{z}_3 &= -\gamma z_3 + z_1 z_2, \end{aligned} \tag{17}$$

and system (17) becomes chaotic in the presence of $(\alpha, \beta, \gamma)^{\mathrm{T}} = (0.4, 12, 5.0)^{\mathrm{T}}$.

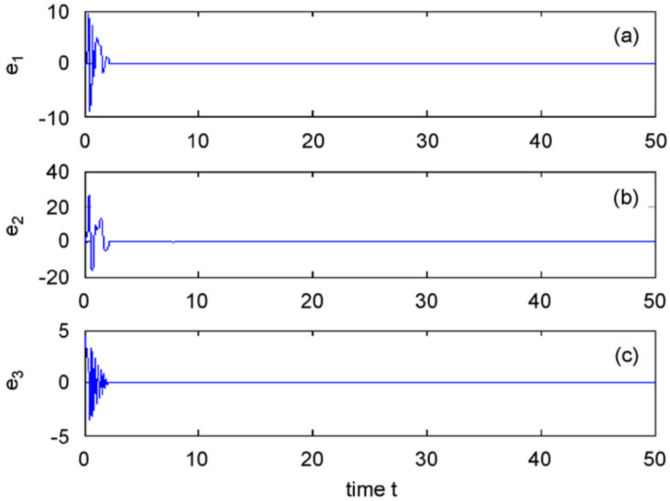

Fig. 4. Time evolution of error signals $\mathbf{e} = \mathbf{x}$–$\mathbf{h}$.

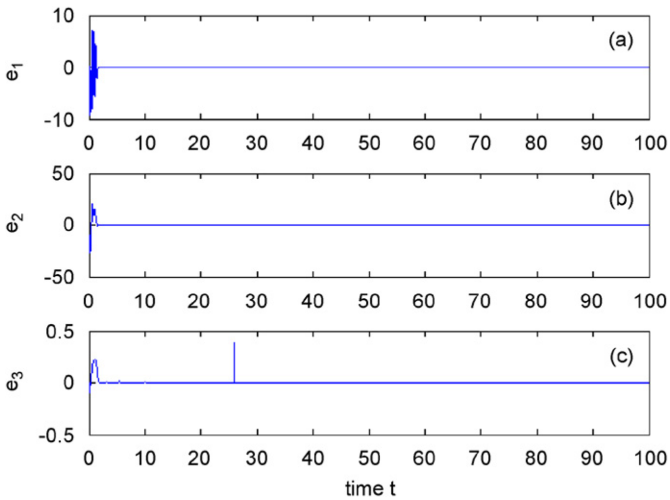

Fig. 5. Time evolution of error signals $\mathbf{e} = \mathbf{x}$–$\mathbf{h}$.

Considering the chattering of phenomenon, the controller becomes

$$
\mathbf{u} = \dot{\mathbf{h}} - l\mathbf{e} - \mathbf{f}(\mathbf{x}) - \mathbf{F}(\mathbf{x})\hat{\boldsymbol{\theta}} - \frac{\hat{k}\mathbf{e}}{||\mathbf{e}|| + 0.0001}
= \begin{pmatrix}
(x_1 - x_2)\hat{\theta}_1 + \alpha z_1 - z_2 z_3 - l(x_1 - z_1) - \dfrac{\hat{k}(x_1 - z_1)}{||\mathbf{e}|| + 0.0001} \\[2ex]
x_2 + x_1 x_3 - x_1\hat{\theta}_2 - \beta z_2 + z_1 z_3 - l(x_2 - z_2) - \dfrac{\hat{k}(x_2 - z_2)}{||\mathbf{e}|| + 0.0001} \\[2ex]
-x_1 x_2 + x_3\hat{\theta}_3 - \gamma z_3 + z_1 z_2 - l(x_3 - z_3) - \dfrac{\hat{k}(x_3 - z_3)}{||\mathbf{e}|| + 0.0001}
\end{pmatrix}, \tag{18}
$$

and

$$\dot{\hat{\boldsymbol{\theta}}} = [\mathbf{F}(\mathbf{x})]^{\mathrm{T}}\mathbf{e} = \begin{pmatrix} (x_2 - x_1)(x_1 - z_1) \\ x_1(x_2 - z_2) \\ -x_3(x_3 - z_3) \end{pmatrix}. \tag{19}$$

The initial values of Eqs. (8), (17) and (19) are given as follows: $\mathbf{x(0)} = (-1,1,3)^{\mathrm{T}}$, $\mathbf{z(0)} = (-1,11,3)^{\mathrm{T}}$, $\hat{\boldsymbol{\theta}}(\mathbf{0}) = (0,0,1)^{\mathrm{T}}$; what is more, we take $\hat{k}(0) = 0$. Fig. 5 shows us numerical results for verifying the validity of this proposed method; Fig. 5(a)–(c) illustrates time evolution of errors ($\mathbf{x}$–$\mathbf{z}$), so we can see that the perturbed chaotic signals of system (8) track the reference signals (17) quickly. Thus different chaotic dynamical systems achieve synchronization as time goes on.

4. Conclusions

In this paper, adaptive tracking control is given for a type of uncertain chaotic dynamical systems with random perturbations based on the invariance principle of differential equations where random disturbances are taken into consideration in the design of controller. This method makes the chaotic system track any desired reference signal. In the end, we take the perturbed Lorenz system as an example to illustrate the validity of this method.

It should be noted that this new scheme is applicable to a fairly large class of chaotic systems such as Chen system, Rössler system, Chua's system, and Liu system when the parameters of these systems are uncertain.

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