Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 315 (2008) 146–156
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Love waves in layered piezoelectric/piezomagnetic structures

Jin-xi Liu[a,*], Dai-Ning Fang[b], Wei-Yi Wei[b], Xiao-Fang Zhao[a]

[a]Department of Engineering Mechanics, Shijiazhuang Railway Institute, Shijiazhuang 050043, China
[b]Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

Received 2 February 2007; received in revised form 12 January 2008; accepted 21 January 2008
Handling Editor: J. Lam
Available online 21 March 2008

Abstract

The propagation of Love waves in layered structures is investigated for two cases: a piezomagnetic (PM) layer on a piezoelectric (PE) half-space and the reverse configuration. The dispersion relations are obtained in explicit form. The numerical examples are provided to illustrate the variations of the phase and group velocities versus the wavenumber for the combinations of different materials. The results show that: (1) the phase and group velocities initiate at the bulk shear wave velocity of the half-space medium and approach the bulk shear wave velocity of the layer with increasing wavenumber; (2) the influence of the magnetic permeabilities of PE materials on the phase velocity can be neglected; (3) for the layered medium consisting of a PM layer and a PE half-space, the properties of PE materials have a great influence on the phase and group velocities at lower wavenumber for the lowest mode. These findings are useful for PE/PM composite media or structures in the microwave technology.

1. Introduction

Composite materials or structures consisting of piezoelectric (PE) and piezomagnetic (PM) phases are able to facilitate the conversion of energy between electric and magnetic fields. Such a phenomenon is called magnetoelectric (ME) effect. This ME effect of the composites is a new product property, and results from the interaction between different material properties of the two phases in composites. Neither PE phase nor PM phase has the ME effect, but the composites of these two phases possess a remarkable ME effect. The ME effect of PE/PM composites was first reported by van Suchtelen [1] and was then studied by van den Boomgaard [2], van Run et al. [3] and van den Boomgaard et al. [4] for $BaTiO_3/CoFe_2O_4$ composites. Since their pioneering works, many efforts have been devoted to the prediction and determination of the ME effect of the PE/PM composites [5–15] both theoretically and experimentally. An overall and detailed review of this research topic can be found in a recent paper of Fiebig [16]. Due to the capability of conversion between electric and magnetic energy, PE/PM composites are potential candidates for use as ME memory elements, sensors, actuators, acoustic wave and microwave devices as well as other electric products [16]. These possible applications call for a better understanding of the static and dynamic behaviors of PE/PM composites or

*Corresponding author.
E-mail address: liujx02@hotmail.com (J.-x. Liu).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2008.01.055

structures, such as magneto-electro-elastic Green's functions [17,18], static deformations, vibrations and wave propagations, etc. Recently, there were several investigations that analyzed the bending deformations and free vibrations of PE/PM multilayered plates and shells [19–25]. Alshits et al. [26] conducted a qualitative investigation into the existence of surface-waves in half-infinite anisotropic elastic media with PE, PM and ME effects. Soh and Liu [27] gave the existence conditions of interfacial shear horizontal (SH) waves in a PE/PM bi-material. Chen et al. [28] investigated the propagation behaviors of harmonic waves in multilayered magneto-electro-elastic plates. Zhou et al. [29,30] investigated the scattering of the SH waves by two collinear interfacial mode III cracks between two different PE/PM materials.

In this paper, we consider the propagation of Love waves in PE/PM layered half-spaces. The main concerns are the dispersion characteristic and the influences of material properties on phase and group velocities. The basic equations of materials that possess simultaneously the coupling effects between mechanical, electric and magnetic fields are summarized in Section 2. The basic equations of PE and PM media are also given as two special cases. The problem to be considered is described in Section 3. The explicit dispersion relations are obtained in Section 4. The phase and group velocities are plotted for different combinations of materials and dispersion behaviors are discussed in Section 5. Finally, several conclusions are drawn in Section 6.

2. Basic equations

For anisotropic elastic materials that possess simultaneously PE, PM and ME effects, the constitutive equations in a fixed rectangular coordinate system x_i ($i = 1, 2, 3$) are described as

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - h_{kij}H_k, \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k + \alpha_{ik}H_k, \\ B_i &= h_{ikl}\varepsilon_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k, \end{aligned} \tag{1}$$

where σ_{ij}, D_i and B_i are the stress, electric displacement and magnetic induction (i.e., magnetic flux), respectively; ε_{ij}, E_i and H_i are the strain, electric field and magnetic field, respectively; c_{ijkl}, κ_{ij} and μ_{ij} are the elastic, dielectric and magnetic permeability coefficients, respectively; e_{kij}, h_{kij} and α_{ij} are the PE, PM and ME coefficients, respectively. The repeated indices imply usual summation convention. The comma denotes coordinate differentiation with respect to x_i. The strain, electric field and magnetic field are related to the elastic displacement u_i, the electric potential φ and magnetic potential ψ by

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2, \quad E_i = -\varphi_{,i}, \quad H_i = -\psi_{,i}. \tag{2}$$

In the usual quasi-static approximation, the equations of motion, electrostatics and magnetostatics are given by

$$\sigma_{ij,i} = \rho\ddot{u}_i, \quad D_{i,i} = 0, \quad B_{i,i} = 0 \tag{3}$$

in which a superimposed dot stands for differentiation with respect to time t.

Most magneto-electro-elastic media, especially composite materials consisting of a PE phase and a PM phase, possess transversely isotropic property. If the x_1x_2-plane is taken as the isotropic plane of materials, then the constitutive equations can be written in the Voigt form as [8,9]

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{32} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{32} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} - \begin{bmatrix} 0 & 0 & h_{31} \\ 0 & 0 & h_{31} \\ 0 & 0 & h_{33} \\ 0 & h_{15} & 0 \\ h_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \end{Bmatrix}, \tag{4}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{32} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix} + \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{11} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \end{Bmatrix}, \tag{5}$$

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & h_{15} & 0 \\ 0 & 0 & 0 & h_{15} & 0 & 0 \\ h_{31} & h_{31} & h_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{32} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \end{Bmatrix}, \tag{6}$$

where $c_{66} = 0.5 \times (c_{11} - c_{12})$.

If interested it is only the coupled problem in the isotropic plane, then the non-zero field quantities are: (u_3, $\varepsilon_{3\beta}$, $\sigma_{3\beta}$), (φ, E_β, D_β) and (ψ, H_β, B_β), $\beta = 1, 2$, which is called anti-plane magneto-electro-elastcity. For this situation, the basic equations reduce to

$$\begin{aligned} \sigma_{3\beta} &= 2c_{44}\varepsilon_{3\beta} - e_{15}E_\beta - h_{15}H_\beta, \\ D_\beta &= 2e_{15}\varepsilon_{3\beta} + \kappa_{11}E_\beta + \alpha_{11}H_\beta, \\ B_\beta &= 2h_{15}\varepsilon_{3\beta} + \alpha_{11}E_\beta + \mu_{11}H_\beta, \end{aligned} \tag{7}$$

$$\varepsilon_{3\beta} = u_{3,\beta}/2, \quad E_\beta = -\varphi_{,\beta}, \quad H_\beta = -\psi_{,\beta}, \tag{8}$$

$$\sigma_{3\beta,\beta} = \rho\ddot{u}_3, \quad D_{\beta,\beta} = 0, \quad B_{\beta,\beta} = 0. \tag{9}$$

Substituting Eq. (7) with Eq. (8) into Eq. (9), one obtains

$$\begin{aligned} c_{44}\nabla^2 u_3 + e_{15}\nabla^2\varphi + h_{15}\nabla^2\psi &= \rho\ddot{u}_3, \\ e_{15}\nabla^2 u_3 - \kappa_{11}\nabla^2\varphi - \alpha_{11}\nabla^2\psi &= 0, \\ h_{15}\nabla^2 u_3 - \alpha_{11}\nabla^2\varphi - \mu_{11}\nabla^2\psi &= 0 \end{aligned} \tag{10}$$

which is a set of the governing differential equations for u_3, φ and ψ, and $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ is the two-dimensional Laplacian operator.

By setting to zero the coefficients h_{15} and α_{11} in Eqs. (7) and (10), the constitutive relations and governing equations of transversely isotropic PE materials under anti-plane deformation are obtained as follows:

$$\begin{aligned} \sigma^e_{3\beta} &= c^e_{44}u^e_{3,\beta} + e_{15}\varphi^e_{,\beta}, \\ D^e_\beta &= e_{15}u^e_{3,\beta} - \kappa^e_{11}\varphi^e_{,\beta}, \\ B^e_\beta &= -\mu^e_{11}\psi^e_{,\beta}, \end{aligned} \tag{11}$$

$$\begin{aligned} c^e_{44}\nabla^2 u^e_3 + e_{15}\nabla^2\varphi^e &= \rho^e\ddot{u}^e_3, \\ e_{15}\nabla^2 u^e_3 - \kappa^e_{11}\nabla^2\varphi^e &= 0, \\ \nabla^2\psi^e &= 0. \end{aligned} \tag{12}$$

Similarly, the constitutive relations and governing equations of transversely isotropic PM materials under anti-plane deformation are:

$$\begin{aligned} \sigma^m_{3\beta} &= c^m_{44} u^m_{3,\beta} + h_{15} \psi^m_{,\beta}, \\ D^m_\beta &= -\kappa^m_{11} \varphi^m_{,\beta}, \\ B^m_\beta &= h_{15} u^m_{3,\beta} - \mu^m_{11} \psi^m_{,\beta}, \end{aligned} \tag{13}$$

$$\begin{aligned} c^m_{44} \nabla^2 u^m_3 + h_{15} \nabla^2 \psi^m &= \rho^m \ddot{u}^m_3, \\ h_{15} \nabla^2 u^m_3 - \mu^m_{11} \nabla^2 \psi^m &= 0, \\ \nabla^2 \varphi^m &= 0. \end{aligned} \tag{14}$$

In Eqs. (11)–(14), the superscripts "e" and "m" in the field quantities and the material constants indicate that they correspond to PE and PM media, respectively.

3. Statement of the problem

The layered structure and the coordinate system are illustrated in Fig. 1. Two kinds of combinations will be considered: one is a PE half-space carrying a PM layer and the other is a PM half-space covered by a PE layer. Both materials are hexagonal (6 mm) crystals (transversely isotropic materials). The x_1x_2-plane is an isotropic plane of both materials. The SH wave propagating in the structure along the x_1-axis possesses only one component of mechanical displacement u_3 accompanied by the electric potential φ and the magnetic potential ψ. They are governed respectively by Eq. (12) for the PE medium and Eq. (14) for the PM medium.

The solution of the SH wave propagation must satisfy the boundary conditions on the layer surface and the continuity conditions along the interface between the layer and the half-space. At the interface $x_2 = 0$, the mechanical displacements, electric potentials and magnetic potentials as well as the normal components of the stress, electric displacement and magnetic induction are continuous, i.e.

$$u^e_3 = u^m_3, \quad \varphi^e = \varphi^m, \quad \psi^e = \psi^m, \quad \sigma^e_{32} = \sigma^m_{32}, \quad D^e_2 = D^m_2, \quad B^e_2 = B^m_2. \tag{15}$$

The surface of the layer is traction free, electrically open and magnetic closed. This requires

$$\sigma^m_{32}(x_1, -h) = 0, \quad D^m_2(x_1, -h) = 0, \quad B^m_2(x_1, -h) = 0 \tag{16}$$

when the layer is PE and

$$\sigma^e_{32}(x_1, -h) = 0, \quad D^e_2(x_1, -h) = 0, \quad B^e_2(x_1, -h) = 0 \tag{17}$$

when the layer is PE.

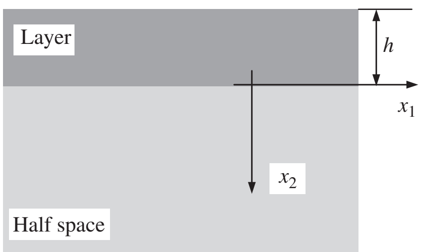

Fig. 1. A schematic of layered piezoelectric–piezomagnetic structure.

4. Dispersion relations

First of all, we consider the propagation of the SH wave in the structure consisting of the PM layer and PE half-space. The solution of Eqs. (12) and (14) can be expressed by the following form:

$$\begin{aligned} u_3^\beta(x_1,x_2,t) &= \tilde{u}_3^\beta(x_2)\cos k(x_1-ct),\\ \varphi^\beta(x_1,x_2,t) &= \tilde{\varphi}^\beta(x_2)\cos k(x_1-ct), \quad (\beta=e,m),\\ \psi^\beta(x_1,x_2,t) &= \tilde{\psi}^\beta(x_2)\cos k(x_1-ct), \end{aligned} \tag{18}$$

where k is the wavenumber, c stands for the phase velocity of the SH wave, $\tilde{u}_3^\beta(x_2)$, $\tilde{\varphi}^\beta(x_2)$ and $\tilde{\psi}^\beta(x_2)$ are undetermined functions. Substitution of Eq. (18) into Eqs. (12) and (14) results in

$$\begin{aligned} &\frac{\mathrm{d}^2\tilde{u}_3^e}{\mathrm{d}x_2^2} - k[1-(c/c_{\rm sh}^e)^2]\tilde{u}_3^e = 0,\\ &\frac{\mathrm{d}^2\tilde{\varphi}^e}{\mathrm{d}x_2^2} - k^2\tilde{\varphi}^e = \frac{e_{15}}{\kappa_{11}^e}\left(\frac{\mathrm{d}^2\tilde{u}_3^e}{\mathrm{d}x_2^2} - k^2\tilde{u}_3^e\right),\\ &\frac{\mathrm{d}^2\tilde{\psi}^e}{\mathrm{d}x_2^2} - k^2\tilde{\psi}^e = 0 \end{aligned} \tag{19}$$

and

$$\begin{aligned} &\frac{\mathrm{d}^2\tilde{u}_3^m}{\mathrm{d}x_2^2} - k^2[1-(c/c_{\rm sh}^m)^2]\tilde{u}_3^m = 0,\\ &\frac{\mathrm{d}^2\tilde{\varphi}^m}{\mathrm{d}x_2^2} - k^2\tilde{\varphi}^m = 0,\\ &\frac{\mathrm{d}^2\tilde{\psi}^m}{\mathrm{d}x_2^2} - k^2\tilde{\psi}^m = \frac{h_{15}}{\mu_{11}^m}\left(\frac{\mathrm{d}^2\tilde{u}_3^m}{\mathrm{d}x_2^2} - k^2\tilde{u}_3^m\right) \end{aligned} \tag{20}$$

with $c_{\rm sh}^\beta = \sqrt{\bar{c}_{44}^\beta/\rho^\beta}$, $\beta = e, m$. $c_{\rm sh}^e$ and $c_{\rm sh}^m$ are the velocities of the bulk shear wave in the PE and PM media, respectively; $\bar{c}_{44}^e = c_{44}^e + e_{15}^2/\kappa_{11}^e$ and $\bar{c}_{44}^m = c_{44}^m + h_{15}^2/\mu_{11}^m$ are the elastic constants with the PE and PM effects accounted for, respectively.

For the Love wave considered in this case, there exists the fact that $c_{\rm sh}^m < c < c_{\rm sh}^e$. So, the solutions of Eqs. (19) and (20) are:

$$\begin{aligned} \tilde{u}_3^e(x_2) &= A_1 \mathrm{e}^{-k^e x_2},\\ \tilde{\varphi}^e(x_2) &= A_2\mathrm{e}^{-kx_2} + \frac{e_{15}}{\kappa_{11}^e}\tilde{u}_3^e,\\ \psi^e(x_2) &= A_3\mathrm{e}^{-kx_2}, \end{aligned} \tag{21}$$

$$\begin{aligned} \tilde{u}_3^m(x_2) &= A_4\cos(k^m x_2) + A_7\sin(k^m x_2),\\ \tilde{\varphi}^m(x_2) &= A_5\cosh(kx_2) + A_8\sinh(kx_2),\\ \psi^m(x_2) &= A_6\cosh(kx_2) + A_9\sinh(kx_2) + \frac{h_{15}}{\mu_{11}^m}\tilde{u}_3^m, \end{aligned} \tag{22}$$

where $k^\beta = k\lambda^\beta$, $\lambda^e = \sqrt{1-(c/c_{\rm sh}^e)^2}$, $\lambda^m = \sqrt{(c/c_{\rm sh}^m)^2 - 1}$, A_i $(i = 1\text{–}9)$ are undetermined constants. For the solution of the PE half-space, we have used the conditions that the mechanical displacement u_3, electrical potential φ and magnetic potential ψ vanish for $x_2 = +\infty$. From Eqs. (18), (21) and (22), we have

$$
\begin{aligned}
u_3^e(x_1,x_2,t) &= A_1 \mathrm{e}^{-k^e x_2}\cos k(x_1-ct),\\
\varphi^e(x_1,x_2,t) &= A_2 \mathrm{e}^{-k x_2}\cos k(x_1-ct)+\frac{e_{15}}{\kappa_{11}^e}u_3^e,\\
\psi^e(x_1,x_2,t) &= A_3 \mathrm{e}^{-k x_2}\cos k(x_1-ct),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
u_3^m(x_1,x_2,t) &= [A_4\cos(k^m x_2)+A_7\sin(k^m x_2)]\cos k(x_1-ct),\\
\varphi^m(x_1,x_2,t) &= [A_5\cosh(kx_2)+A_8\sinh(kx_2)]\cos k(x_1-ct),\\
\psi^m(x_1,x_2,t) &= [A_6\cosh(kx_2)+A_9\sinh(kx_2)]\cos k(x_1-ct)+\frac{h_{15}}{\mu_{11}^m}u_3^m.
\end{aligned}
\tag{24}
$$

Substituting Eqs. (21) and (22) into Eqs. (11) and (13), respectively, one obtains

$$
\begin{aligned}
\sigma_{32}^e &= -(\bar{c}_{44}^e k^e A_1 \mathrm{e}^{-k^e x_2}+e_{15}kA_2\mathrm{e}^{-kx_2})\cos k(x_1-ct),\\
D_2^e &= \kappa_{11}^e kA_2\mathrm{e}^{-kx_2}\cos k(x_1-ct),\\
B_2^e &= \mu_{11}^e kA_3\mathrm{e}^{-kx_2}\cos k(x_1-ct),
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\sigma_{32}^m &= \{\bar{c}_{44}^m k^m[-A_4\sin(k^m x_2)+A_7\cos(k^m x_2)]\\
&\quad +h_{15}k[A_6\sinh(k^m x_2)+A_9\cosh(k^m x_2)]\}\cos k(x_1-ct),\\
D_2^m &= -\kappa_{11}^m k[A_5\sinh(kx_2)+A_8\cosh(kx_2)]\cos k(x_1-ct),\\
B_2^m &= -\mu_{11}^m k[A_6\sinh(kx_2)+A_9\cosh(kx_2)]\cos k(x_1-ct).
\end{aligned}
\tag{26}
$$

Making using of Eqs. (23)–(26) as well as the interface continuity conditions (15) and the boundary conditions (16), we get

$$
\begin{aligned}
&A_4=A_1,\ A_5=A_2+A_1e_{15}/\kappa_{11}^e,\ A_6+A_4h_{15}/\kappa_{11}^m=A_3,\\
&\bar{c}_{44}^m k^m A_7+h_{15}kA_9=-\bar{c}_{44}^e k^e A_1-e_{15}kA_2,\\
&-\kappa_{11}^m A_8=\kappa_{11}^e A_2,\ -\mu_{11}^m A_9=\mu_{11}^e A_3,\\
&\bar{c}_{44}^m k^m[A_4\sin(k^m h)+A_7\cos(k^m h)]\\
&+h_{15}k[A_9\cosh(kh)-A_6\sinh(kh)]=0,\\
&A_8\cosh(kh)-A_5\sinh(kh)=0,\\
&A_9\cosh(kh)-A_6\sinh(kh)=0,
\end{aligned}
\tag{27}
$$

which is a system of the homogeneous linear algebraic equations for A_i ($i=1$–9). For nontrivial solutions of A_i, the determinate of the coefficient matrix must vanish, which leads to

$$
\left[1+\frac{\mu_{11}^m}{\mu_{11}^e}\tanh(kh)\right]\left[\bar{c}_{44}^e\lambda^e-\frac{e_{15}^2}{\kappa_{11}^e}\frac{\tanh(kh)}{\kappa_{11}^e/\kappa_{11}^m+\tanh(kh)}-\bar{c}_{44}^m\lambda^m\tan(k^m h)\right]-\frac{h_{15}^2}{\mu_{11}^m}\tanh(kh)=0. \tag{28}
$$

Eq. (28) is the dispersion relation that determines the wave speed c.

When the layered structure is composed of a PE layer and a PM half-space, the dispersion relation can be obtained in a same procedure as follows:

$$
\left[1+\frac{\kappa_{11}^e}{\kappa_{11}^m}\tanh(kh)\right]\left[\bar{c}_{44}^m\lambda^m-\frac{h_{15}^2}{\mu_{11}^m}\frac{\tanh(kh)}{\mu_{11}^m/\mu_{11}^e+\tanh(kh)}-\bar{c}_{44}^e\lambda^e\tan(k^e h)\right]-\frac{e_{15}^2}{\kappa_{11}^e}\tanh(kh)=0. \tag{29}
$$

It should be mentioned that the wave speed c is less than c_{sh}^m but larger than c_{sh}^e. Eqs. (28) and (29) are the transcendental equations with respect to c and k. The relation between c and k is only determined by using numerical methods.

From Eq. (28), we can obtain the results of two special cases reported in the literature straightforwardly. For example, when h_{15}^2 vanishes and κ_{11}^m approaches infinity, Eq. (28) reduces to

$$\left(c_{44}^H + \frac{e_{15}^2}{\kappa_{11}^H}\right)\sqrt{1-\left(\frac{c}{c_{sh}^H}\right)^2} - c_{44}^L\sqrt{\left(\frac{c}{c_{sh}^L}\right)^2 - 1}\tan\left(kh\sqrt{\left(\frac{c}{c_{sh}^L}\right)^2-1}\right) = \frac{e_{15}^2}{\kappa_{11}^H}, \tag{30}$$

where $c_{sh}^H = \sqrt{(c_{44}^H + e_{15}^2/\kappa_{11}^H)/\rho^H}$ and $c_{sh}^L = \sqrt{c_{44}^L/\rho^L}$. The superscripts "$L$" and "$H$" denotes the material constants and the bulk shear wave velocity corresponding to the layer and the half-space medium, respectively. Eq. (30) is the dispersion relation for quasi-static PE Love waves in a PE half-space carrying an elastic metal layer given in Ref. [31]. When $e_{15}^2 = 0$, Eq. (30) degenerates into

$$c_{44}^H\sqrt{1-\left(\frac{c}{c_{sh}^H}\right)^2} - c_{44}^L\sqrt{\left(\frac{c}{c_{sh}^L}\right)^2 - 1}\tan\left(kh\sqrt{\left(\frac{c}{c_{sh}^L}\right)^2-1}\right) = 0, \tag{31}$$

which is the well-known dispersion equation for a purely elastic layered half-space [32].

5. Numerical examples and discussions

For the propagation of waves in the layered structures, the important thing is to know phase and group velocities for some applications in acoustic devices, such as filters, delay lines and sensors [33,34]. According to work by Zakharenko [34], the group velocity can be calculated by the following formula:

$$c_g = \frac{d\omega}{dk} = c + kh\frac{dc}{d(kh)}, \tag{32}$$

where ω is the circular frequency.

Based on the dispersion equations (28) and (29) as well as formula (32), numerical examples are provided to illustrate the dispersion behaviors of Love waves. The material properties used in numerical calculation are listed in Table 1, where Terfenol-D and $CoFe_2O_4$ are PM and the others are PE. Also, the electromechanical coupling coefficient $K_e = e_{15}/\sqrt{\bar{c}_{44}^e \kappa_{11}^e}$ for the PE materials and magnetomechanical coupling coefficient $K_m = h_{15}/\sqrt{\bar{c}_{44}^m \mu_{11}^m}$ for the PM materials are given in Table 1. We assume that magnetic permeabilities of PZT-4 and PZT-7 to be the same as $BaTiO_3$. Our numerical results have shown that this assumption is reasonable. In addition, magnetic permeability μ_{11} of $CoFe_2O_4$ is taken to be the same as μ_{33}, which is different from Ref. [23]. In Ref. [23], the negative value of the magnetic permeability μ_{11} for $CoFe_2O_4$ is used. Pan [18] and Liu and Chue [33] have explained why the negative μ_{11} is unreasonable, respectively.

Table 1
The properties of piezoelectric and piezomagnetic materials

Material constants	$BaTiO_3$ [35]	PZT-4 [35]	PZT-7 [35]	$CoFe_2O_4$ [23]	Terfenol-D [36]
c_{44} (10^9 N/m^2)	44	25.6	25	45.3	5.99
ρ (10^3 kg/m^3)	5.7	7.5	7.8	5.3	9.23[a]
κ_{11} (10^{-9} C^2/Nm)	9.82	6.46	17.1	0.08	15.04×10^7
μ_{11} (10^{-6} Ns^2/C^2)	5[b]	5	5	157	3.97
e_{15} (C/m^2)	11.4	12.7	13.5	0	0
h_{15} (N/Am)	0	0	0	550	167.66
$\bar{c}_{44}$ (10^9 N/m^2)	57.17	50.55	35.57	48.33	13.06
c_{sh} (10^3 m/s)	3.167	2.596	2.138	3.02	1.189
K_e	0.4809	0.7027	0.5467	–	–
K_m	–	–	–	0.2020	0.7360

[a]Cited from Ref. [15].
[b]Magnetic permeability μ_{11}(5×10^{-6} Ns^2/C^2) of $BaTiO_3$ is used in all the literature available.

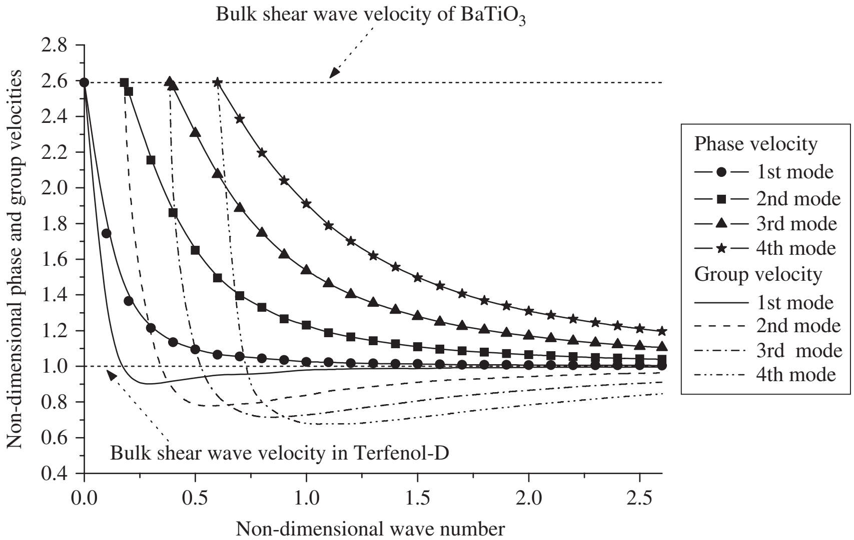

Fig. 2. Non-dimensional phase and group velocities for a $BaTiO_3$ half-space covered by a Terfenol-D layer for the first four modes.

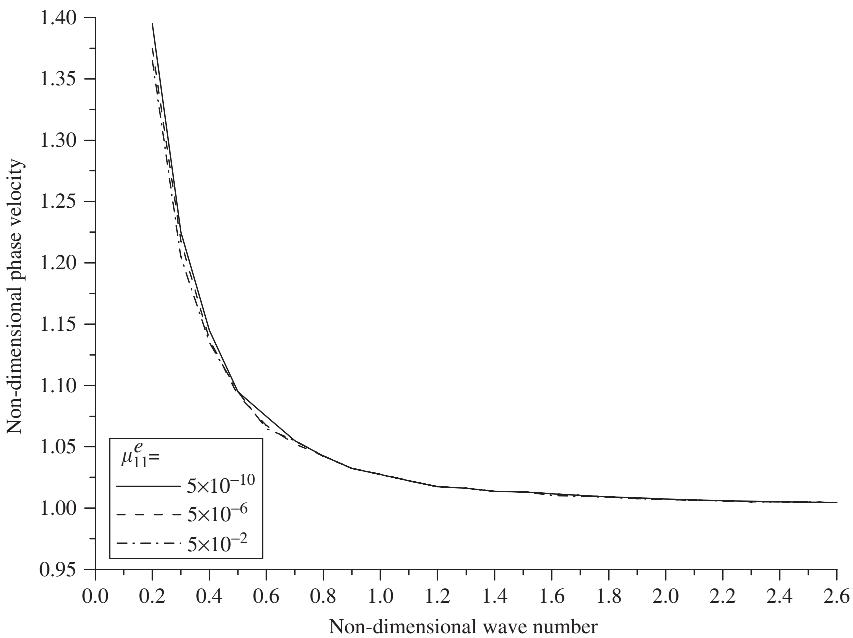

Fig. 3. Dispersive curves of a $BaTiO_3$ half-space covered by a Terfenol-D layer for the lowest mode when magnetic permeability of $BaTiO_3$ are taken as different values.

The dimensionless wave is taken as $\tilde{k} = kh/2\pi$. The dimensionless phase velocity and phase velocity are $\tilde{c} = c/c_{\rm sh}^{\beta}$ and $\tilde{c}_g = c_g/c_{\rm sh}^{\beta}$, where $c_{\rm sh}^{\beta}$ is the bulk shear wave velocity of the layer medium. The dispersive curves for the structure consisting of the $BaTiO_3$ half-space covered by the Terfenol-D layer are plotted in Fig. 2. It can be seen that the dimensionless phase and group velocities initiate at the ratio of the shear wave velocity $c_{\rm sh}^{e}$ of $BaTiO_3$ to the shear wave velocity of Terfenol-D, namely $c_{\rm sh}^{e}/c_{\rm sh}^{m} = 2.589$, and approach unit with the increase of the wavenumber. This means that the phase and group velocities start from the bulk shear wave velocity in $BaTiO_3$ and tend to the bulk shear wave velocity in Terfenol-D. In addition, it is found from Fig. 2 that the phase velocity for each mode monotonously decreases with the increase of the wavenumber. However,

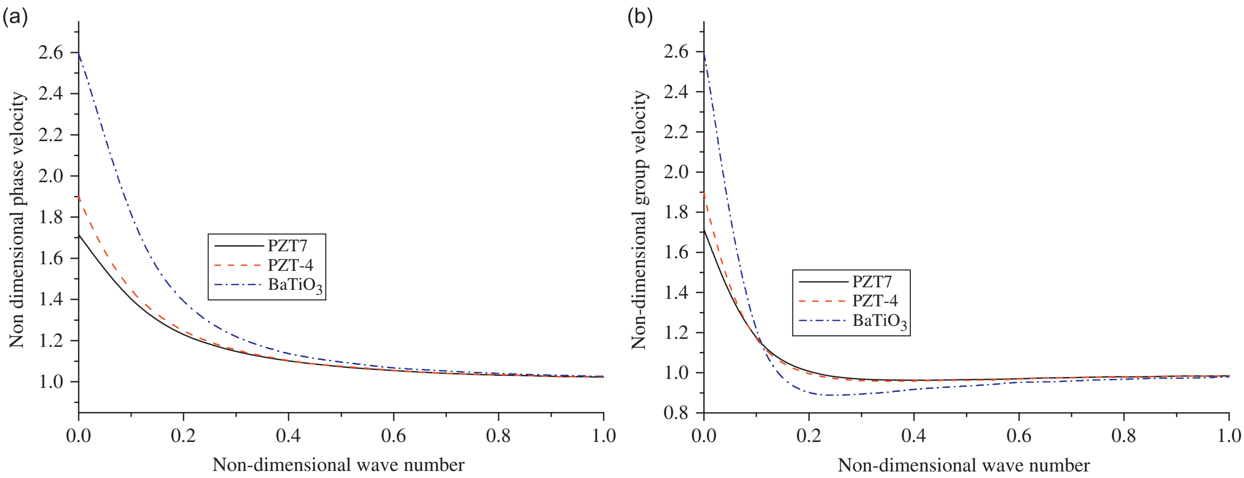

Fig. 4. Non-dimensional phase and group velocities of a Terfenol-D layer on three different types of piezoelectric half-space, namely $BaTiO_3$, PZT-4 and PZT-7 for the lowest mode.

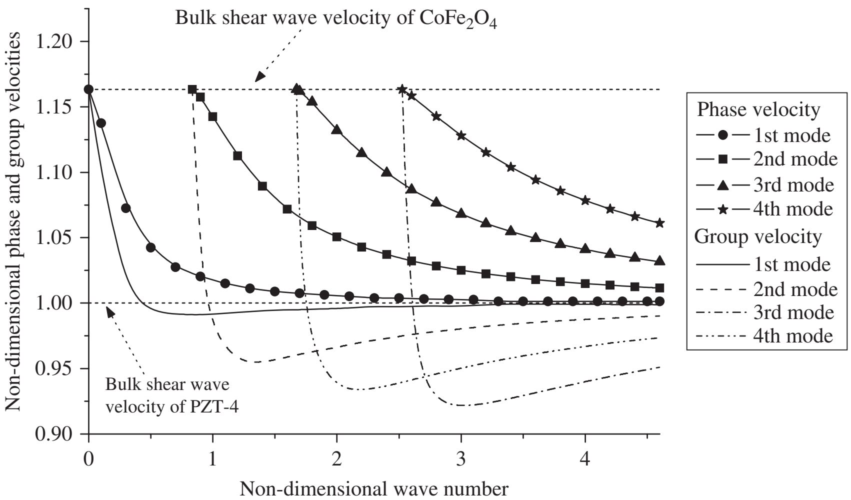

Fig. 5. Non-dimensional phase and group velocities for a $CoFe_2O_4$ half-space covered by a PZT-layer for the first four modes.

the group velocity for each mode decreases from the bulk shear wave velocity of $BaTiO_3$ to its minimum value, and then increases toward the bulk shear wave velocity of Terfenol-D.

Next, we examine the influence of the magnetic permeability of PE materials on the dispersion behaviors. This is very significant because the magnetic permeability of all PE materials is unknown except $BaTiO_3$. The variations of $\tilde{c}$ with $\tilde{k}$ are plotted in Fig. 3 for the lowest mode when the magnetic permeabilities of $BaTiO_3$ are taken as different values. It is clearly shown in Fig. 3 that the influence of the magnetic permeability of $BaTiO_3$ on the dispersion characteristic is rather slight. This influence can be neglected. Accordingly, the magnetic permeability of PZT-4 and PZT-7 are also taken as $5.0 \times 10^{-6}\,\mathrm{NS^2/C^2}$ in the following numerical calculation.

It is anticipated that the material properties of the PE half-spaces would affect the phase and group velocities. In Fig. 4(a) and (b), the phase and group velocities for the lowest mode are plotted, respectively, when three different types of PE half-spaces, namely $BaTiO_3$, PZT-4 and PZT-7, are covered by a Terfenol-D layer. Fig. 4(a) shows that the phase velocities for three different combinations have the relation: $c_{\text{Terfenol-D/BaTiO}_3} > c_{\text{Terfenol-D/PZT}-4} > c_{\text{Terfenol-D/PZT}-7}$ when the wavenumber is smaller. This results from the difference between the bulk shear wave velocities of PE materials (see Table 1). It is seen from Fig. 4(b) that the group velocities for three different combinations have the same relation as the phase velocities, namely $c_{g(\text{Terfenol-D/BaTiO}_3)} > c_{g(\text{Terfenol-D/PZT-4})} > c_{g(\text{Terfenol-D/PZT-7})}$ when $\tilde{k} < 0.1094$. However, when $\tilde{k} < 0.1094$, the group velocity for Terfenol-D/$BaTiO_3$ combination is smaller than the group velocities for Terfenol-D/PZT-4 and Terfenol-D/PZT-7 combinations.

For the case where a $CoFe_2O_4$ half-space is covered by a PZT-4 layer, the phase and group velocities are calculated by using Eqs. (29) and(32). The variations of the dimensionless phase and group velocities with dimensionless wavenumbers are plotted in Fig. 5 for the first four modes. The dispersive characteristic of this case is similar to that of a $BaTiO_3$ half-space carrying a Terfenol-D layer.

6. Conclusions

The propagation of Love waves in the layered PE/PM structures is investigated in this paper. The explicit dispersion equations are derived and the numerical simulations are carried out. From the obtained numerical results, the following conclusions can be drawn.

(1) The magnetic permeabilities of PE materials have a very slight influence. This influence is neglectable when investigating dynamic behavior of wave propagation in PE/PM materials.
(2) The phase and group velocities start from the bulk shear wave velocity of a half-space medium and tend to the bulk shear wave velocity of a layer material as the wavenumber increases.
(3) For the structure consisting of a PM layer and a PE half-space, the properties of the PE have a significant influence on the phase and group velocities at lower wavenumber. The larger the bulk shear wave velocity of the PE material, the higher the phase and group velocities of the wave propagation in the corresponding composite structure at lower wavenumber.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 10472074).

References

[1] J. van Suchtelen, Product properties: a new application of composite materials, *Philips Research Reports* 27 (1972) 28–37.
[2] J. van Denboomgaard, et al., Insitu grown eutectic magnetoelectric composite-material. 1. Composition and unidirectional solidification, *Journal of Materials Science* 9 (1974) 1705–1709.
[3] A.M.J.G. van Run, D.R. Terrell, J.H. Scholing, Insitu grown eutectic magnetoelectric composite-material. 2. Physical-properties, *Journal of Materials Science* 9 (1974) 1710–1714.
[4] J. van Denboomgaard, A.M.J.G. Van Run, J. Van Suchtelen, Piezoelectric–piezomagnetic composites with magnetoelectric effect, *Ferroelectrics* 14 (1976) 727–728.
[5] G. Harshe, J.P. Dougherty, R.E. Newnham, Theoretical modelling of multilayer magnetoelectric composites, *International Journal of Applied Electromagnetics in Materials* 4 (1993) 145–159.
[6] G. Harshe, J.P. Dougherty, R.E. Newnham, Theoretical modelling of 3-0/0-3 magnetoelectric composites, *International Journal of Applied Electromagnetics in Materials* 4 (1993) 161–171.
[7] M. Avellaneda, G. Harshe, Magnetoelectric effect in piezoelectric/magnetostrictive multilayer (2–2) composites, *Journal of Intelligent Material Systems and Structures* 5 (1994) 501–513.
[8] C.W. Nan, Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases, *Physical Review B* 50 (1994) 6082–6088.
[9] Y. Benveniste, Magnetoelectric effect in fibrous composites with piezoelectric and piezomagnetic phases, *Physical Review B* 51 (1995) 16,424–16,427.
[10] J.Y. Li, M.L. Dunn, Micromechanics of magnetoelectroelastic composite materials: average fields and effective behavior, *Journal of Intelligent Material Systems and Structures* 9 (1998) 404–416.
[11] J.Y. Li, Magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials, *International Journal of Engineering Science* 38 (2000) 1993–2011.
[12] J. Ryu, A.V. Carazo, K. Uchino, H.E. Kim, Magnetoelectric properties in piezoelectric and magnetostrictive laminate composites, *Japanese Journal of Applied Physics, Part 1* 40 (2001) 4948–4951.
[13] G. Srinivasan, V.M. Laletsin, R. Hayes, N. Puddubnaya, E.T. Rasmussen, D.J. Fekel, Giant magnetoelectric effects in layered composites of nickel zinc ferrite and lead zirconate titanate, *Solid State Communications* 124 (2002) 373–378.

[14] M.I. Bichurin, D.A. Filippov, V.M. Petrov, V.M. Laletsin, N. Paddubnaya, Resonance magnetoelectric effects in layered magnetostrictive–piezoelectric composites, *Physical Review B* 68 (2003) 132408(1–4).
[15] S.X. Dong, J.F. Li, D. Viehland, Characterization of magnetoelectric laminate composites operated in longitudinal-transverse and transverse-transverse modes, *Journal of Applied Physics* 95 (2004) 2625–2630.
[16] M. Fiebig, Revival of the magnetoelectric effect, *Journal of Physics D-Applied Physics* 38 (2005) R123–R152.
[17] J.X. Liu, X.L. Liu, Y.B. Zhao, Green's functions for anisotropic magnetoelectroelastic solids with an elliptical cavity or a crack, *International Journal of Engineering Science* 39 (2001) 1405–1418.
[18] E. Pan, Three-dimensional Green's functions in anisotropic magneto-electro-elastic bimaterials, *Journal of Applied and Physics (ZAMP)* 53 (2002) 815–838.
[19] E. Pan, Exact solution for simply supported and multilayered magneto-electro-elastic plates, *Journal of Applied Mechanics* 68 (2001) 608–618.
[20] E. Pan, P.R. Heyliger, Free vibrations of simply supported and multilayered magneto-electro-elastic plates, *Journal of Sound and Vibration* 252 (2002) 429–442.
[21] E. Pan, P.R. Heyliger, Exact solutions for magneto-electro-elastic laminates in cylindrical bending, *International Journal of Solids and Structures* 40 (2003) 6859–6876.
[22] P.R. Heyliger, E. Pan, Static fields in magnetoelectroelastic laminates, *AIAA Journal* 42 (2004) 1435–1443.
[23] F. Ramirez, P.R. Heyliger, E. Pan, Free vibration response of two-dimensional magneto-electro-elastic laminated plates, *Journal of Sound and Vibration* 292 (2006) 626–644.
[24] A.R. Annigeri, N. Ganesan, S. Swarnamani, Free vibrations of simply supported layered and multiphase magneto-electro-elastic cylindrical shells, *Smart Materials and Structures* 15 (2006) 459–467.
[25] A.R. Annigeri, N. Ganesan, S. Swarnamani, Free vibrations of clamped–clamped magneto-electro-elastic cylindrical shells, *Journal of Sound and Vibration* 292 (2006) 300–314.
[26] V.I. Alshits, A.N. DarinskiI, J. Lothe, On the existence of surface-waves in half-infinite anisotropic elastic media with piezoelectric and piezomagnetic properties, *Wave Motion* 16 (1992) 265–283.
[27] A.K. Soh, J.X. Liu, Interfacial shear horizontal waves in a piezoelectric–piezomagnetic bi-material, *Philosophical Magazine Letters* 81 (2006) 31–35.
[28] J.Y. Chen, E. Pan, H.L. Chen, Wave propagation in magneto-electro-elastic multilayered plates, *International Journal of Solids and Structures* 44 (2007) 1073–1085.
[29] Z.G. Zhou, L.Z. Wu, B. Wang, The dynamic behavior of two collinear interface cracks in magneto-electro-elastic materials, *European Journal of Mechanics A-Solids* 24 (2005) 253–262.
[30] Z.G. Zhou, B. Wang, The scattering of the harmonic anti-plane shear stress waves by two collinear interface cracks between two dissimilar functionally graded piezoelectric/piezomagnetic material half-infinite planes dynamic loading, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 220 (2006) 137–148.
[31] R.G. Curtis, M. Redwood, Transverse surface waves on a piezoelectric material carrying a metal layer of finite thickness, *Journal of Applied Physics* 44 (1973) 2002–2007.
[32] K.F. Graff, *Wave Motion in Elastic Solids*, Dover, New York, 1991.
[33] A.A. Zakharenko, Dispersive Rayleigh type waves in layered systems consisting of piezoelectric crystals bismuth silicate and bismuth germinate, *Acta Acustica United with Acustica* 91 (2005) 708–715.
[34] A.A. Zakharenko, Analytical studying the group velocity of three-partial Love (type) waves in both isotropic and anisotropic media, *Nondestructive Testing and Evaluation* 20 (2005) 237–254.
[35] S.N. Jiang, Q. Jiang, X.F. Li, S.H. Guo, H.G. Zhou, J.S. Yang, Piezoelectromagnetic waves in a ceramic plate between two ceramic half-spaces, *International Journal of Solids and Structures* 43 (2006) 5799–5810.
[36] T.J.C. Liu, C.H. Chue, On the singularities in a bimaterial magneto-electro-elastic composite wedge under antiplane deformation, *Composite Structures* 72 (2006) 254–265.