Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 315 (2008) 685–699
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Dynamics of a vertical riser with weak structural nonlinearity excited by wakes

Marko Keber, Marian Wiercigroch*

Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen AB24 3UE, Scotland, UK

Accepted 17 March 2008
The peer review of this article was organized by the Guest Editor
Available online 2 May 2008

Abstract

In this paper we investigate the effect of a weak structural nonlinearity on the dynamical behaviour of a vertical offshore riser subjected to vortex-induced vibration (VIV). Coupling of the riser dynamics with the flow of the surrounding fluid is achieved by attaching a wake oscillator to a reduced model of the structure, which is obtained through the application of the invariant manifold technique for the derivation of nonlinear normal modes.

By comparing the free responses of the linear and the nonlinear structure, it was found that the structural nonlinearity has a stiffening effect on the oscillation of the riser, which becomes more pronounced when the internal flow is incorporated into the model. Consequently, in the coupled system, the response is considerably modified for the structure as well as for the fluid variable.

1. Introduction

Marine structures can exhibit different dynamical behaviours due to variable ambient excitation they are subjected to. Offshore risers, a family of such structures that are normally fully submerged, experience primarily vortex-induced vibration (VIV) caused by sea currents. When velocities of these currents are large enough, the difference of pressures acting on the cylindrical structure of a riser creates the condition where vortices begin shedding downstream. This in turn leads to vibration of the flexible riser. Bishop and Hassan [1] observed that an interaction between oscillations of a slender structure and the wake created behind its body takes place, and this can lead to synchronisation of the vortex-shedding frequency with a natural frequency of the riser when the two are sufficiently close. The most undesirable consequence of this lock-in phenomenon is the resulting resonant motion that can induce elevated stresses in the structure, thus shortening the riser's operational life. In order to prolong their use, vortex-suppression devices are being used by the oil and gas industry, which, despite being effective under certain conditions, can prove to be a very costly solution. Furthermore, as Trim et al. [2] showed experimentally for helical strakes, the ability of such devices to suppress motion is dependent on their geometry as well as on the configuration of flow at the part of the riser

*Corresponding author. Tel.: +44 1224 272509; fax: +44 1224 272497.
E-mail address: m.wiercigroch@abdn.ac.uk (M. Wiercigroch).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2008.03.023

that is bare. Results also indicate that best performance is obtained if the vortex-suppression devices are attached along the entire length, which makes the construction and installation of risers more complicated. Therefore, it would be highly desirable to avoid using such devices altogether. This could be achieved through a better understanding of the interactions between the fluid and the structure, especially with respect to the responses of the latter. Additionally, a deeper insight into the dynamics would enable improved predictions of fatigue life, which for offshore risers is largely dominated by vortex-induced vibrations [3].

The purpose of the research presented in this paper was to examine how the behaviour of the coupled system changes in the lock-in when a weak structural nonlinearity is introduced into the model of a vertical riser. Typically, in investigations of fluid–structure interactions applied to offshore risers, nonlinearities are restricted only to the fluid forces, while the structure is modelled as a perfectly linear system. Assuming that the tensile force applied at the top of the riser is very large and therefore appears to hide certain characteristics of nonlinear behaviour, pertaining especially to the modal shape as well as the amplitude and natural frequency [4], approximation of the response with linear normal modes would be perfectly valid. However, Lucor et al. [5], who studied vibration of a tensioned riser, noticed that the motion in the non-uniform flow cannot be captured by just one linear mode due to the mixture of standing and travelling waves in the oscillation of the riser. Similarly, results of experiments with a tensioned riser in a stepped current suggest a multimodal response with significant contributions from several indirectly excited modes that control its dynamics even in lock-in at a natural frequency of the dominant mode [6]. This indicates a nonlinear nature of the structural dynamics that is affected by the nonlinearity in the forcing from the fluid. Therefore, a more detailed description of a the structure is needed. On the other hand, a more precise model will increase the complexity of the entire system, mostly due to the interactions between the structural and fluid nonlinearities. Inevitably, justification for the longer computational time in terms of the contribution of additional nonlinear relation needs to be made in this case.

Often analytical and numerical analyses take advantage of the fact that, when synchronisation occurs, the structure oscillates at a resonant frequency, enabling that only a reduced model of the system is considered in modal space by equating the respective frequencies [7]. Modal decomposition through modal analysis is performed for the structural part of the system, and for the problem at hand, nonlinear normal modes might prove to be a valuable tool. Like the linear modes, the nonlinear ones represent the dynamics of a chosen mode but can accommodate also nonresonant modes as well as connections between them occurring as, for example, internal resonances [8]. Serious study of nonlinear normal modes began in the 1960s with the application firstly to the discrete conservative systems [9]. To date several methods capable of building them have been proposed; among others, techniques based on the harmonic balance and generalised Ritz methods [10], centre manifold theory for the invariant manifold approach proposed by Shaw and Pierre [11], multiple scales techniques [12] for systems with weak nonlinearities, the normal form approach [13]. In contrast to linear theory, the ability to incorporate damping without the need for it to be expressed in the complex domain is common to most of these techniques. They are therefore very suitable for modelling modal dynamics of real structures experiencing travelling waves. However, for the desired characteristics of the motion to be retained in the reduced model, both the structural and fluid parts need to be transferred to a common space, where the final analysis is then carried out.

Sarpkaya [14] and Gabbai and Benaroya [15] give a comprehensive overview of different approaches that have been employed to describe the wake phenomena for fixed as well as flexible cylinders oscillating in lock-in. As shown, some of them are very suitable for transformation to modal space, especially those that specifically formulate the forcing caused by the currents, including not necessarily only contributions from vortex-shedding, but also from other sources like wave motion, to achieve the desired effect on the structure. An example of a relatively simple model for the combined motion of a casing pipe resembling a riser was obtained by You-Gang et al. [16] in this way. Another approach that allows one to build reduced-order models, which account for nonlinearities of the fluid forcing, is based on a wake oscillator governed by the van der Pol-type differential equation [15,17]. A particular feature of this approach is that the equation modelling the change of the lift coefficient is described independently of the structural oscillator. The connection with the latter is introduced into the fluid equation through the forcing term.

Offshore risers experience additional excitation from the flow of the fluid inside the pipe. Since the external and the internal flows act on the structure simultaneously, they should both be considered in the analysis. The work presented here takes into account the influence of internal flow, although some simplifications were made. Details of these are given in the next section where the mathematical model is explained. Further, a reduction procedure for the linear and nonlinear structures as well as the forcing terms is proposed in Section 3. Free responses of the linear and nonlinear systems are obtained through nonlinear normal modes in Section 4, while the forced responses are analysed in Section 5. Finally, a summary with final remarks is given in Section 6.

2. Description of the model

The adopted model for the fluid–structure interactions considered here consists of two parts that are described separately. The structural part, representing a straight but slender, vertically positioned pipe, is modelled as an Euler–Bernoulli beam with flexural stiffness. Although pretension is normally applied in actual applications of risers, an external tensile force is not included in the present model since the aim is only to evaluate the effect of an added nonlinearity. For the model to be as simple as possible, the length-wise mass distribution is also neglected. The structure may move only in the direction normal to flow (Fig. 1 (b)), an assumption that is quite valid when its flowwise displacement is relatively small. For flexible cylinders supported at both ends, experimental investigations show that the cross-flow response dominates over the in-line motion in terms of amplitude [18], therefore having a greater contribution in the fatigue of the riser's material.

The governing differential equation for the continuous structure with purely linear characteristics has the following form:

$$m^* \frac{\partial^2 v(z,t)}{\partial t^2} + EI \frac{\partial^4 v(z,t)}{\partial z^4} + \mu \frac{\partial^2 v(z,t)}{\partial z^2} = f(z,t), \tag{1}$$

where $v(z,t)$ is the lateral displacement of the structure and $f(z,t)$ is the external forcing. The mass parameter m^* includes the mass of the fluid inside the pipe (for a riser with internal flow), the mass of the pipe, and the added mass due to the fluid displacement during oscillation:

$$m^* = \rho_{F_i} A_i + \rho_{\mathrm{ST}} A_o + \rho_{F_o}(A_i + A_o). \tag{2}$$

The coefficient of added mass can be omitted here since the cross-section of the riser is perfectly circular and the surfaces are ideally smooth [19]. An approximation of forcing due to the fluid flowing inside the pipe, which is modelled here as a part of the structural oscillator, neglects internal damping, gravitational, and

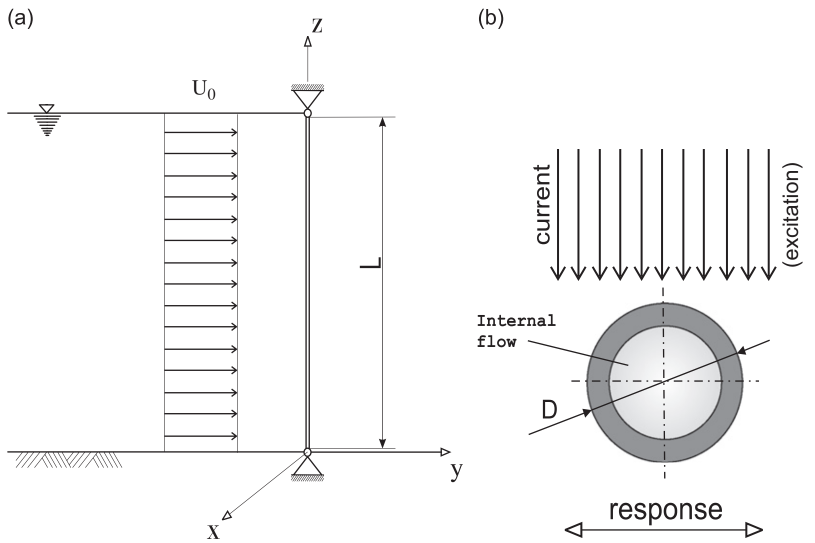

Fig. 1. A physical model of a vertical riser operating in lock-in condition; (a) geometrical configuration of the structure and (b) a cross-section of the riser showing the direction of the response in relation to excitation by a current.

pressurisation effects, but includes the contribution from the centrifugal forces [20]:

$$\mu = \rho_{F_i} A_i U_i^2. \tag{3}$$

Under the prescribed external forcing and the fact that the structure oscillates in lock-in, the shape of the motion will resemble a modal response of a dominant normal mode [18,21]. Since for normal modes the Coriolis forces, arising as a result of the deformation of the riser, do no work [20], they are also left out. However, it should be pointed out that this does not mean their influence on the overall response can be totally dismissed even when the system is gyroscopic conservative. In the present model this is done for the purpose of avoiding a more complex analysis (see for instance the nonlinear analysis of Kuiper and Metrikine [22]), which might obscure certain properties associated with the nonlinearities due to too many parameters affecting each other. Additionally, since the actual spatial distribution of nonlinear displacements is not known at this stage, the authors decided to keep all characteristics the same to enable a straightforward comparison. The only difference between the two studied configurations is expressed in the strain function:

$$\varepsilon(u, v) = \frac{\partial u(z,t)}{\partial z} + \frac{1}{2}\left(\frac{\partial v(z,t)}{\partial z}\right)^2, \tag{4}$$

which couples the transverse $v(z,t)$ and axial $u(z,t)$ displacements. As discussed above, the nonlinear differential equation for lateral motion has a similar form to its linear counterpart, Eq. (1), with one added term:

$$m^* \frac{\partial^2 v(z,t)}{\partial t^2} + EI \frac{\partial^4 v(z,t)}{\partial z^4} + \mu \frac{\partial^2 v(z,t)}{\partial z^2} - EA \frac{\partial}{\partial z}\left(\varepsilon(u,v)\frac{\partial v(z,t)}{\partial z}\right) = f(z,t), \tag{5}$$

but for a full description this model requires the axial motion to be included as well:

$$m^* \frac{\partial^2 u(z,t)}{\partial t^2} - EA \frac{\partial \varepsilon(u,v)}{\partial z} = 0. \tag{6}$$

The same time-varying forcing $f(z,t)$ from the external flow is imposed on the structures in both examples. The right-hand sides of Eqs. (1) and (5) contain, respectively, external damping as well the lift force, which induces the motion in the cross-flow direction:

$$f(z,t) = f_D(z,t) + f_L(z,t). \tag{7}$$

The hydrodynamic damping is the only source of energy dissipation in the system, since a small structural damping coefficient can be assumed for a long slender riser [23]. In linearised form it is expressed as [21]

$$f_D(z,t) = -\frac{1}{2} C_D \rho_{F_o} D U_e(z) \frac{\partial v(z,t)}{\partial z}, \tag{8}$$

where C_D is the damping coefficient, whose value can be taken as 1.2 in the region of Reynolds numbers for well-developed wakes [24]. Because the incident flow is steady with a constant value $U_e(z) = U_0$ at each point along the riser, as shown in Fig. 1(a), the damping the structure experiences is also the same everywhere.

The magnitude of the lift force depends on the value of the reduced lift coefficient $q(t)$, which describes the behaviour of the wake [25]:

$$f_L(z,t) = \tfrac{1}{4} C_{L_0} \rho_{F_o} D (U_e(z))^2 q(t). \tag{9}$$

The lift coefficient C_{L_0}, assumed to have a constant value of 0.3 [24], is the reference for the reduced lift coefficient $q(t)$ governed by the wake oscillator:

$$\frac{d^2 q(\tau)}{d\tau^2} + \lambda \Omega_n (q(\tau)^2 - 1) \frac{dq(\tau)}{d\tau} + \Omega_n^2 q(\tau) = F_n(\tau), \tag{10}$$

where λ is the van der Pol parameter and Ω_n the nondimensional frequency of vortex-shedding. For the right-hand term $F_n(\tau)$ Facchinetti et al. [25] found that inertial type of coupling best describes the connection between the two oscillators:

$$F_n(\tau) = P\frac{d^2\bar{v}_n(\tau)}{d\tau^2}. \tag{11}$$

Like the left-hand side of Eq. (10), the coupling term $F_n(\tau)$ is also nondimensional, with $\bar{v}_n(\tau)$ being the nondimensional acceleration of structural response.

Fluid models based on empirical observations, like the one used in this work (Eq. (10)), approximate flow behaviour on a plane and by themselves do not account for the spatial correlation between wakes. Expressions in Eqs. (10) and (11) therefore require the structural displacement $v(z,t)$ to be defined only in the temporal domain. Dependence on a single variable is obtained by reducing the response of the continuous structure to that of a single-degree-of-freedom system, which is made possible by transforming the original continuous system to the modal space. Details of this reduction process are to be presented in the following section.

3. The reduction procedure

3.1. Reduction of the structure

The decomposition of the system's response into its modal components is achieved through substitution of the Galerking-type discretisation of transverse $v(z,t)$ and axial $u(z,t)$ displacements [26],

$$v(z,t) = \sum_{i=1}^{N} v_i(t)\tilde{v}_i(z), \quad u(z,t) = \sum_{i=1}^{M} u_i(t)\tilde{u}_i(z), \tag{12}$$

into Eqs. (1), (5), and (6). By invoking the orthogonality property of linear normal modes

$$\int_0^L \tilde{v}_i(t)\tilde{v}_j(t)\,dz = \begin{cases} 0, & i \neq j, \\ \dfrac{L}{2}, & i = j, \end{cases} \tag{13}$$

$$\int_0^L \tilde{u}_i(t)\tilde{u}_j(t)\,dz = \begin{cases} 0, & i \neq j, \\ \dfrac{L}{2}, & i = j, \end{cases} \tag{14}$$

the following governing equation is obtained for the resonant motion of the linear structure in the nth normal mode:

$$\ddot{v}_n(t) + \omega_{T_n}^2 v_n(t) = \hat{f}(t), \tag{15}$$

where the nth transverse natural frequency is denoted as ω_{T_n}. The dynamical characteristics are now described in the temporal domain while the spatial distribution of the response $\tilde{v}_n(z)$ is defined by a harmonic function that satisfies the boundary conditions:

$$v(z=0,t) = 0, \quad v(z=L,t) = 0, \tag{16}$$

$$\frac{d^2 v}{dz^2}(z=0,t) = 0, \quad \frac{d^2 v}{dz^2}(z=L,t) = 0. \tag{17}$$

The nonlinear structure is transformed to the same modal space with the harmonic spatial distributions in the transverse and axial directions—$\tilde{v}_n(z)$ and $\tilde{u}_m(z)$, respectively—forming the basis for its modal response. The resulting governing equation for the lateral oscillations is

$$\ddot{v}_n(t) + \omega_{T_n}^2 v_n(t) - K\left(\sum_{i=1}^{N}\sum_{j=1}^{M} v_i(t)u_j(t)\Gamma + \sum_{i=1}^{N}\sum_{j=1}^{M} v_i(t)u_j(t)\Upsilon \right.$$
$$\left. + \frac{3}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} v_i(t)v_k(t)v_l(t)\Psi\right) = \hat{f}(t), \tag{18}$$

where integrals of known spatial functions are of the form shown below:

$$\Gamma = \int_0^L \tilde{v}_i(z)'\tilde{u}_j(z)''\tilde{v}_n(z)\,\mathrm{d}z, \tag{19}$$

$$\Upsilon = \int_0^L \tilde{v}_i(z)''\tilde{u}_j(z)'\tilde{v}_n(z)\,\mathrm{d}z, \tag{20}$$

$$\Psi = \int_0^L \tilde{v}_i(z)'\tilde{v}_k(z)'\tilde{v}_l(z)''\tilde{v}_n(z)\,\mathrm{d}z. \tag{21}$$

The overdot ($\dot{}$) denotes differentiation with respect to time and the prime ($'$) differentiation with respect to the positional variable z. In the axial direction the nonlinear model (Eq. (6)) is reduced to

$$\ddot{u}_m(t) + \omega_{A_m}^2 u_m(t) - K\sum_{i=1}^{N}\sum_{k=1}^{N} v_i(t)v_k(t)\Phi = 0, \tag{22}$$

where ω_{A_m} is the the linear axial natural frequency for the mth mode and Φ is again an integral of the known transverse and axial spatial functions:

$$\Phi = \int_0^L \tilde{v}_i(z)'\tilde{v}_k(z)''\tilde{u}_m(z)\,\mathrm{d}z.$$

Dependence of the chosen nonlinear normal mode on a number of unexcited modes is shown explicitly by the the sums in Eqs. (18) and (22), which also represent how the modal spaces are connected. Interestingly, these connections indicate stiffening in the reduced nonlinear structure that opposes the effect of internal flow. Where movement of fluid inside the pipe is considered, the fluid parameter μ in the reduced model appears only in the natural frequency ω_{T_n} for transverse motion:

$$\omega_{T_n} = \left(\frac{n\pi}{L}\right)\sqrt{\frac{(n\pi/L)^2 EI - \mu}{\rho_{F_i}A_i + \rho_{\mathrm{ST}}A_o + \rho_{F_o}(A_i + A_o)}},$$

$$\omega_{A_m} = \left(\frac{m\pi}{L}\right)\sqrt{\frac{EA_o}{\rho_{F_i}A_i + \rho_{\mathrm{ST}}A_o + \rho_{F_o}(A_i + A_o)}}. \tag{23}$$

For the sake of clarity it should be pointed out that ω_{T_n} and ω_{A_m} are different for a riser without internal flow:

$$\omega_{T_n} = \left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{\rho_{\mathrm{ST}}A_o + \rho_{F_o}(A_i + A_o)}},$$

$$\omega_{A_m} = \left(\frac{m\pi}{L}\right)\sqrt{\frac{EA_o}{\rho_{\mathrm{ST}}A_o + \rho_{F_o}(A_i + A_o)}}. \tag{24}$$

Furthermore, because the response is projected onto planes of linear normal modes, the natural frequencies do not change for the linear and nonlinear structures.

3.2. Reduction of the fluid forces

External forcing and the initial conditions need to be transferred to the modal space along with the differential equation for the structure. Damping in Eq. (8) is reduced to a constant force and for uniform flow it becomes [7]

$$\widehat{f}_{D_n}(t) = -\tfrac{1}{4}LC_D\rho_{F_o}DU_0\dot{v}_n, \tag{25}$$

where L is the length of the structure, C_D coefficient of damping, ρ_{F_o} density of the fluid surrounding the riser, and U_0 is its velocity. The reduced lift force, however, does not have the same form for all normal modes. It is strongly dependent on the order of normal mode n due to the geometry of the structure and the shape of its transverse linear modes [7]. As a consequence, the excitation reduces to zero for the even-ordered modes while for odd values of n it becomes [7]

$$\widehat{f}_{L_n}(t) = \frac{C_{L_0}\rho_{F_o} D U_o^2 L}{2\pi n} q(t). \tag{26}$$

Normally, in analyses of vortex-induced vibration the initial conditions are imposed on the fluid variable $q(t)$ [24], which is done here as well for the coupled system. For the free vibration discussed below, modal initial conditions were used. It was observed that they exhibit the same property as the lift force, namely that when they are reduced to the modal space, the initial velocity and initial displacement for even-ordered modes become zero. For the odd-ordered modes their values are again dependent on n, which means that the velocity will change in accordance with the following expression [7]:

$$\dot{v}_n(t=0) = \frac{2U_{x_0}}{n\pi}. \tag{27}$$

The value for U_{x_0}, representing the physical initial velocity in the cross-flow direction, is set to a maximum of one tenth of the incident velocity U_0 in order to stay within the angular limits for which formulation of forcing (Eqs. (8) and (9)) is still valid [7]. For the initial displacement caused by a point load in the middle of the riser, the formula transforming the physical displacement v_0 to the modal one is

$$v_n(t=0) = \frac{96 v_0}{n^4 \pi^4} \sin\left(\frac{n\pi}{2}\right). \tag{28}$$

Apart from the damping and lift forces expressed by Eqs. (25) and (26), the rest of the fluid model does not need to be transformed to modal space because in the lock-in condition the frequency of vortex-shedding Ω_n in Eq. (10) is synchronised with the natural frequency of the structure ω_{T_n} for the nth mode. The frequencies are the same in both spaces [7].

4. Dynamic responses of the structure

4.1. Free vibration

Initially, we determine the effects of the nonlinearity on the dynamic responses of the riser, i.e. without the excitation from the current. For this, investigations of the system with and without internal flows were undertaken, which also provided an insight into the behaviour of the riser with a reduced flexural stiffness due to the internal flow.

A convergence study was carried out to determine the number of modes to be retained in the nonlinear terms of Eqs. (18) and (22). It was understood that a sufficiently good approximation of the response requires two axial and one transverse mode. An approximation with two modes in the axial as well as the transverse direction, for example, is practically indistinguishable from the chosen one [27]. This is not surprising since both the linear and the nonlinear systems have similar properties, namely, the modes are perfectly symmetrical or antisymmetrical across the midspan of the structure. Additionally, the boundary conditions are homogenous (Eqs. (16) and (17)), thus well defined [26].

In order to effectively analyse the nonlinear dynamics of the coupled system, the invariant manifold approach for deriving nonlinear normal modes [28] was applied to reduce the complexity of the problem even further, yielding a single ordinary ordinary differential equation from the previous two (Eqs. (18) and (22)):

$$\frac{d^2 v_1(t)}{dt^2} + \omega_{T_1} v_1(t) + \alpha v_1(t)^3 + \beta v_1(t)\left(\frac{dv_1(t)}{dt}\right)^2 = 0, \tag{29}$$

where α and β are constants at the nonlinear terms obtained from the invariant manifold technique. The non-directly excited modes, that were captured in these coefficients, were approximated by constraint equations expressed as power series, which were expanded to third order to accommodate the cubic nonlinearities appearing in the reduced-order system, Eq. (18):

$$X_i(x,y) = a_{1i}x + a_{2i}y + a_{3i}x^2 + a_{4i}xy + a_{5i}y^2 + a_{6i}x^3 + a_{7i}x^2y + a_{8i}xy^2 + a_{9i}y^3 \tag{30}$$

$$Y_i(x,y) = b_{1i}x + b_{2i}y + b_{3i}x^2 + b_{4i}xy + b_{5i}y^2 + b_{6i}x^3 + b_{7i}x^2y + b_{8i}xy^2 + b_{9i}y^3 \quad (31)$$

Because vortex-induced vibration is self-limiting, thus will prevent structures from deflecting for more than a maximum of two diameters [29], it can be postulated that the amplitude of the nonlinear structure will not exceed this limit. In fact, it can be expected that the amplitudes of the linear and nonlinear structures will be close. Therefore, large deviation from the linear motion is not expected and a power series expansion in the constraint functions $X(x,y)$ and $Y(x,y)$ can be justified. Correctness of this type of approximation was also confirmed by comparing results obtained by direct integration of the original nonlinear system in Eqs. (18) and (22) with the results of Eq. (29). Also, the projections of the nonlinear modal surfaces (i.e. invariant manifolds) onto the corresponding eigenplanes of the linear model (Figs. 2(b) and 3(b)) indicate a lesser effect of the nonlinearity in the area of interest for the prescribed initial conditions.

For the differences in responses of the linear and nonlinear structures to be seen most easily, first modes were compared. In the constraint functions $X(x,y)$ and $Y(x,y)$, variables x and y represent the displacement $v_1(t)$ and its velocity $\dot{v}_1(t)$, respectively. The invariant manifold surfaces, shown in Fig. 2(a) for the displacement and Fig. 3(a) for the velocity intermodal relationship with the second axial mode, were built for the structure with and without internal flow to analyse the change of shape of the manifolds when the internal flow is added to the structure. It was observed that the shape remained practically the same for both configurations. The differences in coefficients A and B in the constraint equations of second axial mode

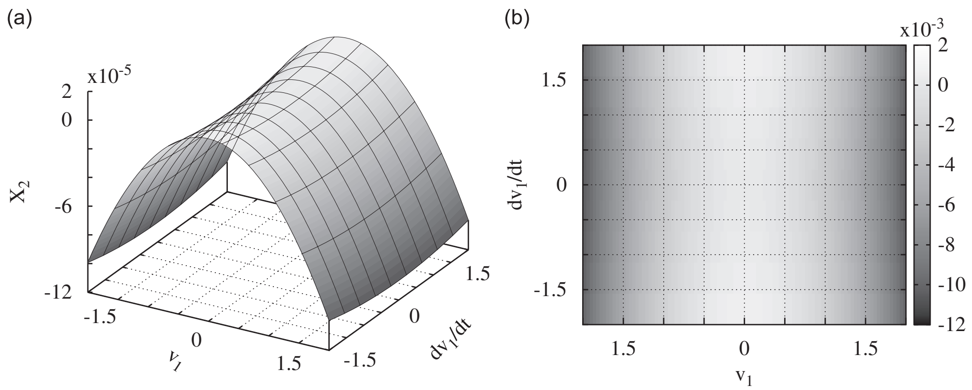

Fig. 2. The relationship between the displacement of the excited and unexcited mode; (a) the invariant manifold in the phase space $(v_1, \dot{v}_1, X_2)$ and (b) its projection onto the eigenplane of the linear mode.

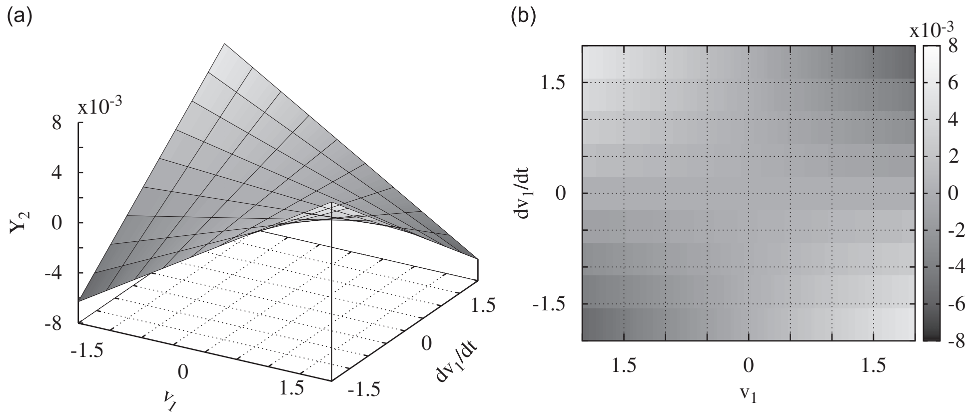

Fig. 3. The relationship between the velocity of the excited and unexcited mode; (a) the invariant manifold in the phase space $(v_1, \dot{v}_1, Y_2)$ and (b) its projection onto the eigenplane of the linear mode.

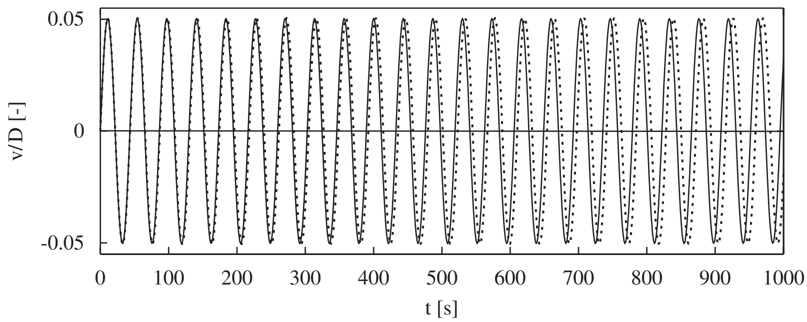

Fig. 4. Time histories of the free responses of the first mode for the linear [- - -] and nonlinear [—] structures without internal flow; $\dot{v}_0 = U_0/10$.

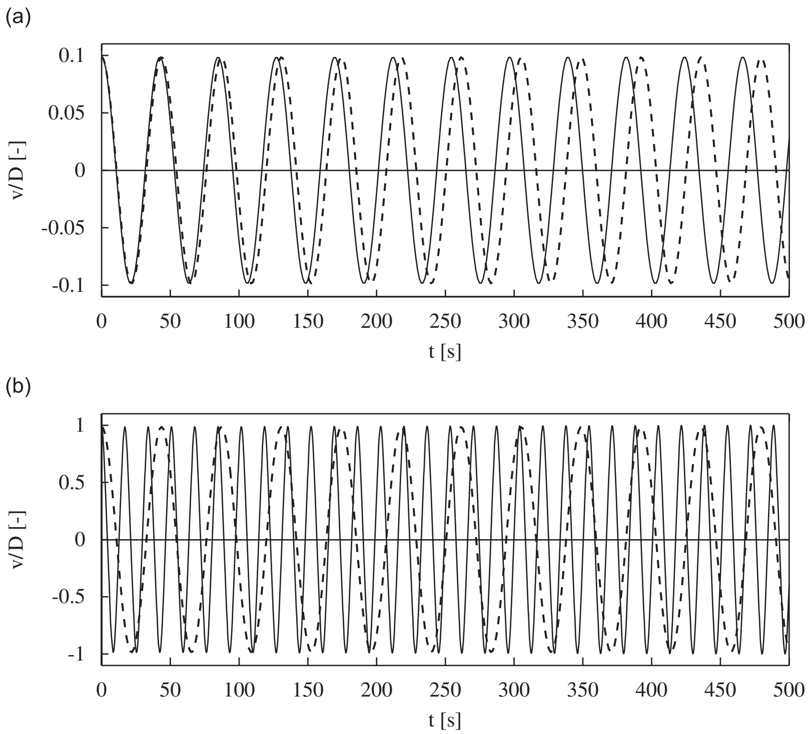

Fig. 5. Time histories of the free responses for the first mode of the linear [- - -] and nonlinear [—] structure without internal flow; (a) $v_0 = D/10$ and (b) $v_0 = D$.

displacement X_2, for instance:

$$X_2 = Av_1(t)^2 + B\dot{v}_1(t)^2, \tag{32}$$

are very small for the structure with and without flow.[1] This suggests that the system behaviour is influenced predominately by the cubic nonlinearity expressed as stiffening. For the free oscillations of the riser without internal flow, stiffening can be observed for the nonlinear case when motion is initiated with a velocity (Fig. 4), and even more so when it is displaced by v_0 (Fig. 5). The differences in the two graphs can be explained by the

[1]The values of A and B are, respectively, -0.00261800 and 1.30877×10^{-7} for the structure without internal flow and -0.00261799 and 1.34997×10^{-7} for the structure with internal flow. Values for B indicate that for a further simplification of the equation of motion, Eq. (29), this term can easily be omitted because of its insignificant influence on the overall motion.

frequency–amplitude relationship that nonlinear systems generally tend to exhibit [30]. Using the Lindstedt-Poincaré technique (the nonlinear structure is undamped), an expansion about ω_{T_1} was made to obtain a nonlinear natural frequency [31]:

$$\omega_{T_{NL}} = \omega_{T_1} + \frac{a^2}{8}\left(\frac{3\alpha}{\omega_{T_1}} + \beta\omega_{T_1}\right). \tag{33}$$

Like the frequency, the nonlinear response function shows the connection between the initial amplitude of vibration a and its frequency:

$$v_{1_{NL}}(t) = a\cos(\omega t + \varphi) - \frac{a^3}{32}\left(-\frac{\alpha}{\omega_{T_1}^2} + \beta\right)\cos(3\omega t + 3\varphi). \tag{34}$$

Contrary to the properties of the manifolds, the characteristics of responses in the time domain change considerably when internal flow is added. Effect of the structural nonlinearity is much more pronounced in the oscillation and again the nonlinearity causes a significant frequency change in responses for the displacement initial condition (Fig. 6), while for the velocity initial condition, it contributes also to a large reduction in amplitude, as shown in Fig. 7. Although a smaller amplitude would be desired when the riser is in operation, higher natural frequencies that the nonlinear structure exhibits can lead to precocious damage of the riser due to an earlier onset of critical fatigue.

4.2. Forced responses

Dynamic responses in the lock-in conditions were simulated by attaching the reduced damping and lift forces (Eqs. (25) and (26), respectively) to the single-degree-of-freedom oscillators describing the linear (Eq. (15)) and the nonlinear structures (Eq. (29)). Again, oscillations associated with the first linear natural

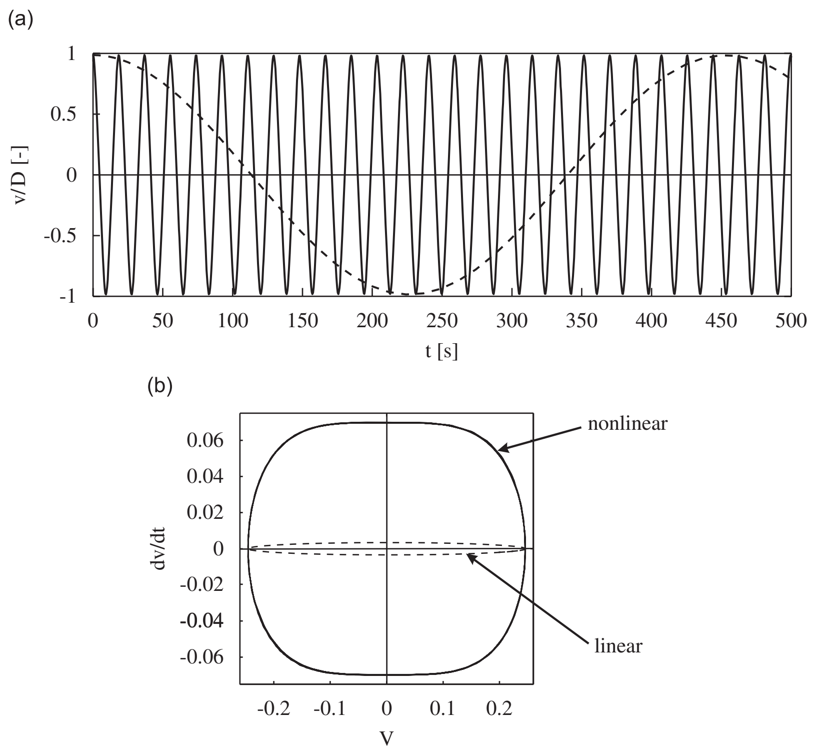

Fig. 6. Comparisons of the free response in the first mode of the linear [- - -] and nonlinear [—] structure with internal flow for $v_0 = D$; (a) a time history plot and (b) a phase plot.

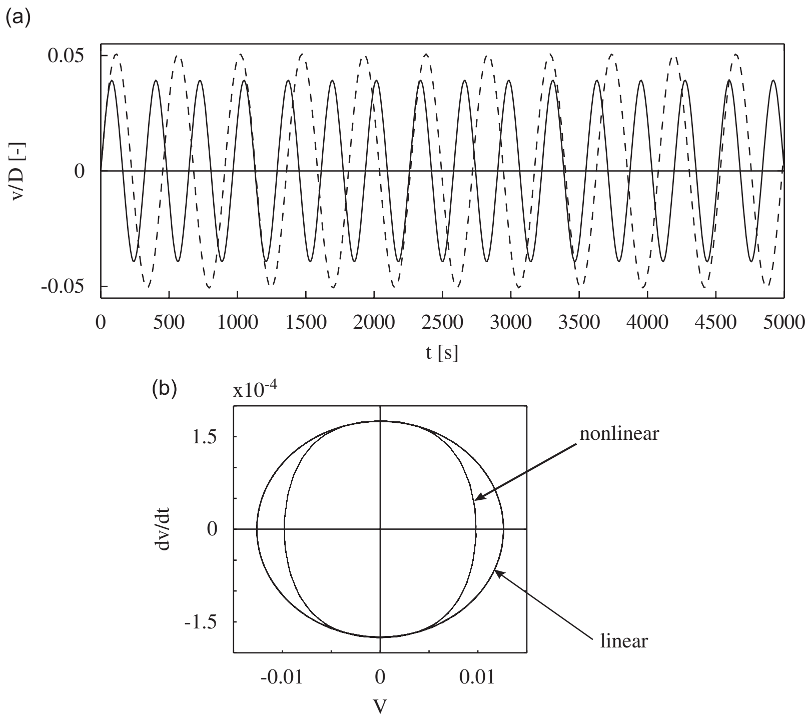

Fig. 7. Comparisons of the free responses for the first mode of the linear [- - -] and nonlinear [—] structure with internal flow for $\dot{v}_0 = U_0/10$; (a) a time history plot and (b) a phase plot.

frequency were looked at and compared, even though in actual applications risers tend to vibrate at much higher resonant frequencies. The velocity of flow U_0, on which fluid forces depend, is determined with the Strouhal law:

$$U_0 = \frac{\omega_{T_1} D}{2\pi St}, \tag{35}$$

where the vortex-shedding frequency is equal to the chosen natural frequency of the structure ω_{T_1}. Thus, the lock-in condition can be imposed automatically. Following the assumptions made in Ref. [24], the Strouhal number St is chosen to be 0.2 and is the same for both cases, as are geometrical and material parameters given in Table 1.

The two important parameters in the wake oscillator λ and P effectively define its behaviour. The values used here were initially the same as those determined by Facchinetti et al. [25], namely $\lambda = 0.3$ and $P = 12$, which were shown to adequately describe the VIVs of linear structures [24]. The results obtained for this configuration showed the predicted difference in frequency and amplitude for the two systems. A much smaller amplitude of the nonlinear response compared to the one of the linear could indicate that the nonlinear system is not in the lock-in condition for these values. Even though this could be a very probable explanation for such a radical change in magnitude of vibration, it should be recalled here that wake oscillators are essentially semi-empirical, which means that parameters λ and P in Eqs. (10) and (11) need to be adjusted to fit the actual system behaviour [25]. Therefore, the reason for the variation most probably hides in the interplay of nonlinearities of the fluid and the structure.

Next, an attempt was made to get the amplitudes of the linear and nonlinear systems as close as possible with the linear response being the reference. The desire was to see what exactly would happen to the entire system if amplitudes were matched. If real structures are presumed to be inherently nonlinear, then predicted nonlinear responses should be similar to experimentally obtained behaviour, especially since vortex-induced vibration is limited in amplitude [17]. Similar amplitudes were obtained when the coupling strength was adjusted to a new value of $P = 2000$. Figs. 8 and 9 show how the nonlinear response changes from small amplitude oscillations with the frequency close to the linear motion when the initial parameters are used, to

Table 1
Values of geometrical and material properties of the riser used in calculations

Length (L)	150 m
Outer diameter (D_o)	0.25 m
Inner diameter (D_i)	0.125 m
Density of riser (ρ_{ST})	7850 kg m^{-3}
Density of fluid outside (ρ_{F_o})	1020 kg m^{-3}
Density of fluid inside (ρ_{F_i})	870 kg m^{-3}
Velocity of internal flow 1 (U_i)	0 ms^{-1}
Velocity of internal flow 2 (U_i)	39.2 ms^{-1}
Young's modulus (E)	2.1×10^{11} Nm^{-2}

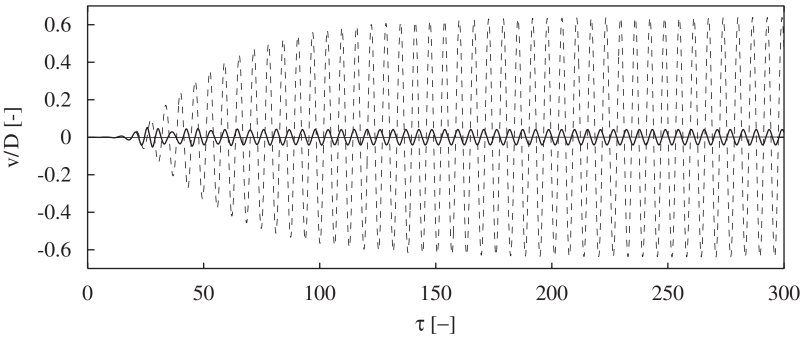

Fig. 8. Time histories of the linear [- -] and nonlinear [—] structures with internal flow when excited by currents in the lock-in ($\lambda = 0.3$ and $P = 12$ for the linear and the nonlinear case).

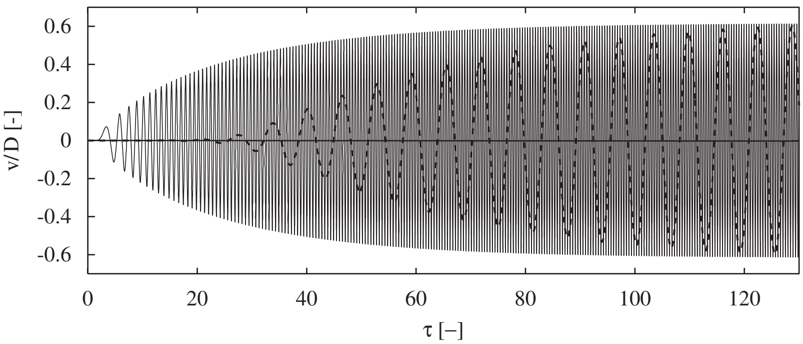

Fig. 9. Time histories of the linear [- - -] and nonlinear [—] structures with internal flow when excited by currents in the lock-in ($\lambda = 0.3$ and $P = 12$ for the linear case and $\lambda = 0.3$ and $P = 2000$ for the nonlinear case).

very high-frequency oscillations when the amplitudes are matched. The latter, interestingly, resembles the oscillations of the linear forced structure without internal flow more than the vibration of a riser with the fluid flowing through it. Comparing the dynamic responses of the two riser models, the same behaviour can be deciphered again. When the nonlinearity is introduced, the motion of the riser with internal flow is affected more, at least as far as the change in amplitude is concerned (Figs. 8 and 10).

5. Dynamics of the wake oscillator

A similar behaviour to that of the structural displacement was expected for the reduced vortex coefficient $q(\tau)$ since the fluid and structure are coupled. The responses of the fluid variable associated with the linear and nonlinear models of the structure with internal flow are presented here to emphasise the change in the dynamic behaviour that happens when the parameters in the governing differential equation of the fluid, Eq. (10), are varied. Understanding this behaviour is important because the reduced lift coefficient directly defines the magnitude of the excitation force acting on the riser.

The oscillations shown in Figs. 11 and 13 correspond to the motion depicted in Figs. 8 and 10, respectively. Where the same parameters λ and P are used for the linear and nonlinear models, the amplitude of the latter becomes smaller and is close to the amplitude of an unexcited van der Pol oscillation. On the other hand, the increase in amplitude for the fluid variable $q(\tau)$ is very large (approximately 25 times) when P is changed (Fig. 12), which implies that the lift force must increase for just as much, when amplitudes of the linear and nonlinear structures are similar. Comparing $q(\tau)$ of the fully nonlinear system with the one where the structure is linear, it can be seen that the magnitude increases by approximately ten times. It is very questionable, however, if this change is realistic or, if it occurs because the current model of the coupled system cannot properly capture the true behaviour. Since the development of the wake oscillator in Eq. (10) was based on

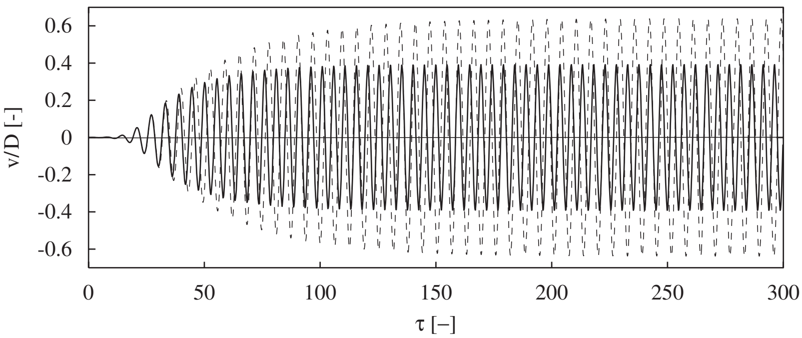

Fig. 10. Time histories of the linear [- - -] and nonlinear [—] structures without internal flow when excited by currents in the lock-in ($\lambda = 0.3$ and $P = 12$ for the linear and the nonlinear case).

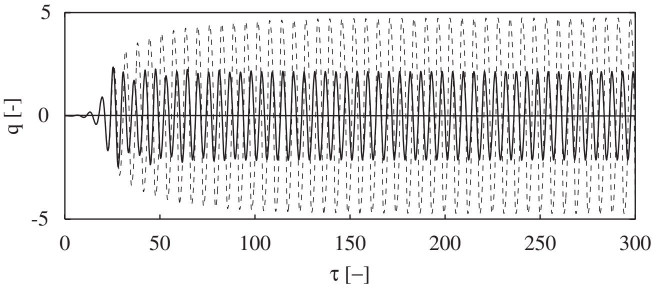

Fig. 11. Oscillations of the fluid variable $q(\tau)$ for the linear [- - -] and nonlinear [—] case with internal flow in the lock-in ($\lambda = 0.3$ and $P = 12$ for the linear and the nonlinear case).

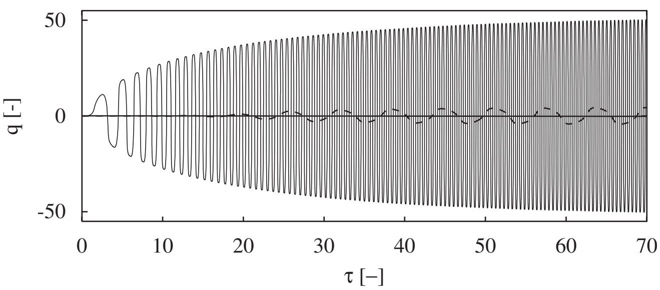

Fig. 12. Oscillations of the fluid variable $q(\tau)$ for the linear [- - -] and nonlinear [—] case with internal flow in the lock-in ($\lambda = 0.3$ and $P = 12$ for the linear case and $\lambda = 0.3$ and $P = 2000$ for the nonlinear case).

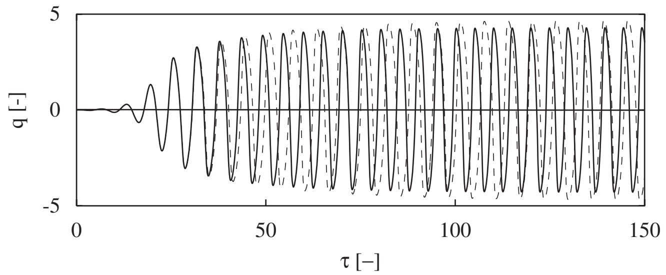

Fig. 13. Oscillations of the fluid variable $q(\tau)$ for the linear [- - -] and nonlinear [—] case without internal flow in lock-in ($\lambda = 0.3$ and $P = 12$ for the linear and the nonlinear case).

laboratory experimental data and was tuned for a rigid cylinder with purely linear characteristics [32], it can be concluded that the abrupt change in the amplitude of $q(\tau)$ is a consequence of the added structural nonlinearity and its interaction with the fluid nonlinearity. Once more, and this time perhaps even more clearly, the need for a modified model of the coupled system becomes apparent (see Figs. 11–13).

6. Conclusions

The undertaken study aimed at investigating the effect of a weak structural nonlinearity on a vertical riser oscillating in the lock-in condition. It was observed that the presented model for fluid–structure interaction bears some similarities to the findings in the recent work on a different system with interacting structural nonlinearities [33]. More importantly, it was demonstrated that the nonlinear effects on the behaviour are substantial when compared with responses of a linear structure, both in free and forced motion.

Comparisons of responses were made for a linear riser with and without internal flow. As expected, both cases exhibit a stiffening effect when the geometric nonlinearity of the structure is introduced in free motion as well as when undergoing vortex-induced vibration. However, the nonlinearity affects the system with the internal flow considerably more. Since the oscillation frequency is much lower for this configuration, the change can be contributed solely to the frequency–amplitude characteristics of the nonlinear model. To gain a further insight, a dedicated parametric investigation and comparison with the experimental data would need to be made in order to validate the model for the nonlinear case. The results presented here give an indication of how a vertical riser with a weak geometrical nonlinearity would tend to behave if excited by a uniform current at a single resonance. Even though its maximal amplitude could be smaller than the one predicted by a linear model, the frequency change would still lead to a shorter fatigue life.

The fluid variable does not give a direct indication of the configuration of wakes behind the cylinder, but it can be inferred from the time histories that it is still well organised with the vortices being shed periodically also for the nonlinear structure with and without internal flow. In fact, the simulated examples of the displacement and the reduced vortex coefficient show that they oscillate with a constant frequency and amplitude in a manner similar to the linear model.

References

[1] R.E.D. Bishop, A.Y. Hassan, The lift and drag forces on a circular cylinder oscillating in a flowing fluid, *Proceedings of the Royal Society of London*, *Series A Mathematical and Physical Sciences* 277 (164) 51–75.

[2] A.D. Trim, H. Braaten, H. Lie, M.A. Tognarelli, Experimental investigation of vortex-induced vibration of long marine risers, *Journal of Fluids and Structures* 21 (2005) 335–361.

[3] C. Le Cunff, F. Biolley, E. Fontaine, S. Etienne, M.L. Facchinetti, Vortex-induced vibrations of risers: theoretical, numerical and experimental investigation, *Oil and Gas Science and Technology—Rev. IFP* 57 (2002) 59–69.

[4] M. Keber, M. Wiercigroch, A reduced order model for vortex-induced vibration of a vertical offshore riser in lock-in, *Proceedings of the IUTAM Symposium on Fluid–Structure Interaction in Ocean Engineering*, Hamburg, Germany, 2007.

[5] D. Lucor, H. Mukundan, M.S. Triantafyllou, Riser modal identification in CFD and full-scale experiments, *Journal of Fluids and Structures* 22 (2006) 95–917.

[6] J.R. Chaplin, P.W. Bearman, F.J. Huera Huarte, R.J. Pattenden, Laboratory measurements of vortex-induced vibrations of a vertical tension riser in a stepped current, *Journal of Fluids and Structures* 21 (2005) 3–24.
[7] M. Keber, M. Wiercigroch, C. Sanches, C.E.N. Mazzilli, Can even modes of a slender structure be excited by wakes? (2008), submitted for publication.
[8] W. Lacarbonara, G. Rega, Resonant non-linear normal modes. Part I: Analytical treatment for structural one-dimensional systems, *International Journal of Nonlinear Mechanics* 38 (2003) 851–872.
[9] C.H. Pak, R.M. Rosenberg, On the existence of normal mode vibrations in nonlinear systems, *Quarterly of Applied Mathematics* (1968) 403–416.
[10] W. Szemplinska-Stupnicka, ''Non-linear normal modes'' and the generalised Ritz method in the problems of vibrations of non-linear elastic continuous systems, *International Journal of Non-Linear Mechanics* 2 (1983) 149–165.
[11] S.W. Shaw, C. Pierre, Non-linear normal modes and invariant manifolds, *Journal of Sound and Vibration* 150 (1991) 170–173.
[12] A.H. Nayfeh, S.A. Nayfeh, On nonlinear modes of continuous systems, *Journal of Vibration and Accoustics* 116 (1994) 129–136.
[13] C. Touzé, O. Thomas, Asymptotic non-linear normal modes for large-amplitude vibration of continuous structures, *Computers and Structures* 82 (2004) 2671–2682.
[14] T. Sarpkaya, A critical overview of the intrinsic nature of vortex-induced vibrations, *Journal of Fluids and Structures* 19 (2004) 389–447.
[15] R.D. Gabbai, H. Benaroya, An overview of modelling and experiments of vortex-induced vibration of circular cyilinders, *Journal of Sound and Vibration* 282 (2005) 575–616.
[16] You-Gang Tang, Su-Xia Zhang, Cong Yi, Nonlinear vibration behaviors of casing pipe in the deep water, *Journal of Sound and Vibration* 297 (2006) 408–413.
[17] R.D. Blevins, *Flow-induced Vibration*, Van Nostrand Reinhold, New York, 1990.
[18] H. Marcollo, J.B. Hinwood, On shear flow single mode lock-in with both cross-flow and in-line lock-in mechanisms, *Journal of Fluids and Structures* 22 (2006) 197–211.
[19] A. Leonard, A. Roshko, Aspects of flow-induced vibration, *Journal of Fluids and Structures* 15 (2001) 415–425.
[20] M. Païdoussis, *Fluid–Structure Interactions; Slender Structures and Axial Flow*, Vol. 1, Academic Press, New York, 1998.
[21] J.K. Vandiver, Dimensionless parameters important to the prediction of vortex-induced vibration of long, flexible cylinders in ocean currents, *Journal of Fluids and Structures* 7 (1993) 423–455.
[22] G.L. Kuiper, A.V. Metrikine, Dynamic stability of a submerged, free-hanging riser conveying fluid, *Journal of Sound and Vibration* 280 (2005) 1051–1065.
[23] S.K. Chakrabarti, *Hydrodynamics of Offshore Structures*, Springer, Berlin, 1987.
[24] R. Violette, E. de Langre, J. Szydlowski, Computation of vortex-induced vibrations of long structures using a wake oscillator model: comparison with DNS and experiments, *Computers & Structures* 85 (2007) 1134–1141.
[25] M.L. Facchinetti, E. de Langre, F. Biolley, Coupling of structure and wake oscillators in vortex-induced vibration, *Journal of Fluids and Structures* 19 (2004) 123–140.
[26] L. Meirovitch, *Analytical Methods in Vibrations*, The Macmillan Company, New York, 1967.
[27] M. Keber, M. Wiercigroch, Comparison of dynamical responses of an offshore riser with linear and nonlinear structural characteristics through nonlinear normal modes, *Proceedings of the OCEANS'07*, Aberdeen, UK, 2007.
[28] S.W. Shaw, C. Pierre, Normal modes for non-linear vibratory systems, *Journal of Sound and Vibration* 164 (1) (1993) 85–124.
[29] A. Bokaian, Lock-in prediction of marine risers and tethers, *Journal of Sound and Vibration* 175 (5) (1994) 607–623.
[30] L. Azrar, R. Benamar, R.G. White, A semi-analytical approach to the non-linear dynamic response problem of S–S and C–C beams at large vibration amplitudes, Part I: General theory and application to the single mode approach to free and forced vibration analysis, *Journal of Sound and Vibration* 224 (2) (1999) 183–207.
[31] A.H. Nayfeh, *Perturbation Methods*, Wiley, New York, 1973.
[32] M.L. Facchinetti, Un Modèle Phénoménologique des Vibrations Induites par Détachement Tourbillonnaire, Ph.D. Thesis, Ecole Polytechnique, Paris, 2003 (in French).
[33] J. Emans, M. Wiercigroch, A.M. Krivtsov, Comulative effect of structural nonlinearities: chaotic dynamics of cantiliver beam system with impacts, *Chaos, Solitons and Fractals* 23 (2005) 1661–1670.