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Smooth orthogonal decomposition for modal analysis of randomly excited systems

U. Farooq, B.F. Feeny*

Department of Mechanical Engineering, Michigan State University, East Lansing, MI 48824, USA

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Abstract

Modal parameter estimation in terms of natural frequencies and mode shapes is studied using smooth orthogonal decomposition for randomly excited vibration systems. This work shows that under certain conditions, the smooth orthogonal decomposition eigenvalue problem formulated from white-noise induced response data can be tied to the unforced structural eigenvalue problem, and thus can be used for modal parameter estimation. Using output response ensembles only, the generalized eigenvalue problem is formed to estimate eigenfrequencies and modal vectors for an eight-degree-of-freedom lightly damped vibratory system. The estimated frequencies are compared against system frequencies obtained from structural eigenvalue problem and estimated modal vectors are checked using the modal assurance criterion. For lightly damped simulations, satisfactory results are obtained for estimating both system frequencies and modal vectors.

1. Introduction

Output-only modal analysis has gained popularity over recent years (see for example Refs. [1–9]). Advantages of output-only analysis over traditional modal analysis are the following. (1) In many real life applications, the nature of input forcing prevents its measurement (for instance earthquake, wind, or traffic loads on structures) and output-only analysis eliminates the need to measure inputs. (2) The construction of complex frequency response functions or transfer matrix functions requires an experienced engineer to correlate various response rows (or columns) to correctly identify the system modes and is cumbersome in case the modes are not well separated. (3) Contrary to traditional modal analysis, in many cases output-only analysis can eliminate the need of testing the structure at various locations (or components).

Output-only methods can be either time or frequency based. Some time domain output-only methods are the Ibrahim time domain method [1], polyreference method [2], eigensystem realization algorithm [3], least square complex exponential method [4], independent component analysis [10,11] and stochastic subspace identification methods [5]. Frequency based output-only methods include the orthogonal polynomial methods

*Corresponding author.
E-mail address: feeny@egr.msu.edu (B.F. Feeny).

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[6,7], complex mode indicator function [8] and frequency domain decomposition [9]. Recent additions to the time domain output-only family are the smooth orthogonal decomposition [12] and state-variable modal decomposition methods [13,14], which have shown good results for modal analysis of free response cases. These methods are variants of proper orthogonal decomposition methods recently studied for structural modal analysis [15–19]. The smooth orthogonal decomposition method is also applicable in blind source separation [20], fatigue damage identification [21,22], and was also presented as a generalized modal analysis scheme [23]. The advantages of using these decomposition based methods are that they do not involve the possibility of oversized state matrices and their spurious modes (as in Ibrahim time domain), estimation of states (for instance in stochastic subspace identification methods) or spectral density functions (as in frequency domain decomposition) or constructing generalized block Hankel matrices (as in eigensystem realization algorithm), and thereby are simpler in construction and induce minimum assumptions. However, these methods have room for development. The current work explores smooth orthogonal decomposition for the modal parameter estimation of systems under random excitation.

2. Smooth orthogonal decomposition

2.1. Smooth orthogonal decomposition and modal analysis for free vibration

The "smooth orthogonal decomposition" [12] can be applied to lightly damped symmetric vibration systems with inhomogeneous mass distributions to find structural modes. First, an $n \times N$ ensemble matrix $\mathbf{X}$ of displacements is obtained from N time samples of n displacement signals. Then an ensemble $\mathbf{V} \approx \dot{\mathbf{X}}$ of velocities is formed. This can be done by finite difference through a matrix $\mathbf{D}$, such that $\mathbf{V} = \mathbf{X}\mathbf{D}^{\mathrm{T}}$ where $\mathbf{X}$ is an ensemble of displacements (in structures case). Next the velocity covariance matrix $\mathbf{S} = \mathbf{V}\mathbf{V}^{\mathrm{T}}/N_v$ is formed, where N_v is the number of velocity samples. If the finite difference covers two adjacent samples, such that $v_i(t_j) = x_i(t_{j+1}) - x_i(t_j)$, then $N_v = N - 1$. If $v_i(t_j) = x_i(t_{j+1}) - x_i(t_{j-1})$, then $N_v = N - 2$, and so on. Keeping the displacement data that correspond to the calculated velocity data, the ensemble $\mathbf{X}$ is pared down to the same dimensions as $\mathbf{V}$. The matrix $\mathbf{R} = \mathbf{X}\mathbf{X}^{\mathrm{T}}/N_v$ is then formed, representing a covariance matrix if the mean of the displacement data is zero.

Then the smooth orthogonal decomposition is based on a generalized eigenvalue problem cast as

$$\lambda \mathbf{R}\,\underline{\psi} = \mathbf{S}\,\underline{\psi}. \tag{1}$$

For a free multi-modal response with light damping, the eigenvalues λ approximate the frequencies squared, and the inverse-transpose of the modal matrix $\mathbf{\Psi}$ approximates the linear modal matrix.

To see this, consider first the symmetric undamped vibration system of the form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \tag{2}$$

which is associated with the eigenvalue problem $-\omega^2\mathbf{M}\,\underline{\phi} + \mathbf{K}\,\underline{\phi} = \mathbf{0}$, which in matrix form is

$$\mathbf{K}\mathbf{\Phi} = \mathbf{M}\mathbf{\Phi}\mathbf{\Lambda}, \tag{3}$$

where $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues and the columns of $\mathbf{\Phi}$ are the eigenvectors. The eigenvalues and eigenvectors provide the modal frequencies and the mode shapes.

The smooth orthogonal decomposition eigenvalue problem Eq. (1) can be written as $\lambda\mathbf{X}\mathbf{X}^{\mathrm{T}}\,\underline{\psi} = \mathbf{V}\mathbf{V}^{\mathrm{T}}\,\underline{\psi}$ or

$$\lambda\mathbf{X}\mathbf{X}^{\mathrm{T}}\,\underline{\psi} = \mathbf{X}\mathbf{D}^{\mathrm{T}}\mathbf{D}\mathbf{X}^{\mathrm{T}}\,\underline{\psi}. \tag{4}$$

Close examination [12] shows that $\mathbf{D}^{\mathrm{T}}\mathbf{D}\mathbf{X}^{\mathrm{T}} \approx -\ddot{\mathbf{X}}^{\mathrm{T}}$. If the system damping is negligible, then from the symmetric vibration model of Eq. (2), we would find $-\ddot{\mathbf{X}}^{\mathrm{T}} = \mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1}$. Hence, Eq. (4) becomes $\lambda\mathbf{X}\mathbf{X}^{\mathrm{T}}\,\underline{\psi} = \mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1}\,\underline{\psi}$. In matrix form,

$$\mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{\Psi}\mathbf{\Lambda} = \mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1}\mathbf{\Psi}. \tag{5}$$

Assuming $\mathbf{X}\mathbf{X}^{\mathrm{T}}$ is invertible (n modes are active), we have $\mathbf{\Psi}\mathbf{\Lambda} = \mathbf{K}\mathbf{M}^{-1}\mathbf{\Psi}$. Taking the inverse-transpose and noting symmetry, $\mathbf{\Psi}^{-\mathrm{T}}\mathbf{\Lambda}^{-1} = \mathbf{K}^{-1}\mathbf{M}\mathbf{\Psi}^{-\mathrm{T}}$, and hence

$$\mathbf{K}\mathbf{\Psi}^{-\mathrm{T}} = \mathbf{M}\mathbf{\Psi}^{-\mathrm{T}}\mathbf{\Lambda}. \tag{6}$$

Comparing Eqs. (6) and (3), the eigenvalue problem of smooth orthogonal decomposition has reduced to the eigenvalue problem Eq. (3) of the undamped vibration system. The smooth orthogonal decomposition modal matrix is thus related to the structural linear modal matrix as $\mathbf{\Phi} = \mathbf{\Psi}^{-\mathrm{T}}$. Chelidze and Zhou [12] derived this relationship starting with an optimization representation of the eigenvalue problem.

Smooth orthogonal decomposition is applicable for symmetric, but otherwise general, mass and stiffness distributions. Smooth orthogonal decomposition directly produces estimates of the modal frequencies from the eigenvalue problem. Insight to modal participation is not directly obtained, but can come from analysis of the modal coordinates, dependent on how modal vectors are normalized. Limitations of smooth orthogonal decomposition are that the smooth orthogonal decomposition is restricted to lightly damped systems, and it has not been justified or studied for random excitations. Also, sufficient numbers of sensed displacements are needed.

2.2. Smooth orthogonal decomposition for systems under random excitation

Previously, proper orthogonal decomposition for modal analysis was justified for random excitation [19]. Here the smooth orthogonal decomposition will be justified for white noise excitation.

2.2.1. Smooth orthogonal decomposition and random excitation

Consider the symmetric vibration system neglecting damping,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t), \tag{7}$$

where $\mathbf{f}(t)$ is the random excitation. In terms of the sampled ensemble matrices, $\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}$, with $\mathbf{F}$ representing the ensemble matrix of the sampled $\mathbf{f}(t)$, and therefore $\mathbf{D}^{\mathrm{T}}\mathbf{D}\mathbf{X}^{\mathrm{T}} \approx -\ddot{\mathbf{X}}^{\mathrm{T}} = \mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1} - \mathbf{F}^{\mathrm{T}}\mathbf{M}^{-1}$. Hence, from the matrix form of Eq. (4)

$$\frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{\Psi}\mathbf{\Lambda} = \frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1}\mathbf{\Psi} - \frac{1}{N}\mathbf{X}\mathbf{F}^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{\Psi}. \tag{8}$$

The elements in the matrix $(1/N)\mathbf{X}\mathbf{X}^{\mathrm{T}}$ represent cross correlations (with zero delay) between responses, and are expected to be nonzero. The elements in the matrix $\mathbf{L} = (1/N)\mathbf{X}\mathbf{F}^{\mathrm{T}}$ represent cross correlations (with zero delay) between responses and random inputs. In other words, the elements L_{ij} are the means of the products $x_i(t)f_j(t)$. If their expected values are zero, then this term can be neglected, and the decomposition eigenvalue problem would then converge, as N gets large, to

$$\frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{\Psi}\mathbf{\Lambda} = \frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{T}}\mathbf{K}\mathbf{M}^{-1}\mathbf{\Psi}, \tag{9}$$

which is the same as Eq. (5), and thus reduces to the undamped structural eigenvalue problem if $\mathbf{X}\mathbf{X}^{\mathrm{T}}$ is invertible. Under this condition, the smooth orthogonal decomposition, even with random excitation, would produce the modal frequencies and mode shapes of the system. Thus we are interested in conditions for which $\mathbf{L} \to \mathbf{0}$ as N gets large.

Elements of $\mathbf{L}$ have the form

$$L_{ij} = \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{m}\int_{-\infty}^{\infty} h_{il}(\tau)f_l(t_k-\tau)\,\mathrm{d}\tau f_j(t_k), \tag{10}$$

where $h_{il}(t)$ is an element of the impulse response matrix, between $\mathbf{f}(t)$ and $\mathbf{x}$. In this form $h_{il}(t)$ is a linear combination of modal coordinate impulse response functions, each sinusoidal with a modal frequency. Interchanging the order of sums,

$$L_{ij} = \sum_{l=1}^{m}\int_{-\infty}^{\infty} h_{il}(\tau)\frac{1}{N}\sum_{k=1}^{N} f_l(t_k-\tau)f_j(t_k)\mathrm{d}\tau = \sum_{l=1}^{m}\int_{-\infty}^{\infty} h_{il}(\tau)C_{jl}^{f}(\tau)\,\mathrm{d}\tau,$$

where $C_{jl}^{f}(\tau)$ is the cross correlation between the forcing functions associated with coordinates j and l.

2.2.2. White noise

Here there are two useful possibilities. One is that the forcing on all coordinates are statistically independent. For example, independent bombardment of each coordinate by random turbulence fluctuations might qualify. Then $C^f_{jl}(\tau) = R^f_j(\tau)\delta_{jl}$, where $R^f_j(\tau)$ is the autocorrelation of the jth forcing term. If the forcing functions are modeled as white noise, then $C^f_{jl}(\tau) = \gamma_j\delta(\tau)\delta_{jl}$, where $\delta(\tau)$ is the Dirac delta function and δ_{jl} is the Kronecker delta.

Another possibility is that each forcing term is dependent, for example in random base excitation. Then $f_j(\tau) = \gamma_j f(\tau)$, and hence $C^f_{jl}(\tau) = \hat{\gamma}_{jl}R_f(\tau)$. If the forcing function is modeled as white noise, then $C^f_{jl}(\tau) = \gamma_{jl}\delta(\tau)$.

In either of these white noise cases, we have the form

$$L_{ij} = \sum_{l=1}^{m}\int_{-\infty}^{\infty} h_{il}(\tau)\gamma_{jl}\delta(\tau) = \sum_{l=1}^{m} h_{il}(0)\gamma_{jl}.$$

For a typical vibration system, the impulse response function will be such that $h_{il}(0) = 0$, whence $L_{ij} = 0$. Thus, for white noise, the response and excitation are uncorrelated, and the matrix form of the smooth orthogonal decomposition eigenvalue problem Eq. (4) represents the undamped structural eigenvalue problem Eq. (3) for large N. As such, the smooth orthogonal decomposition should produce estimates of the modal frequencies and mode shapes of the undamped model under white noise excitation. The natural excitation algorithm (referred to as NExT) [24] also arrives to a similar conclusion albeit in a different way. There, it was shown that for a system subjected to uncorrelated white-noise inputs, the cross correlation between various outputs would be a sum of complex exponential functions of the same form as the sum of impulse response functions of the original system. Thus, NExT would accommodate using output-only methods for modal parameter identification in case of independent (uncorrelated) white noise forcing.

3. Example

We simulated the eight-degree-of-freedom linear vibratory system shown in Fig. 1. The system observes light modal damping and was excited by white noise applied to the first mass with zero initial conditions. The mass (kg) and stiffness (N/m) matrices are given as

$$\mathbf{M} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \tag{11}$$

and the damping matrix is chosen to be $\mathbf{C} = c\mathbf{M}$, where $c = 0.01$ Ns/m.

The system was simulated for 1000 s with the Simulink toolbox in Matlab, which uses a fixed-step Dormand Prince (a member of the family of Runge–Kutta methods) differential equation solver [25] to evaluate the response of the system. White-noise forcing was generated using the Gaussian white-noise generator function

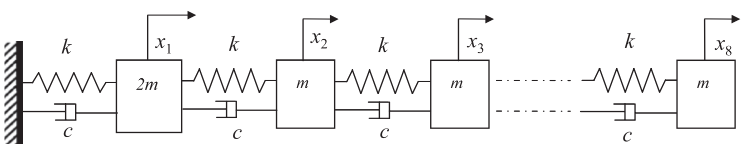

Fig. 1. The mass–spring–damper model. The dashpots are figurative to represent the presence of damping, and do not accurately correspond to the example damping matrix.

Table 1
System frequencies estimated from smooth orthogonal decomposition (SOD) with white noise forcing compared against the structural eigenfrequencies from the eigenvalue problem (EVP) of Eq. (3)

SOD	Structural EVP	Percent error
0.1816	0.1838	1.19
0.5275	0.5266	0.17
0.8133	0.8143	0.12
1.0963	1.0966	0.02
1.3875	1.3859	0.11
1.6449	1.6412	0.22
1.8368	1.8366	0.01
1.9524	1.9586	0.31

Table 2
Estimated modes compared against the system modes using the modal assurance criterion

Mode	1	2	3	4	5	6	7	8
1	0.9993	0.0001	0.0001	0.0001	0.0000	0.0002	0.0004	0.0003
2	0.0003	0.9998	0.0004	0.0005	0.0001	0.0008	0.0019	0.0012
3	0.0003	0.0000	0.9994	0.0013	0.0002	0.0022	0.0051	0.0035
4	0.0003	0.0002	0.0003	0.9999	0.0005	0.0056	0.0127	0.0094
5	0.0000	0.0001	0.0001	0.0004	1.0000	0.0006	0.0022	0.0011
6	0.0003	0.0007	0.0023	0.0057	0.0005	0.9999	0.0279	0.0136
7	0.0008	0.0017	0.0051	0.0136	0.0018	0.0256	0.9997	0.0307
8	0.0005	0.0008	0.0021	0.0070	0.0012	0.0145	0.0296	0.9999

that generates discrete-time normally distributed random numbers with sampling time step matching the solver step size chosen as 0.1, resulting in the generation of 10,000 data points. The forcing was observed to have a mean approaching zero. Both displacement and forcing matrices were saved to the Matlab workspace for further processing. The excitation was applied to the first mass only.

In the decomposition, the $\mathbf{V}$ ensemble was formed with centered finite differences with a total step of two samples, such that difference matrix $\mathbf{D}$ was $N_v \times N$, and $\mathbf{V} = \mathbf{X}\mathbf{D}^{\mathrm{T}}$ was $n \times N_v$, where $N_v = N - 2$. From the data decomposition eigenvalues, estimates of the natural frequencies are compared to the true modal frequencies in Table 1.

The modal assurance criterion [26,27] is a useful tool for testing whether or not the estimated modes are consistent with the system modes. The normalized inner products (squared) between estimated and true modes are seen in Table 2. Values of near unit magnitude indicate modal vectors that nearly line up. For visualization, the modal vectors from smooth orthogonal decomposition and the structural eigenvalue problem are compared in Fig. 2.

With random excitation, results are expected to converge as N increases. For this example, we increased N, with the time step fixed at 0.1, and plotted the estimated frequencies in Fig. 3 (Table 3 shows the percent errors). Increasing N improves the frequency estimates. The period of the lowest-frequency mode is about 34 s. Estimates of this mode converged within about 10,000 samples, or 1000 s (about 30 first-mode cycles of random response).

Important in the convergence is the relative contributions of matrices $(1/N)\mathbf{X}\mathbf{X}^{\mathrm{T}}$, $(1/N)\mathbf{V}\mathbf{V}^{\mathrm{T}}$ and $(1/N)\mathbf{X}\mathbf{F}^{\mathrm{T}}$. The maximum singular values of these matrices are plotted in Fig. 4, indicating that $(1/N)\mathbf{X}\mathbf{F}^{\mathrm{T}}$ approaches zero (while the other matrices' singular values settle to finite values), thereby becoming negligible for large N.

The example problem studied had a maximum damping ratio of $\zeta = 0.027$ in the system corresponding to fundamental frequency of $\omega_1 = 0.1838$. With increasing damping, the results deteriorated as seen in Fig. 5,

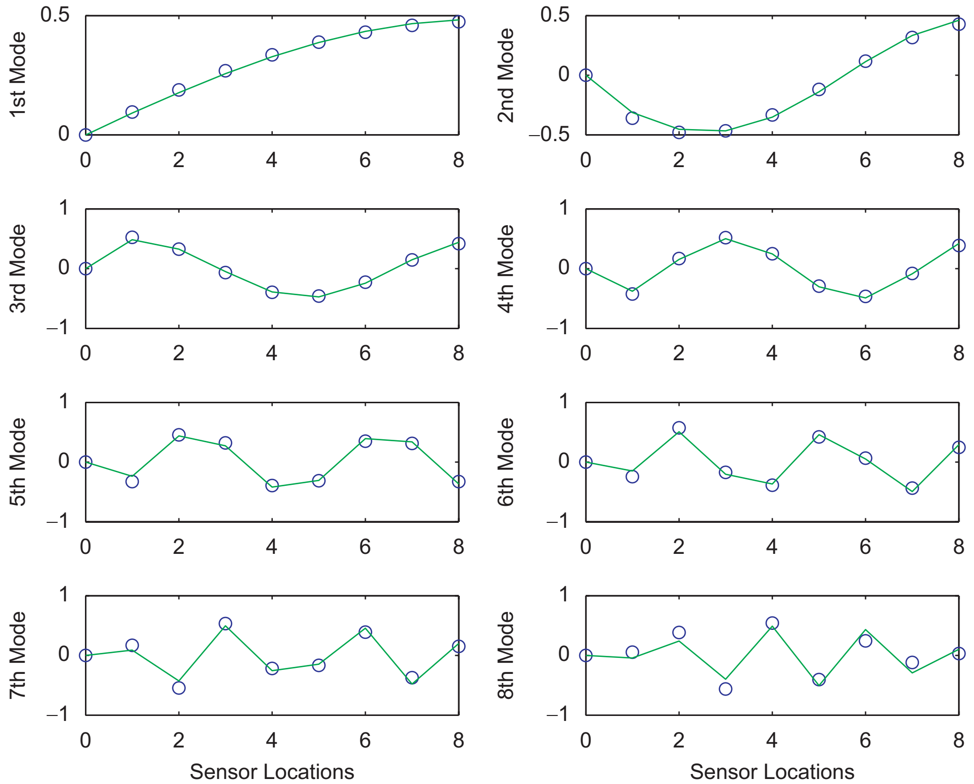

Fig. 2. A comparison between mode shapes from the structural eigenvalue problem (solid lines) and those estimated from the smooth orthogonal decomposition (○ symbols). The sensor locations refer to the mass indices. The location 0 represents the wall attachment. The modes are in ascending order by modal frequency.

even with increased sample size. When the system had a first-mode damping factor of $\zeta = 0.8$, the error in corresponding frequency estimation was ≈ 8 percent. We also see in Fig. 5 that the frequency estimation is very good for the ideal undamped case, for which the theory was developed.

4. Conclusion

The extension of smooth orthogonal decomposition for modal parameter estimation under random excitation has been presented. Analysis suggests that, for undamped systems, if the expected value of the product between response and excitation variables is zero, then the smooth orthogonal decomposition converges to an equivalent representation of the undamped structural eigenvalue problem, and therefore should produce estimated modal frequencies and mode shapes for randomly excited structures. This was justified for white noise excitation, and the convergence to the structural eigenvalue problem was demonstrated in a simulation.

In the simulation example, random excitations were applied to a linear eight-degree-of-freedom structural system while damping was kept light. It was shown that the mean of the product of displacement matrix and forcing vector approaches zero as sufficiently large number of samples are captured. This in turn means that the effect of forcing in formulating the eigenvalue problem becomes negligible. Therefore, the smooth orthogonal decomposition eigenvalue problem from data of the randomly forced problem in essence becomes representative of the free structural eigenvalue problem.

The example problem studied was subjected to light damping. As damping increased, the estimation results slowly deteriorated. While this work focused on white noise excitation, it can be shown that the mean of product of the response and the forcing approaches zero (in reference to Eqs. (8) and (10)) for other classes

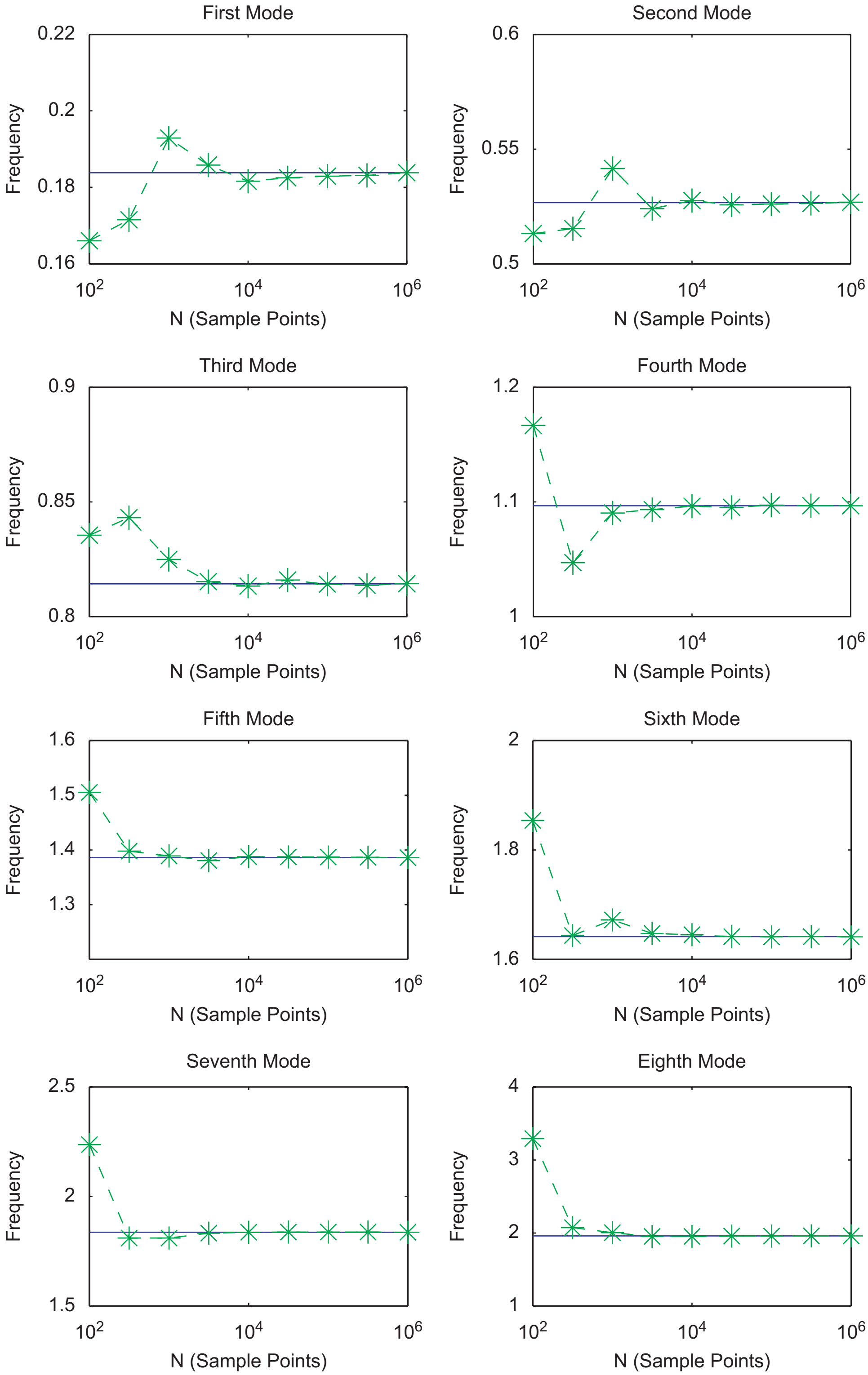

Fig. 3. The frequency estimates improve as N gets large. The solid line (—) represents the true frequency (rad/s) and the asterisks (– –*) show the estimated frequencies (rad/s). The percent error in frequency estimation computed at $N = 10^4$ for modes 1–8 are 1.2, 0.17, 0.12, 0.02, 0.11, 0.22, 0.01 and 0.31 percent, respectively, also shown in Table 1.

Table 3
Percentage error computation for all modes

Sample points	10^2	$10^{2.5}$	10^3	$10^{3.5}$	10^4	$10^{4.5}$	10^5	$10^{5.5}$	10^6
ω_1	9.68	6.69	4.95	1.08	1.19	0.70	0.48	0.38	0.00
ω_2	2.54	2.14	2.82	0.49	0.17	0.17	0.11	0.05	0.03
ω_3	2.60	3.53	1.30	0.12	0.12	0.20	0.02	0.07	0.01
ω_4	6.38	4.51	0.57	0.29	0.02	0.11	0.04	0.00	0.00
ω_5	8.60	0.85	0.21	0.39	0.11	0.08	0.06	0.02	0.00
ω_6	12.94	0.13	1.87	0.37	0.22	0.01	0.00	0.01	0.01
ω_7	21.79	1.42	1.45	0.27	0.01	0.05	0.03	0.04	0.00
ω_8	68.04	5.83	2.25	0.39	0.31	0.14	0.07	0.03	0.01

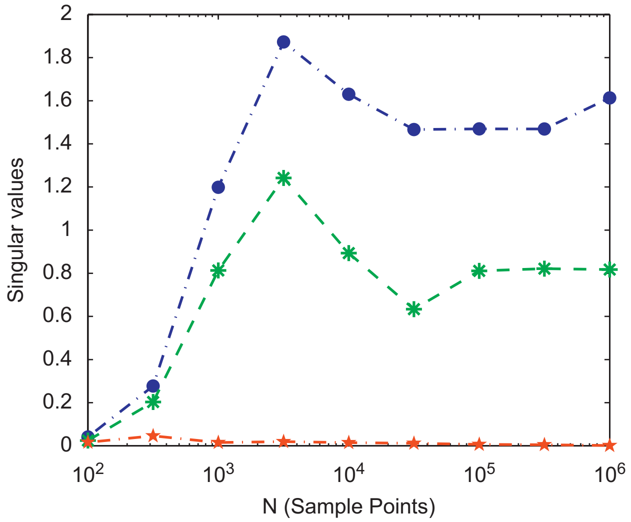

Fig. 4. The maximum singular values of $\mathbf{XX}^T/N$ (m^2), $\mathbf{VV}^T/N$ (m/s)^2 and $\mathbf{XF}^T/N$ (N m). The bulleted line (– · –•) represents values of $\mathbf{XX}^T/N$, the asterisk marker (– –*) represents the values of $\mathbf{VV}^T/N$ and the starred line (– · –⋆) represents values of $\mathbf{XF}^T/N$.

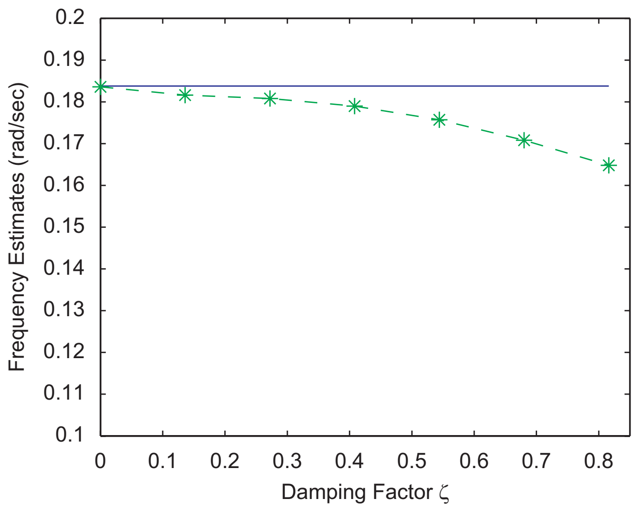

Fig. 5. The undamped natural frequency estimate starts to deteriorate for high damping. The solid line (—) represents the true first-mode frequency and the dashed line (– –*) is the estimated frequency.

of random excitation; this would broaden the applicability of smooth orthogonal decomposition for randomly excited systems.

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